Modeling Open Channel Fluid Flow past a Trapezoidal Cross-section with a Segment Base having Lateral Inflow Channel

ABSTRACT

A flow in a closed conduit is regarded as an open channel flow if it has a free surface. A free surface is the surface of a fluid that is subject to zero parallel shear stress. Floods in flood-stricken areas have been a major threat to the survival of lives and livelihoods in various aspects. For instance, increased pot-holes, road disconnection and tearing off as well as bridges being carried away have led to increased cases of accidents leading to loss of lives. This has led to Government over-stretching budgetary allocations to cater for maintenance and repair of roads and bridges. This study has developed a model for fluid flow past an open channel with a trapezoidal cross-section with a segment base having lateral inflow channel. The turbulent formation between the lateral inflow channel and the main channel are assumed to be negligible and hence the flow is laminar. The model equations governing the fluid flow are non-dimensionalized and solved using finite-difference method. The numerical values are simulated using Matlab software. The results are discussed, analyzed and presented. It is found that an increase in cross-section area of the lateral channel increases the discharge in the main channel leading to an increase in flow velocity. An increase in surface roughness increases shear stress thereby recording a reduced flow velocity. The findings are applicable to engineers in making appropriate drainage channels with optimal dimensions for maximum discharge. Moreover, the findings are applicable in hydro-electric power plants where large volumes of high velocity water are required to turn large turbines for electrical processes.

Key Words: Open Channel, Free Surface, Floods, Cross-Section area, Surface Roughness, Electrical Processes

1. INTRODUCTION

Water flows more rapidly on a steeper gradient but for a constant gradient, the velocity reaches a steady value when the gravitational force is equal to the resistance of the flow [1]. The cross-section of a channel may be open or closed at the top [2]. The structures with closed tops are referred to as closed conduits while those with open tops are called open channels [3].

In order to direct flood water to the desired areas such as farms, dams and lakes, man has constructed open channels of different cross-sections which are the physical systems in which fluid, in this case water, flows with a free surface [4]. The free surface is the interface between two fluids of different densities, such as water and atmospheric air [5].

In Kenya, most road networks lack efficient drainage systems, especially rural roads set up and hence, road carnage, fatalities and economic devastation are all common, particularly when it rains [9]. This has a negative effect on the achievement of Kenya's vision 2030 that aims to create a high-quality, internationally competitive and prosperous Nation. The three main pillars that the government aims to accomplish in line with vision 2030 are anchored on economic, political and social platforms [6]. These three pillars are linked to this study in that inadequate drainage directly affects the economic, political and social aspects of the nation at large. When it rains, transport is disrupted and roads are cut off by runoff and this affects the flow of goods and services [10]. Disease outbreaks and other related health issues pose a danger to the population's health if drainage is insufficient [7]. As a result, this study seeks to find solutions to these drainage related issues in order to contribute to the vision 2030.

The study by Macharia *et al* [3] developed a model on fluid flow past a rectangular cross-section with a lateral inflow channel. The findings reported that increasing the cross-section area of the lateral channel leads to a decrease in the flow velocity of the open rectangular channel.

On the effect of varying the inclination angle of two lateral inflow channels on the flow velocity of the main rectangular channel, Chirchir *et al* [6] developed a model of continuity and momentum equations. The findings established that the flow velocity at the main channel increases at inclination angles between the range of 30^0 and 72^0, for two lateral channels but the flow velocity is at maximum at an inclination angle of 45^0 for each lateral channel to the main channel.

Kinyanjui *et al* [1] developed a model on fluid flow past a circular cross-section. The study reported that for any circular cross-section, increasing the slope of the channel led to an increase in flow velocity. Further, an increase in the radius of the channel led to decrease in the flow velocity of the fluid.

Marangu *et al* [8] developed a model on open trapezoidal channel with a segment base. The study reported that increasing the cross sectional area of the flow leads to decrease in the flow velocity. An increase in channel radius and manning coefficient of surface roughness leads to a decrease in flow velocity. As the channel slope increases, the flow velocity along the channel also increases.

