

Original Research Article

Development of a Scheffe's Model to Predict the Duration of Project Tasks

ABSTRACT

This study was aimed at applying Scheffe's approach to determine task durations with a three-component design, to a second degree. The PERT approach was adopted to estimate the durations of three tasks for a small construction project. The study further applied Scheffe's simplex theorem to develop a linear regression model to predict the task durations using the durations from the PERT approach. The model-predicted results were found to be very close to those estimated by the PERT approach. The model was tested with a two-tailed student t-test and was found adequate and fit with an R^2 value of 0.9986. This proved that Scheffe's approach can also be used to estimate or predict the durations of project tasks for projects with up to 3 tasks.

Keywords: Model prediction, PERT, project task duration, Scheffe's simplex.

1. INTRODUCTION

Every project consists of tasks or activities. These tasks are supposed to be managed and followed squarely to the completion of the project. In order to do so, adequate project management skills and techniques are required from the responsible civil engineer or project manager.

SOFTSTRUCT CONSULTANTS, a small engineering and construction company, domiciled in Port Harcourt, Nigeria, was used as the case study for this research. The company has several construction projects across the country. One of such is an Engineering-Procurement-Construction (EPC) project. The company was awarded the contract by a private client to design and construct a 4-bedroom duplex for residential purposes.

1.1 The PERT Model

The Program Evaluation Research Task (PERT) which was later renamed to Program Evaluation and Review Technique, was first developed by the United States Navy when they were working on a large nuclear submarine project called the Polaris Fleet Ballistic Missile (FBM) program in 1957 [1]. This model usually works in conjunction with a Work Breakdown Structure (WBS) and the Critical Path Method (CPM). The PERT model has long become globally acceptable for estimating the durations of tasks in a given project as several researchers [2]–[5] have applied it in their studies. As derived and developed by [6], the estimated duration of a project task is given by eq. (1).

$$t_e = \frac{t_o + 4t_m + t_p}{6} \quad (1)$$

where,

t_o , t_m , and t_p are the optimistic, most likely and pessimistic durations respectively of the said project task. However, this model has been argued [1] to be applicable mainly to large projects with multiple tasks. Further criticisms have also been

made by [5] that the PERT method neglects the merge event bias, over-estimates the variances of the durations, and under-estimates the durations, which resulted in his development of the M-PERT. In a similar vein, [4] developed a fuzzy PERT by using Delphi method to determine the pessimistic, most likely, and optimistic durations. This study seeks to apply a new approach to determine, estimate, or predict the durations of a project task, with the parameters given in eq. (1) but mostly for projects with small number of tasks. The study also attempts to apply Scheffe's theory, which was discovered by [7] for mixture related parameters. The PERT model is a stochastic one, but the Scheffe's model is an empirical one. The integration of an empirical approach to solving a stochastic problem is one of the basic objectives of this study. Finally, the results from this study shall be compared to the PERT results in order to ascertain whether or not the new model will be useful.

1.2 The Scheffe's Simplex Theory

Several scholarly studies [8]–[15] have been carried out for concrete mixture resulting to the development of mathematical models, most of which were based on Scheffe's theory. None of them was for durations of projects or project tasks.

Scheffe's model is based on the simplex theory or approach [7]. The simplex approach considers a number of components, q , and a degree of polynomial, m . The sum of all the i^{th} components is equal to 1. Hence,

$$\sum_{i=1}^q x_i = 1 \quad (2)$$

$$x_1 + x_2 + \dots + x_q = 1 \quad (3)$$

with $0 \leq x \leq 1$. The model as derived in [15] is given in eq.(5).

When $\{q,m\} = \{3,2\}$:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 \quad (4)$$

and eq. (3) becomes

$$x_1 + x_2 + x_3 = 1 \quad (5)$$

Eq. (4) can be rearranged as:

$$Y = \sum_{i=1}^3 \beta_i x_i + \sum_{1 \leq i \leq j \leq 3} \beta_{ij} x_i x_j \quad (6)$$

Where the response, Y is a dependent variable (Duration of project task). Eq. (6) is the general equation for a $\{3,2\}$ polynomial, and it has 6 terms, which conforms to Scheffe's theory in eq. (3).

Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in i and j . If $x_i=1$ and $x_j = 0$, since $j \neq i$, then

$$Y_i = \beta_i \quad (7)$$

which means

$$\sum_{i=1}^3 \beta_i x_i = \sum_{i=1}^3 Y_i x_i \quad (8)$$

Hence,

$$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad (9)$$

2. RESEARCH METHODOLOGY

The project was divided into 3 milestones or tasks. Conception and design, construction, and finishing/furnishing were the three tasks as shown in Table 1. From the table, the initial estimated finish dates for each task were determined by the use of the PERT model.

Table 1. Project Tasks Summary

S/N	Project Task	Short form	Start date	Likely finish dates (days)			Estimated duration using PERT (days)	Actual finish dates	Actual duration (days)
				Optimistic	Most likely	Pessimistic			
1	Conception and Design	CD	12-Jun-15	22-Oct-15	05-Feb-16	20-May-16	238	30-Mar-16	292
				132	238	343			
2	Construction of Building	CB	27-Dec-16	15-Feb-18	13-Apr-18	26-Dec-17	445	22-May-18	511
				415	472	364			
3	Finishing and furnishing of Building	FB	05-Nov-17	22-Aug-19	06-Jan-20	10-Apr-20	785	12-Mar-20	858
				655	792	887			

The finish dates were converted to ratio form as shown in Table 2.

Table 2. Project task Proportions

cc	Response Y	Actual Components			Estimated durations using PERT (days)
		Optimistic S ₁	Most likely S ₂	Pessimistic S ₃	
CD	Y ₁	0.5546	1	1.4411	238
CB	Y ₂	0.8792	1	0.7711	445
FB	Y ₃	0.8270	1	1.1199	785

Putting the above in matrix form:

$$S = \begin{bmatrix} 0.5546 & 0.8792 & 0.8270 \\ 1 & 1 & 1 \\ 1.4411 & 0.7711 & 1.1199 \end{bmatrix} \quad (10)$$

The corresponding pseudo components are given as follows:

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

With center points

$$X_{12} = [0.5 \ 0.5 \ 0]; \quad X_{13} = [0.5 \ 0 \ 0.5]; \quad \text{and } X_{23} = [0 \ 0.5 \ 0.5].$$

According to [7],

$$S_{ij} = XS_i \quad (12)$$

Substituting, we get:

$$\begin{bmatrix} S_{12} \\ S_{13} \\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.5546 \\ 0.8792 \\ 0.8270 \end{bmatrix} \quad (13)$$

The process of matrix multiplication was repeated for an additional 6 (control) points, which were used for the verification of the formulated model. The regular triangles for the actual and pseudo components were given in Figures. 1 and 2 respectively.

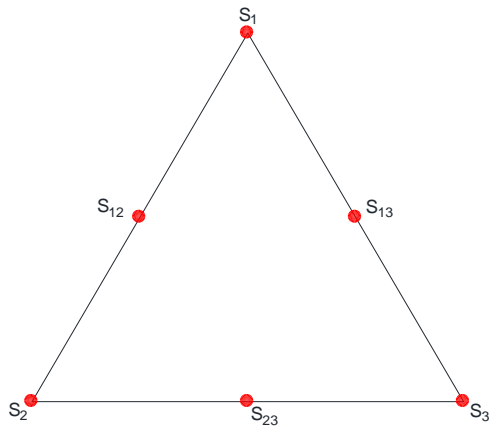


Fig. 1.Simplex Plot for Actual components

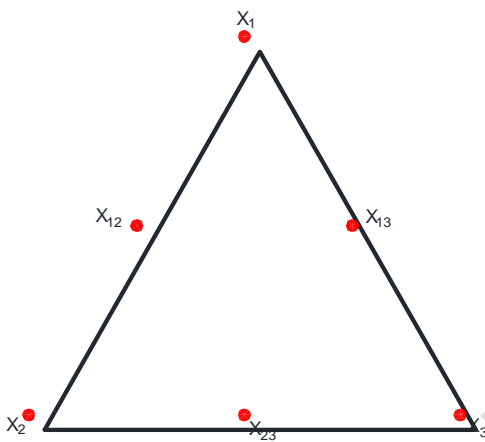


Fig. 2.Simplex Plot for Pseudo components

Table 3.Duration Proportions for Model Calibration

Sample Points	Actual Components			Response Y	Pseudo Components		
	Optimistic	Most likely	Pessimistic		Optimistic	Most likely	Pessimistic
	S ₁	S ₂	S ₃		X ₁	X ₂	X ₃
CD	0.5546	1	1.4412	Y ₁	1	0	0
CB	0.8792	1	0.7712	Y ₂	0	1	0
FB	0.8270	1	1.1199	Y ₃	0	0	1
N1	0.7169	1	1.1062	Y ₁₂	0.5	0.5	0
N2	0.6908	1	1.2806	Y ₁₃	0.5	0	0.5
N3	0.8531	1	0.9456	Y ₂₃	0	0.5	0.5

