Uncertainty Quantification of the Buckley Leverett Equation with Random Oil Viscosity

Abstract: Reservoir simulation is an important but tough task in the petroleum sector. Reservoir properties (rocks and fluids) exhibit uncertainties in their values thus, quantification of uncertainty of reservoir prediction models becomes a necessity. In this paper, an application of uncertainty quantification has being carried out using the Buckley Leverett model for twophase immiscible flow in porous media considering the randomness of oil viscosity in the model. Based on the Monte Carlo method for uncertainty quantification, we use a finite volume scheme to obtain the deterministic solution. Our solutions revealed that water saturation during water flooding process for secondary oil recovery can be affected by randomness of oil viscosity.

KEYWORDS: Buckley Leverett, Uncertainty quantification, Monte Carlo, Finite volume,

1 Introduction

Scientific computer simulations help to design complex engineering systems and saves time and money when compared to building prototype models for the study of such systems. Some areas of application include automobile engines, aerospace engines and reservoir systems. This advanced technology has played a major role in the oil and gas sector. The petroleum sector for example has enjoyed the application of computer simulation in the discovery and recovery of petroleum resources in subsurface reservoirs. But, the possibility of predicting the performance of complex systems through computer simulations with 100% certainty is not obtainable (Li and Tchelepi, 2015) [20] thus, there is the need to carry out uncertainty quantification of prediction models to improve on the prediction confidence of such models.

Uncertainty quantification is the science of quantitative characterization and reduction of uncertainties in mathematical models and real-world systems. Uncertainty quantification characterizes the proximity between predictions and observations of real systems through methods that connects computational models and their true physical systems simulated by the models (sullivan, 2015) [32]. This area of study deals mostly with detailed complex models of physical systems with limited experimental or observed data, (soize,2017) [30]. A well defined problem in uncertainty quantification must have a system's mathematical model with special interest in a given quantity subject to uncertainties from the model parameters, boundary values, initial values or both boundary values and initial values which are handled probabilistically in most cases. Uncertainty quantification has being applied in many real world systems with random behaviours, some of these include space weather predictions with atmospheric drag as the main source of uncertainty (Licata, 2022) [21]. Atmospheric weather and climate prediction where initial condition and model formulations are seen as main sources of uncertainties, (Palmer, 2000) [26]. Climate modelling and projection, (Qian,2016) [27]. Building performance simulation of structural engineering systems with uncertainties in input parameters of model (Chong, 2015) [4], Building energy assessment with both model form and parameter uncertainties (Tian, 2018) [35], aircraft structures, (Diaz 2010) [7]. Machine learning for space weather prediction with uncertainty in dataset and input parameters (Siddique, 2022) [28], stock and bond market with uncertainties in stock volatility and abnormal stock turnover, (Connolly, 2005) [5]. Cardiovascular application models in Medical sciences, (Eck, 2016) [8] as well as Reservoir models with uncertainties in permeability and other fluid parameters (Subbey, 2006) [31]. Isobo et al (2023)[15] investigated the effect of random thermal conductivity on the temperature profile of a copper material with Monte Carlo method with different heat transfer time periods.

1.1 The Buckley Leverett equation

The Buckley Leverett equation(Buckley and Leverett, 1954) [1] describes the flow of two immiscible and incompressible fluids in a horizontal and homogeneous porous medium. The equation stated as

$$\frac{\partial \mathbf{S}}{\partial t} + \nabla \cdot (\frac{q}{A\phi} f(s)) = 0 \tag{1}$$

is a nonlinear first order partial differential equation and solutions are mostly obtained through numerical methods though analytical solutions do exist with strong simplifying assumptions (Deb et al, 2018) [6]. The buckley Leverett equation represents the simplest form of material balance in reservoir engineering using water flooding for secondary oil recovery. Flow in porous media occur under uncertain conditions and prediction results exhibit uncertainties due to the fact that data for both reservoir and fluid properties are difficult to obtain at reservoir conditions (Deb et al, 2018) [6]. Thus, there is the presence of parameter uncertainty from predictions made out of the use of such data for model predictions.

