Testing Fractional Persistence and Nonlinearity in Infant Mortality Rates of Asia Countries

ABSTRACT

The infant mortality rates in 45 Asian countries (1960-2018), obtained from the Federal Reserve Bank of St. Louis database, are investigated using I(d) framework, which allows for simultaneous estimation of the degree of persistence and nonlinearities in infant mortality rates as well as their growth rates. A high degree of persistence in the decreases of mortality rate is found with nonlinear evidence in most of the cases, confirming nonlinear dynamics of mortality rates. In the growth of mortality rates, we find ten countries (Armenia, Indonesia, Israel, Japan, Kuwait, Myanmar, Saudi Arabia, Sri Lanka, Thailand, and UAE) with evidence of mean reversion. Health management in those listed countries needs to kick start interventions that improve the survival rates of infants.

***Keywords**:* Infant mortality rate; Death rate; Fractional persistence; nonlinearity; Asia

1. INTRODUCTION

Infant mortality rate (IMR), the ratio of deaths among children aged less than one year to a thousand, reflects the socio-economic and environmental conditions of the health of mothers and infants in a particular region. This also determines the effectiveness of health systems in such areas of the globe. Factors such as mothers' health, the possibility of preterm birth and birth weight, quality of antenatal, childbirth care, and infant feeding practices all contribute to yearly infant mortality level. One of the sustainable development goals (SDGs) for a child's health is to reduce infant mortality to 12 deaths or fewer per 1000 live births by 2030, and this is being achieved by many nations (OECD/World Health Organization, 2018). Thus, as the global health agenda broadens, there arise up-to-date, accurate measures of mortality for predicting life expectancy at birth, which is needed for policy decisions. IMR measures also inform regional convergence between death rates in countries with similar economic growth as it translates to the provision of health facilities in such region. IMR also depicts stagnation or reversal in mortality (Wang et al., 2017).

The United Nations Children Emergency Fund (UNICEF), the World Bank, the World Health Organization (WHO), the United Nations Population Division (UNDP) and the United Nations Interagency Group for Child Mortality Estimation (UNIGME) formed an ally in 2004 to foster cooperation towards the production of child mortality data sets and monitor the progress of child mortality and survivals across countries (You et al., 2015). The UNIGME relies on estimates obtained from survey data, using common statistical methods across countries to compute child mortality in the interest of comparability.

In Asia, the average IMR among the lower middle and lower-income class countries is about 30 deaths based on 2016 data sets. Many upper-middle income class countries in Asia have reached the SDG goal, reporting an average IMR of 11.5 deaths per 1000 live births. For instance, IMRs are lower among eastern countries such as Hong Kong, China, Japan, Singapore, and Korea, while more than half of the countries in Asia present IMRs higher than target (OECD/World Health Organization, 2018). According to World Health Organization, the global IMR has decreased from 65 to 29 deaths per 1000 live births between 1999 and 2018, with a decrease in the annual infant deaths from 8.7 million to 4.0 million within the same period.

IMRs are time structured, and their analysis is expected to be based on methods in time series analysis. Such time structured series often possess a certain degree of persistence that determines the degree of integration of the series. An integrated series of order 1 (say $I(1)$ series) requires first difference series transformation for it to produce stable/equilibrium series (i.e. $I(0)$), which is the growth series of the IMRs. The unit integration is too restrictive since time series are generally $I(d)$ in which d is some fractional values that depend on the time series. A very serious assumption in time series is to impose stationary $I(0)$ errors, and stationary IMRs implies such that shocks to IMRs will have temporary effects, with the permanent effect of shocks when IMRs are nonstationary $I(1)$ or $I(d > 1)$. In this case, if the IMRs are decreasing as expected, induced shocks on IMRs, which further make the series persist indefinitely, are expected when the health status of infants in those countries improves on an annual basis. Meanwhile, IMRs are mean-reverting [i.e. $I(0 < d < 1)$] if the series change their decreasing path or remain stagnant for a specified period. This applies to the growth rate of IMR, which in this case is the first series unit differencing. An anti-persistence IMR growth rate series implies strong negative growth of infant mortality as expected from any IMR from a country with good child health policy.

The Lee-Carter mortality model (Lee and Carter, 1992) assumes linear dynamics of mortality rates and this model has been challenged in Hill et al. (1999), Booth et al. (2002), Booth (2006) and Shang et al. (2011). These studies emphasize mortality forecasts, having considered the nonlinearity factor in the generating process of the mortality series.

The present study is invariant to the existing literature on time series analysis of mortality rates. It investigates the time dependencies of IMR in Asia. A study of this nature is crucial as it has serious implications for life expectancy. Whether the expected life span of infants would revolve around a cycle, improve, or otherwise depend on the established degree of persistence. Also, thorough knowledge of the trending behavior of IMR in the Asian region is crucial for determining the level of financial commitment to public health and the socio-economic attributed to the same sector. Thus, we employ the fractional persistence approach in both linear and nonlinear frameworks to achieve this objective. The nonlinear specification is based on the method developed in Cuestas and Gil-Alana (2016), using Chebyshev polynomial in time as the nonlinear deterministic term. Originally, Robinson (1994) earlier proposes the linear version of the fractional integration model, which is extended to the nonlinear deterministic setup in Cuestas and Gil-Alana (2016). The study offers the possibility of estimating the fractional integration and nonlinearity parameters jointly in the IMRs in a unified treatment.

The remainder part of the paper is structured as follows: Section 2 presents the time series econometric method applied in the paper. Section 3 presents the data sets and empirical findings, while Section 4 concludes the paper.