Table 4.Duration Proportions for Model Verification

Sample Points	Actual Components			Response Y	Pseudo Components		
	Optimistic	Most likely	Pessimistic		Optimistic	Most likely	Pessimistic
	S ₁	S ₂	S ₃		X ₁	X ₂	X ₃
C1	0.7798	1	1.0608	Y _{C1}	0.25	0.4	0.35
C2	0.7363	1	1.1264	Y _{C2}	0.4	0.35	0.25
C3	0.7795	1	1.1377	Y _{C3}	0.2	0.133 3	0.6667
C4	0.8201	1	0.9414	Y _{C4}	0.15	0.65	0.2
C5	0.6524	1	1.2876	Y _{C5}	0.6667	0.133 3	0.2
C6	0.7536	1	1.1107	Y _{C6}	0.3333	0.333 3	0.3333

3. RESULTS AND DISCUSSION

The actual components in Tables 3 and 4 for N1 to C6 were converted to whole numbers, and the estimated durations calculated using PERT equation as shown in Table 5.

Table 5. Estimated Durations

Sample Points	Likely durations (days)			Estimated durations using PERT (days)
	Optimistic	Most likely	Pessimistic	
N1	247	345	382	335
N2	360	520	667	518
N3	540	633	599	612
C1	409	525	557	511
C2	338	460	518	449
C3	496	636	724	628
C4	410	500	471	480
C5	248	381	490	377
C6	382	506	563	495

From Table 5, and eq. (9), the polynomial coefficients are: $\beta_1 = 238$, $\beta_2 = 445$, $\beta_3 = 785$, $\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$, $\beta_{12} = 4 * 335 - 2 * 238 - 2 * 445 = -24.667$

Similarly, $\beta_{13} = 26.333$, and $\beta_{23} = -11$.

Substituting the above coefficients into eq. (6):

$$Y = 238x_1 + 445x_2 + 785x_3 - 024.667x_1x_2 + 26.333x_1x_3 - 11x_2x_3 \quad (14)$$

Eq. (14) above is the Scheffe's mathematical model to predict the durations of the tasks of the project.

In Table 6, the formulated model in eq. (14) was used to predict the durations.

Table 6. Estimated and Predicted Durations

Sample Points	Response Y	Pseudo Components			Estimated duration using PERT, Y _{est} (days)	Scheffe's Predicted duration, Y _{pred} (days)
		Optimistic X ₁	Most likely X ₂	Pessimistic X ₃		
CD	Y ₁	1	0	0	238	238

CB	Y ₂	0	1	0	445	445
FB	Y ₃	0	0	1	785	785
N1	Y ₁₂	0.5	0.5	0	335	335
N2	Y ₁₃	0.5	0	0.5	518	518
N3	Y ₂₃	0	0.5	0.5	612	612
C1	Y _{C1}	0.25	0.4	0.35	511	510
C2	Y _{C2}	0.4	0.35	0.25	449	445
C3	Y _{C3}	0.2	0.1333	0.6667	628	632
C4	Y _{C4}	0.15	0.65	0.2	480	479
C5	Y _{C5}	0.6667	0.1333	0.2	377	376
C6	Y _{C6}	0.3333	0.3333	0.3333	495	488

Notice that the results from the control points were the only ones that gave slightly different values. This is because the values from the first part were used to calibrate (determine the model constants), while those from the second part (control points) were used to verify the formulated model. Figure 3 shows a graphical comparison between the PERT and Scheffe's results.

