

## Original Research Article

### The Remkan Distribution and its Applications

#### Abstract

Due to the ever growing demand for the development of new lifetime distributions to meet the goodness of fit demand of complex datasets, two-parameter distributions has been proposed in recent times. This study therefore aims to contribute to this demand. We propose a new two-parameter lifetime distribution known as the Remkan distribution. Important mathematical properties of the Remkan distribution such as the moments and other related measures, and moment generating function were derived and the model parameters estimated using the maximum likelihood estimate technique. Finally, the flexibility of the new Remkan distribution was illustrated using a real life dataset and the results showed that the new Remkan distribution was the best amongst other competing two parameter distributions.

*Keywords: Remkan distribution; Lifetime distributions; Two-parameter distributions; Simulation; Moments; Moment generating functions.*

#### 1. Introduction

Lifetime distributions, also known as survival distributions or failure time distributions provide a statistical framework for modeling the time until an event of interest occurs (Chahkandi & Ganjali, 2009; SchÄbe, 1994). They provide a powerful tool for analyzing and interpreting data, enabling researchers and practitioners to make informed decisions based on probabilistic models. Due to the complexities of real life dataset, the need for developing new distributions cannot be overemphasized. This is because there is no one size fit all lifetime distribution for any given complex real life dataset. The quest to get a perfect fit to a real life complex dataset has led to the development of several one-parameter lifetime distributions to model real lifetime situations across several fields including engineering, finance, biology, epidemiology, and social sciences.

Comment [Ma1]: parameters

Examples of the one-parameter lifetime distributions that has been proposed over the years includes the Exponential distribution (Epstein, 1958), Lindley distribution (Lindley, 1958), Akash distribution (Shanker, 2015a), Sujatha distribution (Shanker, 2016), Ishita distribution (Shanker & Shukla, 2017a), Akshaya distribution (Shanker, 2017a). Others include Rama distribution (Shanker, 2017b), Pranav distribution (Shukla, 2018b), Odoma distribution (Odom & Ijomah, 2019), Nwikpe distribution (Nwikpe et al., 2021), Iwueze distribution (Elechi et al., 2022), Juchez distribution (Echebiri & Mbegbu, 2022), Chris-Jerry distribution (Onyekwere & Obulezi, 2022).

In recent years, new two-parameter distributions have emerged in the literature. These new two-parameter distributions have been shown to provide better fit to complex real life datasets than the one-parameter distributions. Some of the recently developed two-parameter distributions includes the Darna distribution (Shraa & Al-Omari, 2019), the Hamza distribution (Aijaz et al., 2020), the Samade distribution (Aderoju, 2021), the Alzoubi distribution (Benrabia & Alzoubi, 2021), and the Copoun distribution (Uwaeme et al., 2023).

It is important to note that these distributions are a mixture of the Exponential and Gamma distributions. These two distributions are known to have their weaknesses. The weakness of the Exponential distribution is that the hazard rate function is constant; hence, it cannot handle datasets with monotone non-decreasing hazard rates (Elechi et al., 2022; Epstein, 1958; Ronald et al., 2011; Shukla, 2018c, 2018a). Furthermore, the weakness of the Gamma distribution is that the survival rate function cannot be expressed in closed form (Elechi et al., 2022; Shanker, 2015a, 2015b). The limitations of these two distributions are what the aforementioned one-parameter and two-parameter distributions address, providing distributions whose survival rate

function can be expressed in closed form and hazard rate functions capable of handling datasets with monotone non-decreasing hazard rates.

In this study, we propose a new two-parameter distribution called the Remkan distribution. The subsequent sections of the paper will be arranged as follows. Section 2 discusses the new distribution, section 3 discusses the mathematical properties of the Remkan distribution, section 4 discusses the maximum likelihood estimate of the Remkan distribution, Section 5 discusses the simulation study, section 6 looks at the real life application of the new distribution, and section 7 concludes the paper.

## 2. The Remkan Distribution

This section will introduce the probability density function (pdf) and the cumulative distribution function (cdf) of the Remkan distribution and illustrate the different shapes of the Remkan distribution.