Many researchers have studied the Buckley Leverett equation since its introduction in 1954 by Buckley and Leverett. Langtangen et al (1992)[18] studied the simultaneous one-dimensional flow of water and oil in a heterogeneous medium modeled by the Buckley-Leverett equation. A variety of heterogeneity profiles were studied with Buckley-Leverett method by Chang and Yortsos (1992)[2] where it was shown that Capillary heterogeneity significantly affects the saturation distributions, which closely follow the heterogeneity variation. Frid (1995) [10] solved the initial boundary-value problem for the regularized Buckley-Leverett system, which described the flow of two immiscible incompressible fluids through a porous medium Siddiqui et al (1996)[29] investigated the validity of the three-phase extension of the Buckley-Leverett theory experimentally using three immiscible liquids. Method of characteristics (MOC) solutions to the threephase BuckleyLeverett problem was presented with and without gravity by Guzman and Fayers (1997)[12]. Terez and Firoozabadi (1999) [34] examined water injection in water-wet fractured porous media and its modeling using the Buckley–Leverett theory. A stochastic analysis of immiscible two-phase flow with Buckley–Leverett displacement was presented in heterogeneous reservoirs by Dongxiao and Tchelepi (1999)[41]. Kaasschieter (1999)[17] derived an entropy inequality from a regularization procedure, where the physical capillary pressure term is added to the Buckley-Leverett equation. An extension of the Buckley–Leverett (BL) equation describing two-phase flow in porous media was discussed by Van et al (2007)[37]. Mustafiz et al (2008)[23] used a semi-analytical technique and the Adomian decomposition method (ADM) to unravel the true nature of the one-dimensional, two-phase flow. Sumnu-Dindoruk and Dindoruk (2008)[33] solved the resulting nonisothermal two-phase convective flow equation in porous media analytically, including a tracer component. Chemetov and Neves (2013) [3] proposed a new approach to the mathematical modeling of the Buckley–Leverett system, for describing twophase flows in porous media. Wang and Kao (2013)[39] extended the second and third order classical central schemes for the hyperbolic conservation laws to solve the modified BuckleyLeverett (MBL) equation which is of pseudo-parabolic type. Wang and Kao (2014)[40] numerically verified that the convergence rate is consistent with the theoretical derivation by BuckleyLeverett (BL) equation. Larsen et al(1990) [19] examined the relationship between backflow of water from the invaded zone and changed in skin owing to reduced water saturation and the associated change in mobility for homogeneous reservoirs with Buckley-Leverett methods.

For more rigorous investigations, stochastic variables were introduced into the Buckley Leverett equation leading to the stochastic Buckley Leverett equation. Cedric and Hamdi (2022) [11] used the parameterized physics informed neural network approach to quantify uncertainties in reservoir engineering problems with random porosity and permeability. A machine learning based hybrid multifidelity multilevel Monte Carlo method was applied by Nagoor and Ahmed (2019)[16] to quantify uncertainty in a stochastic BLE with random permeability. Valdez et al (2020) [36] used polynomial chaos method to study the effect of input uncertainties for different random permeability models. Fagerlund et al (2015)[9] investigated failure probabilities with random permeability using Monte Carlo method. A bayesian framework for UQ was introduced by James et al (2003) to handle uncertainties from random viscosity ratio and permeability field. A random time dependent flux function was proposed by Wang et al (2013)[38] which the approached with their proposed cumulative distribution function method. Fayadhoi et al (2017)[14] considered random porosity and permeability and proposed the frozen streamline distribution method for uncertainty quantification

To the best of our knowledge, no one has considered investigating uncertainty introduced into the Buckley Leverett equation due to randomness of a particular type of oil viscosity, most especially, the bonny light crude produced from the southern part of Nigeria.

In this study, we apply the Monte Carlo simulation method for uncertainty quantification to a subsurface reservoir model, the Buckley Leverett equation, which describes the flow of two immiscible fluids in a horizontal channel. Our aim is to investigate the effect of the randomness of viscosity of oil on the saturation profile of water during secondary recovery process of crude oil in a reservoir system. Our objectives include the formulation of a stochastic Buckley Leverett equation with water saturation as quantity of interest, characterization of the random parameter to obtain its probability distribution for the propagation of uncertainty into the system and quantification of the uncertainty of the results through Monte Carlo method (Liu et al, 2023) [22]. The rest of the paper has problem formulation followed by the method of solution, results presentation and discussion and ends with conclusion.

2 Problem formulation

Given the model in equation (1), for horizontal flow with negligible capillary pressure (Wang et al, 2013) fractional flow of water is given as

$$f_w(S) = \frac{1}{1 + \frac{K_{ro}\mu_w}{K_{rw}\mu_o}} \tag{2}$$

where,$f_w(S)$ is fractional flow of water phase which is a function of saturation, K_{ro} is the relative permeability of oil, K_{rw} is relative permeability of water, mu_w is viscosity of water and mu_o is viscosity of oil.

