Influence of Crack Inclination Angle and Thickness on Mode-Mixityusing FEM

Abstract:

Mode mixity is common in fatigue growth problems to assess the integrity of components by obtaining the fracture parameters as stress intensity factor, T-stress, J-integrals etc. Present study aims to investigate the effects of fracture parameters in a modified Compact Tension (CT) specimen using finite element methods (FEM) with ANSYS2022 software. Crack orientation angle (α) and thickness (t) have their effects on crack growth which affect the life of a mechanical component. With increasing the crack inclination angle (0°, 15°, 30°, 45° and 60°),an increase in mode III stress intensity factor is noticed while mode I showed a significant decrement. Pure mode I is notice at 0°. With further increase in the angle mode-mixity (mode I and mode III) is obtained. Also, with an increase in the thickness (8mm, 10mm and 12mm) of component there is a noticeable decrease in all modesof stress intensity factor.This mode-mixity play a crucial role in the cracked model of the specimen.

Keywords: Mode-mixity, crack orientation,stress intensity factor (SIF), Finite element method (FEM)

1. Introduction:

The study of fracture mechanics is crucial to understanding how a material will behave under different types of loading. The structural integrity of the component is impacted by the existence of cracks, which may sometimes result in the material failing completely. Various ship and marine structures and components have intricate structures that are exposed to a variety of loading conditions, including tension, compression, shear, torsion, and bending. Therefore, if these components have any internal cracks within the material, the cracks will begin to spread and eventually cause the component to fail since it includes all three modes of fracture failure, or the mode mixity case [25]. Discontinuities will appear and a macroscopic crack will develop, which will lead to a catastrophic failure. So, the emergence of cracks is the cause of material failure. Both static and dynamic loading on the materials could be the cause of the cracks. Over the last few decades, defects and microcracks already present in the material have caused structural failures. Examples of these include the failure of the Boston Molasses Tank, the Liberty Ship, and the fuselage of Aloha Airlines [23]. So, it becomes essential to investigate about 3D behavior of cracks under different loading conditions. Many studies focused on the fatigue crack growth in mode I (opening mode) or on mixed-mode I/II (opening and sliding mode) to investigate their fractured behavior [19, 24] However, many unstable failures are the combinations of mode I/II/III. The loadings are mainly cyclic, it is essential to evaluate the fatigue damages in the components or the structures. Due to these, very concise studies are done on the mixed mode I+III. Three point bending specimen was used by [1] to study the case of fatigue crack growth (FCG)in mixed mode I+III of inclined lateral crack. By twisting the surface of crack, variation of fracture mode was observed to pure mode I. Results of Mixed-mode I+III in angled- slit three point bend specimen fatigue crack propagation was presented by [2]. Observation showed that crack growth occurs due to mode I and developed a correlation among crack growth rates. However, correlation was observed by local angle of twist in crack front [3]. Materials having high yield strength and low toughness in mode I fracture show high toughness in mixed mode. Also, with an increase in the mode I fracture toughness to yield stress ratio, ratio of mixed mode to mode I fracture toughness significantly decreases and becomes lower than mode I fracture toughness [4,5]. Energy parameters was proposed which is useful in fatigue life calculation under torsion, bending or synchronous bending with torsion [6]. Experimental results on different stress ratio of fatigue crack growth in aluminium alloys was presented by Rozumek [7] under mode I+III. Specimen is tested under MZGS-100 machine to test its strength under torsion, cyclic bending and proportional bending with torsion [8]. In mixed mode I+III, J-integral is useful to investigate the fatigue crack growth rate under proportional bending with torsion [9,10]. Many studies are also focused on the modified Single Edge Notch (SEN) specimen having inclined notches. Also, when three point bend loading is applied in such types of specimen, all three modes (mode I, mode II and mode III) must be in consideration [2, 11, 12]. Under torsion and bending, in these specimens, equivalent stress intensity factor and direction of crack growth is calculated using S-criterion [13]. Many researchers have used the modified CT (MCT) specimen having inclined cracks for computing fatigue crack growth (FCG) under mixed mode loading. Under this study, mode II effect is neglected. Crack twisting and crack