**Definition:** A random variable  $X$  is said to have a Remkan distribution (RD) with parameters  $\eta$  and  $\phi$  if its probability density function, is given by

$$g(x; \eta, \phi) = \frac{\eta^2}{(\eta + 2\phi + 6)} [1 + \phi\eta x^2 + \eta^2 x^3] e^{-\eta x}; \quad x > 0, \eta > 0, \phi > 0 \quad (1)$$

**Remark 1:** The pdf in equation 1 is a three-component density of an

Exponential( $\eta$ ), Gamma(3,  $\eta$ ), and Gamma(4,  $\eta$ ) distribution with mixing proportions  $\tau_1 =$

$$\frac{\eta}{\eta + 2\phi + 6}, \tau_2 = \frac{2\phi}{\eta + 2\phi + 6} \text{ and } \tau_3 = \frac{6}{\eta + 2\phi + 6} \text{ such that}$$

$$g(x; \eta, \phi) = \pi_1 g_1(x; \eta) + \pi_2 g_2(x; \eta) + \pi_3 g_3(x; \eta) \quad (2)$$

The corresponding cdf of equation 1 is given by

**Comment [Ma2]:** parameters!

**Comment [Ma3]:** Remkan distribution (RD)  
Replace all others with RD below

**Comment [Ma4]:** Define all the parameters inside the equations

**Comment [Ma5]:**

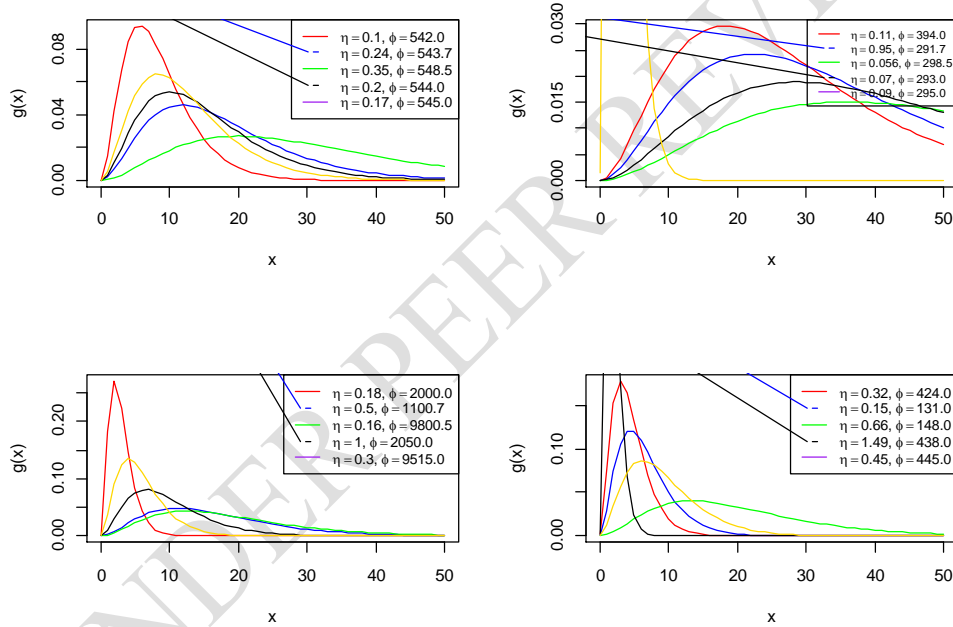
**Comment [Ma6]:** Exponential

**Comment [Ma7]:** Why capital letters here?

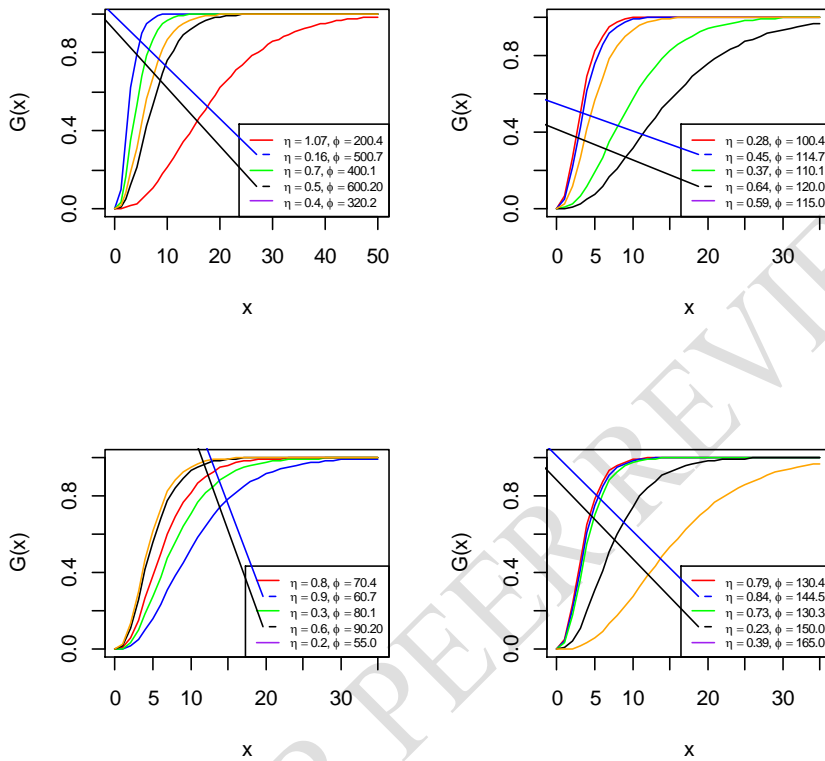
$$G(x; \eta, \phi) = 1 - \left[ 1 + \frac{\phi \eta^3 x^3 + (3 + \phi) \eta^2 x^2 + (6 + 2\phi) \eta x}{\eta + 2\phi + 6} \right] e^{-\eta x} \quad (3)$$

**Remark 2:** It can be easily seen that the pdf in equation 1 is a proper pdf.

The graphical plots of the theoretical density and distribution function (for some selected but different real points of  $\eta$  and  $\phi$ ) of a Remkan distribution (RD) are shown in the Figure 1 and Figure 2 below.



