Original Research Article

Development of a Scheffe's Model to Predict the Duration of Project Tasks

ABSTRACT

This study was aimed at applying Scheffe's approach to determine task durations with a three-component design, to a second degree. The PERT approach was adopted to estimate the durations of three tasks for a small construction project. The study further applied Scheffe's simplex theorem to develop a linear regression model to predict the task durations using the durations from the PERT approach. The model-predicted results were found to be very close to those estimated by the PERT approach. The model was tested with a two-tailed student t-test and was found adequate and fit with an R^2 value of 0.9986. This proved that Scheffe's approach can also be used to estimate or predict the durations of project tasks for projects with up to 3 tasks.

Keywords: Model prediction, PERT, project task duration, Scheffe's simplex.

1. INTRODUCTION

Every project consists of tasks or activities. These tasks are supposed to be managed and followed squarely to the completion of the project. In order to do so, adequate project management skills and techniques are required from the responsible civil engineer or project manager.

SOFTSTRUCT CONSULTANTS, a small engineering and construction company, domiciled in Port Harcourt, Nigeria, was used as the case study for this research. The company has several construction projects across the country. One of such is an Engineering-Procurement-Construction (EPC) project. The company was awarded the contract by a private client to design and construct a 4-bedroom duplex for residential purposes.

1.1 The PERT Model

The Program Evaluation Research Task (PERT) which was later renamed to Program Evaluation and Review Technique, was first developed by the United States Navy when they were working on a large nuclear submarine project called the Polaris Fleet Ballistic Missile (FBM) program in 1957 [1]. This model usually works in conjunction with a Work Breakdown Structure (WBS) and the Critical Path Method (CPM). The PERT model has long become globally acceptable for estimating the durations of tasks in a given project as several researchers [2]–[5] have applied it in their studies. As derived and developed by [6], the estimated duration of a project task is given by eq. (1).

$$t_e = \frac{t_o + 4t_m + t_p}{6} \tag{1}$$

where,
t_o, t_m, and t_p are the optimistic, most likely and pessimistic durations respectively of the said project task. However, this model has been argued [1] to be applicable mainly to large projects with multiple tasks. Further criticisms have also been

made by [5] that the PERT method neglects the merge event bias, over-estimates the variances of the durations, and under-estimates the durations, which resulted in his development of the M-PERT. In a similar vein, [4] developed a fuzzy PERT by using Delphi method to determine the pessimistic, most likely, and optimistic durations. This study seeks to apply a new approach to determine, estimate, or predict the durations of a project task, with the parameters given in eq. (1) but mostly for projects with small number of tasks. The study also attempts to apply Scheffe's theory, which was discovered by [7] for mixture related parameters. The PERT model is a stochastic one, but the Scheffe's model is an empirical one. The integration of an empirical approach to solving a stochastic problem is one of the basic objectives of this study. Finally, the results from this study shall be compared to the PERT results in order to ascertain whether or not the new model will be useful.

1.2 The Scheffe's Simplex Theory

Several scholarly studies [8]–[15] have been carried out for concrete mixture resulting to the development of mathematical models, most of which were based on Scheffe's theory. None of them was for durations of projects or project tasks.

Scheffe's model is based on the simplex theory or approach [7]. The simplex approach considers a number of components, *q*, and a degree of polynomial, *m*. The sum of all the i^{th} components is equal to 1. Hence,

$$\sum_{i=1}^{q} x_i = 1 \qquad (2)$$

$$x_1 + x_2 + \cdots + x_q = 1 \qquad (3)$$

with $0 \leq x \leq 1$. The model as derived in [15] is given in eq.(5).

When {*q,m*} = {*3,2*}:

$$Y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 \qquad (4)$$

and eq. (3) becomes

$$x_1 + x_2 + x_3 = 1 \qquad (5)$$

Eq. (4) can be rearranged as:

$$Y = \sum_{i=1}^{3} \beta_i x_i + \sum_{1 \leq i \leq j \leq 3} \beta_{ij} x_i x_j \qquad (6)$$

Where the response, Y is a dependent variable (Duration of project task). Eq. (6) is the general equation for a {*3,2*} polynomial, and it has 6 terms, which conforms to Scheffe's theory in eq. (3).

Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in *i* and *j*. If x_i =1 and x_j = 0, since $j \neq i$,
then

$$Y_i = \beta_i \qquad (7)$$

which means

$$\sum_{i=1}^{3} \beta_i x_i = \sum_{i=1}^{3} Y_i x_i \qquad (8)$$

Hence,

$$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \qquad (9)$$

2. RESEARCH METHODOLOGY

The project was divided into 3 milestones or tasks. Conception and design, construction, and finishing/furnishing were the three tasks as shown in Table 1. From the table, the initial estimated finish dates for each task were determined by the use of the PERT model.

Table 1.Project Tasks Summary

S/N	Project Task	Short form	Start date	Likely finish dates (days)			Estimated duration using PERT (days)	Actual finish dates	Actual duration (days)
				Optimistic	Most likely	Pessimistic			
1	Conception and Design	CD	12-Jun-15	22-Oct-15	05-Feb-16	20-May-16	238	30-Mar-16	292
				132	238	343			
2	Construction of Building	CB	27-Dec-16	15-Feb-18	13-Apr-18	26-Dec-17	445	22-May-18	511
				415	472	364			
3	Finishing and furnishing of Building	FB	05-Nov-17	22-Aug-19	06-Jan-20	10-Apr-20	785	12-Mar-20	858
				655	792	887			

The finish dates were converted to ratio form as shown in Table 2.

Table 2. Project task Proportions

cc	Response Y	Actual Components			Estimated durations using PERT (days)
		Optimistic S_1	Most likely S_2	Pessimistic S_3	
CD	Y_1	0.5546	1	1.4411	238
CB	Y_2	0.8792	1	0.7711	445
FB	Y_3	0.8270	1	1.1199	785

Putting the above in matrix form:

$$S = \begin{bmatrix} 0.5546 & 0.8792 & 0.8270 \\ 1 & 1 & 1 \\ 1.4411 & 0.7711 & 1.1199 \end{bmatrix} \quad (10)$$

The corresponding pseudo components are given as follows:

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

With center points

X_{12} = [0.5 0.5 0]; X_{13} = [0.5 0 0.5]; and X_{23} = [0 0.5 0.5].

According to [7],

$$S_{ij} = XS_i \quad (12)$$

Substituting, we get:

$$\begin{bmatrix} S_{12} \\ S_{13} \\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} * \begin{bmatrix} 0.5546 \\ 0.8792 \\ 0.8270 \end{bmatrix} \quad (13)$$

The process of matrix multiplication was repeated for an additional 6 (control) points, which were used for the verification of the formulated model. The regular triangles for the actual and pseudo components were given in Figures. 1 and 2 respectively.

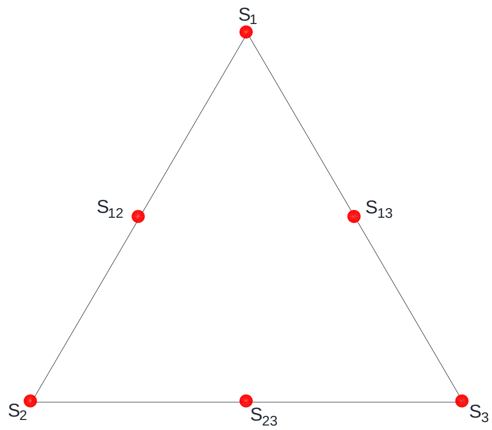

Fig. 1.Simplex Plot for Actual components

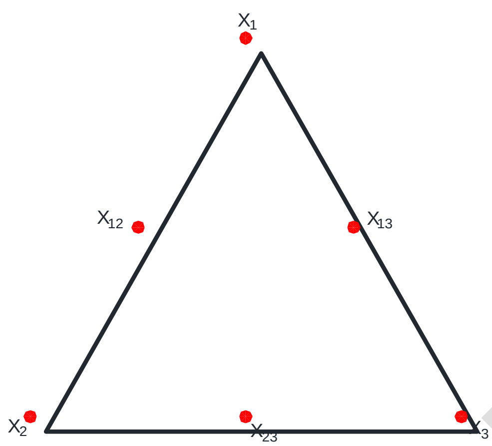

Fig. 2.Simplex Plot for Pseudo components

Table 3.Duration Proportions for Model Calibration

Sample Points	Actual Components			Response Y	Pseudo Components		
	Optimistic S_1	Most likely S_2	Pessimistic S_3		Optimistic X_1	Most likely X_2	Pessimistic X_3
CD	0.5546	1	1.4412	Y_1	1	0	0
CB	0.8792	1	0.7712	Y_2	0	1	0
FB	0.8270	1	1.1199	Y_3	0	0	1
N1	0.7169	1	1.1062	Y_{12}	0.5	0.5	0
N2	0.6908	1	1.2806	Y_{13}	0.5	0	0.5
N3	0.8531	1	0.9456	Y_{23}	0	0.5	0.5