The basic assumptions (Wang et al, 2013)[38] are

1. the two fluids are immiscible and incompressible
2. Fluid flow is in one dimension (horizontal)
3. Negligible capillarity and gravitational effect
4. Homogeneous rock formation
5. Constant viscosity of water
6. uniformly distributed random viscosity of oil
7. Water is injected from left at $x = 0$ at constant rate

Based on assumption number 6, equation (1) becomes a stochastic equation with final form in one dimension given as

$$\frac{\partial S_w}{\partial t} + \frac{q_T}{A\phi}\frac{\partial f_w(S)}{\partial x} = 0 \tag{3}$$

with initial and boundary conditions given as
$S_w(x, 0) = 0$
$S_w(0, t) = 1$
$\mu_o \sim U[1, 2.94]$
where q_T is the total flow rate, A is the cross sectional area of core sample and ϕ is the porosity of the medium.

We applied the Monte Carlo method for uncertainty quantification Fagerlund et al (2015)[9]. We used the finite volume method to obtain a numerical scheme for the deterministic solution for each point.
First, we adopt the Power law representation of the relative permeabilities, see Nwaigwe and Sudi (2021) [25] and Holden and Risebro (1991) [13] for more details.

$$k_{rw}(S) = S_w^2 \tag{4}$$

giving us the flux function as

$$f_w(S) = \frac{S^2}{S^2 + (\frac{\mu_w}{\mu_o})(1-S)^2} \tag{5}$$

The finite volume numerical scheme.
Given the differential form of the model

$$\frac{\partial S_w}{\partial t} + \frac{v}{\phi}\frac{\partial f_w(S)}{\partial x} = 0 \tag{6}$$

We convert to the integral form

$$\frac{\partial}{\partial t}\int_{x_{i-(\frac{1}{2})}}^{x_{i+(\frac{1}{2})}} S dx + \frac{v}{\phi}\int_{x_{i-(\frac{1}{2})}}^{x_{i+(\frac{1}{2})}} \frac{\partial f_w(S)}{\partial x} dx = 0 \tag{7}$$

We discretize the time and space variables as

$$\frac{\Delta x}{\Delta t}(S_i^{n+1} - S_i^n) + \frac{v}{\phi}[f^n(S_{i+(\frac{1}{2})} - f^n(S_{i-(\frac{1}{2})}] = 0 \tag{8}$$

which gives us

$$S_i^{n+1} = S_i^n - \frac{v}{\phi}\frac{\Delta t}{\Delta x}[f^n(S_i^n, S_{i+1}^n) - f^n(S_{i-1}^n, S_i^n)] \tag{9}$$

Which was implemented with the FV solver (Nwaigwe, 2016) [24] in python platform. At every point, viscosity values are generated from the uniform distribution to account for the randomness of viscosity of Bonny light crude oil during to computation of the input parameters. This process propagates uncertainty into the model. Thus, at every point, we get a collection of solutions and by the Monte Carlo method, we take the ensemble average.

$$\bar{S} = \frac{1}{N}\sum_{j=1}^{N} S_j(x, t) \tag{10}$$

where N is the total number of random values generated for a solution at each point. Our final solution is the distribution of the mean saturation of water over the spatial domain $x \in (0, 1)$ for particular time steps.
the variance is given as

$$\sigma_S^2 = E[(S - \bar{S})^2] = \frac{1}{N}\sum_{j=1}^{N}(S_j - \bar{S})^2 \tag{11}$$

The input parameter dataset is as given in the table below

parameter	value
porosity	0.3
velocity($\frac{q_w}{A}$)	0.5
μ_w	1
μ_o	U[1, 2.94]

Table 1: reservoir data from Nembe well 01

3 Results

Given the dataset and the finite volume scheme, we carry out our numerical experiment for 150 random variables at every point on the spatial domain. Our results for the uncertainty quantification problem are the mean and variance of the stochastic solutions. We compute means and corresponding variances and present our results graphically below.