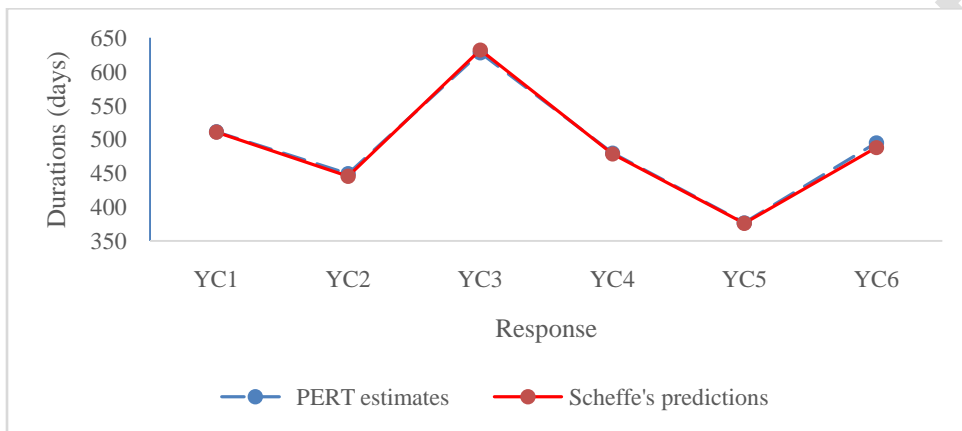


Fig. 3. Graphical comparison of PERT and Scheffe's durations

A two-tailed student t-test was carried out at 95% confidence level, which implies $100 - 95 = 5\%$ significance.

Let D be difference between the eestimated and predicted responses.

The mean of the difference,

$$D_a = \frac{1}{n} \sum_{i=1}^n D_i \quad (15)$$

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1} \right) \sum_{i=1}^n (D - D_a)^2_i \quad (16)$$

$$t_{calculated} = \frac{D_a \sqrt{n}}{S} \quad (17)$$

Where n = number of observations with degree of freedom n - 1.

$$S^2 = \frac{72.745}{6-1}$$

$$S = \sqrt{14.549} = 3.814$$

$$t_{\text{calculated}} = 1.021$$

Table 7. Student t-table for estimated and predicted durations

Sample points	Task durations		t-test		
	$Y_{\text{estimated}}$	$Y_{\text{predicted}}$	$D = Y_{\text{est}} - Y_{\text{pred}}$	$D_a - D$	$(D - D_a)^2$
C1	511.000	510.306	0.694	0.896	0.80
C2	449.000	445.176	3.824	-2.234	3
C3	627.500	632.054	-4.554	6.145	4.99
C4	480.000	478.555	1.445	0.145	0
C5	377.000	375.841	1.159	0.432	37.7
C6	495.000	488.025	6.975	-5.384	55
TOTAL			9.543		0.02
AVERAGE D_a			1.590		1
					0.18
					6
					28.9
					89
					72.7
					45

From the t-table, $t_{(\beta, v)}$ can be determined where $v = 6 - 1 = 5$, and $\beta =$ significance level. $t_{(0.975, 5)} = 2.571$. Table 7 was used to determine the parameters with which the $t_{\text{calculated}}$ was determined.

Since $t_{\text{calculated}} < t_{(0.975, 5)}$, and lies between -2.571 and 2.571, therefore there is no significant difference between the PERT estimated and Scheffe's predicted responses, H_0 is accepted, and H_a is rejected. The model is ascertained to be adequate. It is also found to be fit, since from Fig. 4, the R^2 value is 0.9986. This also means that the predicted model values are highly correlated to the estimated values.

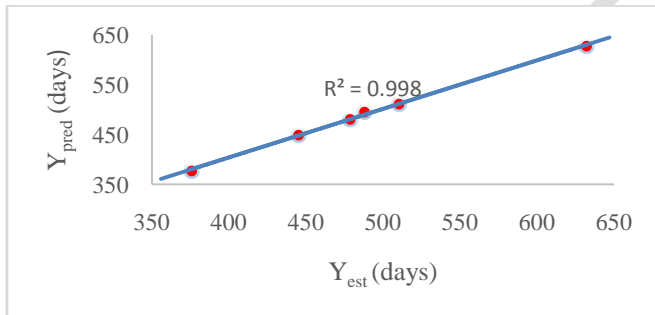


Fig. 4. Scatterplot of PERT and Scheffe's durations

4. CONCLUSION

The estimated durations with PERT is a stochastic approach and has been globally acceptable for determination of task durations since the 1950s. However, an empirical approach (Scheffe's simplex) has been successfully adopted to develop a multiple linear regression model to degree 2 to predict the task durations, given the same optimistic, most likely, and pessimistic durations as in the PERT. The model has successfully predicted the durations and has been found fit and adequate, with R^2 value of 0.9986, after being subjected to a two-tailed student t-test. This shows that the Scheffe's simplex approach can also be applied to task durations, while resulting in very high accuracy. This study, however was carried out for a project with only three tasks. Further studies are hereby recommended for a larger number of tasks.

CONSENT (WHEREEVER APPLICABLE)

No written consent note of any sort is required for this manuscript.