The study by Mose *et al* [7] developed a model on fluid flow past an open channel with elliptic cross-section. The findings established that an increase in channel friction resulted to decrease in fluid flow velocity. An increase in hydraulic radius leads to an increase in fluid flow depth.

In order to minimize floods, Engineers have designed channels of different cross-sections to convey maximum discharge to designated areas but still the problem of flooding continue to persist. The analysis of this study focuses on appropriate cross sectional area and surface roughness of the lateral channel to align with the main channel in order to maximize discharge of water from flooded areas, which is a frequent occurence in rainy seasons. A lot of study appears to have been done in open channels of rectangular, elliptic and circular cross-sections. The study on trapezoidal channel with a base segment and a lateral inflow channel, on the other hand, has received little attention. As a result, this study aims at developing a model of a trapezoidal cross-section with a segment base having a lateral inflow channel that will have maximum discharge of water. The results of this study is highly applicable in the design of drainage systems for road construction, sewer building, street drainage, dams, and airport construction in Kenya and elsewhere.

2. RESEARCH OBJECTIVES

i. To develop a model for fluid flow past an open channel with a trapezoidal cross-section with a segment base having lateral inflow channel.

ii. To determine the effect of surface roughness of the lateral channel to the flow velocity of the main channel

iii. To determine the effect of cross-section area of the lateral channel on the flow velocity of the main channel

3. BASIC ASSUMPTIONS

i. The flow is natural and thus is due to gravitational forces only.

ii. The fluid is considered incompressible hence density is invariant.

iii. The flow is laminar and unsteady

iv. The cross-section area of the main channel is twice that of lateral channel.

v. The fluid in consideration is Newtonian.

4. GEOMETRIC REPRESENTATION OF THE MODEL

Figure 1 illustrates the geometric model of the trapezoidal channel with a segment base, having the trapezoidal lateral inflow channel at an angle. The discharge in the main channel and the lateral inflow channel is denoted by Q and q respectively, while θ and L represent the inclination angle and length respectively of the lateral inflow channel.

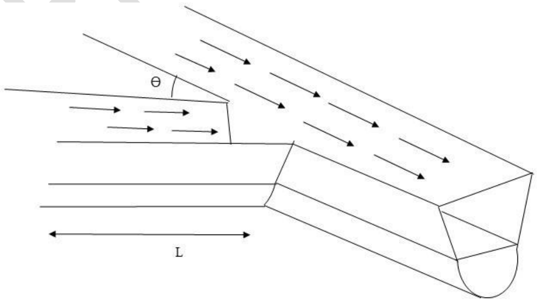

Figure 1: Geometric representation of the model

5. THE MATHEMATICAL FORMULATION OF THE MODEL

5.1 Continuity Equation

The continuity equation is based on the principal of conservation of mass, which states that mass cannot be created nor destroyed. For any given shape of an open channel with a lateral inflow channel, the continuity equation for unsteady incompressible fluid flow is given by equation (1.1)

$$V\frac{\partial y}{\partial x} + \frac{A}{T}\frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} = \frac{q}{TL}\operatorname{Sin}\theta \qquad (1.1)$$

Consider Figure 2, the cross section of a trapezoidal channel with a segment base.

