Fixed Point Theorem in Multidimensional Partially Ordered set in Neutrosophic Metric Spaces

Research Article

Abstract

In this paper, the author gave the idea of Ξ- isotone map and g-monotone property using Ξ-isotone mapping the author proved that there is a conicidence point for two self mappings in partially ordered set in neutrosophic metric spaces. Additionally if Ξ is weakly compatible then the two mappings have a common fixed point

Keywords: Common fixed point, Coincidence point, Neutrosophic metric spaces, ϕ-weakly compatible, Ξ-isotone, g-monotone

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1 Introduction

The concept of $\mathfrak{A}$-metric space was first developed in 2015 by Abbas, Ali, and Suleiman [1]. Vishal Gupta and Kanwar [20] created the concept of $\mathfrak{V}$-fuzzy metric space, improved the description of $\mathfrak{G}$-fuzzy metric space, and generated the linked fixed point result in a partially ordered $\mathfrak{V}$-fuzzy metric space during the generalization process. The concept of $\mathfrak{V}$- fuzzy metric space was discussed by Poovaragavan D., Jeyaraman M.[8,9].

In 1983, Attanssov. K [2] introduced the concept of Intuitionistic fuzzy set. As a generalization of the fuzzy metric space created by George and Veeramani [3], Park [7] established the concept of intuitionistic fuzzy metric space with the help of continuous ζ-norm and continuous ζ-conorm in 2004. Later Jeyaraman et.al., [4,5] discussed about the generalised intuitionistic fuzzy metric spaces. Martinez-Moreno [6,10 & 11] established multidimensional coincidence results for compatible mappings in partially ordered fuzzy metric spaces in 2012 and 2014.

Neutrosophy is an extension of the intuitionistic fuzzy set which was introduced by Florentin Smarandache. F. Smarandache and Wadei F. Al-Omeri et.al.,[12-19], published many papers in Neutrosophic cone metric spaces, Neutrosophic topological spaces and Neutrosophic graphs etc.. Also, M. Jeyaraman[20] et.al., published papers on Generalised Neutrosophic Metric Spaces. It asserts that there exists a continuum-power spectrum of neutralities between an idea and its opponent. The research community was motivated by neutrosophy, which adds neutralities to intuitionistic fuzzy sets, and the topic is currently flourishing with a wide range of studies, analyses, computing methods, and applications.

In generalized neutrosophic metric spaces, we demonstrate a common fixed point theorem for ϕ-compatible systems. This paper presents several common fixed points and multidimensional coincidence theorems for ϕ-compatible in partially ordered neutrosophic metric spaces. The results are used to frame the proper requirements to ensure the presence of the common fixed point and multidimensional coincidence point outcomes.

2 Preliminaries

Definition 2.1 (20)**.** Consider a non-empty set $\mathfrak{X}$. A triple $(\mathfrak{X}, \mathscr{V}, *)$ is known as a $\mathscr{V}$- fuzzy metric space (highlighted by $\mathscr{V} - \mathscr{F}\mathscr{M}\mathscr{S}$), where $*$ is a continuous ζ-norm and fuzzy set $\mathscr{V}$ on $\mathfrak{X}^n \times (0, \infty)$ meets the aforementioned requirements: For every $\zeta, \eta > 0$:

(i) $(\mathscr{V}\mathscr{F} - 1)\mathscr{V}(\varpi, \varpi, \varpi, \ldots, \varpi, v, \zeta) > 0$ for all $\varpi, \nu \in \mathfrak{X}$ with $\varpi \neq v$,

(ii) $(\mathscr{V}\mathscr{F} - 2)\mathscr{V}(\varpi_1, \varpi_1, \varpi_1, \ldots, \varpi_1, \varpi_2, \zeta) \geq \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta)$ for all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}$ with $\varpi_1 \neq \varpi_2 \neq \varpi_3 \neq \cdots \neq \varpi_n$,

(iii) $(\mathscr{V}\mathscr{F} - 3)\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 1$ if and only if $\varpi_1 = \varpi_2 = \varpi_3 = \cdots = \varpi_n$,

(iv) $(\mathscr{V}\mathscr{F} - 4)\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = \mathscr{V}(p(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n), \zeta)$, where p is a permutation function,

(v) $(\mathscr{V}\mathscr{F} - 5)\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta + \eta) \geq \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_{n-1}, w, \zeta)$ $* \mathscr{V}(w, w, w, \ldots, w, \varpi_n, \eta)$,

(vi) $(\mathscr{V}\mathscr{F} - 6) \lim_{\zeta \to \infty} \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 1$,

(vii) $(\mathscr{V}\mathscr{F} - 7)\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

Example 2.1. *Define a continuous ζ-norm as $\alpha * \beta = \alpha\beta$ and let $\mathfrak{X} = \mathcal{R}$ and $(\mathfrak{X}, \mathfrak{A})$ be $\mathfrak{A}$- metric space. Define $\mathfrak{B} : \mathfrak{X}^n \times (0, \infty) \to \ell$ such that $\mathfrak{B}(\varpi, \varpi, \varpi, , \varpi, \zeta) = \left[e^{\frac{\mathfrak{A}(\varpi_1, \varpi_2, \ldots, \varpi_n)}{\zeta}} \right]^{-1}$ for all $\varpi_1, \varpi_2, , , \varpi_n \in \mathfrak{X}$ and $\zeta > 0$. Then $(\mathfrak{X}, \mathfrak{B}, *)$ is a $\mathfrak{B}$- fuzzy metric space.*

Definition 2.2. Let $\mathscr{H}$ and Ξ are two self maps on the partially ordered set $(\mathfrak{X}^p, \preceq)$.Then $\mathscr{H}$ is called an Ξ-isotone map, if for every $\mathscr{A}_1, \mathscr{A}_2 \in \mathfrak{X}^p, \Xi(\mathscr{A}_1) \preceq_p \Xi(\mathscr{A}_2) \Rightarrow \mathscr{H}(\mathscr{A}_1) \preceq_p \mathscr{H}(\mathscr{A}_2)$. Consider $\mathscr{A}, \mathscr{B}$ is the partition of $\varpi_p = \{1, 2, \ldots, p\}$, i.e., the union of $\mathscr{A}$ and $\mathscr{B}$ is ϖ_p and $\mathscr{A}, \mathscr{B}$ are disjoint non-empty sets. $\Omega_{A,B} = \{\rho : \Lambda_p \to \Lambda_p : \rho(\mathscr{A}) \subseteq \mathscr{A}$ and $\rho(B) \subseteq B\}$
and $\Omega'_{A,B} = \{\rho : \Lambda_p \to \Lambda_p : \rho(A) \subseteq B$ and $\rho(B) \subseteq \mathscr{A}\}$. Consider a partially ordered space $(\mathfrak{X}, \preceq)$, let $\varpi, \omega \in \mathfrak{X}$ and $\kappa \in \Lambda_p$. The following notation will be used: $\varpi \preceq_\kappa \omega \Longleftrightarrow \begin{cases} \varpi \preceq \omega & \text{if} \quad \kappa \in \mathscr{A} \\ \varpi \succeq \omega & \text{if} \quad \kappa \in \mathscr{B} \end{cases}$

Definition 2.3. Consider a partially ordered space $(\mathfrak{X}, \preceq)$ and the mappings $\mathscr{F} : \mathfrak{X}^p \to \mathfrak{X}$and $g : \mathfrak{X} \to \mathfrak{X}$. Invoking that $\mathscr{F}$ has the mixed g-monotone property if $\mathscr{F}$ is g-monotone increasing in arguments of $\mathscr{A}$ and g-monotone non-decreasing in arguments of $\mathscr{B}$.
That is, for every $\varpi_1, \varpi_2, \ldots, \varpi_p, \omega, \varsigma \in \mathfrak{X}$ and for all κ,
$g(\omega) \preceq g(\varsigma) \Rightarrow \mathscr{F}(\varpi_1, \ldots, \varpi_{\kappa-1}, \omega, \varpi_{\kappa+1}, \ldots, \varpi_p) \preceq_\kappa \mathscr{F}(\varpi_1, \ldots, \varpi_{\kappa-1}, \varsigma, \varpi_{\kappa+1}, \ldots, \varpi_p)$.