front rotation is easily visible in MCT specimen. Based on J-integral [4,14], under mode I and mixed mode I+III, at varying temperature toughness test is carried using MCT. A significant decrease in Armco iron’s fracture toughness was noticed while increasing mode III cycling loading [15] whereas for cyclic mode I and static mode III, a strong decrease is rate of FCG was noticed with experimental data [16]. Three dimensional specimen can be modeled under mixed mode loading using S-FEM [17] for FCG problems. It is an adaptive multilevel superposition technique for linear elastic behavior. Under mixed mode I +III, instability of crack front theoretically analyzed [18] as instability causes surface of crack and crack front to deviate from their straight and planar shapes. In various studies, mixed-mode I+III FCG, fracture cycle (N) versus crack length (a) and variation of crack growth rate were noticed[19, 24]. A modified CT specimen is used to study mixed modeI + III fatigue growth propagation both mathematically and experimentally [19]. Certain characteristics like thickness, initial angle and length of crack, load ratio have an impact on fatigue behaviors. Using Franc3d, fracture growth path and surface, crack front length, twisting of crack surface, variation in stress intensity parameter, and fatigue life were calculated and compared with experimental data. Mode I impact decreases as the angle and length of the initial crack increase, but mode III effects increases. The fracture's Mode II is essentially nonexistent in modified CT specimen [19]. The fatigue crack growth test of specimens is made of aluminum alloy 7N01 with varying thicknesses under three-point bending is shown in this study by [20]. The finite element method and an interaction integral method were combined to create a three-dimensional stress intensity factor calculation model based on the thickness size impact. The findings indicate that fatigue crack development life of the aluminum alloy 7N01 declines as thickness of the specimen increases. In all phases of steady-state crack growth, the numerical solution is greater than the analytical solution, and the difference between the two solutions widens as specimen thickness increases, according to a comparison between the numerical and analytical solutions of the stress intensity factor. Among them, specimen with a thickness of 10 mm has a difference range of 2.531%–6.414%, while the specimen with a thickness of 15 mm has a difference range of 3.961%–8.908%. The analytical method provides a conservative solution for stress intensity factor when thickness is not taken into account [20]. In comparison to the conventional three-point bending specimen, the three-point bending inclined crack specimen might produce mixed-mode I/II/III fractures through the examination of SIFs [21]. The mixed-mode I/III fracture varied when the horizontal deflection angle α changed. The mixed-mode I/II fracture was influenced by the value of the horizontal deflection angle β. By analyzing the biaxiality ratio B, it was demonstrated that the T-stress was a significant factor in mixed-mode fractures. [21]. [27] suggested a single edge cracked circular (SECC) specimen that can be used under fatigue and static loads on metallic and non-metallic materials. According to experimental findings, pure mode III fracture toughness is over 1.46 times higher than the mode I value. The cause for the finding is the growth of inelastic zone surrounding the crack point in pure mode III. The mode III component grows as the roughness of fractured surfaces increases.

This research is basically based upon the investigation on mode-mixity in a modified CT specimen. From the literature reviewed it is known that very precise study is done on mode mixity(Iand III)and thickness variation using Ansys Workbench to evaluate the fracture parameters in 3D geometries. Variation of SIF values on crack front during static loading is obtained which providesdependency of fracture parameters on the specimen. A good agreement between a 3D specimen and fracture parameters is developed with this investigation.

2. Materials and Methodology

In the research, an aluminium alloy 7075 is chosen for the investigation. Geometry of a specimen and meshed specimen is shown in figure 1 and dimensions are listed in the table 1. In the present work , for different conditions, analysis was done on various crack orientations(α = 0°, 15°, 30°, 45° and 60°) as shown in figure 2 and thickness (8mm, 10mm and 12mm),respectively.Whole specimen is tetrahedron meshed. Meshing of specimen is done on Workbench 2022 as shown in figure 4. Benefit of using a tetrahedral mesh is that it can accurately follow the surface contour [26]. Higher-order elements, such as quadratic interpolating 10-node tetrahedral elements, should be employed in place of 4-node elements, which can only interpolate linearly, in order to get good results.