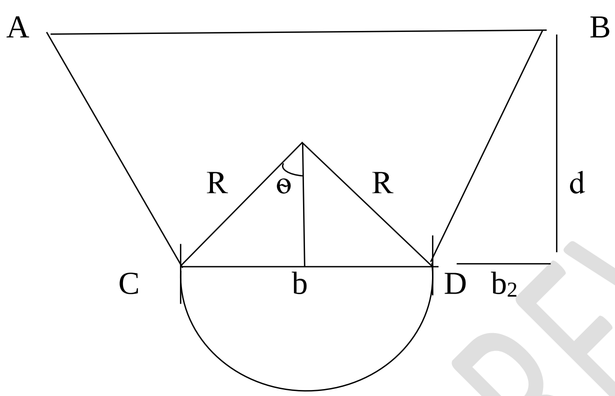

Figure 2: Cross-section of a trapezoidal channel with a segment base

The cross-sectional area of trapezium ABCD (figure 2) is given by equation 1.2

$$A_1 = (bd + b_2d^2) \qquad (1.2)$$

while that of the bottom segment is given by equation 1.3

$$A_2 = R^2(\theta - \sin\theta\cos\theta) \qquad (1.3)$$

Total area of the cross-section is thus given by equation 1.4

$$A = (bd + b_2d^2) + R^2(\theta - \sin\theta\cos\theta) \qquad (1.4)$$

Let the length b increase by a factor b_1, and Radius R increase by a factor R_1. The new cross section area, A is given by equation 1.5

$$A = [(b + b_1)d + b_2d^2)] + [(R + R_1)^2(\theta - \sin\theta\cos\theta)] \qquad (1.5)$$

Let the angle of inclination be defined by $\frac{\pi}{m}$ where m is the set of positive integers and the length of the channel be γ+L where γ is the set of positive integers. The lateral discharge along the direction of flow will be defined by $\frac{1}{2}\frac{q}{(\gamma+L)T}\ sin\ \frac{\pi}{m}$. Therefore, the modified continuity equation becomes equation 1.6

$$V\frac{\partial y}{\partial x}+\frac{[(b+b_1)d+b_2d_2)]+[(R+R1)^2(\theta-\sin\theta\cos\theta)]}{T}\frac{\partial V}{\partial x}+\frac{\partial y}{\partial t}=\frac{1}{2}\frac{q}{(\gamma+L)T}\mathrm{Sin}\frac{\pi}{m} \qquad (1.6)$$

5.2 Momentum Equation

The equation is derived from the Newton's second law of motion that relates the sum of forces acting on the fluid element to the rate of change of momentum. According to the conservation law, the momentum equation is defined by equation 1.7

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial v}{\partial x}+g(S_f-\ S_0)=\frac{q}{AL}\mathrm{Sin}\ \theta(u\cos\theta-V) \qquad (1.7)$$

Where A is the cross section area and u is the flow velocity of the channel.

With previous assumptions and that the cross-section area of the lateral channel is half the cross-section area of the main channel, the modified momentum equation becomes equation (1.8)

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial v}{\partial x}+g(S_f-\ S_0)=$$

$$\frac{q}{[(b+b_1)d+b_2d_2)]+[(R+R_1)^2(\theta-\sin\theta\cos\theta)]\ (\gamma+L)}\mathrm{Sin}\ \frac{\pi}{m}(u\cos\frac{\pi}{m}-V) \qquad (1.8)$$

6. NUMERICAL METHOD OF SOLUTION

In this study, the model equations are non-linear hence cannot be solved analytically. The equations are first expressed in dimensionless form and then solved using finite difference method. A numerical iterative scheme is regarded as being consistence if it produces a system of algebraic equations that can be demonstrated to be equivalent to the original partial differential governing equations as the grid spacing approaches zero. The stability of a numerical scheme is associated with its ability to minimize errors as iterations progresses. The property of a numerical scheme to produce results which tends to exact solutions as the grid approaches zero is referred to as convergence. Finite difference method has the advantage over other methods because of its highly consistency, convergence and stability. Numerical values are simulated using Matlab software, where the velocity profiles of the main channel are plotted against time as various parameters are varied. The results are demonstrated graphically for various values of the parameters involved in the study.

6.1 Model Equations in finite difference form

In order to express the model equations in finite difference form, the Taylor series approximations are taken as equations 1.9a, 1.9b, 1.9c, 1.9d.

$$\frac{\partial v}{\partial t} = \frac{v(i,j+1) - v(i,j)}{\Delta t} \qquad (1.9a)$$

$$\frac{\partial y}{\partial t} = \frac{y(i,j+1) - y(i,j)}{\Delta t} \qquad (1.9b)$$

$$\frac{\partial v}{\partial x} = \frac{v(i+1,j) - v(i-1,j)}{2\Delta x} \qquad (1.9c)$$

$$\frac{\partial y}{\partial x} = \frac{y(i+1,j+1) - y(i-1,j)}{2\Delta x} \qquad (1.9d)$$