Example 2.2. *Let $g : \mathfrak{X} \to \mathfrak{X}, F : \mathfrak{X}^p \to \mathfrak{X}$ are mixed g-montone property. Let $g(\omega) = \frac{(p-1)}{p}\omega$ and*
$$\mathfrak{F}(\varpi_1, \varpi_2, \ldots, \omega, \varpi_p) = \begin{cases} \frac{\varpi_1 - \varpi_2 + \varpi_3 - \varpi_4 \ldots, -\varpi_p}{2p} & \text{if } p \text{ is even} \\ \frac{\varpi_1 - \varpi_2 + \varpi_3 - \varpi_4, \ldots, +\varpi_p}{2p} & \text{if } p \text{ is odd} \end{cases}$$
$\mathfrak{X} = \mathcal{R}, p = 4, \varpi_1 = 1, \varpi_2 = 2, \varpi_3 = 3, \varpi_4 = 4, \omega = 5, \eta = 6$ *For the above function $\mathfrak{F}$ satisfies the mixed g-montone property.*

Definition 2.4. The two self-maps $\mathscr{H}$ and Ξ on $\mathfrak{X}$ are called weakly compatible if $\mathscr{H}\Xi\varpi = \Xi\mathscr{H}\varpi$ for every $\varpi \in \mathfrak{X}$ like that $\mathscr{H}\varpi = \Xi\varpi$.

Definition 2.5. Consider a p-tuple of mappings $\Phi = (\rho_1, \rho_2, \ldots, \rho_p)$ from $\{1, 2, \ldots, p\}$ into itself. The mappings $\mathscr{F} : \mathfrak{X}^p \to \mathfrak{X}$ and $g : X \to X$ are called Φ-weakly compatible if
$g\mathscr{F}(\varpi_{\rho_\kappa(1)}, \varpi_{\rho_\kappa(2)}, \ldots, \varpi_{\rho_\kappa(p)}) = \mathscr{F}(g\varpi_{\rho_\kappa(1)}, g\varpi_{\rho_\kappa(2)}, \ldots, g\varpi_(\rho_\kappa(p))),$
whenever $g\varpi_i = \mathscr{F}(\varpi_{\rho_\kappa(1)}, \varpi_{\rho_\kappa(2)}, \ldots, \varpi_{\rho_\kappa(p)})$ for every κ and some $(\varpi_1, \varpi_2, \ldots, \varpi_p) \in \mathfrak{X}^p$.

Definition 2.6. Consider the function $\mathscr{H} : \mathfrak{X} \to \mathfrak{X}$ and $\Xi : \mathfrak{X} \to \mathfrak{X}$. A point $\varpi \in \mathfrak{X}$ is called

- Fixed point if $\mathscr{H}(\varpi) = \varpi$.
- Coincidence point if $\mathscr{H}(\varpi) = \Xi(\varpi)$.
- Common fixed point if $\mathscr{H}(\varpi) = \Xi(\varpi) = \varpi$.

Definition 2.7. Consider the function $\mathscr{F} : \mathfrak{X}^p \to \mathfrak{X}$ and $g : \mathfrak{X} \to \mathfrak{X}$. A point $(\varpi_1, \varpi_2, \ldots, \varpi_p) \in \mathfrak{X}^p$ is said to be

(i) Φ-coincidence point if $\mathscr{F}(\varpi_{\rho_\kappa(1)}, \varpi_{\rho_\kappa(2)}, \ldots, \varpi_{\rho_\kappa(p)}) = g\varpi_\kappa$ for all $\kappa \in \{1, 2, \ldots, p\}$ and $(\rho_1, \rho_2, \ldots, \rho_p)$ be a p-tuple mappings from $\{1, 2, \ldots, p\}$ into itself.

(ii) Φ- common fixed point if $\mathscr{F}(\varpi_{\rho_\kappa(1)}, \varpi_{\rho_\kappa(2)}, \ldots, \varpi_{\rho_\kappa(p)}) = g\varpi_\kappa = \varpi_\kappa$ for all $\kappa \in \{1, 2, \ldots, p\}$ and $(\rho_1, \rho_2, \ldots, \rho_p)$ be a p-tuple mappings from $\{1, 2, \ldots, p\}$ into itself.

Definition 2.8. Let Φ_w represent the collection of all functions $\phi : \mathbb{R} \to \mathbb{R}$ satisfying the condition, for every $\zeta > 0$ there exist $s \geq \zeta$ like that $\lim_{p\to\infty} \phi^p(s) = 0$.

Lemma 2.3. *Let $\phi \in \Phi_w$, then for each $\zeta > 0$ there exist $s \geq \zeta$ like that $\phi(s) < \zeta$.*

Proposition 2.1. *If $\mathfrak{X} \preceq_p \mathscr{Y}$, it follows that,*
$(\varpi_{\rho(1)}, \varpi_{\rho(2)}, \ldots, \varpi_{\rho(p)}) \preceq (\omega_{\rho(1)}, \omega_{\rho(2)}, \ldots, \omega_{\rho(p)})$ *if* $\rho \in \Omega_{\mathscr{A},\mathscr{B}}$ *and*
$(\varpi_{\rho(1)}, \varpi_{\rho(2)}, \ldots, \varpi_{\rho(p)}) \succeq (\omega_{\rho(1)}, \omega_{\rho(2)}, \ldots, \omega_{\rho(p)})$ *if* $\rho \in \Omega'_{\mathscr{A},\mathscr{B}}$.

3 Neutrosophic Metric Spaces

If $u_1, u_2, u_3, \ldots, u_n \in [0, 1]$ then $*_{\kappa=1}^n u_\kappa = u_1 * u_2 * \cdots * u_n$ and $\diamond_{i=1}^n u_\kappa = u_1 \diamond u_2 \diamond \ldots \diamond u_n$.

Definition 3.1. Consider a nonempty set $\mathfrak{X}$. A 7-tuple $(\mathfrak{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ is known as a Neutrosophic Metric Space [NMS], where $*$ is a continuous ζ-norm, $\diamond, \odot$ is a continuous ζ-conorms and $\mathscr{V}, \mathscr{W}$ and $\mathscr{T}$ are fuzzy sets on $\mathfrak{X}^n \times (0, \infty)$ meets the aforementioned requirements:
For each $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, w \in \mathfrak{X}, \zeta, \eta > 0$,

(i) $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) + \mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) + \mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) \leq 3$,

(ii) $\mathscr{V}(\varpi, \varpi, \varpi, \ldots, \varpi, v, \zeta) > 0$, for all $\varpi, v \in \mathfrak{X}$ with $\varpi \neq v$,

(iii) $\mathscr{V}(\varpi_1, \varpi_1, \varpi_1, \ldots, \varpi_1, \varpi_2, \zeta) \geq \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta)$, for all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}$ with $\varpi_1 \neq \varpi_2 \neq \varpi_3 \neq \cdots \neq \varpi_n$,

(iv) $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 1$ if and only if $\varpi_1 = \varpi_2 = \varpi_3 = \cdots = \varpi_n$,

(v) $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = \mathscr{V}(p(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n), \zeta)$, where p is a permutation function,

(vi) $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta + \eta) \geq \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_{n-1}, , w, \zeta) * \mathscr{V}(w, w, w, \ldots, w, \varpi_n, \eta)$,

(vii) $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \cdot) : (0, \infty) \to [0, 1]$ is continuous,

(viii) $\mathscr{V}$ is a increasing function on $\mathbb{R}^+$, $\lim\limits_{\zeta \to \infty} \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 1$ and $\lim\limits_{\zeta \to 0} \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 0$, for all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}, \zeta > 0$,

(ix) $\mathscr{W}((\varpi, \varpi, \varpi, \ldots, \varpi, v, \zeta) < 1$ for all $\varpi, v \in \mathfrak{X}$ with $\varpi \neq v$,

(x) $\mathscr{W}(\varpi_1, \varpi_1, \varpi_1, \ldots, \varpi_1, \varpi_2, \zeta) \leq \mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta)$ for all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}$ with $\varpi_1 \neq \varpi_2 \neq \varpi_3 \neq \cdots \neq \varpi_n$,