**Figure 1:** The graphical plots of the probability density function (for some selected but different real points of  $\eta$  and  $\phi$ ) of a Remkan distribution (RD).



**Figure 2: The graphical plots of the cumulative distribution function (for some selected but different real points of  $\eta$  and  $\phi$ ) of a Remkan distribution (RD)**

The curves displayed in Figure 1 are not bell-shaped, but are positively skewed, unimodal, and right tailed. In addition, the curve shows that increasing the value of  $\phi$  leads to a considerable increase in the peak of the curve. In addition, the curves displayed in Figure 2 shows that the cumulative distribution function converges to one.

**Comment [Ma8]:** Explain why?

**Comment [Ma9]:** Why? show that

### 3. Mathematical properties

In this section, we derive and present some of the mathematical properties of the Remkan distribution (RD) such as the moment and other related measures, and moment generating function.

#### 3.1 Moments of the Remkan Distribution

We derive the  $r$ th moment of the Remkan distribution (RD) in this subsection.

Comment [Ma10]:  $r^{\text{th}}$

##### Theorem 1

Given a random variable  $X$ , following the Remkan distribution (RD), the  $k$ th order moment about origin,  $E(X^k)$  of the Remkan distribution is given by

Comment [Ma11]: the  $k^{\text{th}}$

$$\mu'_k = E(X^k) = \frac{\eta^2}{\eta+2\phi+6} \left[ \frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}} \right] \quad (4)$$

##### Proof:

The  $k$ th crude or uncorrected moments of a random variable and can be written as

Comment [Ma12]:  $k^{\text{th}}$

$$\mu'_k = E(X^k) = \int_0^\infty x^k g(x; \eta, \phi) dx \quad (5)$$

$$= \int_0^\infty x^k \cdot \frac{\eta^2}{(\eta+2\phi+6)} [1 + \phi\eta x^2 + \eta^2 x^3] e^{-\eta x} dx \quad (6)$$

$$= \frac{\eta^2}{(\eta+2\phi+6)} \left[ \int_0^\infty x^k e^{-\eta x} dx + \phi\eta \int_0^\infty x^{k+2} e^{-\eta x} dx + \eta^2 \int_0^\infty x^{k+3} e^{-\eta x} dx \right] \quad (7)$$