2. METHODOLOGY FRAMEWORK

2.1 Linear Model

Conventional practice in the linear modelling of trending time series follows the function given as:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, ..., \tag{1}$$

where, for the sake of this study, y_t denotes infant mortality rate, and x_t denotes the detrended disturbance term. The parameter β_1 is the trend coefficient that measures the average reduction (yearly) in the mortality rate. Expectedly, β_1 should be significant and negative. However, making valid and accurate statistical inferences about β_1 requires valid statistical inference about β_1 in Eq. (1); it is vital to correctly determine the exact structure of the disturbance term, x_t. Thus, the integration order of x_t must be significantly zero (i.e., $x_t \approx I(0)$), which suggests that the observations are not dependent. At most, the observations should exhibit weak dependence, such as the one expressed through the first-order Autoregressive (AR(1)) process as shown thus:

$$x_t = \varphi x_{t-1} + \varepsilon_t, \qquad t = 1, 2, ..., \tag{2}$$

where $|\varphi|$ less than one and ε_t follow the white noise process.

However, the test statistic is likely to suffer from the problem of size distortions when the null hypothesis that β_1 equals zero is tested against its alternative of β_1 less than zero. If the coefficient of the autoregressive component, φ in Eq. (2) tends towards unity (see Park and Mitchell, 1980; Woodward and Gray, 1993). Therefore, the way to correct this is to make φ equal to one, thereby making the process to be integrated into the first order (i.e. $x_t \approx I(1)$). Then, this makes the statistical inference to be stationary having been subjected to primary differences, i.e. $x_t - x_{t-1}$. Combining Eqs. (1) and (2) when φ equals one gives:

$$(1-B)\, y_t = \beta_1 + x_t; \qquad t = 1, 2, ..., \tag{3}$$

where B is the lag operator ($Bx_t = x_{t-1}$).[1]

To this end, a time series $\{x_t,\ t = 0, \pm 1, \ldots\}$ is defined to be integrated of order d, represented by $I(d)$ if:

$$(1-B)^d x_t = u_t, \qquad t = 1, 2, ..., \tag{4}$$

with $u_t \approx I(0)$), (see Granger and Joyeux, 1980; Hosking, 1981). The binomial representation of the expression on the left-hand side of Eq. (4) is given as:

$$(1-B)^d = \sum_{j=0}^{d} \binom{d}{j} (-1)^j B^j = 1 - d\,B + \frac{d\,(d-1)}{2} B^2 - ..., \tag{5}$$

[1] Vogelsang (1998) constructed a t-statistic based on (3) in the presence of serial correlation.

where the value of *d* determines the degree of association among observations in a distant time.
A higher *d* value suggests a stronger association. Furthermore, the fractional parameter *d* is notable for the determination of the level of persistence of the time series being considered. A short memory process is inferred if *d* equals zero (i.e., $u_t = x_t = I(0)$), although it could also imply a weak AR process sometimes. Mean reversion and covariance stationarity hold if d is greater than zero and less than 0.5, indicating that the effect of shocks will last longer than the case of $I(0)$ before it disappears. If d is equal to or greater than 0.5 and less than one, the process becomes nonstationary mean-reverting. In other words, it still exhibits mean reversion but loses its covariance stationary property. The implication of this is that although shocks will disappear in the long-run, it will be slow. The last scenario is when d is equal to or greater than one, which implies a non-mean reversion. In this case, the effect of the shocks is not transitory but permanent, unless strong policy measures are undertaken to restore normalcy.

2.2Non-Linear Model

Meanwhile, the literature has convincingly revealed that mortality rates often exhibit nonlinear dynamic trend patterns (see Hill et al., 1999; Booth et al., 2002; etc.). Hence, Eq. (1) is transformed into a nonlinear form as developed by Cuestas and Gil-Alana (2016):

$$y_t = f(\theta; t) + x_t, \quad t = 1, 2, \ldots \qquad (6)$$

where the nonlinear function, which relies on the unknown parameter vector *θ* with m-dimension, is captured by $f(.)$.

Accordingly, the proposed trend function depends on the Chebyshev polynomials in time whose performance in fractionally integrated frameworks cannot be underplayed. Eq. (6) is therefore re-expressed as:

$$y_t = \sum_{i=0}^{m} \beta_i P_{i,T}(t) + x_t, \quad t = 1, 2, \ldots, \qquad (7)$$

where *T* denotes the sample size; and *m* is the Chebyshev polynomial order given as:

$$P_{0,T}(t) = 1, \qquad (8)$$

and

$$P_{i,T}(t) = \sqrt{2}\cos\left(i\,\pi\,(t-0.5)/T\right), \quad t = 1, 2, \ldots, T; \quad i = 1, 2, \ldots \qquad (9)$$

as explored in Cuestas and Gil-Alana (2016).

The degree of nonlinearity is deducted from the value of *m,* such that a higher value of *m* denotes higher nonlinear structure. Specifically, if m equals zero, then only an intercept is contained in the model. If *m* equals one, both intercept and a linear trend are contained in the model, thereby reverting to the linear model in Eq. (1). That the value of *m* is greater than one (i.e., *m* > 1) makes the model nonlinear. Cuestas and Gil-Alana (2016) argue that *m* equalsthreeis not enough to infer the nonlinear dynamics of the model. Hence, we consider only the second- and third-order polynomial degrees to judge nonlinearity in this study.

