

Review Article

MATHEMATICAL APPLICATIONS FOR SUSTAINABLE AGRICULTURAL DEVELOPMENT: A REVIEW

ABSTRACT

This review article explores the pivotal role of mathematics in fostering sustainable development within the realm of agriculture. Drawing upon diverse sources, the paper investigates the multifaceted applications of mathematical principles in various facets of agricultural practices. From soil analysis to financial decision-making, crop modeling, and statistical approaches for yield tests, mathematics emerges as an indispensable tool for enhancing efficiency, productivity, and environmental sustainability in agriculture. The article delves into the significance of number systems, mathematical models, and experimental designs, providing a comprehensive overview of how mathematical applications contribute to informed decision-making and innovation in agricultural science. The exploration extends to statistical approaches for yield tests, emphasizing the importance of well-designed experiments in yielding robust and reliable results.

Keywords: Mathematics; Agriculture; Sustainability; Mathematical models; Experimental designs; Statistical.

1. Introduction

Johnson and Rising [1] described mathematics as a creation of the human mind, concerned primarily with ideas, processes and reasoning. In all spheres of life, including agriculture, application of mathematics has been found to be key [2]. Mathematics is called “The Queen of Science”.

Agriculture is an integral part of science which involves the cultivation of land, raising and rearing of animals for the purpose of production of food for man, feed for animals and raw materials for industry [3]. Development in a sustainable way is defined as a continuous process of generating and more efficiently allocating resources (without degradation) for achieving greater socially satisfying needs.

2. Requirements for Sustainable Development in Agriculture

According to Adedayo [4], the following are requirements of agricultural development:

2.1 Marketing

There should be a high demand or consumption for agricultural product; marketing entails getting products from farmers to the consumers. It enlarges agricultural production as well as facilitating industrial growth and bringing greater wealth to the nation. Marketing involves transportation, storage, processing and packaging.

2.2 New Farm Technology

Agricultural development requires a research programme that is continuously generating new agricultural techniques.

2.3 Purchasable Input

Agricultural inputs like seeds, fertilizers, etc. should be available at affordable prices to farmers, it should also be supplied at the right time and in the right place.

2.4 Incentives to Farmers

Farmers should be motivated towards producing more through the provision of price subsidy, fair sharing of farm produce in case of share cropping and the goods and services required by farmers should be made available for improving their standard of living.

2.5 Production Credit

Credit to farmers is necessary to be able to procure and adapt necessary technologies. A major source of credit to farmers is by government and commercial banks which are usually not available to farmers because of lack of collateral security and default in repayment.

These requirements demand one form of mathematics application or the other, especially in the area of new farm technology known to be the springboard of agricultural development. While acknowledging this fact, Ajai and Imoko [5] pointed out that without mathematics there is no science, without science there is no modern technology and without modern technology in our farms and farming activities, the society stands to suffer greatly [6].

3. Role of Mathematics for the Sustainable Development of Agriculture

Mathematics plays a crucial role in the sustainable development of agriculture by providing valuable tools and techniques for analyzing data, making informed decisions, and optimizing various aspects of agricultural practices. To be a farmer it is more important to have problem solving, decision making and money management abilities. They use advanced mathematical technology to calibrate machines and irrigation pumps. The areas in agriculture where mathematical skill is vital include:

3.1 Soil Analysis

One of the fundamental aspects of sustainable agriculture is understanding the properties and characteristics of soil. Mathematics facilitates soil analysis by enabling precise measurements and calculations. Mathematical models are used to interpret soil composition, texture, nutrient levels, and pH values, helping farmers determine optimal fertilization and irrigation practices. Through data analysis and statistical methods, mathematicians can identify soil patterns, predict nutrient deficiencies, and recommend appropriate soil amendments, thus maximizing crop yield and minimizing resource wastage.