Recall,  $\frac{\Gamma(\theta)}{\phi^\theta} = \int_0^\infty z^{\theta-1} e^{-\phi z} dz$ , and  $\Gamma(s) = (s-1)!$ . Hence,

$$E(X^k) = \frac{\eta^2}{(\eta+2\phi+6)} \left[ \frac{\Gamma(k+1)}{\eta^{k+1}} + \frac{\phi\eta\Gamma(k+3)}{\eta^{k+3}} + \frac{\eta^2\Gamma(k+4)}{\eta^{k+4}} \right] \quad (8)$$

$$\therefore E(X^k) = \frac{\eta^2}{(\eta+2\phi+6)} \left[ \frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}} \right] \quad (9)$$

Which completes the proof.

In particular, the first four (4) moments about the origin of the Remkan distribution (RD) is obtained by substituting the values of  $k= 1, 2, 3,4$  as follows;

$$\mu'_1 = E(X) = \frac{\eta+6\phi+24}{\eta(\eta+2\phi+6)} \quad (10)$$

$$\mu'_2 = E(X^2) = \frac{2\eta+24\phi+120}{\eta^2(\eta+2\phi+6)} \quad (11)$$

$$\mu'_3 = E(X^3) = \frac{6\eta+120\phi+720}{\eta^3(\eta+2\phi+6)} \quad (12)$$

$$\mu'_4 = E(X^4) = \frac{24\eta+720\phi+5040}{\eta^4(\eta+2\phi+6)} \quad (13)$$

Furthermore, the first four (4) moments about the mean of the Remkan distribution (RD) is obtained by substituting the values of  $k= 1, 2, 3,4$  as follows;

$$\sigma^2 = \mu_2 = \frac{\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144}{\eta^2(\eta+2\phi+6)^2} \quad (15)$$

$$\mu_3 = \frac{2[864+\eta^3+6\eta^2(33+5\phi)+36\eta(\phi^2+9\phi+9)+24\phi^3+288\phi^2+864\phi]}{\eta^3(\eta+2\phi+6)^3} \quad (16)$$

$$\mu_4 = \frac{9\eta^4+\eta^3(384\phi+2808)+\eta^2(1224\phi^2+12384\phi+20736)+\eta(1728\phi^3+22752\phi^2+81216\phi+93312)+720\phi^4+11520\phi^3+58752\phi^2+124416\phi+93312}{\eta^4(\eta+2\phi+6)^4} \quad (17)$$

The coefficient of variation,coefficient of skewness, coefficient of kurtosis, and index of dispersion of the Remkan distribution (RD), respectively, are given by

$$C.V = \frac{\sqrt{\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144}}{\eta+2\phi+6} \quad (18)$$

**Comment [Ma13]:** what is the importance to write (4)?

**Comment [Ma14]:** Show how did you get the eqs. (10) – (21)? Why there are no references?

$$\sqrt{\beta_1} = \frac{2[864+\eta^3+6\eta^2(33+5\phi)+36\eta(\phi^2+9\phi+9)+24\phi^3+288\phi^2+864\phi]}{(\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144)^{\frac{3}{2}}} \quad (19)$$

$$\beta_2 = \frac{9\eta^4+\eta^3(384\phi+2808)+\eta^2(1224\phi^2+12384\phi+20736)+\eta(1728\phi^3+22752\phi^2+81216\phi+93312)+720\phi^4+11520\phi^3+58752\phi^2+124416\phi+93312}{(\eta^2+16\phi\eta+12\phi^2+84\eta+96\phi+144)^2} \quad (20)$$

$$\gamma = \frac{\eta^2+\eta(84+16\phi)+\phi(96+12\phi)+144}{\eta(\eta+2\phi+6)(\eta+6\phi+24)} \quad (21)$$

**Table 1: CV of RD for varying values of parameters  $\eta$  and  $\phi$**

$\eta \backslash \phi$	0.2	0.5	1	2	3	4
0.2	2.2347	1.8790	1.7004	1.5854	1.5409	1.5172
0.5	1.8366	1.4927	1.3041	1.1691	1.1119	1.0797
1	1.5743	1.2561	1.0764	0.9404	0.8791	0.8430
2	1.3546	1.0622	0.8972	0.7689	0.7085	0.6717
3	1.2468	0.9662	0.8101	0.6881	0.6299	0.5939
4	1.1797	0.9053	0.7549	0.6377	0.5815	0.5465

For a given value of  $\phi$ , CV decreases as the value of  $\eta$  increases.