2.3Method of Estimation

For the linear model, the fractional parameter *d* following Eqs. (1) and (4) is estimated using the Lagrange Multiplier (LM) approach of Robinson (1994) in line with the Whittle function in the frequency domain. For any range of values of *d,* the null hypothesis tested by this method is:

$$H_o : d = d_o, \tag{10}$$

The test statistic is computed as:

$$\hat{R} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \tag{11}$$

where

$$\hat{a} = -\frac{2\pi}{T}\sum_j^{*}\psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1} I_{\hat{u}}(\lambda_j), \tag{12}$$

$$\hat{A} = \frac{2}{T}\left\{\sum_j^{*}\psi(\lambda_j)\psi(\lambda_j)' - \sum_j^{*}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)'\left[\sum_j^{*}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right]^{-1}\sum_j^{*}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)'\right\} \tag{13}$$

with

$$\psi(\lambda_j) = \log\left|\sin\frac{\lambda_j}{2}\right|, \qquad \lambda_j \; \frac{2\pi j}{T}. \tag{14}$$

and

$$\hat{\varepsilon}(\lambda_j) = \arg\min_{\tau} \sigma^2(\tau), \tag{15}$$

* as used in Eqs. (12) and (13) refer to all bounded discrete frequencies in the spectrum. Due to the white noise process of u_t and $Var(u_t) = \sigma^2$, then the spectral density function of u_t becomes $\frac{\sigma^2}{2\pi}$. Also, $g(.)$ in Eq. (12) becomes equal to one, and $\hat{\varepsilon}(\lambda_j) = 0.$ In addition, Robinson (1994) shows that if a very mild regularity condition is assumed. If it is up to second-order moments, then:

$$\hat{R} \;\to_d\; \chi_1^2 \tag{16}$$

as $T \to \infty$.

Moreover, the above method is modified by Cuestas and Gil-Alana (2016) to incorporate nonlinearity, which leads to the replacement of Eq. (1) by Eq. (7). The combination of Eqs. (4) and (7), therefore, yields:

$$y_t^{*} = \sum_{i=0}^{m} \beta_i P_{i,T}^{*}(t) + x_t, \quad (1-B)^d x_t = u_t, \quad t = 1, 2, ..., \tag{17}$$

where $y_t^* = (1 - B)^d y_t$; and $P_{i,Tt}^*(t) = (1 - B)^d P_{iT}(t)$, thus, making u_t in Eq. (17) to follow I(0) process. Following Cuestas and Gil-Alana (2016), the value of θ in Eqs. (7) and (17) is obtained using the least square methods.

3. DATA AND EMPIRICAL RESULTS

Infant mortality rate data sets for 45 Asian countries are analyzed in this paper. These data sets are retrieved from the Federal Reserve Bank of St Louis Economic Research Division database website at https://fred.stlouisfed.org. The countries considered are Afghanistan, Armenia, Azerbaijan, Bahrain, Bangladesh, Bhutan, Brunei, Cambodia, China, India, Indonesia, Iran, Iraq, Israel, Japan, Jordan, Kazakhstan, Korea, Kuwait, Kyrgyzstan, Laos, Lebanon, Malaysia, Maldives, Mongolia, Myanmar, Nepal, Oman, Pakistan, Palestinian, Philippine, Qatar, Saudi Arabia, Singapore, Sri Lanka, Syria, Tajikistan, Thailand, Timor-Leste, Turkey, Turkmenistan, UAE, Uzbekistan, Vietnam, and Yemen (see Table 1).

Table 1 presents the sample periods of each country's IMR, with starting and ending IMR for each country's case as well as the percentage reduction in IMR over the historical period. The countries have different starting periods with the most extended series having samples beginning in 1960, and the shortest series having samples beginning in 1985. By looking at the starting IMRs, Brunei presents the lowest IMR in 1983 (0.0116), and its IMR as of 2018 is 0.0098, ranking 17th among other Asian countries in terms of lower IMR in that year. Yemen presents IMR of 0.2794 in 1962, of the lowest rank in that year; this country is able to lower its IMR to 0.0429 in 2018, ranking 42nd in that year. The percentage reductions are computed for each country and ranked to determine the best-performing country's IMR over the years; these results are presented in the last column of Table 1. Brunei, with the lowest initial IMR in 1983, indicates the lowest percentage reduction in IMR (15.5%) compared to other Asian countries' IMRs, while Maldives indicates the highest percentage reduction in IMR (96.5%) over the period from 1964 to 2018. Based on the percentage reduction, we have the five countries with the slow growth of IMRs, namely Brunei (15.5%), Korea (50.5%), Turkmenistan (62.4%), the Philippines (66.2%) and Pakistan (69.2%), while other highly developed Asian nations such as China, Japan, Saudi Arabia, and Singapore have reduced their IMRs to about 90% and above.

Table 1: Countries examined; sample periods and % growth rate in IMR

Country	Time period	Starting IMR	Ending IMR	% Reduction
Afghanistan	1961-2018	0.2365 (44)	0.0479 (43)	79.7 (13)
Armenia	1976-2018	0.0693 (12)	0.0110 (19)	84.1 (18)
Azerbaijan	1982-2018	0.0855 (16)	0.0920 (45)	77.5 (12)
Bahrain	1960-2018	0.1337 (30)	0.0061 (6)	95.4 (43)
Bangladesh	1960-2018	0.1736 (37)	0.0251 (34)	85.5 (20)
Bhutan	1969-2018	0.1874 (40)	0.0248 (33)	86.8 (23)
Brunei	1983-2018	0.0116 (1)	0.0098 (17)	15.5 (1)
Cambodia	1975-2018	0.1777 (38)	0.0240 (32)	86.5 (22)
China	1969-2018	0.0837 (15)	0.0074 (12)	91.2 (35)
India	1960-2018	0.1614 (34)	0.0299 (36)	81.5 (16)