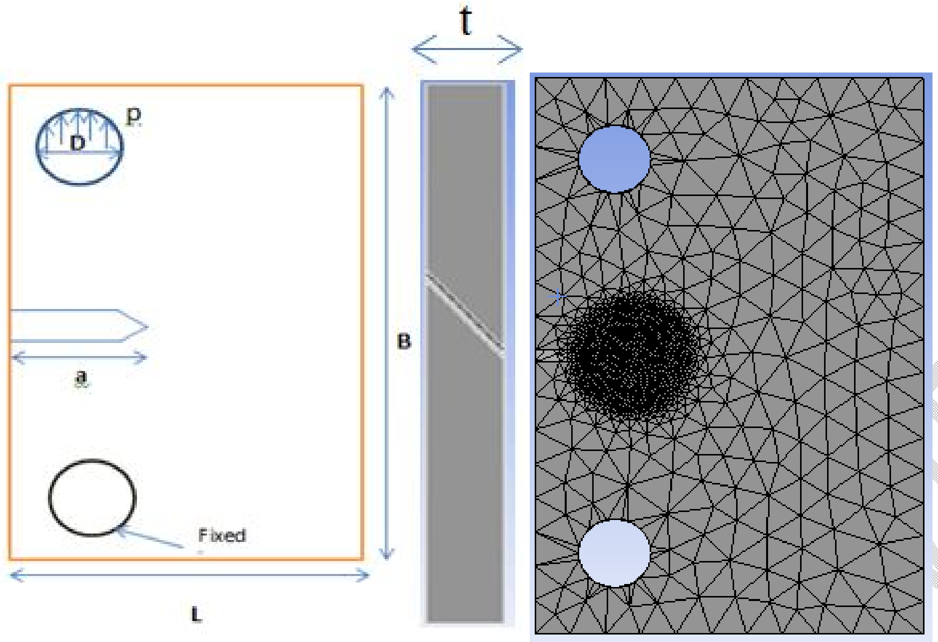
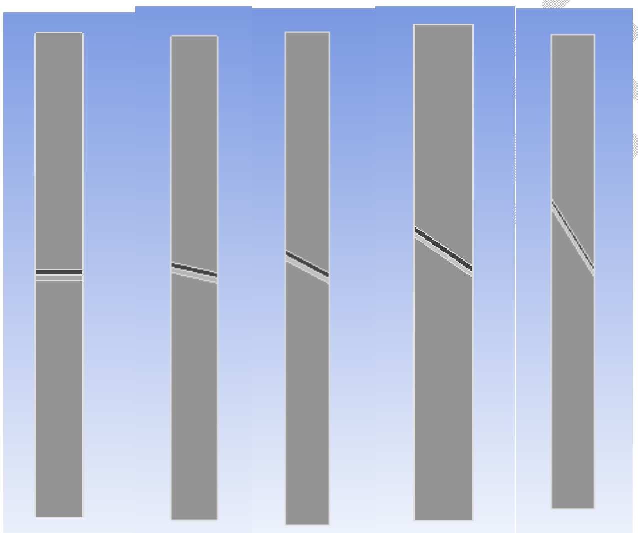


Figure 1: Shows the geometry and meshed specimen



(a) 0° (b)**15°** (c) **30°**(d) **45°**(e)**60°**

Figure 2: (a), (b), (c), (d), and (e) shows different side views of crack inclination angle of the specimen

Also, the mechanical properties required for the specimen is listed in the table 2. These properties are according to the ASTM standards.Mechanical properties includes type of material with its young’s modulus, poison’s ratio, density etc. and composition of AA7075 is listed in the table 3.