Mathematical skill is needed in soil analysis to assist the agriculturalist in the:

- a. Calculation of existing amount of soil nutrient
- b. Calculation of amount of soil nutrient to be added to the soil

3.2 Evaluation of Retailer Performance

Mathematics plays a vital role in evaluating the performance of retailers in the agricultural sector. By employing statistical analysis and data modeling techniques, mathematical tools allow for the assessment of various factors such as supply chain efficiency, inventory management, pricing strategies, and customer demand. Farmers and policymakers can use these mathematical models to identify inefficiencies, optimize distribution channels, reduce post-harvest losses, and improve overall profitability while minimizing environmental impact. Mathematics plays a role in investments and advertisement, calculation of revenues generated and investments made as percentage of sales [7].

3.3 Application Rates

Determining the appropriate application rates for fertilizers, pesticides, and other agricultural inputs is crucial for sustainable farming. Mathematics provides the necessary framework for optimizing application rates, taking into account factors such as soil fertility, crop type, growth stage, and environmental considerations. Mathematical models, such as regression analysis and optimization algorithms, help in calibrating application rates to ensure optimal resource utilization, minimize negative impacts on ecosystems, and reduce the risk of over-application or under-application. In fertilizer compounding and applications, mathematics is involved and its application to the field must be calculated as under and over applications are counterproductive. It will enable the agriculturist to calculate the nozzle size needed, the amount of pressure, the flow rate and quantity of water needed. In chemical control of pests,

diseases and weeds, a lot but simple mathematics is involved. This is to avoid loss of crops that might occur during preparation (mixing) and application of such chemicals. A wrongly mixed and applied chemical could cause damping-off, burning and killing of the crop foliage. The feeding area is calculated through a mathematical equation given below:

$$\text{Feeding area} = \text{Intra} \times \text{Inter row spacing}$$

3.4 Finance Decision

Effective financial management is essential for the sustainability of agricultural operations. Mathematics assists in financial decision-making processes by providing tools for budgeting, risk analysis, and investment planning. Farmers can use mathematical models to evaluate the profitability of different crops, assess the impact of market fluctuations on their revenue, and make informed decisions about resource allocation. Additionally, mathematical techniques like linear programming can optimize resource allocation and production planning, thereby maximizing profitability while minimizing resource waste. Mathematics skills will assist the agriculturist in the evaluation of investment in trucks, storage tanks and manufacturing.

3.5 Crop Modeling and Yield Prediction

Mathematics plays a crucial role in developing crop models and predicting yields. By incorporating various factors such as weather patterns, soil characteristics, crop genetics, and management practices, mathematical models can simulate crop growth and predict yields under different scenarios. These predictive models assist farmers in making informed decisions regarding planting dates, irrigation schedules, and pest management strategies. By optimizing resource allocation and reducing yield variability, these models contribute to sustainable agricultural practices and ensure food security.

3.6 Optimization of Resource Use

Mathematics enables the optimization of resource utilization in agriculture, promoting sustainable practices. Through mathematical techniques like linear programming, farmers can determine the optimal allocation of resources such as water, land, and fertilizers, taking into account economic, environmental, and social constraints. This optimization process helps farmers achieve higher productivity while minimizing resource waste, improving resource efficiency, and reducing the environmental footprint of agricultural operations.

3.7 Number System

Number systems and related concepts play a crucial role in driving sustainable development in agriculture. They provide a foundation for accurate measurements, efficient resource allocation, and informed decision-making. Table 1 highlights the importance of various number system components in the context of sustainable agriculture, including conversions, number concepts, percentage measurements, and linear programming.

3.8 Others

Farmers use numbers every day for a variety of tasks, from measuring and weighing, to land marking.

Field experimentation is another core area where mathematics applications are used. According to Adedayo [4] it is a systematic and logical process of comparing two or more factors on the field to identify the best in certain characteristics. It is an organized agricultural research endeavour aimed at gaining new facts and knowledge which will bring about increased productivity and agricultural development. It involves the conduct of field experiment where the field work is actually carried out and the application of statistical method for collecting; organizing, presenting, analyzing and interpreting data are employed. Here different fertilizers effects on crops for optimum yields as well as plant population implications are studied. Soil

suitability for crops and other various soil tests are carried out using field experimentation. All these are realizable through full mathematics applications.