Indonesia	1960-2018	0.1487 (33)	0.0211 (29)	85.8 (21)
Iran	1971-2018	0.1255 (28)	0.0124 (20)	90.1 (33)
Iraq	1960-2018	0.1294 (29)	0.0225 (30)	82.6 (17)
Israel	1974-2018	0.0248 (2)	0.0030 (3)	87.9 (27)
Japan	1960-2018	0.0304 (4)	0.0018 (1)	94.1 (39)
Jordan	1960-2018	0.1070 (22)	0.0139 (22)	87.0 (24)
Kazakhstan	1971-2018	0.0682 (11)	0.0088 (15)	87.1 (25)
Korea	1985-2018	0.0277 (3)	0.0137 (21)	50.5 (2)
Kuwait	1960-2018	0.0977 (19)	0.0067 (10)	93.1 (37)
Kyrgyzstan	1975-2018	0.0878 (17)	0.0169 (26)	80.8 (15)
Laos	1978-2018	0.1448 (32)	0.0376 (39)	74.0 (8)
Lebanon	1960-2018	0.0566 (8)	0.0064 (7)	88.7 (30)
Malaysia	1960-2018	0.0673 (10)	0.0067 (10)	90.0 (32)
Maldives	1964-2018	0.2101 (41)	0.0074 (12)	96.5 (45)
Mongolia	1978-2018	0.1179 (26)	0.0140 (23)	88.1 (29)
Myanmar	1968-2018	0.1226 (27)	0.0368 (38)	70.0 (6)
Nepal	1960-2018	0.2161 (42)	0.0267 (35)	87.6 (26)
Oman	1963-2018	0.2163 (43)	0.0098 (18)	95.5 (44)
Pakistan	1960-2018	0.1857 (39)	0.0572 (44)	69.2 (5)
Palestinian[2]	1975-2018	0.0767 (14)	0.0173 (27)	77.4 (11)
Philippine	1960-2018	0.0666 (9)	0.0225 (30)	66.2 (4)
Qatar	1969-2018	0.0532 (6)	0.0058 (4)	89.1 (31)
Saudi Arabia	1972-2018	0.1094 (23)	0.0060 (5)	94.5 (40)
Singapore	1960-2018	0.0354 (5)	0.0023 (2)	93.5 (38)
Sri Lanka	1960-2018	0.0706 (13)	0.0064 (7)	90.9 (34)
Syria	1960-2018	0.1169 (24)	0.0140 (23)	88.0 (28)
Tajikistan	1972-2018	0.1177 (25)	0.0304 (37)	74.2 (9)
Thailand	1960-2018	0.1013 (20)	0.0078 (14)	92.3 (36)
Timor-Leste	1985-2018	0.1628 (35)	0.0393 (41)	75.9 (10)
Turkey	1960-2018	0.1723 (36)	0.0091 (16)	94.7 (41)
Turkmenistan	1977-2018	0.1046 (21)	0.0393 (40)	62.4 (3)
UAE	1960-2018	0.1341 (31)	0.0065 (9)	95.2 (42)
Uzbekistan	1979-2018	0.0965 (18)	0.0191 (28)	80.2 (14)
Vietnam	1964-2018	0.0563 (7)	0.0165 (25)	70.7 (7)
Yemen	1962-2018	0.2794 (45)	0.0429 (42)	84.6 (19)

In parentheses the ranking based on the reduction in infant mortality rates.

Since IMRs evolved as a nonlinear decreasing series, we consider a model in Eq. (17) for the analysis of the time series persistence and nonlinearity properties. The fractional integration framework, as earlier described allows model intercept β_0 and nonlinear parameters of order 3 (β_1, β_2 and β_3), with the significance of at least one of the nonlinear parameters implying nonlinear dynamics of IMR. The results are presented in Table 2. Even though Cuestas and Gil-Alana (2016) and Yaya and Gil-Alana (2020) have noted the dominance of persistence over nonlinearity property when both properties are investigated simultaneously, these are bound to happen in occasional cases. In the results in Table 2, a

[2] In the case of Palestine due to ongoing war, IMRs were only recorded for occupied territory.

few cases of insignificant IMRs are Bahrain, India, Israel, Kazakhstan, Kyrgyzstan, Laos, Qatar, Syria, Thailand, and Turkmenistan. The estimates of persistence in the 45 countries are greater than one in most cases, and they reach two in more than half (50%) of the cases. This explains the strict persistent decreases in IMRs of those countries. In few cases, we observe evidence of mean reversion such that IMR has tendencies to reverse its course, and this evidence is found in Armenia where I(d = 1) hypothesis is sternly rejected against I(d < 1).

Table 2: Estimated coefficients in the nonlinear I(d) model given by Eq. (17)