Table 4.Duration Proportions for Model Verification

Sample Points	Actual Components			Response Y	Pseudo Components		
	Optimistic	Most	Pessimistic		Optimistic	Most	Pessimistic

	likely				likely		
	S_1	S_2	S_3		X_1	X_2	X_3
C1	0.7798	1	1.0608	Y_{C1}	0.25	0.4	0.35
C2	0.7363	1	1.1264	Y_{C2}	0.4	0.35	0.25
C3	0.7795	1	1.1377	Y_{C3}	0.2	0.1333	0.6667
C4	0.8201	1	0.9414	Y_{C4}	0.15	0.65	0.2
C5	0.6524	1	1.2876	Y_{C5}	0.6667	0.1333	0.2
C6	0.7536	1	1.1107	Y_{C6}	0.3333	0.3333	0.3333

3. RESULTS AND DISCUSSION

The actual components in Tables 3 and 4 for N1 to C6 were converted to whole numbers, and the estimated durations calculated using PERT equation as shown in Table 5.

Table 5.Estimated Durations

Sample Points	Likely durations (days)			Estimated durations using PERT (days)
	Optimistic	Most likely	Pessimistic	
N1	247	345	382	335
N2	360	520	667	518
N3	540	633	599	612
C1	409	525	557	511
C2	338	460	518	449
C3	496	636	724	628
C4	410	500	471	480
C5	248	381	490	377
C6	382	506	563	495

From Table 5, and eq. (9), the polynomial coefficients are: β_1 = 238, β_2 = 445, β_3 = 785,$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$, $\beta_{12} = 4 * 335 - 2 * 238 - 2 * 445 = -24.667$

Similarly, β_{13} = 26.333, and β_{23} = -11.

Substituting the above coefficients into eq. (6):

$$\boldsymbol{Y = 238x_1 + 445x_2 + 785x_3 - 024.667x_1x_2 + 26.333x_1x_3 - 11x_2x_3} \quad (14)$$

Eq. (14) above is the Scheffe's mathematical model to predict the durations of the tasks of the project.

In Table 6, the formulated model in eq. (14) was used to predict the durations.

Table 6.Estimated and Predicted Durations

Sample Points	Response Y	Pseudo Components			Estimated duration using PERT, Y_{est} (days)	Scheffe's Predicted duration, Y_{pred} (days)
		Optimistic X_1	Most likely X_2	Pessimistic X_3		
CD	Y_1	1	0	0	238	238
CB	Y_2	0	1	0	445	445

FB	Y_3	0	0	1	785	785
N1	Y_{12}	0.5	0.5	0	335	335
N2	Y_{13}	0.5	0	0.5	518	518
N3	Y_{23}	0	0.5	0.5	612	612
C1	Y_{C1}	0.25	0.4	0.35	511	510
C2	Y_{C2}	0.4	0.35	0.25	449	445
C3	Y_{C3}	0.2	0.1333	0.6667	628	632
C4	Y_{C4}	0.15	0.65	0.2	480	479
C5	Y_{C5}	0.6667	0.1333	0.2	377	376
C6	Y_{C6}	0.3333	0.3333	0.3333	495	488

Notice that the results from the control points were the only ones that gave slightly different values. This is because the values from the first part were used to calibrate (determine the model constants), while those from the second part (control points) were used to verify the formulated model. Figure 3 shows a graphical comparison between the PERT and Scheffe's results.