**Table 2: Skewness of RD for varying values of parameters  $\eta$  and  $\phi$**

$\eta \backslash \phi$	0.2	0.5	1	2	3	4
0.2	0.0309	0.0111	0.0038	0.0009	0.00037	0.00018
0.5	0.0098	0.0048	0.0023	0.00086	0.0004	0.00024
1	0.0039	0.0021	0.0012	0.0006	0.0003	0.00021
2	0.0018	0.0008	0.0005	0.0003	0.0002	0.00014
3	0.0013	0.0005	0.0003	0.0002	0.0001	0.00010
4	0.0010	0.0004	0.0002	0.0001	0.00009	0.00007

For a given value of  $\phi$ ,  $\sqrt{\beta_1}$  decreases as the value of  $\eta$  increases. Also, since  $\sqrt{\beta_1} > 0$ , RD is always positively skewed.

**Table 3: Kurtosis of RD for varying values of parameters  $\eta$  and  $\phi$**

$\eta \backslash \phi$	0.2	0.5	1	2	3	4
0.2	3.7139	3.2686	3.0239	2.4711	2.0527	1.7547
0.5	6.1218	3.7139	3.3202	3.1316	2.9091	2.6806
1	11.9188	5.1868	3.7139	3.3202	3.2254	3.1316
2	26.9814	9.4135	5.1868	3.7139	3.4177	3.3102
3	44.7241	14.6245	7.1468	4.3673	3.7139	3.4789

**Comment [Ma15]:** There are no any mentioned phrases before about Tables 1- 4!!!

4	63.8215	20.5343	9.4135	5.1868	4.1273	3.7139
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For  $0.2 \leq \phi \leq 0.5$ , and  $2 \leq \eta \leq 4$ ,  $\beta_2 < 3$ , and the RD is platykurtic, while the rest is leptokurtic, since  $\beta_2 > 3$ , RD can be both platykurtic or leptokurtic.

**Table 4: Index of dispersion of RD for varying values of parameters  $\eta$  and  $\phi$**

$\eta \backslash \phi$	0.2	0.5	1	2	3	4
0.2	5.4032	2.3406	1.2992	0.7486	0.5461	0.4351
0.5	5.4493	2.3394	1.2857	0.7337	0.5333	0.4245
1	5.4927	2.3317	1.2652	0.7125	0.5152	0.4093
2	5.5146	2.3079	1.2310	0.6798	0.4872	0.3857
3	5.5011	2.2824	1.2039	0.6558	0.4667	0.3682
4	5.4763	2.2588	1.1823	0.6375	0.4510	0.3547

As long as  $0 \leq \eta \leq 1$ , and  $0 \leq \phi \leq 5$ , the nature of RD is over dispersed ( $\sigma^2 > \mu_1'$ ) and for

$1 \leq \eta \leq 5$ , and  $0 \leq \phi \leq 5$ , the nature of RD is under dispersed ( $\sigma^2 < \mu_1'$ ).

### 3.2 Moment generating function

Here, we propose the moment generating function for the Remkan distribution (RD) on this subsection.

#### Theorem 2

Given a random variable  $X$ , following the Remkan distribution (RD), the moment generating function of  $X$ ,  $M_X(t)$  of the Remkan distribution (RD) is given by

$$M_X(t) = E(e^{tx}) = \frac{\eta^2}{(\eta+2\phi+6)} \sum_{k=0}^{\infty} \binom{t}{\eta}^k \left[ \frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}} \right] \quad (22)$$

#### Proof:

The moment generating function of  $X$ ,  $M_X(t)$  can be written as

$$M_X(t) = E(e^{tx}) = \int_0^{\infty} e^{tx} g(x; \eta, \phi) dx \quad (23)$$

$$= \int_0^{\infty} e^{tx} \cdot \frac{\eta^2}{(\eta+2\phi+6)} [1 + \phi\eta x^2 + \eta^2 x^3] e^{-\eta x} dx \quad (24)$$

$$= \frac{\eta^2}{(\eta+2\phi+6)} \int_0^{\infty} e^{tx} \cdot [1 + \phi\eta x^2 + \eta^2 x^3] e^{-\eta x} dx \quad (25)$$