Country	d	β_0	β_1	β_2	β_3
Afghanistan	2.19 (2.01, 2.37)	**107.43 (32.1)**	**-5.02 (-4.39)**	1.19 (2.13)	-0.05 (-0.06)
Armenia	0.65 (0.40, 0.90)	-2.72 (-1.06)	**16.09 (22.7)**	**2.03 (4.51)**	**1.32 (3.80)**
Azerbaijan	2.66(2.35, 2.97)	**73.01 (2.41)**	-19.16 (-0.92)	**-7.81 (-2.44)**	**-2.86 (-2.57)**
Bahrain	2.64 (2.39, 2.89)	**112.17 (9.15)**	0.54 (0.04)	-3.63 (-0.63)	0.47 (0.24)
Bangladesh	2.49 (2.25, 2.73)	**100.61 (15.7)**	**-4.96 (-9.33)**	**-10.09 (-2.46)**	**-5.16 (-3.48)**
Bhutan	2.16 (2.00, 2.32)	**105.16 (38.4)**	**-8.75 (-3.68)**	0.39 (0.22)	0.56 (0.78)
Brunei	1.63 (1.3, 1.94)	**3.04 (7.70)**	0.10 (0.63)	0.36 (1.52)	**-0.40 (-3.49)**
Cambodia	2.54 (2.21, 2.87)	**516.56 (2.81)**	-241.83 (-1.56)	**-41.38 (-2.34)**	-4.04 (-0.71)
China	2.29 (2.11, 2.47)	**113.88 (4.35)**	**-36.49 (-2.18)**	**-9.07 (-2.96)**	0.26 (0.23)
India	2.12 (1.87, 2.37)	**70.35 (3.68)**	1.13 (0.10)	-1.26 (-0.49)	-0.60 (-0.64)
Indonesia	1.59 (1.41, 1.77)	**78.65 (27.30)**	**-1.62 (-2.82)**	3.11 (1.73)	0.11 (0.12)
Iran	2.36 (2.11, 2.61)	**77.21 (18.2)**	**-7.85 (-6.67)**	2.96 (1.09)	**2.45 (2.37)**
Iraq	2.00 (1.99, 2.01)	**79.17 (49.8)**	**-571.74 (-96.4)**	**2.09 (2.43)**	**1.39 (3.89)**
Israel	1.40 (1.11, 1.69)	**41.34 (1.15)**	0.58 (1.70)	-0.16 (-0.67)	-0.12 (-1.13)
Japan	1.12 (0.92, 1.32)	79.66 (1.24)	0.12 (1.62)	**0.43 (2.07)**	**0.20 (2.83)**
Jordan	2.00 (1.99, 2.01)	**70.44 (21.4)**	**-717.92 (-63.0)**	1.62 (1.04)	0.73 (1.03)
Kazakhstan	2.93 (2.50, 3.36)	237.83 (1.48)	-132.55 (-1.17)	-14.49 (-1.73)	-0.12 (-0.05)
Korea	2.40 (2.16, 2.64)	**59.66 (390.0)**	**-4.96 (-8.11)**	0.23 (0.20)	7.83 (0.47)
Kuwait	1.34 (1.10, 1.58)	**116.71 (2.85)**	0.56 (0.91)	**3.93 (3.98)**	0.47 (1.26)
Kyrgyzstan	2.96 (2.69, 3.23)	**187.70 (2.77)**	-98.36 (-1.92)	-6.13 (-0.60)	0.35 (0.12)
Laos	2.09 (1.91, 2.27)	**64.75 (49.2)**	-8.07 (-1.55)	-0.38 (-0.45)	-0.001 (-0.004)
Lebanon	1.00 (0.99, 1.01)	**-54382.5 (-14.1)**	**14.59 (32.9)**	-0.10 (-0.45)	-0.09 (-0.61)
Malaysia	2.01 (2.00, 2.02)	**-5518.88 (537)**	**-325.99 (-27.3)**	**2.44 (2.00)**	0.68 (1.32)
Maldives	2.01 (1.85, 2.17)	**117.54 (5.05)**	9.27 (0.63)	3.84 (1.14)	**2.83 (2.03)**
Mongolia	2.35 (2.11, 2.59)	**67.57 (23.7)**	**-5.13 (-6.23)**	-1.42 (-0.78)	-0.36 (-0.51)
Myanmar	0.80 (0.56, 1.04)	-6.62 (-0.91)	**23.56 (19.9)**	0.24 (0.36)	**2.95 (5.94)**
Nepal	2.33 (2.17, 2.49)	**103.47 (4.10)**	7.83 (0.47)	-1.86 (-0.56)	-1.15 (-0.94)
Oman	2.17 (2.03, 2.31)	**148.38 (43.3)**	**-19.47 (-3.86)**	**10.13 (4.63)**	**2.01 (2.25)**
Pakistan	1.00 (0.99, 1.01)	**-229016.0 (-17.5)**	**25.79 (14.5)**	-0.003 (-0.004)	**3.58 (5.96)**

Palestinian	2.00 (1.99, 2.01)	**47.32 (355.0)**	**695.46 (88.7)**	-0.37 (-0.44)	-0.001 (-0.004)
Philippine	2.03 (1.85, 2.21)	**49.74 (4.42)**	-9.22 (-1.29)	-3.44 (-2.00)	**-3.41 (-4.93)**
Qatar	2.11 (1.91, 2.31)	**36.98 (213.0)**	-7.37 (-1.57)	0.23 (0.20)	-0.001 (-0.004)
Saudi Arabia	2.17 (2.03, 2.31)	**79.68 (437.0)**	**-12.33 (-4.03)**	-0.38 (-0.45)	-0.001 (-0.004)
Singapore	1.00 (0.99, 1.01)	**-82370.0 (-10.0)**	**6.31 (8.63)**	**2.72 (7.59)**	**0.79 (3.26)**
Sri Lanka	1.33 (1.11, 1.55)	**26.54 (2.73)**	12.20 (2.00)	**3.98 (2.00)**	-0.34 (-0.29)
Syria	2.08 (1.94, 2.22)	**77.68 (29.8)**	-17.01 (-1.35)	1.21 (0.73)	0.77 (1.10)
Tajikistan	3.20 (2.98, 3.42)	**969.60 (13.5)**	**-619.57 (-13.2)**	-27.19 (-0.80)	-4.66 (-0.42)
Thailand	2.05 (1.91, 2.19)	**61.08 (34.6)**	-10.64 (-0.86)	2.18 (1.93)	0.71 (1.49)
Timor-Leste	2.46 (2.19, 2.73)	**85.05 (48.3)**	**-8.35 (-11.5)**	0.07 (0.07)	-0.77 (-1.82)
Turkey	2.44 (2.20, 2.68)	**112.92 (17.8)**	**-4.34 (-7.02)**	-2.22 (-0.55)	-1.65 (-1.10)
Turkmenistan	2.41 (2.12, 2.70)	**108.63 (2.82)**	-42.64 (-1.71)	-6.08 (-1.59)	0.78 (0.59)
UAE	1.00 (0.99, 1.01)	**-273157.0 (-12.9)**	**2.79 (10.6)**	**14.54 (13.0)**	**7.13 (9.50)**
Uzbekistan	2.85 (2.54, 316)	**61.25 (10.2)**	**-6.24 (-15.5)**	-7.28 (-1.83)	0.07 (0.05)
Vietnam	2.08 (1.86, 2.30)	**23.12 (12.6)**	0.99 (0.38)	0.23 (0.20)	**-1.47 (-2.97)**
Yemen	2.74 (2.45, 3.03)	**138.42 (6.73)**	**-4.70 (-5.76)**	12.49 (0.93)	6.97 (1.58)

Note: Values of d with the corresponding confidence intervals are given in the second column. In the columns 3-6,bold figure denotes significance of parameter estimate for nonlinear deterministic model at 5% level with t statistic for corresponding estimate in parenthesis.