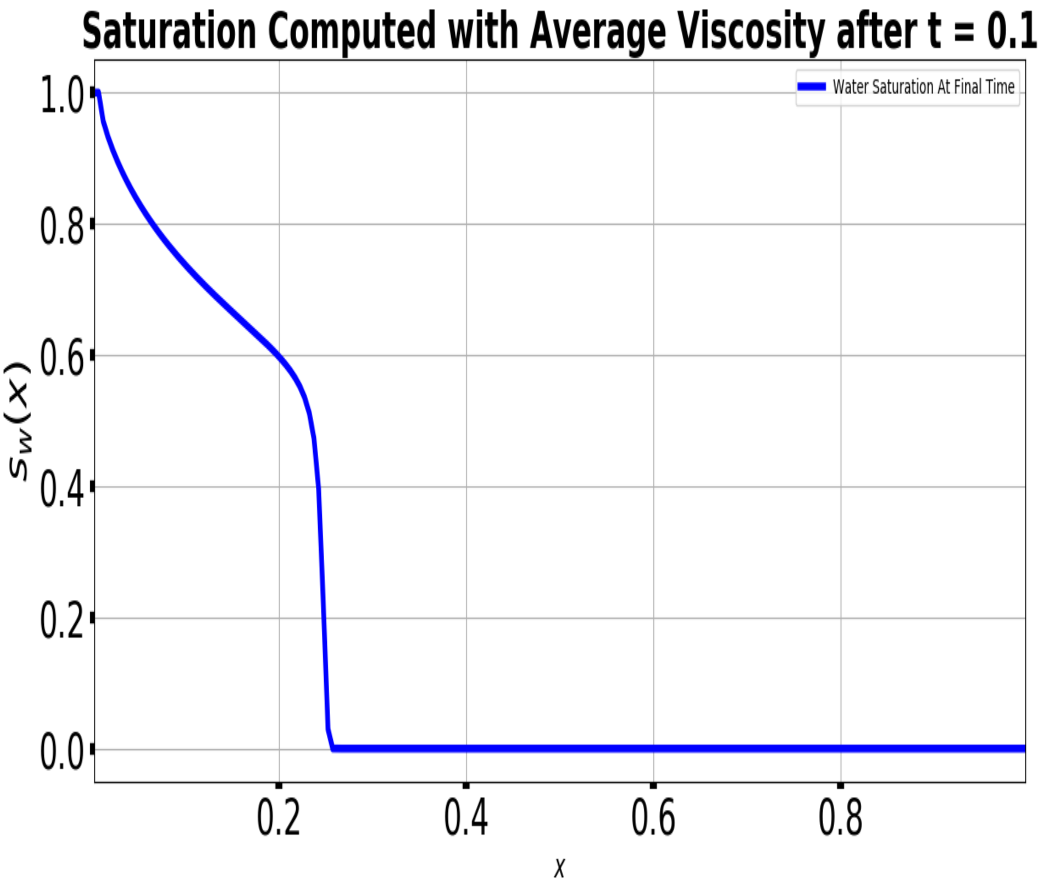

Figure 1: Deterministic solution's saturation distribution after 0.1 time units

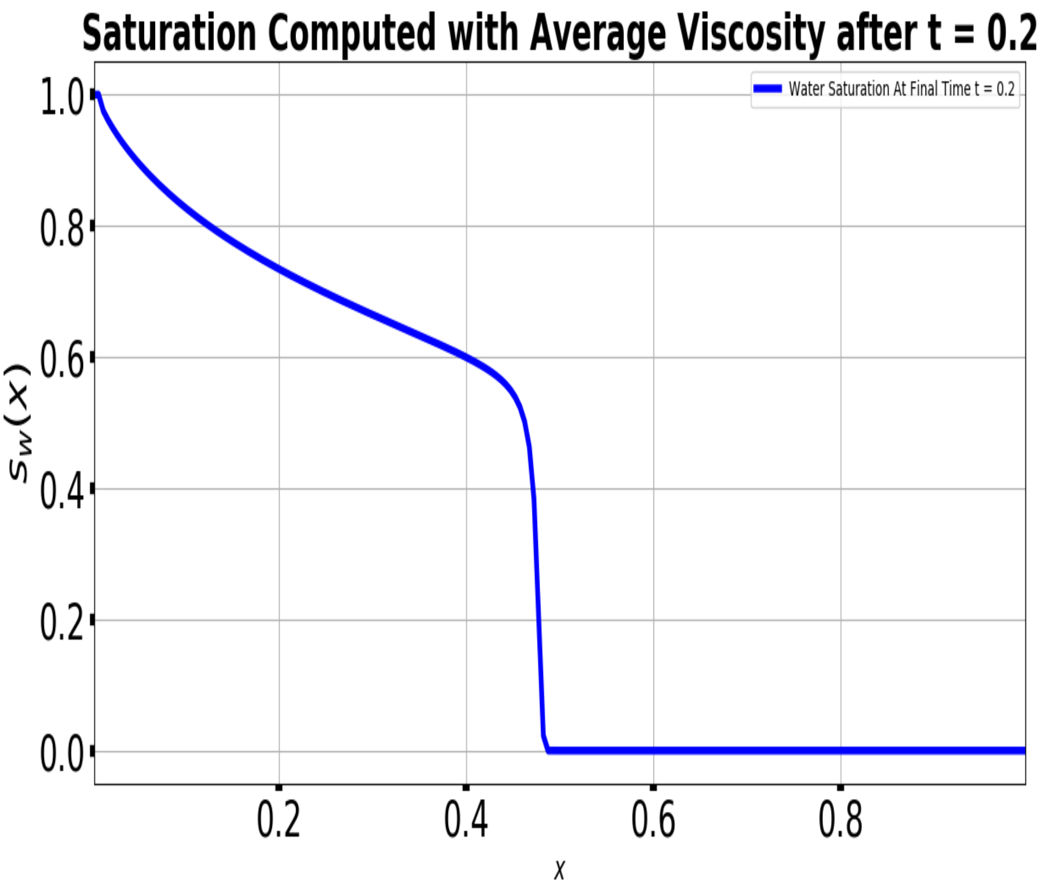