More recent advances in the mathematical sciences have helped improve our ability to predict the weather, to measure the effects of environmental hazards and to study the origin of the universe [8]. This is very important in the current trend of climate change phenomena in the entire world today.

Akissani and Muntari [9] pointed out that overpopulation on the farm causes overcrowding, poor ventilation and reduction in yield as a result of competition for the environmental resources; and that low population densities is not desirable as there is emergence of weeds. Plants population is calculated using the formula:

$$\text{Plant population} = \text{Area of land} / \text{Feeding area}$$

Amodu and Okpanachi [10] underscore the relevance and usefulness of various mathematical models. For instance, the Malthusian model on population growth against growth in food production is still a guide for decisions on food security today. Likewise, mathematical model on economic growth against agricultural development can spur government policy and action on agriculture. Again, mathematical models on world population growth against food security have placed agriculture on the top list of global affairs.

Table 1 Importance of Various Number Systems in Agriculture

Number system and concept	Importance in sustainable development in agriculture
Conversion	Facilitates conversion of units for accurate measurements, e.g., converting acres to hectares or pounds to kilograms. Enables efficient resource allocation and planning by converting quantities into standardized units. Allows for the comparison of data from different sources by converting measurements to a common scale. Supports data integration and analysis across systems or regions by converting diverse formats or units.
Number Concept	Helps analyze and interpret agricultural data, such as crop yield, livestock population, or land area, to make informed decisions. Supports forecasting and predictive modeling by understanding trends and patterns in agricultural data. Enables effective budgeting and financial planning based on numerical information. Facilitates risk assessment and management through statistical analysis of agricultural data.
Percentage Measurement	Assists in assessing agricultural productivity and efficiency by calculating growth rates, yield improvements, or resource utilization percentages. Facilitates monitoring and evaluation of sustainable agricultural practices, such as water or energy consumption reduction, by quantifying the percentage change. Helps in understanding market dynamics, pricing, and profit margins by calculating percentage markups or discounts. Supports benchmarking and goal setting by comparing performance against targets or industry standards.
Linear Programming	Optimizes resource allocation and utilization in agriculture, such as determining the most efficient combination of crops, livestock, and land to maximize output.

Helps in identifying the optimal allocation of resources, such as fertilizers, water, or pesticides, to minimize waste and environmental impact.

Supports decision-making in supply chain management, such as determining the most cost-effective transportation routes or storage capacities.

Enables scenario analysis and simulation modeling for evaluating the potential impact of various agricultural strategies.

Nworah [3] revealed that in agricultural mechanics, calculus is used when adjusting aircraft that apply spray material to crops. It can be used to calculate amounts when the application has multiple variables.

In measuring discharge in irrigation, mathematics is also applied. The discharge of water from a pipe or a channel according to Uguru [11] is the volume of water flowing per second. It is measured in cubic metre per second (m^3/s).

Discharge (m^3/s) = area (m^2) \times velocity of flow (m/s).

Fadare *et al.* [12] pointed out the use of mathematics for the following agricultural activities: plotting the demand and supply schedule and curve for decision making, amount of fertilizers to be applied per area of land and per stand of a crop, oestrus cycle, incubation period and gestation periods in livestock, loss and profit accounts and percentages, diameter of soil components, interest on agricultural loans and credit facilities, number of eggs laid by fowls, litres of honey harvested in a bee box, making of in-let and out-let in a fish pond and their correct depth and height, quantity of water to be used for vaccine, etc.

These are just few specific examples of mathematics applications for sustainable development in agriculture.

4. Statistical Approaches for Yield Tests

Field plot techniques are widely used in various scientific disciplines, such as agriculture, ecology, forestry, and environmental science, to gather data and analyze patterns and trends in natural or experimental settings. Field plots involve the establishment of predefined sampling areas where measurements, observations, and experiments are conducted to collect data on specific variables of interest. Field plot techniques provide valuable insights into the distribution, abundance, composition, and dynamics of organisms or physical features in their natural habitats. These techniques often involve the use of standardized protocols to ensure consistency and comparability of data across different studies. The commonly used field plot techniques are given in Table 2.