Substituting these equations in our model Continuity equation we get

$$\text{v(i,j)}\,\frac{y(i+1,j)-y(i-1,j)}{2\Delta x} + \left(\frac{(\mathrm{b}+\mathrm{b}_1)\mathrm{d}+b_2d_2)] + [(\mathrm{R}+R_1)^2(\theta-\sin\theta\cos\theta)]\,(\gamma+\mathrm{L})}{T}\right)\frac{v(i+1,j)-v(i-1,j)}{2\Delta x} +$$

$$\frac{y(i,j+1)-y(i,j)}{\Delta t} = \frac{1}{2}\,\frac{q}{(\gamma+L)T}\sin\frac{\pi}{m} \qquad (1.10)$$

Rearranging equation 1.10 yields equation 1.11

$$y(i,j+1) = \Delta t\left\{-\mathrm{v(I,j)}\,\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}\right\} -$$

$$\Delta t\left\{\left(\frac{(\mathrm{b}+\mathrm{b}_1)\mathrm{d}+b_2d_2)]+[(\mathrm{R}+R_1)^2(\theta-\sin\theta\cos\theta)]\,(\gamma+\mathrm{L})}{T}\right)\frac{v(i+1,j)-v(i-1,j)}{2\Delta x} + \frac{1}{2}\,\frac{q}{(\gamma+L)T}\,\sin\frac{\pi}{m}\right\} +$$

$$\mathrm{y(i, j)} \tag{1.11}$$

Substituting equation 1.9 in our model Momentum equations we get equation 1.12

$$\frac{v(i,j+1)-v(i,j)}{\Delta t} + \mathrm{v(i,j)}\,\frac{v(i+1,j)-v(i-1,j)}{2\Delta x} + \mathrm{g}\,\frac{y(i+1,j)-y(i-1,j)}{2\Delta x} + \mathrm{g}(S_f - S_0) =$$

$$\frac{1}{2}\,\frac{q}{(\mathrm{b}+\mathrm{b}_1)\mathrm{d}+b_2d_2)]+[(\mathrm{R}+R_1)^2(\theta-\sin\theta\cos\theta)]\,(\gamma+\mathrm{L})}\,\mathrm{Sin}\,\frac{\pi}{m}\left\{\left(\mathrm{u}\cos\frac{\pi}{m} - \mathrm{V(i,j)}\right\}\right. \tag{1.12}$$

Rearranging the above equation yields

$$v(i,j+1) = \Delta t\left\{-\mathrm{v(i,j)}\,\frac{v(i+1,j)-v(i-1,j)}{2\Delta x} - \mathrm{g}\,\frac{y(i+1,j)-y(i-1,j)}{2\Delta x} - \mathrm{g}(S_f - S_0)\right\} +$$

$$\Delta t\left\{\frac{1}{2}\,\frac{q}{(\mathrm{b}+\mathrm{b}_1)\mathrm{d}+b_2d_2)]+[(\mathrm{R}+R_1)^2(\theta-\sin\theta\cos\theta)]\,(\gamma+\mathrm{L})}\,\mathrm{Sin}\,\frac{\pi}{m}\left[\mathrm{u}\cos\frac{\pi}{m} - \mathrm{V(i,j)}\right]\right\} + v(i,j) \tag{1.13}$$

The initial and boundary conditions were taken as;

Initial conditions:

y(0,x) = 0.5 v (0,x) = 0.1

boundary conditions:

y (t, x_{initial}) = 1 v(t, x_{initial}) = 1

y (t, x_{final}) = 12 v(t, x_{initial}) = 20

The following constants were also taken

T = 1, L= 1, q = 0.3, $\theta = \frac{\pi}{3.33} = 54.5$ R = 0.4,

B = 0.5, S_0= 0.002, g = 9.82, d=2

7. RESULTS AND DISCUSSIONS

The Matlab software is used to simulate the equation and which appear in appendix. This was done by varying i and j at various nodal points. Then the graphs were plotted using the values of the velocity against time at a certain location. Various flow parameters of cross-section area and surface roughness of the lateral channel are varied to determine how they affect the fluid velocity in the main channel. The graphs are plotted, analyzed, discussed and then conclusions drawn.