(xi) $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 0$ if and only if $\varpi_1 = \varpi_2 = \varpi_3 = \cdots = \varpi_n$

(xii) $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = \mathscr{W}(p(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n), \zeta)$, where p is a permutation function,

(xiii) $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta + \eta) \leq \mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_{n-1}, w, \zeta) \diamondsuit (w, w, w, \ldots, w, \varpi_n, \eta)$,

(xiv) $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \cdot) : (0, \infty) \to [0, 1]$ is continuous,

(xv) $\mathscr{W}$ is a decreasing function on $\mathbb{R}^+$, $\lim\limits_{\zeta \to \infty} \mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 0$ and $\lim\limits_{\zeta \to 0} \mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 1$, for all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}, \zeta > 0$,

(xvi) $\mathscr{T}(\varpi, \varpi, \varpi, \ldots, \varpi, v, \zeta) < 1$ for all $\varpi, v \in \mathfrak{X}$ with $\varpi \neq v$,

(xvii) $\mathscr{T}(\varpi_1, \varpi_1, \varpi_1, \ldots, \varpi_1, \varpi_2, , \zeta) \leq \mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta)$ for all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}$ with $\varpi_1 \neq \varpi_2 \neq \varpi_3 \neq \cdots \neq \varpi_n$,

(xviii) $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 0$ if and only if $\varpi_1 = \varpi_2 = \varpi_3 = \cdots = \varpi_n$,

(xix) $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = \mathscr{T}(p(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n), \zeta)$, where p is a permutation function,

(xx) $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta + \eta) \leq \mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_{n-1}, w, \zeta) \odot \mathscr{T}(w, w, w, \ldots, w, \varpi_n, \eta)$,

(xxi) $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \cdot) : (0, \infty) \to [0, 1]$ is continuous,

(xxii) $\mathscr{T}$ is a decreasing function on $\mathbb{R}^+$, $\lim\limits_{\zeta \to \infty} \mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 0$ and $\lim\limits_{\zeta \to 0} \mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = 1$, for all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}, \zeta > 0$,

In this case, the triple $(\mathscr{V}, \mathscr{W}, \mathscr{T})$ is called NMS.

Example 3.1. *Let $(\mathscr{X}, \mathscr{A})$ be a $\mathscr{A}$- metric space. For all $\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n \in \mathfrak{X}$ and every $\zeta > 0$, consider $(\mathscr{V}, \mathscr{W}, \mathscr{T})$ to be fuzzy sets on $\mathfrak{X}^n \times (0, \infty)$ defined by $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = \frac{\zeta}{\zeta + \mathscr{A}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)}$, $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = \frac{\mathscr{A}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)}{\zeta + \mathscr{A}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)}$ and $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta) = \frac{\mathscr{A}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)}{\zeta}$, denote $u * v = uv, u \diamondsuit v = \min\{u + v, 1\}$ and $u \odot v = \min\{u + v, 1\}$. Then $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamondsuit, \odot)$ is a NMS.*

Lemma 3.2. *Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamondsuit, \odot)$ be a NMS. Then $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta)$ is non-decreasing, $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta)$ is non-increasing and $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n, \zeta)$ is decreasing with respect to ζ.*

Proof. Since $\zeta > 0$ and $\zeta + \eta > 0$ for $\eta > 0$, consider
$\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta + \eta) \geq \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta) * \mathscr{V}(\varpi_n, \varpi_n, \varpi_n \dots \varpi_n, \varpi_n, \eta)$
which implies $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta + \eta) \geq \mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta)$
Therefore, $\mathscr{V}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta)$ is non-decreasing with respect to ζ.
Also,
$\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta+\eta) \leq \mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta) \diamond \mathscr{W}(\varpi_n, \varpi_n, \varpi_n \dots \varpi_n, \varpi_n, \eta)$
which implies $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta + \eta) \leq \mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta)$.
Therefore, $\mathscr{W}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta)$ is non-increasing with respect to ζ.
$\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta+\eta) \leq \mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta) \diamond \mathscr{T}(\varpi_n, \varpi_n, \varpi_n \dots \varpi_n, \varpi_n, \eta)$
which implies $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta + \eta) \leq \mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta)$
Therefore, $\mathscr{T}(\varpi_1, \varpi_2, \varpi_3, \dots \varpi_{n-1}, \varpi_n, \zeta)$ is decreasing with respect to ζ. □

Definition 3.2. Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a NMS. A sequence $\{\varpi_s\}$ is said to converge to a point $\varpi \in \mathfrak{X}$ if $\mathscr{V}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi, \zeta) \to 1, \mathscr{W}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi, \zeta) \to 0$ and $\mathscr{T}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi, \zeta) \to 0$ as $s \to \infty$ for every $\zeta > 0$, that is, for every $\epsilon > 0$, there exists $n \in \mathbb{N}$ such that for every $s \geq n$, we have $\mathscr{V}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi, \zeta) > 1-\epsilon, \mathscr{W}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi, \zeta) < \epsilon$ and $\mathscr{T}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi, \zeta) < \epsilon$, we write $\lim_{s \to \infty} \varpi_s = \varpi$.

Definition 3.3. Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be NMS. A sequence $\{\varpi_s\}$ in $\mathfrak{X}$ is said to be a Cauchy sequence if $\mathscr{V}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi_p, \zeta) \to 1, \mathscr{W}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi_p, \zeta) \to 0$ and $\mathscr{T}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi_p, \zeta) \to 0$ as $s, p \to \infty$ for every $\zeta > 0$, that is, for every $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for every $s, p \geq n_0$, we have
$\mathscr{V}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi_p, \zeta) > 1 - \epsilon, \mathscr{W}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi_p, \zeta) < \epsilon$ and
$\mathscr{T}(\varpi_s, \varpi_s, \varpi_s, \dots, \varpi_s, \varpi_p, \zeta) < \epsilon$.

Definition 3.4. A NMS $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ is known as complete if each Cauchy sequence in $\mathfrak{X}$ is convergent sequence.

4 Main Results

Definition 4.1. Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a NMS and $\mathscr{H}$ and Ξ two self maps on $\mathfrak{X}$ and are called compatible if and only if
$\lim_{p \to \infty} \mathscr{V}(\mathscr{H}(\Xi(\bar{\omega}_p)), \mathscr{H}(\Xi(\bar{\omega}_p)) \dots \mathscr{H}(\Xi(\bar{\omega}_p)), \Xi(\mathscr{H}(\bar{\omega}_p)), \zeta) = 1,$
$\lim_{p \to \infty} \mathscr{W}(\mathscr{H}(\Xi(\bar{\omega}_p)), \mathscr{H}(\Xi(\bar{\omega}_p)) \dots \mathscr{H}(\Xi(\bar{\omega}_p)), \Xi(\mathscr{H}(\bar{\omega}_p)), \zeta) = 0$ and
$\lim_{p \to \infty} \mathscr{T}(\mathscr{H}(\Xi(\bar{\omega}_p)), \mathscr{H}(\Xi(\bar{\omega}_p)) \dots \mathscr{H}(\Xi(\bar{\omega}_p)), \Xi(\mathscr{H}(\bar{\omega}_p)), \zeta) = 0,$ each $\zeta > 0,$
whenever $\{\bar{\omega}_p\} \in \mathfrak{X}$ such that $\lim_{p \to \infty} \mathscr{H}(\bar{\omega}_p) = \lim_{p \to \infty} \Xi(\bar{\omega}_p) = \bar{\omega}$ for some $\bar{\omega} \in \mathfrak{X}$.

Lemma 4.1. *Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a NMS and $\{\nu_p\}$ be a sequence in $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$. If there exists a function $\phi \in \Phi_w$ such that*

(4.1.1) $\phi(\zeta) > 0$, *for all* $\zeta > 0$,

(4.1.2) $\mathscr{V}(\nu_p, \nu_p, \dots, \nu_p, \nu_{p+1}, \phi(\zeta)) \geq \mathscr{V}(\nu_{p-1}, \nu_{p-1}, \dots, \nu_{p-1}, \nu_p, \zeta)$,
$\mathscr{W}(\nu_p, \nu_p, \dots, \nu_p, \nu_{p+1}, \phi(\zeta)) \leq \mathscr{W}(\nu_{p-1}, \nu_{p-1}, \dots, \nu_{p-1}, \nu_p, \zeta)$ *and*
$\mathscr{T}(\nu_p, \nu_p, \dots, \nu_p, \nu_{p+1}, \phi(\zeta)) \leq \mathscr{T}(\nu_{p-1}, \nu_{p-1}, \dots, \nu_{p-1}, \nu_p, \zeta)$,
for every $p \in \mathbb{N}$ and $\zeta > 0$, then $\{\nu_p\}$ is a Cauchy sequence.