As the fractional parameter *d* exceeds one in virtually all the countries, and many reach two, Table 2 shows that there is evidence of a high degree of persistence in most of the countries. Thus, we deem it fit to extend our empirical consideration to the persistence of the growth rate of the mortality series. We first take the differences of the series against the immediate past value to obtain the general growth rate of the mortality series.[3] Then, we conduct the fractional integration analysis of the newly generated series. Meanwhile, the linear model is now considered since growth rate series are not expected to exhibit nonlinear dynamics. Therefore, the linear model, following from Eqs. (1) and (4), is represented as:

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1-L)^d x_t = u_t, \quad t = 1, 2, ..., \qquad (18)$$

where u_t is assumed to follow a white noise process.

As a common empirical practice in literature, Eq. (18) is estimated under three scenarios: the first is when the model is assumed to have no deterministic terms. The second assumes only intercept, while the third assumes the significance of both intercept and linear time trends.

The results are presented in Table 3 with outcomes of the best model in bold font as judged by the t-values. We first observed that the fractional differencing parameter is not significant in two countries, namely Armenia and Myanmar, suggesting that persistence cannot be attributed to their mortality growth rates. For other countries, the significance is not found for the time trend, except in Israel, Kuwait, Palestinian, Thailand, and UAE. The

[3] The growth rate of the mortality series is mathematically computed as: gt=100*(rt-rt-1rt-1), where gt is the growth rate, and rt and rt-1, respectively denote present mortality rate and one period-lagged mortality rate.

insignificance of the time trends in most of the countries is supported by the disclosure of Yaya and Gil-Alana (2020) that growth rate series of IMR is not expected to give significant linear trend under the linear specification of Robinson (1994) to indicate that IMR growth is constant over time. In contrast, a significant positive slope in IMR growth rate implies slower growth than initial IMR decline, leading to a faster reduction towards the end of the sample. For negative significant linear trend slope, this means that there is a retarding decline in IMR; this is called mean reversion, and it signals danger. Out of the seven (7) countries with significant time trends, three states (Armenia, Myanmar, and Thailand) indicate a negative trend, while the remaining four countries (Israel, Kuwait, Palestinian, and UAE) indicate a positive trend. Except for Palestinian, the growth rate of IMR for the other six countries shows evidence of mean reversion. For the remaining states, the model with no deterministic terms or an intercept is favored, with most countries going with the former.

Notwithstanding, 35 countries of the 45 Asian countries have *d* values being I(d ≥ 1). For other countries, however, the values of *d* are lower than one, leading to the rejection of the unit root null hypothesis of I(d = 1) against the alternative of mean reversion, I(d < 1). Those remaining ten countries with mean reversion evidence are Armenia, Indonesia, Israel, Japan, Kuwait, Myanmar, Saudi Arabia, Sri Lanka, Thailand, and UAE. A graphical illustration of the mean reversion tendency of these countries is reported in Fig. 1 in which the growth rate series of IMR is plotted with actual IMR series. It is observed that the trends of the growth rate series are not explosive but appear to fluctuate around a mean value.

In countries where mean reversion is established, there must be ideal and calculative policy measures to keep mortality rates low because the occurrence of positive shocks that reduce IMRs may only last for a short period before there is a reverse. On the other hand, the effect of adverse shocks that increase IMRs in countries where persistence is found can be permanent unless strong policy measures are formulated. We provide a summary of the results in Table 4 for conciseness.

Table 3: Estimates of d on the growth rate series based on the model given by Eq. (18)

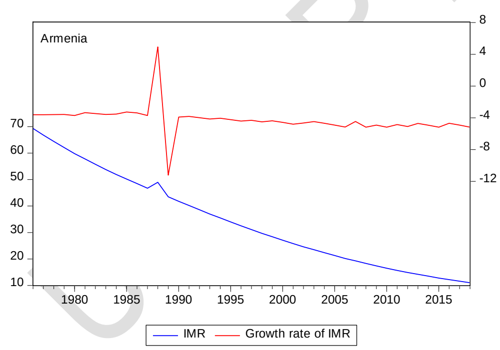
Country	No det. Terms	An intercept	A linear time trend
Afghanistan	1.09 (0.97, 1.21)	**1.22 (1.02, 1.42)**	1.19 (0.97, 1.41)
Armenia	-0.10 (-0.30, 0.10^{ns}	-0.10 (-0.30, 0.10) ^{ns}	**-0.75 (-1.02, -0.48)** ^{ns}
Azerbaijan	**1.89 (1.60, 2.18)**	1.91 (1.62, 2.20)	1.91 (1.62, 2.20)
Bahrain	**1.30(1.06, 1.54)**	1.38 (1.13, 1.63)	1.37 (1.10, 1.64)
Bangladesh	**1.29 (1.13, 1.45)**	1.33 (1.15, 1.51)	1.30 (1.10, 1.50)
Bhutan	**1.16 (0.98, 1.34)**	1.23 (1.03, 1.43)	1.23 (1.03, 1.43)
Brunei	**0.90 (0.66, 1.14)**	0.90 (0.65, 1.15)	0.80 (0.51, 1.09)
Cambodia	1.80 (1.51, 2.09)	**2.23 (1.84, 2.62)**	2.23 (1.84, 2.62)
China	**1.39 (1.15, 1.63)**	1.40 (1.16, 1.64)	1.40 (1.16, 1.64)
India	1.18 (1.06, 1.30)	**1.35 (1.17, 1.53)**	1.27 (1.05, 1.49)
Indonesia	0.20 (0.04, 0.36)	**0.20 (0.04, 0.36)**	-0.01 (-0.21, 0.19)
Iran	**1.12 (0.88, 1.36)**	1.12 (0.88, 1.36)	1.09 (0.84, 1.34)
Iraq	1.01 (0.91, 1.11)	**1.13 (0.95, 1.31)**	1.12 (0.94, 1.30)
Israel	0.66 (0.42, 0.90)	0.66 (0.42, 0.90)	**0.48 (0.21, 0.75)**
Japan	0.36 (0.18, 0.54)	**0.36 (0.18, 0.54)**	0.10 (-0.12, 0.32)
Jordan	1.00 (0.94, 1.06)	**1.00 (0.99, 1.01)**	1.23 (1.05, 1.41)
Kazakhstan	1.72 (1.50, 1.94)	**1.98 (1.73, 2.23)**	1.98 (1.73, 2.23)
Korea	**0.88 (0.68, 1.08)**	0.87 (0.67, 1.07)	0.87 (0.67, 1.07)
Kuwait	0.53 (0.35, 0.71)	0.53 (0.35, 0.71)	**0.41 (0.21, 0.61)**