7.1 A graph of velocity against time with surface roughness n varying

Figure 3 illustrates the effects of the surface roughness n of the lateral channel to the flow velocity on the main channel.

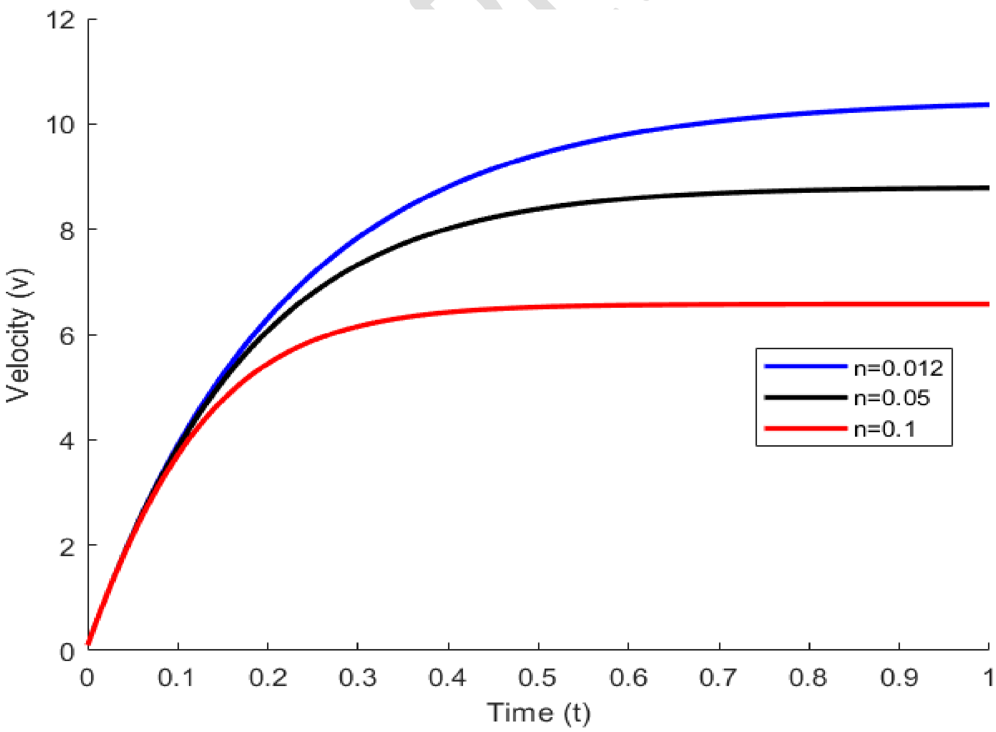

Fig 3. Effects of the surface roughness n of the lateral channel to the flow velocity on the main channel.

From Figure 3, an increase in surface roughness from n= 0.012 to n= 0.1 on the lateral channel leads to a decrease in the flow velocity of the main channel from 10 to 6. This is because increase surface roughness of the lateral channel leads to an increase in shear stress of the fluid particles with the side walls and the bottom of the channel which in turn increases the resistance of the flow rate due to friction leading to a reduction in the flow velocity of the lateral channel. A decrease in the flow velocity of the lateral channel leads to an decrease in the flow velocity of the main channel, since as the velocity of the fluid flow in the lateral channel decreases, less fluid particles at a given time will collide with the fluid particles in the main channel resulting to less bombardments between fluid particles and hence the kinetic energy of the fluid particles is lowered. The reduced kinetic energy leads to the reduction in the flow velocity of the main channel. More so, the reduced velocity of the lateral channel reduces the discharge rate to the Main channel which in turn leads to a reduced fluid flow velocity.