Proof. Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a NMS. We have,
$\lim_{\zeta \to \infty} \mathscr{V}(\nu_1, \nu_2, \nu_3, \dots, \nu_n, \zeta) = 1, \lim_{\zeta \to \infty} \mathscr{W}(\nu_1, \nu_2, \nu_3, \dots, \nu_n, \zeta) = 0$ and
$\lim_{\zeta \to \infty} \mathscr{T}(\nu_1, \nu_2, \nu_3, \dots, \nu_n, \zeta) = 0$ it suggests that for every $\epsilon > 0$, there exist $\zeta_0 > 0$ like that

$\mathscr{V}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_0) > 1 - \epsilon, \mathscr{W}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_0) < \epsilon$ and $\mathscr{T}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_0) < \epsilon$.
Now we have $\phi \in \Phi_w$, there exists $\zeta_1 \geq \zeta_0$ such that $\lim_{p\to\infty} \phi^p(\zeta_1) = 0$.
Therefore, for $\zeta > 0$, there is $p_0 \in \mathbb{N}$ such that $\lim_{p\to\infty} \phi^p(\zeta_1) \leq \zeta$, for all $p \geq p_0$ from condition (4.1.1), $\phi^p(\zeta) > 0$, for all $p \in \mathbb{N}$ and $\zeta > 0$. It follows by induction and condition (4.1.2) we get
$\mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi^p(\zeta)) \geq \mathscr{V}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta)$,
$\mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi^p(\zeta)) \leq \mathscr{W}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta)$ and
$\mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi^p(\zeta)) \leq \mathscr{T}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta)$, for each $p \in \mathbb{N}$ and $\zeta > 0$.
Utilizing Lemma (3.2), we have

$$\begin{aligned}
\mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \zeta) &\geq \mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi^p(\zeta_1)) \geq \mathscr{V}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_1)\\
&\geq \mathscr{V}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_0) > 1 - \epsilon,\\
\mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \zeta) &\leq \mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi^p(\zeta_1)) \leq \mathscr{W}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_1)\\
&\leq \mathscr{W}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_0) < \epsilon \text{ and}\\
\mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \zeta) &\leq \mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi^p(\zeta_1)) \leq \mathscr{T}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_1)\\
&\leq \mathscr{T}(\nu_0, \nu_0, \ldots, \nu_0, \nu_1, \zeta_0) < \epsilon.
\end{aligned}$$

That is, as $p \to \infty$, $\mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \zeta) \to 1, \mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \zeta) \to 0$ and $\mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \zeta) \to 0$ for any $\epsilon > 0$ and $\zeta > 0$.
For $r \in \mathbb{N}$ and $\zeta > 0$, we have

$$\begin{aligned}
\mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+r}, \zeta) \geq{}& \mathscr{V}\left(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \frac{\zeta}{r}\right) * \mathscr{V}\left(\nu_{p+1}, \nu_{p+1}, \ldots \nu_{p+1}, \nu_{p+2}, \frac{\zeta}{r}\right)\\
&* \cdots * \mathscr{V}\left(\nu_{p+r-1}, \nu_{p+r-1}, \ldots, \nu_{p+r-1}, \nu_{p+r}, \frac{\zeta}{r}\right),\\
\mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+r}, \zeta) \leq{}& \mathscr{W}\left(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \frac{\zeta}{r}\right) \diamond \mathscr{W}\left(\nu_{p+1}, \nu_{p+1}, \ldots \nu_{p+1}, \nu_{p+2}, \frac{\zeta}{r}\right)\\
&\diamond \ldots \diamond \mathscr{W}\left(\nu_{p+r-1}, \nu_{p+r-1}, \ldots, \nu_{p+r-1}, \nu_{p+r}, \frac{\zeta}{r}\right) \; and\\
\mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+r}, \zeta) \leq{}& \mathscr{T}\left(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \frac{\zeta}{r}\right) \odot \mathscr{T}\left(\nu_{p+1}, \nu_{p+1}, \ldots \nu_{p+1}, \nu_{p+2}, \frac{\zeta}{r}\right)\\
&\odot \cdots \odot \mathscr{T}\left(\nu_{p+r-1}, \nu_{p+r-1}, \ldots, \nu_{p+r-1}, \nu_{p+r}, \frac{\zeta}{r}\right).
\end{aligned}$$

Letting, $p \to \infty$, we get,
$\mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+r}, \zeta) \geq 1 * 1 * \cdots * 1 = 1, \mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+r}, \zeta) \leq 0 \diamond 0 \diamond \ldots \diamond 0 = 0$ and $\mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+r}, \zeta) \leq 0 \odot 0 \odot \cdots \odot 0 = 0$. Thus the sequence $\{\nu_p\}$ is a Cauchy sequence. □

Theorem 4.2. *Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a complete NMS and $(\mathfrak{X}, \preceq)$ be a partially ordered set. $\mathscr{H} : \mathfrak{X} \to \mathfrak{X}$ and $\Xi : \mathfrak{X} \to \mathfrak{X}$ be two maps such that*
(4.2.1) $\mathscr{H}(\mathfrak{X}) \subseteq \Xi(\mathfrak{X})$.
(4.2.2) $\mathscr{H}$ is a Ξ-isotone mapping.
(4.2.3) Assume that a function $\phi \in \Phi_w$ exists, such that
$\mathscr{V}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}) \ldots \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \geq \mathscr{V}(\Xi(\bar{\omega}), \Xi(\bar{\omega}) \ldots \Xi(\bar{\omega}), \Xi(\omega), \zeta)$,
$\mathscr{W}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}) \ldots \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \leq \mathscr{W}(\Xi(\bar{\omega}), \Xi(\bar{\omega}) \ldots \Xi(\bar{\omega}), \Xi(\omega), \zeta)$ *and*
$\mathscr{T}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}) \ldots \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \leq \mathscr{T}(\Xi(\bar{\omega}), \Xi(\bar{\omega}) \ldots \Xi(\bar{\omega}), \Xi(\omega), \zeta)$,
for all $\bar{\omega}, \omega \in \mathfrak{X}, \zeta > 0$ and $\Xi(\bar{\omega}) \preceq \Xi(\omega)$.
$\mathscr{H}$ and $\Xi(\bar{\omega})$ are continuous and compatible maps.
If there exists $\varpi_0 \in \mathfrak{X}$ such that $\Xi(\bar{\omega}_0) \approx \mathscr{H}(\bar{\omega}_0)$, then $\mathscr{H}$ and Ξ have a coincidence point.