Table 1: Dimensions of a specimen

Length (L) (mm)	Breadth (B) (mm)	Thickness (t) (mm)	Crack length (a)(mm)	Diameter of hole (D) (mm)	Pressure (p) (Mpa)
80	120	10	10	15	100

Table 2: Mechanical Properties of specimen

Properties	**Material**	**Young’s Modulus (GPa)**	**Poisson’s ratio (ν)**	**Density (kg/m^3)**	**Software**
Specimen	Aluminium 7075	71.7	0.33	2810	Workbench 2022 (R1)

Table 3: Chemical Composition of AA7075

Material	**Al**	**Cr**	**Cu**	**Mg**	**Fe**	**Si**	**Zn**	**Ti**	**Mn**
AA7075	Bal	0.2%	1.43%	2.4%	0.25%	0.3%	5.6%	0.01%	0.04%

3. Numerical modeling and Analysis

3.1 Validation

The validation is done from the work of**Maity et.al. (2023) [28].**Fig 3 shows the histogram representing values of all three modes using Ansys 2022 at 0°. As when the crack inclination angle is 0°, pure mode occurs i.e. mode II and mode III are 0.The percentage error was noticed around 5.1%.

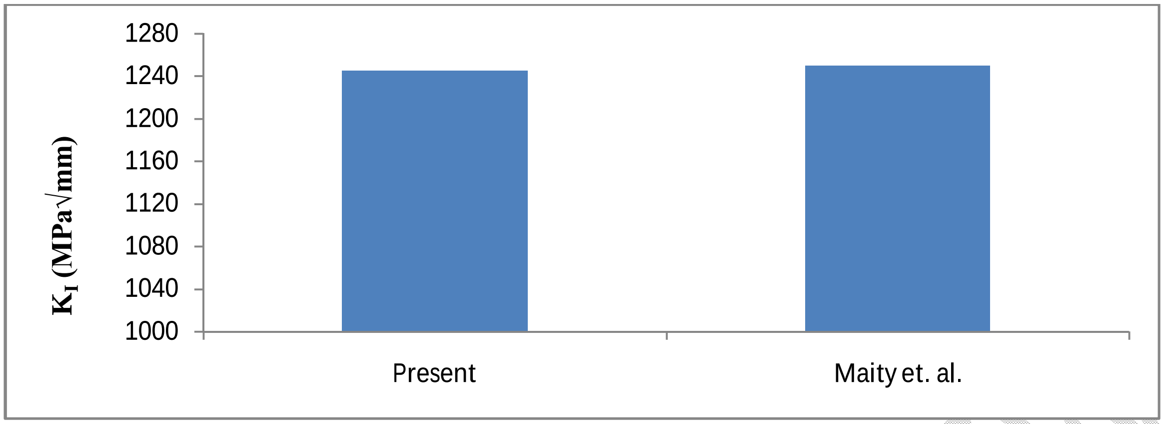


Figure 3: Shows the validation of mode I SIF at 0°

In this paper, solid works is used for modeling of geometries and Ansys is used for analysis. Finite element method is a numerical technique for obtaining approximation solution to problems governing partial differential equations. These problems are boundary value problems. It is a most reliable and valuable tool in the field of research engineering [22]. Ansys Workbench 2022 is used in the research work to model FCG under mixed-mode loading.A flow chart is presented below.