From the manning formula V= $\frac{R^{\frac{2}{3}}S^{\frac{1}{2}}}{n}$, Velocity is inversely proportional to the manning coefficient of roughness and hence the findings agree with this, that increase in surface roughness of the lateral channel leads to a decrease in the flow velocity of the main channel. In this regard, when selecting the most efficient design, engineers should consider minimizing the surface roughness of the channel as much as possible in order that maximum flow rate is achieved. Since concrete is widely used in construction of channels due to its durability and cost friendly, the lining of the surface should be made as smooth as possible since a smooth finish concreate surface would allow faster flow of water hence maximum discharge.

7.2 A graph of Velocity against time with Cross-section Area varying

Figure 4 illustrates the effects of the cross section area of the lateral channel to the flow velocity on the main channel

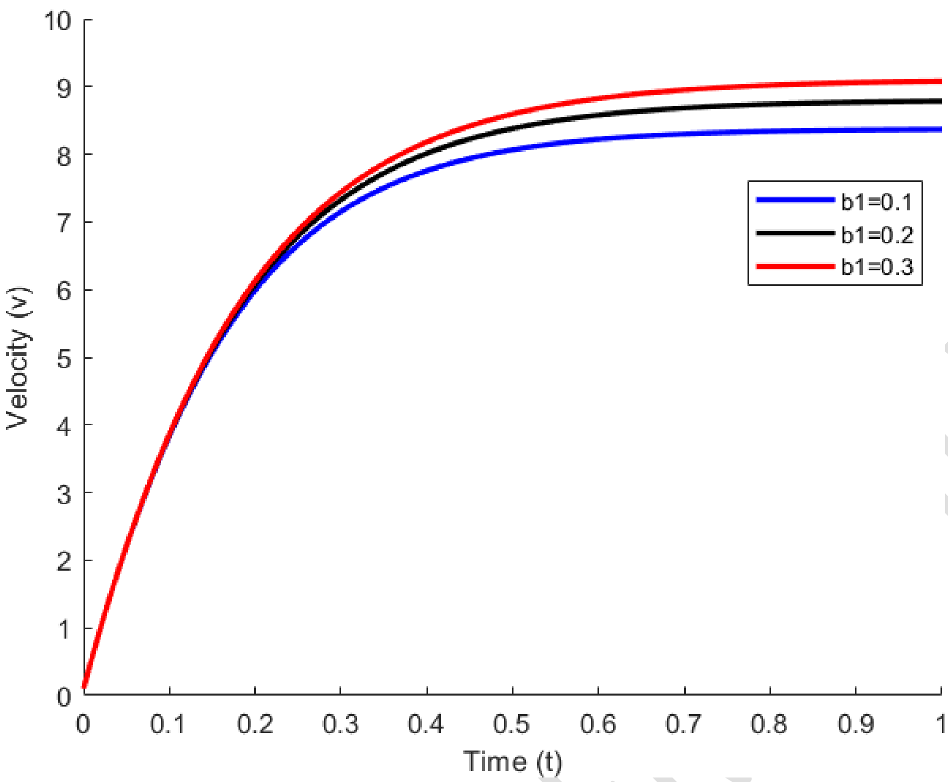

Figure 4: A graph of velocity against against time with b varying

From equation 1.5, the cross section area, A of the model is given by $A = [(b + b_1)d + b_2d^2)] + [(R + R_1)^2(\theta - \sin\theta\cos\theta)]$. The base length of the trapezoidal shape is denoted by b and therefore the cross-section area is directly proportional to base length. From Fiure 4, when base length b is increased from $b_1 = 0.1$ to 0.3, the cross-section area of the channel is also increased. An increase in the cross-section area of the channel leads to increase in fluid flow velocity in the main channel. This is because increasing the cross sectional area of the lateral channel increases the flow discharge to the main channel. Since discharge is given by $Q = AV$, where A and V are cross-section area and velocity respectively, when the flow area is large, discharge is high. Increased flow discharge to the main channel increases the bombardment of the fluid particles of the lateral channel to the particles in the main channel since they collide with each other. This sets an increased kinetic energy which in turn increases the velocity profile of the fluid flow in the main channel.