Proof. Choose a point $\bar{\omega}_0 \in \mathfrak{X}$ such that $\Xi(\bar{\omega}_0) \approx \mathscr{H}(\bar{\omega}_0)$. Given that, $\mathscr{H}(\mathfrak{X}) \subseteq \Xi(\mathfrak{X})$. So, we choose $\varpi_1 \in \mathfrak{X}$ such that $\Xi(\varpi_1) = \mathscr{H}(\varpi_0)$.
Continuing in this manner, we assemble a sequence
$\{\bar{\omega}_p\} \in \mathfrak{X}$ for $p \in \mathbb{N} \cup \{0\}$ like that $\Xi(\bar{\omega}_{p+1}) = \mathscr{H}(\bar{\omega}_p)$. Since $\Xi(\bar{\omega}_0) \approx \mathscr{H}(\bar{\omega})$, we suppose that $\Xi(\bar{\omega}_0) \preceq \mathscr{H}(\bar{\omega}_0)$.
Assume that $\Xi(\bar{\omega}_{p-1}) \preceq \Xi(\bar{\omega}_p)$ and we have $\mathscr{H}$ is Ξ-isotone mapping which suggests $\mathscr{H}(\bar{\omega}_{p-1}) \preceq \mathscr{H}(\bar{\omega}_p)$. We set $\Xi(\varpi_0) = \nu_0 \preceq \mathscr{H}(\Xi_0) = \nu_1$ and $\mathscr{H}(\bar{\omega}_{p-1}) = \nu_p \preceq \mathscr{H}(\bar{\omega}_p) = \bar{\omega}_{p+1}$.
Thus, the sequence $\{\nu_p\}$ is an increasing sequence. From (4.2.3), we get

$$\begin{aligned}
\mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi(\zeta)) &= \mathscr{V}(\mathscr{H}_{p-1}, \mathscr{H}_{p-1} \ldots \mathscr{H}_{p-1}, \mathscr{H}_p, \phi(\zeta)) \\
&\geq \mathscr{V}(\Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_{p-1}) \ldots \Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_p), \zeta) \\
&= \mathscr{V}(\nu_{p-1}, \nu_{p-1}, \ldots, \nu_{p-1}, \nu_p, \zeta), \\
\mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi(\zeta)) &= \mathscr{W}(\mathscr{H}_{p-1}, \mathscr{H}_{p-1} \ldots \mathscr{H}_{p-1}, \mathscr{H}_p, \phi(\zeta)) \\
&\leq \mathscr{W}(\Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_{p-1}) \ldots \Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_p), \zeta) \\
&= \mathscr{W}(\nu_{p-1}, \nu_{p-1}, \ldots, \nu_{p-1}, \nu_p, \zeta) \text{ and} \\
\mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi(\zeta)) &= \mathscr{T}(\mathscr{H}_{p-1}, \mathscr{H}_{p-1} \ldots \mathscr{H}_{p-1}, \mathscr{H}_p, \phi(\zeta)) \\
&\leq \mathscr{T}(\Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_{p-1}) \ldots \Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_p), \zeta) \\
&= \mathscr{T}(\nu_{p-1}, \nu_{p-1}, \ldots, \nu_{p-1}, \nu_p, \zeta)
\end{aligned}$$

for all $p \in \mathbb{N} \cup \{0\}$ and $\zeta > 0$. Clearly $\phi(\zeta) > 0$ for every $\zeta > 0$. From Lemma (4.1), we determine that $\{\nu_p\}$ is a Cauchy sequence. Since $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a complete NMS, there exists a point $v \in \mathfrak{X}$ such that $\lim_{p\to\infty} \mathscr{V}_p = v$. That is, $\lim_{p\to\infty} \mathscr{H}(\bar{\omega}_p) = \lim_{p\to\infty} \Xi(\bar{\omega}_p) = v$. Since $\mathscr{H}$ and Ξ are compatible,
$\lim_{p\to\infty} \mathscr{V}(\mathscr{H}(\Xi(\bar{\omega}_p)), \mathscr{H}(\Xi(\bar{\omega}_p)), \ldots, \mathscr{H}(\Xi(\bar{\omega}_p)), \Xi(\mathscr{H}(\bar{\omega}_p)), \zeta) = 1$,
$\lim_{p\to\infty} \mathscr{W}(\mathscr{H}(\Xi(\bar{\omega}_p)), \mathscr{H}(\Xi(\bar{\omega}_p)), \ldots, \mathscr{H}(\Xi(\bar{\omega}_p)), \Xi(\mathscr{H}(\bar{\omega}_p)), \zeta) = 0$ and
$\lim_{p\to\infty} \mathscr{T}(\mathscr{H}(\Xi(\bar{\omega}_p)), \mathscr{H}(\Xi(\bar{\omega}_p)), \ldots, \mathscr{H}(\Xi(\bar{\omega}_p)), \Xi(\mathscr{H}(\bar{\omega}_p)), \zeta) = 0$, for all $\zeta > 0$.
Since $\mathscr{H}$ and Ξ both are continuous maps,
$\mathscr{V}(\mathscr{H}(v), \mathscr{H}(v), \ldots, \mathscr{H}(v), \Xi(v), \zeta) = 1$,
$\mathscr{W}(\mathscr{H}(v), \mathscr{H}(v), \ldots, \mathscr{H}(v), \Xi(v), \zeta) = 0$ and
$\mathscr{T}(\mathscr{H}(v), \mathscr{H}(v), \ldots, \mathscr{H}(v), \Xi(v), \zeta) = 0$,
for all $\zeta > 0$, which suggests that, $\Xi(v) = \mathscr{H}(v)$. Hence v is a coincidence point of $\mathscr{H}$ and Ξ in $\mathfrak{X}$. □

Theorem 4.3. *Let $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a complete NMS and $(\mathfrak{X}, \preceq)$ be a partially ordered set. $\mathscr{H} : \mathfrak{X} \to \mathfrak{X}$ and $\Xi : \mathfrak{X} \to \mathfrak{X}$ be two maps such that*
(4.3.1) $\mathscr{H}(\mathfrak{X}) \subseteq \Xi(\mathfrak{X})$
(4.3.2) $\mathscr{H}$ is a Ξ-isotone mapping
(4.3.3) Assume that a function $\phi \in \Phi_w$ exists, such that
$\mathscr{V}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}) \ldots \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \geq \mathscr{V}(\Xi(\bar{\omega}), \Xi(\bar{\omega}) \ldots \Xi(\bar{\omega}), \Xi(\omega), \zeta)$,
$\mathscr{W}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}) \ldots \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \leq \mathscr{W}(\Xi(\bar{\omega}), \Xi(\bar{\omega}) \ldots \Xi(\bar{\omega}), \Xi(\omega), \zeta)$ *and*
$\mathscr{T}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}) \ldots \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \leq \mathscr{T}(\Xi(\bar{\omega}), \Xi(\bar{\omega}) \ldots \Xi(\bar{\omega}), \Xi(\omega), \zeta)$,
for all $\bar{\omega}, \omega \in \mathfrak{X}, \zeta > 0$ and $\Xi(\bar{\omega}) \preceq \Xi(\omega)$.
(4.3.4) $\mathfrak{X}$ has the following property
(a) If $\{\bar{\omega}_p\}$ is a increasing sequence such that $\bar{\omega}_p \to \bar{\omega}$ then $\bar{\omega}_p \leq \bar{\omega}$ for all $p \in \mathbb{N}$.
(b) If $\{\bar{\omega}_p\}$ is a decreasing sequence such that $\bar{\omega}_p \to \bar{\omega}$ then $\bar{\omega}_p \geq \bar{\omega}$ for all $p \in \mathbb{N}$.
(4.3.5) $\Xi(\mathfrak{X})$ is closed
If there exists $\bar{\omega}_0 \in \mathfrak{X}$ such that $\Xi(\bar{\omega}_0) \approx \mathscr{H}(\bar{\omega}_0)$, then $\mathscr{H}$ and Ξ have a coincidence point.

Proof. Consider a point $\bar{\omega}_0 \in \mathfrak{X}$ like that $\Xi(\bar{\omega}_0) \approx \mathscr{H}(\bar{\omega}_0)$. Given that, $\mathscr{H}(X) \subseteq \Xi(\mathfrak{X})$. So, we choose $\bar{\omega}_1 \in \mathfrak{X}$ such that $\Xi(\bar{\omega}_1) = \mathscr{H}(\bar{\omega}_0)$.
Proceeding like this way, we assemble a sequence
$\{\bar{\omega}_p\} \in \mathfrak{X}$ where $p \in \mathbb{N} \cup \{0\}$ like that $\Xi(\bar{\omega}_{p+1}) = \mathscr{H}(\bar{\omega}_p)$. Since $\Xi(\bar{\omega}_0) \approx \mathscr{H}(\bar{\omega}_0)$, we suppose that $\Xi(\bar{\omega}_0) \preceq \mathscr{H}(\bar{\omega}_0)$. Assume that $\Xi(\bar{\omega}_{p-1}) \preceq \Xi(\bar{\omega}_p)$ and we have $\mathscr{H}$ is Ξ-isotone mapping which suggests $\mathscr{H}(\bar{\omega}_{p-1}) \preceq \mathscr{H}(\bar{\omega}_p)$. Set $\Xi(\bar{\omega}_0) = \nu_0 \preceq \mathscr{H}(\bar{\omega}_0) = \nu_1$ and $\mathscr{H}(\bar{\omega}_{p-1}) = \nu_p \preceq \mathscr{H}(\bar{\omega}_p) = \nu_{p+1}$. Thus, the sequence $\{\nu_p\}$ is an increasing sequence. From (4.3.3), we get