The choice of field plot technique depends on the specific research objectives, the characteristics of the study area, and the variables being measured. It is important to carefully plan and design field plots, considering factors such as plot size, shape, spacing, and replication, to ensure statistically valid and representative data collection.

Table 2 The Comparative Analysis of Different Field Plot Techniques

Technique	Key Features	Applications
Transects	Linear sampling paths; assesses spatial distribution	Vegetation surveys, animal movements, habitat mapping
Quadrats	Fixed-area frames; estimates population density and composition	Plant ecology, community structure, biomass estimation
Circular Plots	Circular sampling areas; measures vegetation attributes	Forest inventory, plant diversity assessment
Nested Plots	Multiple plots within larger plots; assesses hierarchical patterns	Ecological studies, habitat heterogeneity analysis
Point Sampling	Systematically selected points; estimates cover or density	Vegetation monitoring, soil characterization
Mark-Recapture	Capturing and marking individuals; estimates population size	Population ecology, wildlife conservation

5. Basic principles of a well-designed experiment

It is a planned inquiry to discover new facts or to confirm or deny the results of previous investigations. The basic principles of a well-designed experiment are randomization, replication and local control. These three principles are, in a way, complementary to each other in trying to increase the accuracy of the experiment and to provide a valid test of significance, retaining at the same time the distinctive features of their roles in any investigation.

Before conducting an experiment, an experimental unit is to be defined e.g., a leaf, a tree, or a collection of adjacent trees may be an experimental unit. An experimental unit is also sometimes referred to as a plot. A collection of plots is termed a block. Observations made on experimental units vary considerably. These variations are partly produced by manipulating certain variables of interest generally called treatments, built-in and manipulated deliberately in the experiment to study their influences. Besides the variations produced in the observations due to these known sources, the variations are also produced by a large number of unknown sources such as uncontrolled variation in extraneous factors related to the environment, genetic variations in the experimental material other than that due to treatments, etc. They are there, unavoidable and inherent in the very process of experimentation. These variations because of their undesirable influences are called experimental errors thereby meaning not an arithmetical error but variations produced by a set of unknown factors beyond the control of the experiment. The experimental observations by extraneous factors may be either systematic or random in their mode of incidence. The errors arising due to equipment or the error due to observer fatigue are examples of systematic errors. On the other hand, the unpredictable variation in the number of leaves collected in litter traps under a particular treatment in a related experiment is random in nature. It is clear that any number of repeated measurements would not overcome systematic error whereas it is very likely that the random errors would cancel out with repeated measurements.

5.1 Randomization

Randomization is a fundamental principle that helps eliminate bias and ensure the validity of experimental results. It involves assigning subjects or treatments randomly to different experimental conditions. By doing so, researchers minimize the effects of confounding

variables and create balanced groups, thus enhancing the internal validity of the study. Randomization can be achieved through various methods, such as simple randomization, block randomization, and stratified randomization. Simple randomization involves assigning subjects randomly to different groups, whereas block randomization ensures that each treatment appears an equal number of times within blocks of subjects with similar characteristics. Stratified randomization, on the other hand, involves dividing subjects into subgroups based on certain characteristics and then randomly assigning treatments within each subgroup.

5.2 Replication

Replication involves conducting an experiment multiple times with different samples or subjects to assess the consistency and reliability of the results. It enhances the external validity of the study by increasing the generalizability of the findings beyond the specific sample used in the experiment. Replication can be performed through two main approaches: exact replication and conceptual replication. Exact replication involves reproducing the entire study with an independent sample and attempting to obtain the same results. Conceptual replication, on the other hand, focuses on testing the same research question using different methods, measures, or populations.

5.3 Local control (error control)

Local control refers to the control of extraneous variables within the experimental setting. It involves manipulating and monitoring the variables that are likely to affect the outcome of the study, allowing researchers to establish a cause-and-effect relationship between the independent and dependent variables. Local control can be achieved through various techniques, including random assignment, blocking, matching, and statistical control. Random assignment distributes potential confounders evenly across treatment groups. Blocking involves grouping subjects based on specific characteristics (e.g., age, gender) and then randomly assigning treatments within each block. Matching involves pairing subjects with similar characteristics and assigning them to different treatments. Statistical control, often achieved through techniques like analysis of covariance (ANCOVA), involves including confounding variables as covariates in the analysis. The replication is used with local control to reduce the experimental error.