7.3 A graph of Velocity against time with Segment Radius R varying

The cross section area of the channel is directly proportional to Radius of the base segment and thus Figure 5 illustrates the effects of the increasing the radius of the base segment to the flow velocity on the main channel when R is varied.

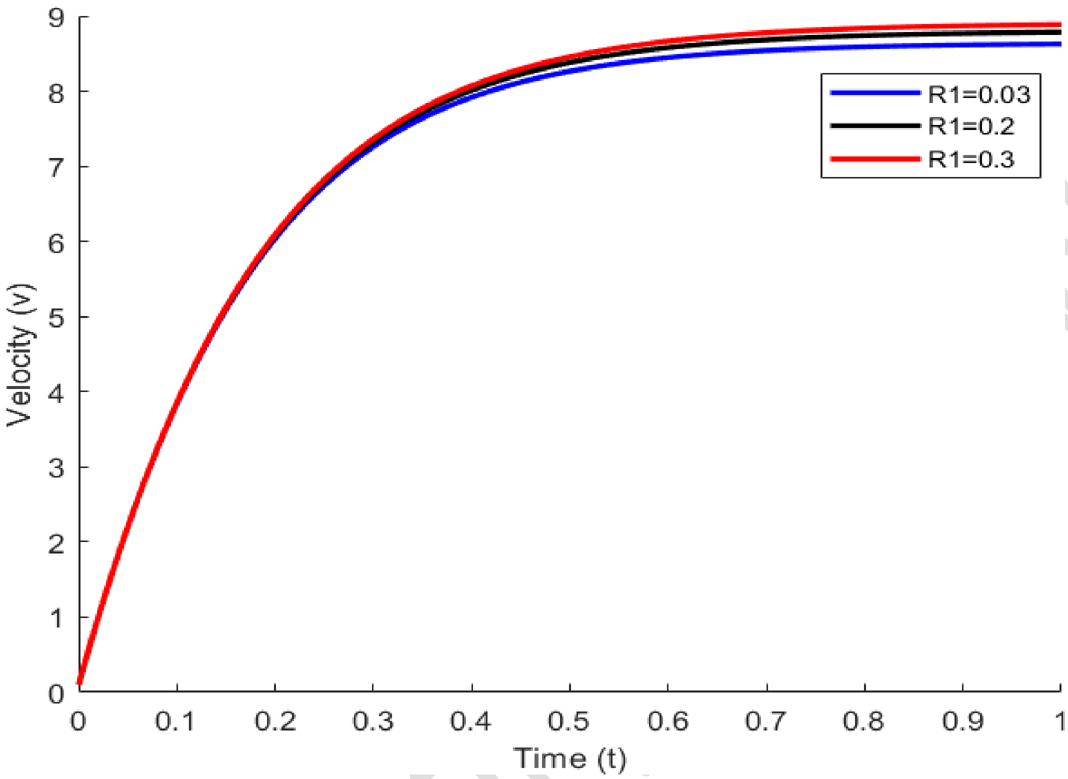

Figure 5: A graph of velocity against against time with R varying.

The crosssection area, A of the model is given by $A = [(b + b_1)d + b_2d^2)] + [(R + R_1)^2(\theta - \sin\theta\cos\theta)$. The radius of the segment base is denoted by R and hence is directly proportional to cross-section area. An increase in R leads to an increase in A and vise versa. From Figure 5, when the segment radius R is increased from $R_1 = 0.03$ to 0.3, the cross-section area of the channel is also increased. More so, the depth is increased. The larger the cross-section area and depth, the more the water can flow through it at a given time and hence the more the discharge to the main channel. Flow discharge is directly proportional to flow velocity and thus, more discharge means more fluid particles are colliding with the particles in the main channel which increases the kinetic energy leading to an increase in fluid flow velocity in the main channel.