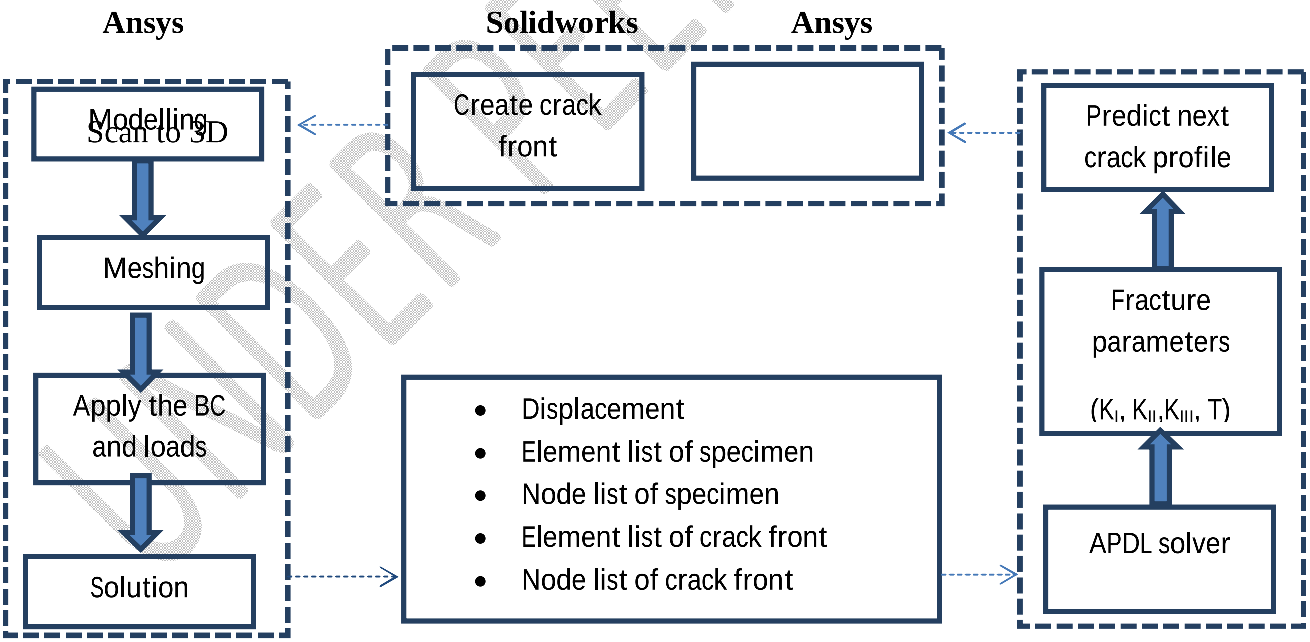


Figure 4: A detailed flowchart of steps involved in prediction of fracture parameters using Ansys

Mathematical relationship

Direction of crack growth was obtained by using maximum tangential stress criterion (MTS) [25]. Extension of crack at different positions on crack front was obtained by equivalent stress intensity factor (K_{eq}) and is represented by the following equation.

$$\mathbf{K_{eq}} = \sqrt{\mathbf{K_I^2} + (\boldsymbol{\alpha}\mathbf{K_{II}})^2 + (\boldsymbol{\beta}\mathbf{K_{III}})^2} \qquad \mathbf{.....(1)}$$

Where, $\alpha = \frac{K_{IC}}{K_{IIC}}$; $\beta = \frac{K_{IC}}{K_{IIIC}}$ and these are independent parameters [3,26].

In LEFM, a relationship among SIF and displacement at vicinity of crack tip is obtained as [27]:

$$\begin{pmatrix} \boldsymbol{K_I} \\ \boldsymbol{K_{II}} \\ \boldsymbol{K_{III}} \end{pmatrix} = \frac{\boldsymbol{G}}{\mathbf{1}+\boldsymbol{k}}\sqrt{\frac{\mathbf{2}\boldsymbol{\pi}}{\boldsymbol{r}}}\begin{pmatrix} \Delta\boldsymbol{u} \\ \Delta\boldsymbol{v} \\ \Delta\boldsymbol{w}\,(\mathbf{1}+\boldsymbol{k}) \end{pmatrix} \mathbf{.....(2)}$$

$$k = \begin{cases} 3-4v & for\ plane\ strain\ and\ 3D\ problems \\ \dfrac{3v}{1+v} & for\ plane\ stress\ problems \end{cases}$$