6. Field plot experimental designs

In the field evaluation of promising materials, there are different methods of field plot experiments. It depends upon the size of the test materials and availability of land for experiments besides, other factors. Among different field plot experimental designs, the most commonly used field experimental designs are RCBD, ARCBD and LSD (Table 3).

6.1 Randomized complete block design

If a large number of treatments are to be compared, then a large number of experimental units are required. This will increase the variation among the responses. In such a case when the experimental material is not homogeneous and there are v treatments to be compared, then it may be possible to:

- Group the experimental material into blocks of sizes v units.
- Blocks are constructed such that the experimental units within a block are relatively homogeneous and resemble each other more closely than the units in the different blocks.
- If there are b such blocks, we say that the blocks are at b levels. Similarly, if there are v treatments, we say that the treatments are at v levels. The responses from the b levels of blocks and v levels of treatments can be arranged in a two-way layout. The observed data set is arranged as follows:

	Treatments (Factor B)						Block totals	
	1	2	...	j	...	v		
Blocks (Factor A)	1	y_{11}	y_{12}	...	y_{1j}	...	y_{1v}	B_1
	2	y_{21}	y_{22}	...	y_{2j}	...	y_{2v}	B_2

	i	y_{i1}	y_{i2}	...	y_{ij}	...	y_{iv}	B_i

	b	y_{b1}	y_{b2}	...	y_{bj}	...	y_{bv}	B_b
Treatment totals	T_1	T_2	...	T_j	...	T_v	Grand total (G)	

Fig .1 **Randomized complete block design**

A two-way layout is called a randomized block design (RBD) or a randomized complete block design (RCBD) if, within each block, the v treatments are randomly assigned to v experimental units such that each of the $v!$ ways of assigning the treatments to the units has the same probability of being adopted in the experiment and the assignment in different blocks are statistically independent.

6.2 Augmented Randomized Complete Block Design (ARCB)

Augmented RCBD is a function for analysis of variance of an augmented randomised block design and the generation as well as comparison of the adjusted means of the treatments/genotypes. ARCB has both replicated and un-replicated treatments. The replicated treatments are tested in each block as in an RCBD but, un-replicated treatments occur in only one block so each block has a different set of un-replicated treatments. It has an advantage of flexibility with large numbers of treatments.

6.3 Latin Square Design (LSD)

In Latin square design (LSD), the experimental material is divided into rows and columns, each having the same number of experimental units which is equal to the number of treatments. The treatments are allocated to the rows and the columns such that each treatment occurs once and only once in each row and in each column.

A Latin square of order p is an arrangement of p symbols in $2 p$ cells arranged in p rows and p columns such that each symbol occurs once and only once in each row and in each column. For example, to write a Latin square of order 4, choose four symbols i.e. A, B, C, and D. These letters are Latin letters that are used as symbols. Write them in a way such that each of the letters out of A, B, C, and D occurs once and only once in each row and each column. For example, as

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Formatted: Font: Italic

Table 3 The key differences between Randomized Complete Block Design (RCBD), Augmented Randomized Complete Block Design (ARCB), and Latin Square Design (LSD)

Property	RCBD	ARCB	LSD
Structure	The experimental units are grouped into homogeneous blocks, and treatments are randomly assigned within each block.	Similar to RCBD, but additional treatments (controls or standards) are included in each block alongside the main treatments.	Treatments are arranged in a square grid, with each treatment appearing once in each row and column.
Blocking	Blocks are formed based on a relevant blocking factor, which helps account for variation within the experimental units.	Similar to RCBD, blocks are formed based on a blocking factor. In addition, each block contains both the additional treatments and the main treatments.	Does not use blocking in the traditional sense. Instead, it controls for both row and column effects by arranging treatments in a square grid.
Treatment Replication	Each treatment appears once within each block, allowing for comparisons within homogeneous groups.	Each block contains both the additional treatments and the main treatments, ensuring all treatments are represented within each block.	Each treatment appears once in each row and column, providing balanced representation across the entire design.
Objective	Designed to reduce the effect of variation by accounting for known sources of variability through blocking.	Extends RCBD by including additional treatments to improve efficiency and statistical power, particularly when comparing against control or standard treatments.	Controls for both row and column effects to investigate the combined influence of two factors simultaneously.
Applicability	Suitable when there is a known or expected source of variability that can be controlled through blocking.	Useful when it is essential to include additional treatments alongside the main treatments to improve experimental design.	Appropriate when the primary interest lies in investigating the combined effects of two factors that can be arranged in a square grid.