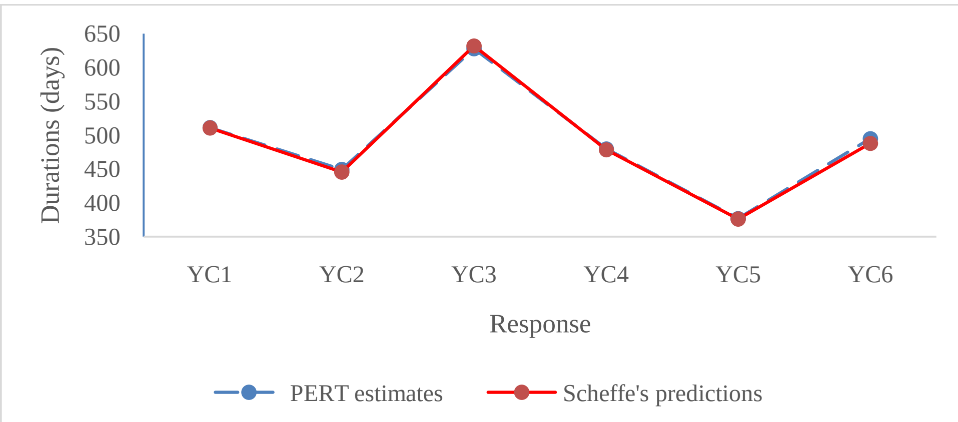

Fig. 3.Graphical comparison of PERT and Scheffe's durations

A two-tailed student t-test was carried out at 95% confidence level, which implies 100 – 95 = 5% significance.

Let D be difference between the eestimated and predicted responses.

The mean of the difference,

$$D_a = \frac{1}{n}\sum_{i=1}^{n} D_i \quad (15)$$

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n} (D - D_a)^2{}_i \quad (16)$$

$$t_{calculated} = \frac{D_a\sqrt{n}}{S} \quad (17)$$

Where n = number of observations with degree of freedom n – 1.

$$S^2 = \frac{72.745}{6-1}$$

$$S = \sqrt{14.549} = 3.814$$

$$t_{calculated} = 1.021$$

Table 7.Student t-table for estimated and predicted durations

Sample points	Task durations		t-test		
	$Y_{estimated}$	$Y_{predicted}$	$D=Y_{est}-Y_{pred}$	D_a-D	$(D-D_a)^2$
C1	511.000	510.306	0.694	0.896	0.803
C2	449.000	445.176	3.824	-2.234	4.990
C3	627.500	632.054	-4.554	6.145	37.755
C4	480.000	478.555	1.445	0.145	0.021
C5	377.000	375.841	1.159	0.432	0.186
C6	495.000	488.025	6.975	-5.384	28.989
TOTAL			9.543		72.745
AVERAGE D_a			1.590		

From the t-table, $t_{(\beta,v)}$ can be determined where v = 6 – 1 = 5, and β = significance level. $t_{(0.975,5)}$ = 2.571. Table 7 was used to determine the parameters with which the $t_{calculated}$ was determined.

Since $t_{calculated} < t_{(0.975,5)}$, and lies between -2.571 and 2.571, therefore there is no significant difference between the PERT estimated and Scheffe's predicted responses, H_0 is accepted, and H_a is rejected. The model is ascertained to be adequate. It is also found to be fit, since from Fig. 4, the R^2 value is 0.9986. This also means that the predicted model values are highly correlated to the estimated values.

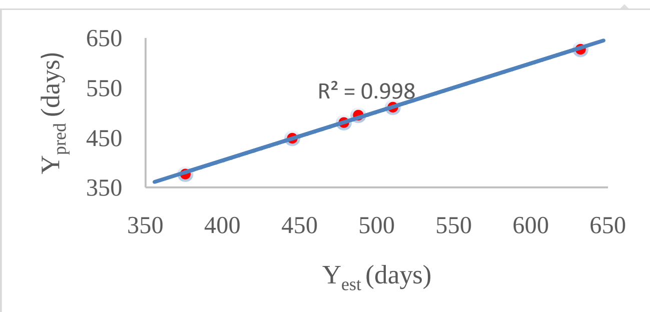

Fig. 4.Scatterplot of PERT and Scheffe's durations

4. CONCLUSION

The estimated durations with PERT is a stochastic approach and has been globally acceptable for determination of task durations since the 1950s. However, an empirical approach (Scheffe's simplex) has been successfully adopted to develop a multiple linear regression model to degree 2 to predict the task durations, given the same optimistic, most likely, and pessimistic durations as in the PERT. The model has successfully predicted the durations and has been found fit and adequate, with R^2 value of 0.9986, after being subjected to a two-tailed student t-test. This shows that the Scheffe's simplex approach can also be applied to task durations, while resulting in very high accuracy. This study, however was carried out for a project with only three tasks. Further studies are hereby recommended for a larger number of tasks.

CONSENT (WHEREEVER APPLICABLE)

No written consent note of any sort is required for this manuscript.

ETHICAL APPROVAL (WHEREEVER APPLICABLE)

No ethical approval of any sort is required for this manuscript.

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DEFINITIONS, ACRONYMS, ABBREVIATIONS

Here is the Definitions section. This is an optional section.

PERT: Program Evaluation and Review Technique