$$\begin{aligned}
\mathscr{V}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi(\zeta)) &= \mathscr{V}(\mathscr{H}_{p-1}, \mathscr{H}_{p-1} \ldots \mathscr{H}_{p-1}, \mathscr{H}_p, \phi(\zeta)) \\
&\geq \mathscr{V}(\Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_{p-1}) \ldots \Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_p), \zeta) \\
&= \mathscr{V}(\nu_{p-1}, \nu_{p-1}, \ldots \nu_{p-1}, \nu_p, \zeta), \\
\mathscr{W}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi(\zeta)) &= \mathscr{W}(\mathscr{H}_{p-1}, \mathscr{H}_{p-1} \ldots \mathscr{H}_{p-1}, \mathscr{H}_p, \phi(\zeta)) \\
&\leq \mathscr{W}(\Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_{p-1}) \ldots \Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_p), \zeta) \\
&= \mathscr{W}(\nu_{p-1}, \nu_{p-1}, \ldots \nu_{p-1}, \nu_p, \zeta) \text{ and} \\
\mathscr{T}(\nu_p, \nu_p, \ldots, \nu_p, \nu_{p+1}, \phi(\zeta)) &= \mathscr{T}(\mathscr{H}_{p-1}, \mathscr{H}_{p-1} \ldots \mathscr{H}_{p-1}, \mathscr{H}_p, \phi(\zeta)) \\
&\leq \mathscr{T}(\Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_{p-1}) \ldots \Xi(\bar{\omega}_{p-1}), \Xi(\bar{\omega}_p), \zeta) \\
&= \mathscr{T}(\nu_{p-1}, \nu_{p-1}, \ldots \nu_{p-1}, \nu_p, \zeta)
\end{aligned}$$

for every $p \in \mathbb{N} \cup \{0\}$ and $\zeta > 0$. Clearly $\phi(\zeta) > 0$ each $\zeta > 0$. From Lemma (4.1), we decide that $\{\nu_p\}$ is a Cauchy sequence. Since $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ be a complete NMS, there exists a point $\upsilon \in \mathfrak{X}$ such that $\lim_{p\to\infty} \mathscr{V}_p = \upsilon$. That is, $\lim_{p\to\infty} \mathscr{H}(\bar{\omega}_p) = \lim_{p\to\infty} \Xi(\bar{\omega}_p) = \upsilon$. Since $\Xi(\mathfrak{X})$ is closed, there exists $\nu_0 \in \mathfrak{X}$ such $\lim_{p\to\infty} \mathscr{H}(\bar{\omega}_p) = \lim_{p\to\infty} \Xi(\bar{\omega}_p) = \Xi(\nu_0) = \upsilon$.
$\Xi(\varpi_p)$ is a non-decreasing sequence. So, $\Xi(\bar{\omega}_p) \leq \Xi(\nu_0)$ for all $p \in \mathbb{N}$.
Using Lemma (3.2) and Lemma (2.3) we get

$$\begin{aligned}
\mathscr{V}(\mathscr{H}(\bar{\omega}_p), \mathscr{H}(\bar{\omega}_p), \ldots, \mathscr{H}(\bar{\omega}_p), \mathscr{H}(\nu_0), \zeta) &\geq \mathscr{V}(\mathscr{H}(\bar{\omega}_p), \mathscr{H}(\bar{\omega}_p), \ldots, \mathscr{H}(\bar{\omega}_p), \mathscr{H}(\nu_0), \phi(s)) \\
&\geq \mathscr{V}(\Xi(\bar{\omega}_p), \Xi(\bar{\omega}_p), \ldots, \Xi(\bar{\omega}_p), \Xi(\nu_0), s) \\
&\geq \mathscr{V}(\Xi(\bar{\omega}_p), \Xi(\bar{\omega}_p), \ldots, \Xi(\bar{\omega}_p), \Xi(\nu_0), \zeta), \\
\mathscr{W}(\mathscr{H}(\bar{\omega}_p), \mathscr{H}(\bar{\omega}_p), \ldots, \mathscr{H}(\bar{\omega}_p), \mathscr{H}(\nu_0), \zeta) &\leq \mathscr{W}(\mathscr{H}(\bar{\omega}_p), \mathscr{H}(\bar{\omega}_p), \ldots, \mathscr{H}(\bar{\omega}_p), \mathscr{H}(\nu_0), \phi(s)) \\
&\leq \mathscr{W}(\Xi(\bar{\omega}_p), \Xi(\bar{\omega}_p), \ldots, \Xi(\bar{\omega}_p), \Xi(\nu_0), s) \\
&\leq \mathscr{W}(\Xi(\bar{\omega}_p), \Xi(\bar{\omega}_p), \ldots, \Xi(\bar{\omega}_p), \Xi(\nu_0)), \zeta) \text{ and} \\
\mathscr{T}(\mathscr{H}(\bar{\omega}_p), \mathscr{H}(\bar{\omega}_p), \ldots, \mathscr{H}(\bar{\omega}_p), \mathscr{H}(\nu_0), \zeta) &\leq \mathscr{T}(\mathscr{H}(\bar{\omega}_p), \mathscr{H}(\bar{\omega}_p), \ldots, \mathscr{H}(\bar{\omega}_p), \mathscr{H}(\nu_0), \phi(s)) \\
&\leq \mathscr{T}(\Xi(\bar{\omega}_p), \Xi(\bar{\omega}_p), \ldots, \Xi(\bar{\omega}_p), \Xi(\nu_0), s) \\
&\leq \mathscr{T}(\Xi(\bar{\omega}_p), \Xi(\bar{\omega}_p), \ldots, \Xi(\bar{\omega}_p), \Xi(\nu_0)), \zeta)
\end{aligned}$$

for all $\zeta > 0$ and $p \in \mathbb{N}$, taking $p \to \infty$ in above inequality we get $\mathscr{H}(\bar{\omega}_p) \to \mathscr{H}(\nu_0)$ and we conclude that the limit is unique because of this $\mathscr{H}(\nu_0) = \Xi(\nu_0)$. Thus ν_0 is a coincidence point of $\mathscr{H}$ and Ξ. □

Theorem 4.4. *A unique coincidence point exists between $\mathscr{H}$ and Ξ if $\mathfrak{X}$ is a totally ordered set, in addition to the assumptions of Theorems (4.2) and (4.3). Furthermore, if Ξ is weakly compatible with $\mathscr{H}$ then $\mathscr{H}$ and Ξ have a one and only common fixed point.*

Proof. Consider that $\varpi, \nu \in \mathfrak{X}$ are coincidence points of $\mathscr{H}$ and Ξ. Since, every coincidence points $\varpi, \nu \in \mathfrak{X}$, there exists a point $\omega \in \mathfrak{X}$ such that $\Xi(\omega)$ is comparable to $\Xi(\varpi)$ and $\Xi(\nu)$. Let $\omega_0 = \omega$ then establish a sequence $\Xi(\omega_p)$. The sequence $\Xi(\omega_p)$ and its limit defined,similar as in Theorem (4.2) and Theorem (4.3), so we have $\Xi(\omega_{p+1}) = \mathscr{H}(\omega_p)$ and $\Xi(\omega_0) = \mathscr{H}(\omega_1)$. We have $\lim_{\zeta\to\infty} \mathscr{V}(\Xi(\omega), \Xi(\omega) \ldots \Xi(\omega), \Xi(\nu), \zeta) = 1$,