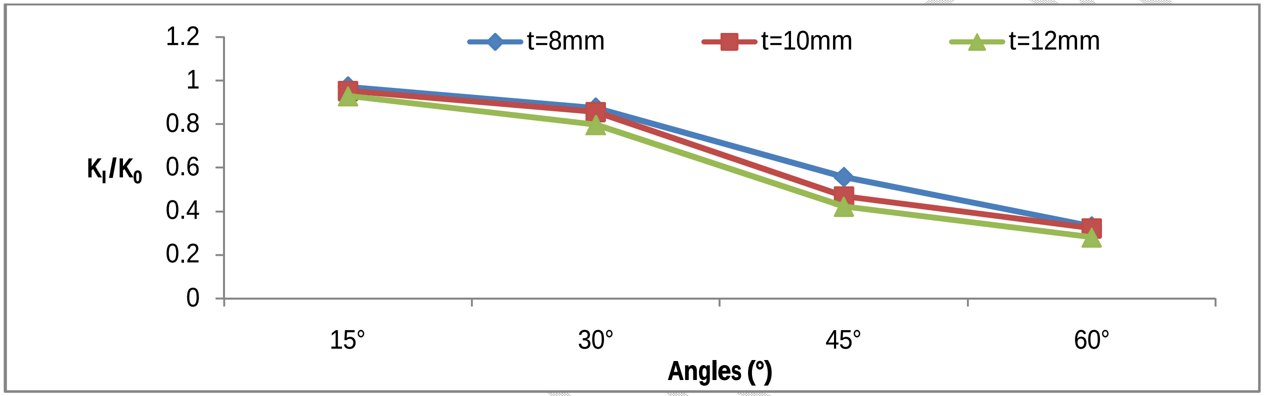
Where, Δu, Δv, Δw are local motions, respectively (in-plane opening, in-plane shearing and out of plane shear) with respect to other. $\boldsymbol{v}$, G and r arepoisson's ratio, shear modulus and distance from crack tip, respectively.

Results and discussion:

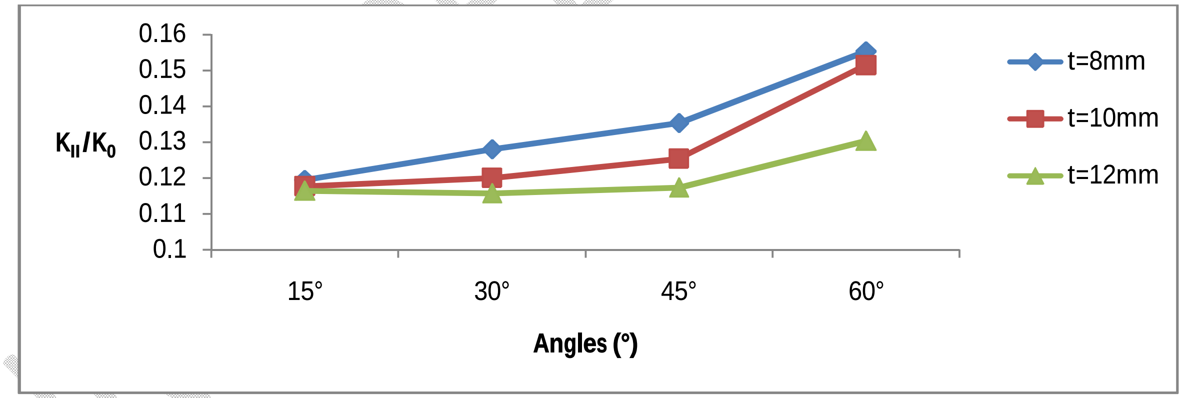
Effect of crack inclination angle and thickness on stress intensity factor in mode mixity:

The effect of crack tip plasticity is considered for the ranges of SIF. It induces compressive residual stresses and causes retarding the crack growth by reducing the ranges of SIF. For the mode mixity I/III, maximum normal stress and maximum shear stress are the two criterion and mode of fracture is a function of material strength ratio (σ_c/τ_c) where, $\sigma_c = K_I^c\sqrt{2\pi r_c}$ and $\tau_c = \sqrt{2\pi r_c}$ in which r_c is the critical or characteristic distance, poisson's ratio, and mode mixity K_I/K_{III}[19].One mode may occur in each place along the crack front, depending on the values of the parameters specified. Under plane strain circumstances, regardless of mode mixity, only shear failure is possible for materials with $\sigma_c/\tau_c < 0.167$, which represent extremely ductile materials. Shear or tensile failure may occur in materials with $0.167 < \sigma_c/\tau_c < 1.0$, depending on the mode mixity; only tensile failure, or very brittle materials, can be achieved in materials with $\sigma_c/\tau_c > 1.0$, regardless of mode mixity So, it can be seen that fatigue life of a specimen increases by increasing the crack angle[19].