Recall that  $e^{tx} = \sum_{k=0}^{\infty} \frac{(tx)^k}{k!}$ . Substituting, we obtain

$$= \frac{\eta^2}{(\eta+2\phi+6)} \sum_{k=0}^{\infty} \frac{t^k}{k!} \int_0^{\infty} x^k \cdot [1 + \phi\eta x^2 + \eta^2 x^3] e^{-\eta x} dx \quad (26)$$

$$= \sum_{k=0}^{\infty} \frac{t^k}{k!} E(X^k) \quad (27)$$

$$= \frac{\eta^2}{6\eta(\phi+\eta)} \sum_{k=0}^{\infty} \binom{t}{\eta}^k \cdot \frac{k!}{k!} + \frac{\phi\eta^4}{6\eta^4(\phi+\eta)} \sum_{k=0}^{\infty} \binom{t}{\eta}^k \cdot \frac{(k+3)!}{k!} \quad (28)$$

$$\therefore M_X(t) = E(e^{tx}) = \frac{\eta^2}{(\eta+2\phi+6)} \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[ \frac{k!}{\eta^{k+1}} + \frac{\phi(k+2)!}{\eta^{k+2}} + \frac{(k+3)!}{\eta^{k+2}} \right] \quad (29)$$

Which completes the proof.

#### 4. Maximum likelihood estimation

Given a random sample of size  $n$ ,  $X_1, X_2, X_3, \dots, X_n$  from the RD distribution. The log-likelihood function of parameters can be obtained by

$$LL(x; \eta\phi) = \prod_{i=1}^n \ln g_{RD}(x) \quad (30)$$

$$LL(x; \eta\phi) = \prod_{i=1}^n \ln \left\{ \frac{\eta^2}{(\eta+2\phi+6)} [1 + \phi\eta x_i^2 + \eta^2 x_i^3] e^{-\eta x_i} \right\} \quad (31)$$

$$LL(x; \eta\phi) = \ln \left\{ \left( \frac{\eta^2}{(\eta+2\phi+6)} \right)^n \prod_{i=1}^n ([1 + \phi\eta x_i^2 + \eta^2 x_i^3] e^{-\eta x_i}) \right\} \quad (32)$$

$$LL(x; \eta\phi) = 2n \ln(\eta) - n \ln(\eta + 2\phi + 6) + \sum_{i=1}^n \ln(1 + \phi\eta x_i^2 + \eta^2 x_i^3) - \eta \sum_{i=1}^n x_i \quad (33)$$

We take the partial differentiation of (33) w.r.t  $\eta$  and  $\phi$  and solve the nonlinear likelihood equations obtained so as to maximize the log-likelihood as shown below;

$$\frac{\partial LL}{\partial \eta} = \frac{2n}{\eta} - \frac{n}{(\eta+2\phi+6)} + \sum_{i=1}^n \frac{(2\eta x_i^3 + \phi x_i^2)}{(\eta^2 x_i^3 + \eta \phi x_i^2)} \quad (34)$$

$$\frac{\partial LL}{\partial \phi} = -\frac{2n}{(\eta+2\phi+6)} + \sum_{i=1}^n \frac{(\eta x_i^2)}{(\eta^2 x_i^3 + \eta \phi x_i^2 + 1)} \quad (35)$$

The nonlinear equations above are then equated to zero and solved simultaneously to obtain the estimates of the parameters. The solutions cannot be solved analytically hence we solve numerically using the Adequacy package of Marinho et al. (2019) with “BFGS” algorithm. The R software (R Team, 2017) is used for this analysis.

**Comment [Ma16]:** Give a reference!

## 5. Simulation study

In this section, we present the results of the simulation study carried out on the new Remkan distribution to determine the efficiency of the maximum likelihood estimate. The acceptance/rejection algorithm was adopted for this simulation study (Law, 2014; Shanker, 2017c).