$\lim_{\zeta\to\infty} \mathscr{W}(\Xi(\omega), \Xi(\omega) \dots \Xi(\omega), \Xi(\nu), \zeta) = 0$ and
$\lim_{\zeta\to\infty} \mathscr{T}(\Xi(\omega), \Xi(\omega) \dots \Xi(\omega), \Xi(\nu), \zeta) = 0$, it suggests that for each $\epsilon \in (0, 1)$ there exists t_1 such that
$\mathscr{V}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\nu), \zeta_1) > 1 - \epsilon$,
$\mathscr{W}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\nu), \zeta_1) < \epsilon$ and
$\mathscr{T}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\nu), \zeta_1) < \epsilon$.
As $\phi \in \Phi_w$, so there exists $s \geq \zeta_1$ such that $\lim_{p\to\infty} \phi^p(s) = 0$. It suggests that, there exists $p_0 \in \mathbb{N}$ such that $\phi^p(s) < t$ every $p \geq p_0$ and $\zeta > 0$. Consider,

$$\begin{aligned}
\mathscr{V}(\Xi(\omega_p), \Xi(\omega_p) \dots \Xi(\omega_p), \Xi(\varpi), \zeta) &\geq \mathscr{V}(\Xi(\omega_p), \Xi(\omega_p) \dots \Xi(\omega_p), \Xi(\varpi), \phi^p(s)) \\
&= \mathscr{V}(\mathscr{H}(\omega_{p-1}), \mathscr{H}(\omega_{p-1}) \dots \mathscr{H}(\omega_{p-1}), \mathscr{H}(\varpi), \phi^p(s)) \\
&\geq \mathscr{V}(\Xi(\omega_{p-1}), \Xi(\omega_{p-1}) \dots \Xi(\omega_{p-1}), \Xi(\varpi), \phi^{p-1}(s)) \\
&\geq \dots \geq \mathscr{V}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\varpi), s) \\
&\geq \mathscr{V}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\varpi), \zeta_1) \\
&\geq 1 - \epsilon, \\
\mathscr{W}(\Xi(\omega_p), \Xi(\omega_p) \dots \Xi(\omega_p), \Xi(\varpi), \zeta) &\leq \mathscr{W}(\Xi(\omega_p), \Xi(\omega_p) \dots \Xi(\omega_p), \Xi(\varpi), \phi^p(s)) \\
&= \mathscr{W}(H(\omega_{p-1}), \mathscr{H}(\omega_{p-1}) \dots \mathscr{H}(\omega_{p-1}), \mathscr{H}(\varpi), \phi^p(s)) \\
&\leq \mathscr{W}(\Xi(\omega_{p-1}), \Xi(\omega_{p-1}) \dots \Xi(\omega_{p-1}), \Xi(\varpi), \phi^{p-1}(s)) \\
&\leq \dots \leq \mathscr{W}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\varpi), s) \\
&\leq \mathscr{W}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\varpi), \zeta_1) \\
&\leq \epsilon \text{ and} \\
\mathscr{T}(\Xi(\omega_p), \Xi(\omega_p) \dots \Xi(\omega_p), \Xi(\varpi), \zeta) &\leq \mathscr{T}(\Xi(\omega_p), \Xi(\omega_p) \dots \Xi(\omega_p), \Xi(\varpi), \phi^p(s)) \\
&= \mathscr{T}(\mathscr{H}(\omega_{p-1}), \mathscr{H}(\omega_{p-1}) \dots \mathscr{H}(\omega_{p-1}), \mathscr{H}(\varpi), \phi^p(s)) \\
&\leq \mathscr{T}(\Xi(\omega_{p-1}), \Xi(\omega_{p-1}) \dots \Xi(\omega_{p-1}), \Xi(\varpi), \phi^{p-1}(s)) \\
&\leq \dots \leq \mathscr{T}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\varpi), s) \\
&\leq \mathscr{T}(\Xi(\omega_0), \Xi(\omega_0) \dots \Xi(\omega_0), \Xi(\varpi), \zeta_1) \\
&\leq \epsilon,
\end{aligned}$$

for all $p \geq p_0$ and $\zeta > 0$.
Hence, $\lim_{p\to\infty} \Xi(\omega_p) = \Xi(\nu)$ and similarly we can easily show that $\lim_{p\to\infty} \Xi(\omega_p) = \Xi(\bar{\omega})$ and from the uniqueness of limit we get $\Xi(\nu) = \Xi(\varpi)$.
Now, let $\mathscr{H}(\nu) = \Xi(\nu) = e$ and $\mathscr{H}$, Ξ are weakly compatible mappings.
That is, $\mathscr{H}(e) = \mathscr{H}(\Xi(\nu)) = \Xi(\mathscr{H}(\nu)) = \Xi(e)$.
So e is a coincidence point it suggests that $\mathscr{H}(e) = \mathscr{H}(\nu) = e$. Thus e is a coincidence and common fixed point of $\mathscr{H}$ and Ξ. Now, imagine that there is $e^{'}(\neq e) \in \mathfrak{X}$ like that $\mathscr{H}(e^{'}) = \Xi(e^{'}) = e^{'}$.
Then $e = \Xi(e) = \Xi(e^{'}) = e^{'}$. Thus, $\mathscr{H}$ and Ξ have a unique common fixed point. □

Example 4.5. *Let $\mathfrak{X} = [0, 1]$ and $(\mathfrak{X}, \preceq)$ be a partially ordered set. Let $\mathscr{H}$ and Ξ are two self mappings in $\mathfrak{X}$ such that $\mathscr{H}(\bar{\omega}) = \frac{\bar{\omega}^2}{2} + \frac{1}{2}$ and $\Xi(\bar{\omega}) = \bar{\omega}$ for all $\bar{\omega} \in \mathfrak{X}$. Then we can easily get condition $\mathscr{H}(\mathfrak{X}) \subseteq \Xi(\mathfrak{X})$ and $\mathscr{H}$ is a Ξ-isotone mapping. Let $\phi(\zeta) = \frac{\zeta}{2}$, for all $\zeta > 0$.*
Define $\mathscr{V} : \mathfrak{X}^p \times (0, \infty)$ such that
$\mathscr{V}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p, \zeta) = \frac{\zeta}{\zeta + \mathscr{A}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p)}$,
$\mathscr{W}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p, \zeta) = \frac{\mathscr{A}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p)}{\zeta + \mathscr{A}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p)}$ *and*
$\mathscr{T}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p, \zeta) = \frac{\mathscr{A}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p)}{\zeta}$
where $\mathscr{A}(\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p, \zeta) = \sum_{\kappa=1}^{p} \sum_{\kappa<j} |\bar{\omega}_\kappa - \omega_j|$ *for all* $\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3 \dots \bar{\omega}_p \in \mathfrak{X}$ *and* $\zeta > 0$.