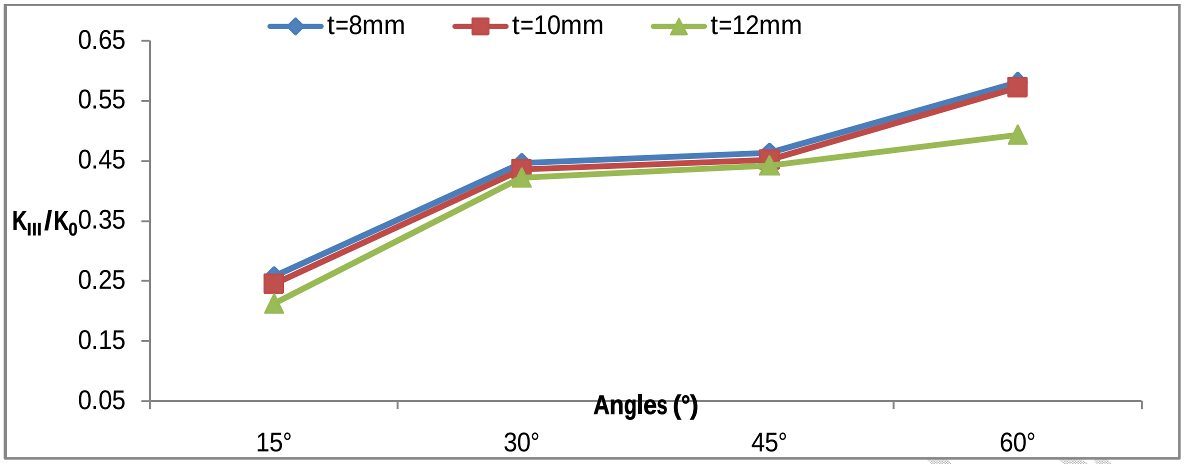
Variation of stress intensity factor (SIF) on the variation of crack inclination angles, is noticed belowin the figures. The value of all modes K_I, K_{II} and $K_{III}$are calculated for crack angles (α=15°, 30°, 45° and 60°) at a crack length of 10mm as shown in **figure**. Variation of stress intensity factor for all the modes of SIF of thickness (t = 8mm, 10mm, and 12mm) are shown.



(a)



(b)



(c)

Figure 5: Variation of stress intensity factor K_I, K_{II} and $K_{III}$with the crack inclination angle (α) and thickness (t)

On observing from the graph abovesome results can be drawn: for all conditions, K_{II} values are small and negligible as compared with the other modes i.e. K_I and K_{III} for all thickness and angles. For mode I and mode III variation of SIF seems to be symmetric along crack front. K_{II} seems to show similar behavior as K_{III}. For a_0=10mm, with the increase in the crack angles (α), K_I effect decreases and K_{III} effect increases. It also shows that K_{II} is not dependent on the length of pre-crack [19].

Also, when then the crack inclination angle is 0°, pure mode I is noticed. As we further increase the crack inclination angle mode mixity (mode I and mode III) is noticed and this mode mixity will effect the life of a component.

Conclusion:

In the present paper, mixed-mode fatigue crack growth is studied and investigated numerically in a modified CT specimens. The investigation was done on different crack orientation and with different thickness to obtain different fracture parameters i.e. stress intensity factor, von-mises stress or equivalent stress. Various conclusion are obtained and are listed below:

1. With an increase in the inclination angles mode III effect increases while mode I effect shows adecrement.

2. The fatigue life of a component can be increased by increasing crack angle and decreasing length of pre crack length.
3. Also, with an increase in the thickness of the sheet all modes shows decrement.
4. Along the crack front, by increasing the angle of the crack in the initial step, K_I increases and K_{III} decreases along crack front.

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