ETHICAL APPROVAL (WHEREEVER APPLICABLE)

No ethical approval of any sort is required for this manuscript.

REFERENCES

- [1] D. G. Malcolm, J. H. Roseboom, C. E. Clark, and W. Fazar, "Application of a Technique for Research and Development Program Evaluation on JSTOR," *Oper Res*, vol. 7, no. 5, pp. 646–669, 1959, Accessed: Apr. 23, 2023. [Online]. Available: <https://www.jstor.org/stable/167013>
- [2] W. Agyei, "Project Planning And Scheduling Using PERT And CPM Techniques With Linear Programming: Case Study," *INTERNATIONAL JOURNAL OF SCIENTIFIC & TECHNOLOGY RESEARCH*, vol. 4, p. 8, 2015, Accessed: May 05, 2023. [Online]. Available: www.ijstr.org
- [3] K. Benaiah Bagshaw, "PERT and CPM in Project Management with Practical Examples," *American Journal of Operations Research*, vol. 11, pp. 215–226, 2021, doi: 10.4236/ajor.2021.114013.
- [4] T. A. Saadon, A. Samman, R. M. Ramo, A. Brahemi, A. Prof, and A. Lecturer, "FUZZY PERT FOR PROJECT MANAGEMENT," *Int J Adv Eng Technol*, vol. 7, pp. 1150–1160, 2014.
- [5] P. Ballesteros-Pérez, "M-PERT: Manual Project-Duration Estimation Technique for Teaching Scheduling Basics," *J Constr Eng Manag*, vol. 143, no. 9, p. 04017063, Jun. 2017, doi: 10.1061/(ASCE)CO.1943-7862.0001358.
- [6] U.S. Dept. of the Navy, "Program Evaluation Research Task, Summary Report, Phase 1," Washington DC, 1958.
- [7] H. Scheffe, "Experiments with Mixtures," *Journal of Royal Statistical Series B*, vol. 25, no. 2, pp. 235–263, 1958.
- [8] C. U. Anya, "Models for Predicting the Structural Characteristics of Sand-Quarry Dust Blocks," Ph.D Thesis, University of Nigeria, Nsukka, 2015.
- [9] Y. M. Gamil and I. H. Bakar, "The Development of Mathematical Prediction Model to Predict Resilient Modulus for Natural Soil Stabilised by POFA-OPC Additive for the Use in Unpaved Road Design," in *Soft Soil Engineering International Conference*, Materials Science and Engineering, 2015, pp. 1–11.
- [10] P. N. Onuamah, "Development-and-Optimization-of-Mechanical-Strength-Model-of-Cement-Laterite-Sand-Hollow-Sandcrete-Blocks.docx," *Int J Sci Eng Res*, vol. 6, no. 5, pp. 645–655, 2015.
- [11] E. M. Mbadike and N. N. Osadere, "Five Component Concrete Mix Optimization of Aluminum Waste Using Scheffe's Theory," *International Journal of Computational Engineering Research*, vol. 4, no. 4, pp. 23–31, 2014.
- [12] E. M. Mbadike and N. N. Osadebe, "Application of Scheffe's model in optimization of compressive strength of lateritic concrete," *J. Civ. Eng. Constr. Technol.*, vol. 4, no. 9, pp. 265–274, 2013, doi: 10.5897/JCECT2013.0288.
- [13] C. E. Okere, D. O. Onwuka, S. U. Onwuka, and J. I. Arimanwa, "Simplex-Based Concrete Mix Design," *IOSR Journal of Mechanical and Civil Engineering*, vol. 5, no. 2, pp. 46–55, 2013.
- [14] N. N. Osadebe, C. C. Mbajorgu, and T. U. Nwakonobi, "AN OPTIMIZATION MODEL DEVELOPMENT FOR LATERIZED- CONCRETE MIX PROPORTIONING IN BUILDING," *Nigerian Journal of Technology*, vol. 26, no. 1, pp. 37–46, 2007.
- [15] K. M. Oba and O. O. Ugwu, "A Scheffe's Predictive Model for Modulus of Elasticity of Sawdust Ash - Sand Concrete," *International Journal of Engineering and Management Research*, vol. 11, no. 1, pp. 9–17, 2021, doi: 10.2139/ssrn.3781416.

DEFINITIONS, ACRONYMS, ABBREVIATIONS

Here is the Definitions section. This is an optional section.

PERT: Program Evaluation and Review Technique