Let $\bar{\omega} * \omega = \min\{\bar{\omega}, \omega\}, \bar{\omega} \diamond \omega = \max\{\bar{\omega}, \omega\}$ *and* $\bar{\omega} \odot \omega = \max\{\bar{\omega}, \omega\}$ *for all* $\bar{\omega}, \omega \in \mathfrak{X}$.
Then $(\mathscr{X}, \mathscr{V}, \mathscr{W}, \mathscr{T}, *, \diamond, \odot)$ *is a complete NMS.*
Now,
$\mathscr{V}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}), \ldots, \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) = \frac{\phi(\zeta)}{\phi(\zeta)+|\mathscr{H}(\bar{\omega})-\mathscr{H}(\omega)|} = \frac{\zeta}{\zeta+|\bar{\omega}^2-\omega^2|}$,
$\mathscr{W}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}), \ldots, \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) = \frac{|\mathscr{H}(\bar{\omega})-\mathscr{H}(\omega)|}{\phi(\zeta)+|\mathscr{H}(\bar{\omega})-\mathscr{H}(\omega)|} = \frac{|\bar{\omega}^2-\omega^2|}{\zeta+|\bar{\omega}^2-\omega^2|}$ *and*
$\mathscr{T}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}), \ldots, \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) = \frac{|\mathscr{H}(\bar{\omega})-\mathscr{H}(\omega)|}{\phi(\zeta)} = \frac{|\bar{\omega}^2-\omega^2|}{\zeta}$.
$\mathscr{V}(\Xi(\bar{\omega}), \Xi(\bar{\omega}), \ldots, \Xi(\bar{\omega}), \Xi(\omega), \zeta) = \frac{\zeta}{\zeta+|\bar{\omega}-\omega|}$,
$\mathscr{W}(\Xi(\bar{\omega}), \Xi(\bar{\omega}), \ldots, \Xi(\bar{\omega}), \Xi(\omega), \zeta) = \frac{|\bar{\omega}-\omega|}{\zeta+|\bar{\omega}-\omega|}$,
$\mathscr{T}(\Xi(\bar{\omega}), \Xi(\bar{\omega}), \ldots, \Xi(\bar{\omega}), \Xi(\omega), \zeta) = \frac{|\bar{\omega}-\omega|}{\zeta}$, *for all* $\bar{\omega}, \omega \in \mathfrak{X}$ *and* $\zeta > 0$.
We get,
$\mathscr{V}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}), \ldots, \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \geq \mathscr{V}(\Xi(\bar{\omega}), \Xi(\bar{\omega}), \ldots, \Xi(\bar{\omega}), \Xi(\omega), \zeta)$,
$\mathscr{W}(H(\bar{\omega}), \mathscr{H}(\bar{\omega}), \ldots, \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \leq \mathscr{W}(\Xi(\bar{\omega}), \Xi(\bar{\omega}), \ldots, \Xi(\bar{\omega}), \Xi(\omega), \zeta)$ *and*
$\mathscr{T}(\mathscr{H}(\bar{\omega}), \mathscr{H}(\bar{\omega}), \ldots, \mathscr{H}(\bar{\omega}), \mathscr{H}(\omega), \phi(\zeta)) \leq \mathscr{T}(\Xi(\bar{\omega}), \Xi(\bar{\omega}), \ldots, \Xi(\bar{\omega}), \Xi(\omega), \zeta)$.
Consider the possibility that $\bar{\omega}_0 = 0$ *such that* $\Xi(\bar{\omega}_0) = 0 \leq \mathscr{H}(\bar{\omega}_0)$, *we can create a sequence* $\nu_0 = \Xi(\bar{\omega}_0)$ *and* $\bar{\omega}_{p+1} = \Xi(\bar{\omega}_{p+1}) = \mathscr{H}(\bar{\omega}_p)$ *for* $p \in \mathbb{N} \cup \{0\}$ *and a sequence*
$\{\nu_p\} = \{\nu_0 = 0, \nu_1 = \frac{1}{2}, \nu_2 = \frac{5}{8}, \nu_3 = \frac{89}{128}, \ldots\}$ *this sequence* $\{\nu_p\}$ *is a non-trivial sequence.*
By Theorem (4.4), $\mathscr{H}$ *and* Ξ *have a distinct common fixed point, as shown, i.e.,*$v = 1$.

Corollary 4.6. *In addition to hypothesis of Theorem (4.2) and Theorem (4.3), suppose that* $\mathscr{H} : \mathfrak{X} \to \mathfrak{X}$ *be a mapping such that* $\mathscr{H}$ *is increasing and non-discontinuous mapping and* $\Xi : \mathfrak{X} \to \mathfrak{X}$ *be an identity map, then* $\mathscr{H}$ *and* Ξ *has a common fixed point. If* $\mathfrak{X}$ *is a totally ordered set, then* $\mathscr{H}$ *and* Ξ *has a unique common fixed point.*

5 Conclusion

By using certain coincidence point and common fixed point results for a pair of mappings, we want to propose some multidimensional coincidence and common fixed point theorems for ϕ-contraction in partially ordered neutrosophic metric spaces in this paper. We also give an illustration of the applicability of our key findings.

References

[1] Abbas. M, Ali. B Suleiman. YI, “Generalized coupled common fixed point results in partially ordered A-metric spaces”, Fixed point theory and applications, 64, 2015, 1-24.

[2] Attanssov. K. “Intuitionistic fuzzy sets”, Fuzzy sets and systems, Vol.20, 1986, 87-96.

[3] George. A and Veeramani. P, “On Some results in fuzzy metric spaces”, Fuzzy sets and Systems, 64, 1994, 395-399.

[4] Jeyaraman. M, Malligadevi. V, Mohanraj. R, PoomKumam and Kanokwan Sitthithakerngkiet, “Fixed point theorems in Generalized Intuitionistic Fuzzy Metric Spaces”, JMCS, Vol. 2,2016, 82-94.

[5] Jeyaraman. M, Poovaragavan. D, “Common Fixed Point Theorems for weakly commuting of type (J) In Generalized Intuitionistic Fuzzy Metric Spaces”, Notes on Intuitionistic Fuzzy Sets, Vol. 25, 2019, No. 3, 26-41.

[6] Martinez-Moreno. J, Roldan. Roldan. A and Y.J. Cho, "Multidimensional coincidence point results for compatible mappings in partially ordered fuzzy metric spaces", Fuzzy Sets Syst., vol.251,no.16, 71-82, 2014.

[7] Park. J.H. "Intuitionistic fuzzy metric spaces", Chaos Solutions Fractals, Vol. 22, 2004, 1039-1046.

[8] Poovaragavan. D, Jeyaraman. M, "Common Fixed Point Results in V- Fuzzy metric Spaces using ω-compatible maps", Advances and Applications in Mathematical Sciences, Vol.21(4), 2137 - 2144.

[9] Poovaragavan. D, Jeyaraman. M, "Multidimensional Common Fixed Point Theorems in V- Fuzzy Metric Spaces", Bull. Int. Math. Virtual Inst., 12(2)(2022), 219-226.

[10] Roldan. A, Martinez-Moreno. J and Roldan. C, "Multidimensional fixed point theorems in partially ordered complete metric spaces", J. Math. Anal. Appl., vol.96 pp. 536-545, 2012.

[11] Roldan. A, Martinez-Moreno. J, Roldan. C and Karapinar. E, "Some remarks on multidimensional fixed point theorems", Fixed Point Theory, Vol. 15, no. 2, 545-558, 2014.

[12] Smarandache, F., "Neutrosophic set, a generalization of intuitionistic fuzzy sets, Int. J. Pure Appl. Math., 24(2005), 287-297.

[13] Wadei F. Al-Omeri, Saeid Jafari & F. Smarandache, "Neutrosophic Fixed Point Theorems and Cone Metric Spaces", Neutrosophic Sets & Systems, Vol. 31, 2020, 250-265.

[14] Wadei F. Al-Omeri, Saeid Jafari & F. Smarandache, "$(\phi-\psi)$ - Weak Contractions in Neutrosophic Cone Metric Spaces via Fixed Point Theorems", Hindawi Journal Mathematical Problems in Engineering , 2020, Article ID 9216805, pages 1-8.

[15] Wadei F. Al-Omeri, "Neutrosophic g^* closed sets in Neutrosophic Topological Spaces", International Journal of Neutrosophic Science, Vol. 20(4),, 2023, 210-222.

[16] Ghareeb. A, Al-Omeri Wadei F., "New degrees for functions in (L, M) - fuzzy topological spaces based on (L, M)-fuzzy semiopen and (L, M)-fuzzy pre open operators", Journal of Intelligent and Fuzzy Systems, vol. 36, 2019, no. 1, pp. 787-803,

[17] Wadei Faris Al-Omeri, "On Mixed b-Fuzzy Topological Spaces",International Journal of Fuzzy Lofic and Intelligent Systems, Vol. (3), 2020, 242-246.

[18] Wadei Faris IL-Omeri, M. Kaviyarasu and Rajeshwari M, "Identifying Internet Streaming Services using Max Product of Complement in Neutrosophic Graphs", International Journal of Neutrosophic Science (IJNS) Vol. 23, 2024,No. 01, PP. 257-272.

[19] Wadei Faris Al-Omeri, "Fuzzy Totally Continuous Mappings based on Fuzzy m-Open Sets inSostaks Sense", International Journal of Applied and Computational Mathematics, 10(2), 2024, 73.

[20] Suganthi M., Jeyaraman M., "A Generalized Neutrosophic Metric Space and Coupled Coincidence Point Results, Neutrosophic Sets and Systems, 42(2021), 253-269.

[21] Vishal Gupta, Manu Verma and Mohammad Saeed Khan, "Some Modified Fixed Point Results in V- Fuzzy Metric Spaces, Advances in Fuzzy Systems, Article ID 6923937, 10 pages, 2019.

[22] WutipholSintunavarat and PoomKumam, "Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces", Journal of Applied Mathematics, Article ID 637958, 14 pages, 2011.

[23] Zadeh L.A., "Fuzzy Sets", Inform. And Control, Vol. 8, 1965, 338-353.