8. **CONCLUSIONS**

This study primarily sets to determine the effect of the surface roughness and surface area of the lateral channel to the flow velocity on the main channel. The model that is developed is solved using the finite difference iterative scheme. To generate numerical values, simulations are carried out using the MATLAB Mathematical software. The results are then presented in graphs. The following are the summary of the findings;

i. Increasing the surface roughness of the lateral channel decreases the flow velocity of the main channel

ii. Increasing the cross-section area of the lateral channel increases the discharge to the main channel which leads to increase in the flow velocity in the main channel

9. RECOMMENDATIONS

There is still room for further extension of this study by establishing the effect of other parameters such as the energy coefficient, top width, bottom slope, on the flow rate in the main channel. More so, it is recommended that future study be carried out on:

i. The flow of the same orientation with two or more outflow lateral channels.
ii. The flow of the same orientation past channels of other shapes with two or more inflow lateral channels.
iii. The effect of length and cross-section area of the lateral channel to the flow rate at the main channel when the flow is assumed to be turbulent.

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APPENDIX: MATLAB CODE FOR SIMULATING VELOCITY PROFILES FOR VARIOUS FLOW PARAMETERS

```
function OpenChan_v2() clear all;clc;
%% ==============constants============== g=9.81; L=1; T=1;d=2; R=0.4;
b=0.5; So=0.002;
%%==============parameters============== b1=0.2;          % 0.1; 0.2; 0.3;
theta=pi/3.3;                    % pi/2; pi/3.3; pi/4 b2=b-2*R*sin(theta);
R1=0.2;              % 0.03; 0.2; 0.3
gamma=0.2;              % 0.05; 0.2; 0.5
n=0.05;              % 0.012; 0.05; 0.1
m=18;           % 3; 4; 6; 18;
u=15;           % 10; 15; 20;
%% ==============parameters==============
color=':b'; % blue(b); black(k); red(r)

x0=0;xN=2;N=51; dx=(xN-x0)/(N-1); x=x0:dx:xN; t0=0;tK=2;K=151; dt=(tK-t0)/(K-
1); t=t0:dt:tK;

%% constitutive relations
A=((b+b1)*d+b2*d*d)+((R+R1)^2) * (theta-sin(theta)*cos(theta)); h=sqrt(b2*b2+d*d);
P=2*h+2*R*theta; Rs=A/P;
q=(A/2)*u; %verify the other 1/2 in equation

%% boundary and initial conditions y0=0.5; v0=0.1;
y=zeros(N,K); v=zeros(N,K);
%% evaluation of finite difference scheme
```

```
for j=1:K-1 for i=2:N-1
v(:,1)=v0; y(:,1)=y0; %IC

y(i,j+1)=0.5*(y(i-1,j)+y(i+1,j))-dt*(v(i,j)*(y(i+1,j)-
y(i-1,j))/(2*dx)... +(A/T)*((v(i+1,j)-v(i-1,j))/(2*dx))- (q/(T*(gamma+L)))*sin(pi/m));

v(i,j+1)=0.5*(v(i-1,j)+v(i+1,j))-dt*(v(i,j)*(v(i+1,j)-
v(i-1,j))/(2*dx)...  +g*(y(i+1,j)-y(i-1,j))/(2*dx)+
g*((n*n/(2*(Rs^(4/3)))) * 0.5 * (v(i - 1, j)^2 + v(i + 1, j)^2) - So)...
-(q/(A*(gamma+L)))*sin(pi/m)*(u*cos(pi/m)-v(i,j)));

v(1,:)=v(2,:); y(1,:)=y(2,:); %BC at x=x0

v(N,:)=v(N-1,:); y(N,:)=y(N-1,:); %BC at x=xN

end
end
figure(1);
hold on; plot(t,y(N,:),color,'linewidth',2); xlabel('Time (t)'); ylabel('Depth (y)');
hold off figure(2);
hold on; plot(t,v(N,:),color,'linewidth',2); xlabel('Time (t)'); ylabel('Velocity (v)');
hold off figure(3)
hold on; plot3(y(N,:),v(N,:),t,color,'linewidth',2); xlabel
('Depth (y)');ylabel('Velocity (v)'); view(2); hold off
end
```