### The acceptance/rejection algorithm:

In order to simulate from the density  $g(x_k; \Phi)$ , it is assumed that we have the envelope density  $\mathfrak{S}$  from which we can simulate from, and that we have some  $k < \infty$  such that  $\sup_x \frac{g(x_k; \Phi)}{\mathfrak{S}(x_k; \Phi)} \leq k$ .

Step 1: Simulate  $X$  from  $\mathfrak{S}$

Step 2: Generate  $Y \sim U(0, k\mathfrak{S}(x_k; \Phi))$  where  $k = \frac{\eta^2}{(\eta+2\phi+6)}$

Step 3: If  $Y < g(x_k; \Phi)$ , then return  $X$ , otherwise go to step 1.

The simulation study will be based on generating  $N = 10,000$  samples of size  $n = 50, 100, 150, 200$  for  $\Phi = 0.5, 1, 1.5$  and  $2$  using the algorithm above. Then the following measures will be calculated.

- i. Average bias of the simulated estimate

$$\text{Average bias} = \frac{1}{N} \sum_{i=1}^N (\hat{\Phi}_i - \Phi). \quad (36)$$

where  $\hat{\Phi}_i$  is the maximum likelihood estimate.

- ii. Average mean square error (MSE)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (\hat{\Phi}_i - \Phi)^2. \quad (37)$$

The average bias and average mean square error (MSE) for each of the ML estimate has been calculated and shown in Table (3), where MSE has been shown in bracket.

Comment [Ma17]: Table (5)

**Table 5: The average bias and MSEs of parameter estimates with MLE for different values of parameters( $\eta, \phi$ ).**

$n$	Par ( $\eta$ )	Average bias	MSE	Par ( $\phi$ )	Average bias	MSE
50	0.5	0.5061	0.2764	2.0	-1.1514	1.3371
	1	0.0062	0.0196	2.5	-1.6484	2.7284
	1.5	-0.5531	0.3212	3.0	-2.1501	4.6341
	2.0	-1.1486	1.3308	3.5	-2.6481	7.0234
100	0.5	0.5084	0.2682	2.0	-1.1507	1.3298
	1	0.00421	0.0095	2.5	-1.6499	2.7277
	1.5	-0.5490	0.3091	3.0	-2.1495	4.6258
	2.0	-1.1503	1.3289	3.5	-2.6476	7.0152

150	0.5	0.5072	0.2641	2.0	-1.1516	1.3300
	1	0.00419	0.0065	2.5	-1.6512	2.7302
	1.5	-0.5498	0.3074	3.0	-2.1498	4.6252
	2.0	-1.1504	1.3272	3.5	-2.6471	7.0108
200	0.5	0.5065	0.2615	2.0	-1.1524	1.3309
	1	0.0055	0.0049	2.5	-1.6507	2.7278
	1.5	-0.5502	0.3065	3.0	-2.1483	4.6180
	2.0	-1.1498	1.3249	3.5	-2.6473	7.0113

**Source: Author's compilation, 2023**

## 6. Application

This section discusses the flexibility and superiority of the Remkan distribution to some competing distributions using real life data sets. The dataset employed represents the lifetime data relating to time (in months from 1st January, 2013 to 31st July, 2018) of 105 patients who were diagnosed with hypertension and received at least one treatment-related to hypertension in the hospital where death is the event of interest. It was reported by (Umeh & Ibenegbu, 2019) and used by (Enogwe et al., 2020). The data is shown below.

45, 37, 14, 64, 67, 58, 67, 55, 64, 62, 9, 65, 65, 43, 13, 8, 31, 30, 66, 9, 10, 31, 31, 31, 46, 37, 46, 44, 45, 30, 26, 28, 45, 40, 47, 53, 47, 41, 39, 33, 38, 26, 22, 31, 46, 47, 66, 61, 54, 28, 9, 63, 56, 9, 49, 52, 58, 49, 53, 63, 16, 67, 61, 67, 28, 17, 31, 46, 52, 50, 30, 33, 13, 63, 54, 63, 56, 32, 33, 37, 7, 56, 1, 67, 38, 33, 22, 25, 30, 34, 53, 53, 41, 45, 59, 59, 60, 62, 14, 57, 56, 57, 40, 44, 63.

