Review Form 1.7

Journal Name:	**Asian Research Journal of Mathematics**
Manuscript Number:	**Ms_ARJOM_119259**
Title of the Manuscript:	**Irredundant and almost irredundant sets in M2**
Type of the Article	**Original Research Article**

General guideline for Peer Review process:

This journal's peer review policy states that **NO** manuscript should be rejected only on the basis of '**lack of Novelty'**, provided the manuscript is scientifically robust and technically sound.
To know the complete guideline for Peer Review process, reviewers are requested to visit this link:

(https://www.journalarjom.com/index.php/ARJOM/editorial-policy)

Review Form 1.7

PART 1: **Review Comments**

	Reviewer's comment	**Author's comment** *(if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)*
Compulsory REVISION comments **Is the manuscript important for scientific community?** (Please write few sentences on this manuscript) **1. Is the title of the article suitable?** (If not please suggest an alternative title) **Is the abstract of the article comprehensive?** **2. Are subsections and structure of the manuscript appropriate?** **3. Do you think the manuscript is scientifically correct?** **4. Are the references sufficient and recent? If you have suggestion of additional references, please mention in the review form.** **(Apart from above mentioned 6 points, reviewers are free to provide additional suggestions/comments)**	**The article is interesting, and pleasant to read ; it is most of the time very clearly written and the notions are introduced in a natural way.** **In the title, replace $\mathbb M_2$ by $\mathbb M_2(\mathbb C)$.** **Yes. Replace all "set" by "subset".** **Yes.** **Not in its current state.** **Yes**	
Minor REVISION comments **Is language/English quality of the article suitable for scholarly communications?**	Yes	
Optional/General comments	The author studies 2 notions of "irredundant" generating subsets of the matrix algebra $M_2(C)$, namely *-irredundancy introduced in (4) [This is unclear in the paper] and almost irredundancy introduced in (3). The author shows that these 2 notions "are not the same". It is known that the maximum size of an irredundant subset S of generators of the algebra $M_2(C)$ is 3. The author shows here that if the conjugate transpose * is allowed in the algebra operations, this maximum size becomes 2 (Proposition 2.2) and is best possible (Remark 2.1). The author also claims that $M_2(C)$ has an in nite almost irredundant set (Proposition 3.1, with a gap in the proof of Lemma 3.1 though). The article is interesting, and pleasant to read ; it is most of the time very clearly written and the notions are introduced in a natural way. However Proposition 2.2 considers only $M_2(C)$, a very special case: the author should absolutely go on and also study the same problem in $M_n(C)$ when $n > 2$. Bellow is a list of comments; I shall make a recommendation about publication only after having seen a revised version that considers all the items of the list. ˆ In the title, replace M_2 by $M_2(C)$. ˆ In the Keywords: irredundancy. ˆ page 2, line 2, replace ", we say" by " for every $x \in S$, we say". ˆ p. 2, l. 5: replace "results" by "result". ˆ p. 2, l. -17: replace "involution" by "an involution".	

	ˆ p. 2, l. -14: "when an involution" ˆ p. 2, l. -11: "integrating an involution" ˆ p. 2, l. -8: add a coma after "$A^* = A$". ˆ p. 2, l. -8: replace "$AA^* = A^*A$" by "$A^*A = AA^* = Id$". ˆ p. 3, l. 1: replace "Let " by "Let F =". ˆ p. 3, l. 15: add one line to explain why $\{B_1, C_1, A_2, ..., A_n\}$ is *-irredundant, maybe by saying that $alg^*(B_1, C_1, A_3, ...A_n)$ equals $alg^*(A_1, A_3, ...A_n)$. ˆ p. 3, l. 17: replace the full sentence "By Claim 1 and Claim 2, ..." by something more pre cise, like "By Claim 2, we may say without loss of generality that say $\{B_1, A_2, . . . , A_n\}^*$- generates $M_2(C)$; it follows that $B_1 \not\in alg^*(A_2, . . . , A_n)$; by Claim 1, the set $\{B_1, A_2, . . . , A_n\}$ is also *-irredundant". ˆ p. 3, Proposition 2.1 should not be stated this way since the "Moreover, if $\|S\| = 3$" part is an empty statement (you show that it cannot occur). Maybe cite (5, Theorem 2.1) instead. ˆ p. 3, Propositions 2.1 and 2.2: When you use (5, Theorem 2.1), bear in mind that since elements of S are self-adjoint, and since U is unitary, then A, B and C are also self adjoint: so one has $c = e = 0$, and your proof of Proposition 2.2 can be simpli ed : A, B and C are all diagonal matrices. ˆ p. 3: In the statement of Proposition 2.1, "USU" should be "U^*SU". ˆ p. 4: in Remark 2.1, where did "we observed that" ? ˆ p. 3-4: last but not least : you should investigate what happens for $M_n(C)$ when $n > 2$. ˆ p. 4, l. 6: replace "*-irredundance" by "*-irredundance introduced in (3)" (even if you have already said it in the introduction) ˆ p. 4: around De nition 3.1, you should precise somewhere that almost irredundancy in (3) was de ned in the general context of a C^*-algebra, and that you restrict it here both to $M_2(C)$ and to sets of self-adjoint elements. Explain why you restrict it to sets of self-adjoint elements. ˆ p. 4: in Lemma 3.1, there is no need to de ne an ε, just say that $\tau(A_i) < 1$ holds for every $1 \leq i \leq n$. ˆ p. 4, l. -3: I do not understand this inequality. Lemma 3.1 is correct if you can show that $\tau(A_1 \cdots A_n)$ is always non negative, but I am not sure about that. At the very least, this inequality needs an explanation. ˆ p. 5, proof of Lemma 3.2: I would reduce a bit the size of this proof; for example: "Consider a sequence of distinct points $(\theta_i)_{i\in N}$ in $[0; \frac{\pi}{2}]$ and de ne $x_i = \cos(\theta_i)$ for each $i \in N$. We claim that the family of points $(x_i)_{i\in N}$ has the desirable properties: one has $f(x_i, x_j) = \cos(\theta_i)\cos(\theta_j) + \sin(\theta_i)\sin(\theta_j) = \cos(\theta_i - \theta_j)$. It follows that $f(x_i, x_j) < 1$ when $i \neq j$, and $f(x_i, x_i) = 1$ for each $i \in N$, as required." ˆ p. 5, l. -14: replace "projection" by "orthogonal projection". ˆ p. 5, l. -11: be consistent with your notations, replace "tr" by "trace". ˆ p. 6, l. 10 of Conclusion: you mention in the introduction that *-irredundance has been introduced in (4) in 2020 but you say here that it is considered also in (8) in 1984. Please clarify this point. Are you making a di erence between the concept and its name?	

Review Form 1.7

PART 2:

	Reviewer's comment	**Author's comment** *(if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)*
Are there ethical issues in this manuscript?	*(If yes, Kindly please write down the ethical issues here in details)*	

Reviewer Details:

Name:	**Milliet Cedric**
Department, University & Country	**Lycée Charlie Chaplin, France**