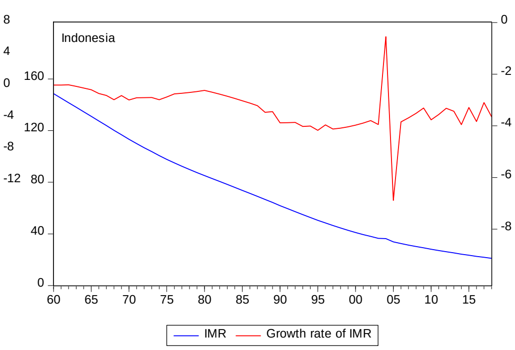
Kyrgyzstan	1.54 (1.25, 1.83)	**1.77 (1.44, 2.10)**	1.77 (1.44, 2.10)
Laos	**0.98 (0.80, 1.16)**	0.98 (0.78, 1.18)	0.87 (0.62, 1.12)
Lebanon	**1.09 (0.91, 1.27)**	1.09 (0.89, 1.29)	1.08 (0.88, 1.28)
Malaysia	**0.91 (0.71, 1.11)**	0.92 (0.72, 1.12)	0.87 (0.65, 1.09)
Maldives	**0.87 (0.67, 1.07)**	0.87 (0.67, 1.07)	0.87 (0.67, 1.07)
Mongolia	1.29 (1.09, 1.49)	**1.40 (1.16, 1.64)**	1.40 (1.16, 1.64)
Myanmar	-0.05 (-0.23, 0.13) ^{ns}	-0.05 (-0.23, 0.13) ^{ns}	**-0.40 (-0.62, -0.18)** ^{ns}
Nepal	1.04 (0.92, 1.16)	**1.22 (1.02, 1.42)**	1.21 (1.01, 1.41)
Oman	**1.20 (1.02, 1.38)**	1.21 (1.03, 1.39)	1.19 (1.01, 137)
Pakistan	1.07 (0.95, 1.19)	**1.25 (1.07, 1.43)**	1.25 (1.07, 1.43)
Palestinian	0.98 (0.82, 1.14)	1.00 (0.99, 1.01)	**0.81 (0.54, 1.08)**
Philippine	**1.28 (1.08, 1.48)**	1.31 (1.09, 1.53)	1.31 (1.09, 1.53)
Qatar	**0.85 (0.63, 1.07)**	0.85 (0.63, 1.07)	0.85 (0.63, 1.07)
Saudi Arabia	**0.58 (0.34, 0.82)**	0.58 (0.34, 0.82)	0.55 (0.31, 0.79)
Singapore	**0.91 (0.66, 1.16)**	0.91 (0.66, 1.16)	0.87 (0.60, 1.14)
Sri Lanka	**0.02 (-0.20, 0.24)**	0.02 (-0.20, 0.24)	0.01 (-0.21, 0.23)
Syria	**0.94 (0.74, 1.14)**	0.94 (0.74, 1.14)	0.87 (0.65, 1.09)
Tajikistan	**2.35 (1.98, 2.72)**	2.37 (2.00, 2.74)	2.37 (2.00, 2.74)
Thailand	0.65 (0.47, 0.83)	0.65 (0.47, 0.83)	**0.42 (0.20, 0.64)**
Timor-Leste	**1.38 (1.03, 1.73)**	1.38 (1.03, 1.73)	1.38 (1.03, 1.73)
Turkey	**0.95 (0.79, 1.11)**	0.96 (0.80, 1.12)	0.87 (0.67, 1.07)
Turkmenistan	**1.56 (1.29, 1.83)**	1.63 (1.34, 1.92)	1.63 (1.34, 1.92)
UAE	0.83 (0.67, 0.99)	0.83 (0.67, 0.99)	**0.73 (0.55, 0.91)**
Uzbekistan	1.54 (1.32, 1.76)	**1.93 (1.62, 2.24)**	1.93 (1.62, 2.24)
Vietnam	**1.15 (0.95, 1.35)**	1.19 (0.97, 1.41)	1.19 (0.97, 1.41)
Yemen	1.71 (1.46, 1.96)	**2.01 (1.70, 2.32)**	2.02 (1.73, 2.31)

In bold indicates selected results based on the significant deterministic terms of no intercept and trend, intercept only and intercept with trend. "ns" implies d values for Armenia and Myanmar are not significant. These indicates that the series are invertible, which is similar to mean reversion. Tests were computed at 5% level of significant.

Table 4: How persistent are the IMR growth rates in Asia?