This dataset is then fitted with the Remkan distribution (RD) and compared with Two-parameter Lindley distribution (TPLD) (Shanker & Mishra, 2013), Two-parameter Akash distribution (TPAD) (Shanker & Shukla, 2017b), Two-parameter Rama distribution (TPRD) (Edith et al.,

2019), Two-parameter Sujatha distribution (TPAD)(Tesfay & Shanker, 2018), and Samade distribution (SD) (Aderoju, 2021) with corresponding pdfs.

$$f_{TPLD}(x; \Phi) = \frac{\eta^2}{\eta\phi+1}(\phi+x)e^{-\eta x} \quad (38)$$

$$f_{TPAD}(x; \Phi) = \frac{\eta^3}{\phi\eta^2+2}(\phi+x^2)e^{-\eta x} \quad (39)$$

$$f_{TPRD}(x; \Phi) = \frac{\eta^4}{\phi\eta^3+6}(\phi+x^3)e^{-\eta x} \quad (40)$$

$$f_{TPSD}(x; \Phi) = \frac{\eta^3}{\eta^2+\phi\eta+2}(1+\phi x+x^2)e^{-\eta x} \quad (41)$$

$$f_{SD}(x; \Phi) = \frac{\eta^4}{\eta^4+6\phi}(\eta+\phi x^3)e^{-\eta x} \quad (42)$$

This comparison is done using some measures for testing the goodness of fit of a distribution. The measures used are the parameter estimates, the log likelihood, the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC)  $-2\ln L$ , Akaike Information Criterion (AIC) (Club, 2016), Bayesian Information Criterion (BIC) (Pollock et al., 1999), Consistent Akaike information criterion (CAIC), Hannan-Quinn information criterion (HQIC). In general, the smaller the values of AIC, BIC, CAIC, and HQIC the better the fit to the data.

$$AIC = 2k - 2 \ln L \quad (43)$$

$$BIC = k \ln n - 2 \ln L \quad (44)$$

$$CAIC = AIC + \frac{2k(k+1)}{(n-k-1)} \quad (45)$$

$$HQIC = 2k \ln(\ln(n)) - 2 \ln L \quad (46)$$

where  $k$  is the number of parameters,  $n$  is the sample size of the dataset, and  $L$  is the likelihood function.

**Table 6: Goodness of fit for the Hypertention Data**

Distribution	MLE's	S.E	-2ln L	AIC	BIC	CAIC	HQIC
RD	$\hat{\eta} = 0.0936$ $\hat{\phi} = -0.0235$	0.0060 0.5267	460.1171	924.2342	929.5422	924.3519	923.3097
TPLD	$\hat{\eta} = 0.001$ $\hat{\phi} = 0.047$	1.1021 0.0035	463.7301	931.4603	931.5579	931.5579	933.6112
Samade	$\hat{\eta} = 0.0948$ $\hat{\phi} = 7428.0$	0.00463 1.049	471.0754	938.1509	932.843	938.0333	939.0755
TPRD	$\hat{\eta} = 381.2325$ $\hat{\phi} = 0.0916$	318.2483 0.0049	472.6460	949.2920	949.4096	949.4096	951.4428
TPSD	$\hat{\eta} = 0.0705$ $\hat{\phi} = 0.6089$	0.0044 2.3069	464.6302	933.2603	938.5683	933.378	932.3358
TPAD	$\hat{\eta} = 0.0700$ $\hat{\phi} = 10.4281$	0.0039 2.9755	463.7305	931.4609	936.7688	931.5786	930.5363

The parameter estimates and their goodness of fit of the different models for the dataset are presented in Table 1. From the results, the Remkan distribution (RD) performed better than the competing distributions.

## 7. Conclusion

This paper proposed a new two-parameter distribution known as the Remkan distribution (RD). The mathematical properties of the Remkan distribution (RD) such as the moments and other related measures, and moment generating functions was derived and presented. The parameter estimates of the Remkan distribution (RD) was obtained using the method of maximum likelihood estimates. Finally, the flexibility of the Remkan distribution (RD) was illustrated

Comment [Ma18]: Table. 6?

using a real lifetime data. The result showed that the Remkan distribution (RD) performed better than the other competing two-parameter distributions.

**Comment [Ma19]:** This section is not conclusions, its just an abstract, you need to give a real abstract.

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