Formatted Table

6. Conclusion

This comprehensive review underscores the paramount importance of mathematical applications in advancing sustainable development within the agricultural domain. The myriad applications of mathematics, ranging from soil analysis and financial decision-making to crop modeling and experimental designs, collectively contribute to optimizing resource utilization, enhancing productivity, and mitigating environmental impact. The integration of statistical approaches, such as randomized complete block designs and Latin square designs, further strengthens the reliability of agricultural experiments and ensures the validity of results. As the

global agricultural landscape confronts challenges like climate change and increasing population, the adoption of mathematical tools becomes imperative for informed decision-making and innovative solutions. Moving forward, the harmonious integration of mathematics with agricultural science is indispensable for achieving and maintaining sustainable agricultural practices worldwide.

References

- [1] Johnson DA, Rising GR. Guidelines for teaching mathematics. (2nd ed). Belmont, Wardsworth Publishing Company. 1972.
- [2] Enemali IA, Adah OC. Empowering individuals and communities through agricultural education: The role of mathematics education. *Journal of Vocational and Technical Educators*. 2015a; 4(4): 46-50.
- [3] Nworah FC. Mathematics application for agricultural development in Nigeria. In Sadiku, J. S (ed.) *Proceedings of September 2015 Annual National Conference of Mathematical Association of Nigeria (M.A.N.)*. 2015; pp: 72-77.
- [4] Adedayo RO. Agricultural economics for colleges and secondary schools. Ado-Ekiti, Adedayo Printing Nigeria Limited. 1999; pp: 65-171.
- [5] Ajai IJ, Imoko TB. Mathematics as a tool for sustainable national development and for addressing Nigerian challenges in the 21st century. *Nasher Journal*. 2011, 9(2): 95-101.
- [6] Enemali IA, Adah OC. Quality assurance in educational administration in the teaching of farm mathematics for national integration in Nigeria. *Journal of Education and Practice*. 2015b; 6(23): 52-56.
- [7] Malami B, Ayyub MH, Halilu A. Mathematics education as a tool for national development. A paper presented at the 50th Annual National Conference of Mathematical Association of Nigeria (M.A.N.) held at St. Patrick's College, Asaba, Delta State. 31st August-5th September. 2013.
- [8] Aguele LI, Usman KO. Mathematics education for dynamic economy in Nigeria in the 21st century. 2007; Retrieved from <http://www.krepublishers.com/02>.
- [9] Akissani I, Muntari I. The role mathematics in agricultural development. In Sadiku, J. S (ed.) *Proceedings of September 2015 Annual National Conference of Mathematical Association of Nigeria (M.A.N.)*. 2015; pp: 317-324.
- [10] Amodu A, Okpanachi D. Effect of mathematical applications on agricultural development in Kogi State, Nigeria. In Sadiku, J.S (ed.) *Proceedings of September 2015 Annual National Conference of Mathematical Association of Nigeria (M.A.N.)*. 2015; pp: 296-303.
- [11] Uguru MI. Crop production: Tools, techniques and practice. Nsukka, Fulladu Publishing Company. 1996; pp: 2432.
- [12] Fadare AO, Ayeni AA. Enhancing the teaching and learning of commercial mathematics for entrepreneurship skills development. A paper presented at the 5th Annual National Conference of School of Sciences, Federal College of Education (Special), 7th-10th April. 2015.

UNDER PEER REVIEW