Mean reversion (d < 1)	Unit roots (d = 1)	I(d > 1)
Armenia	Bhutan	Afghanistan
Indonesia	Brunei	Azerbaijan
Israel	Iran	Bahrain
Japan	Iraq	Bangladesh
Kuwait	Jordan	Cambodia
Myanmar	Korea	China
Saudi Arabia	Laos	India
Sri Lanka	Lebanon	Kazakhstan
Thailand	Malaysia	Kyrgyzstan
UAE	Maldives	Mongolia
	Palestinian	Nepal
	Qatar	Oman
	Singapore	Pakistan
	Syria	Philippine
	Turkey	Tajikhstan
	Vietnam	Timor-Leste
		Turkmenistan
		Uzbekistan
		Yemen





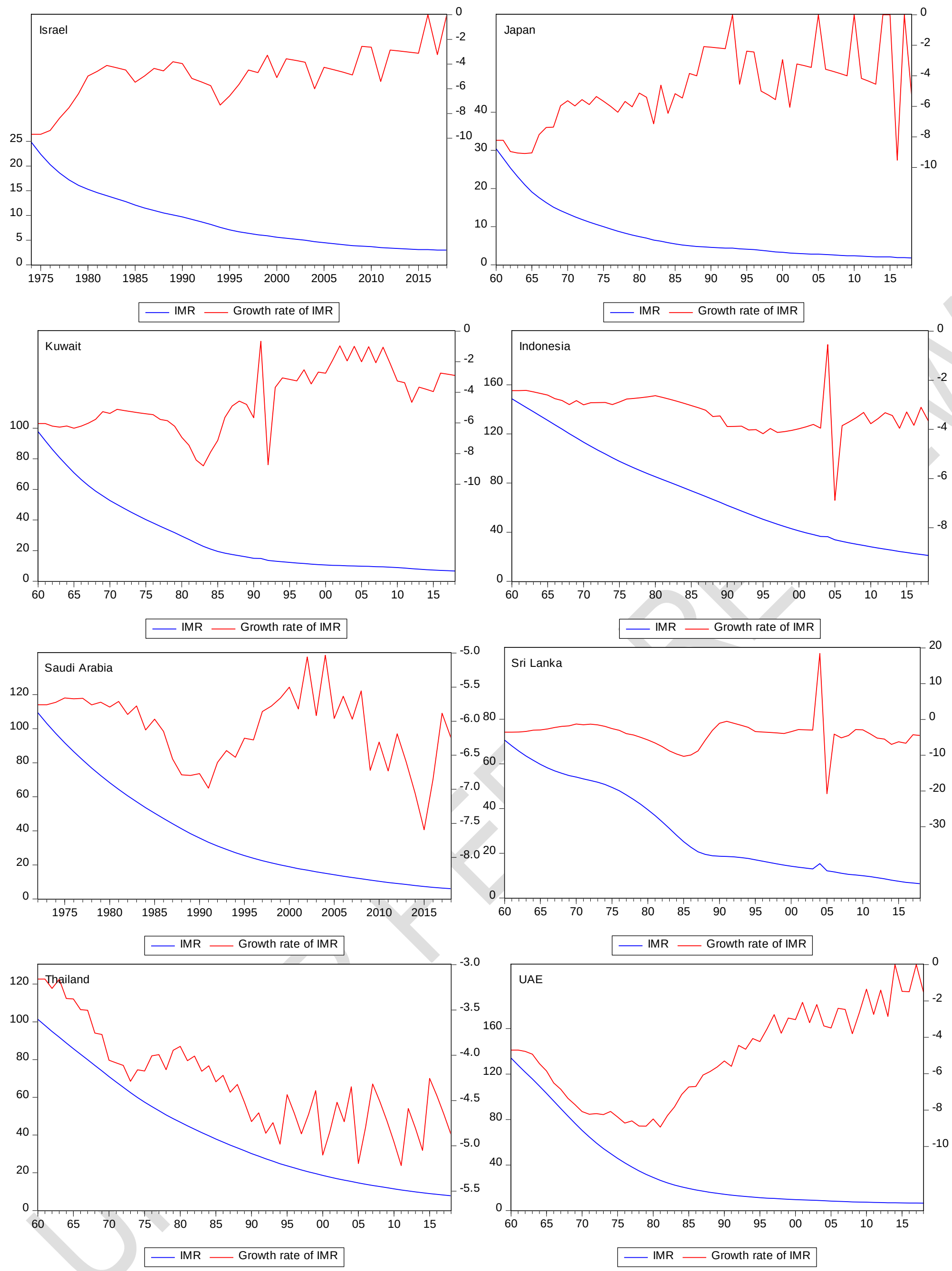


Figure 1: Graphs of IMR and their growth rates for those ten countries with mean reversion evidences

4. CONCLUSION

The present paper investigates infant mortality rates (IMR) in 45 Asian countries by examining time-series persistence and nonlinearity features in the IMRs. The analysis is conducted in fractional I(*d*) setup, which is more flexible and appealing than the standard methods of integer time series differencing. Chebyshev polynomial in time is used to mimic the nonlinear dynamics of mortality rate series, and the results show evidence of nonlinearity of IMR except in few cases including Bahrain, India, Israel, Kazakhstan, Kyrgyzstan, Laos, Qatar, Syria, Thailand and Turkmenistan. IMR persists strongly, with persistence estimates all greater than one, reaching two in many cases, except for Armenia where this is found to be less than one. Further probing into the mortality rate estimation leads to the computation of the growth rate to check if growths of the mortality rate would indicate varying persistence levels. The results find evidence of mean reversion in the growth of IMR in Armenia, Indonesia, Israel, Japan, Kuwait, Myanmar, Saudi Arabia, Sri Lanka, Thailand, and UAE. At the same time, the remaining 35 Asian countries will continue to experience consistent decline/negative growth in their IMRs over the years, given the current health management strategy in those Asian countries.

Special attention is therefore required in the health management of those listed ten countries where mean reversion is found, noting that positive shocks effect on the IMR growth rate will disappear by themselves over time unless policy actions are taken.

Another alternative modelling approach that simultaneously captures persistence of mortality rate and nonlinearity is given in Caporale and Gil-Alana (2007) based on smooth transition nonlinearity, but this approach is more challenging to implement.

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