Figure 2: Deterministic solution's saturation distribution after 0.2 time units

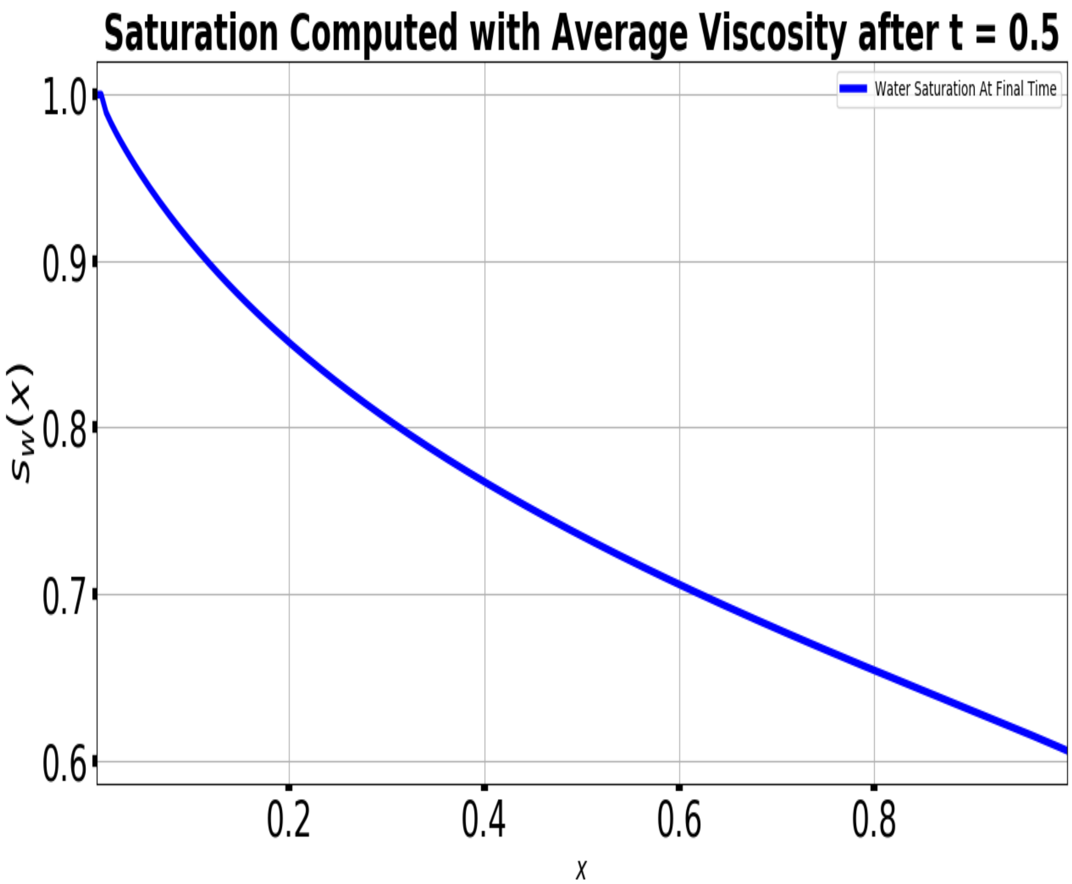

Figure 3: Deterministic solution's saturation distribution after 0.5 time units

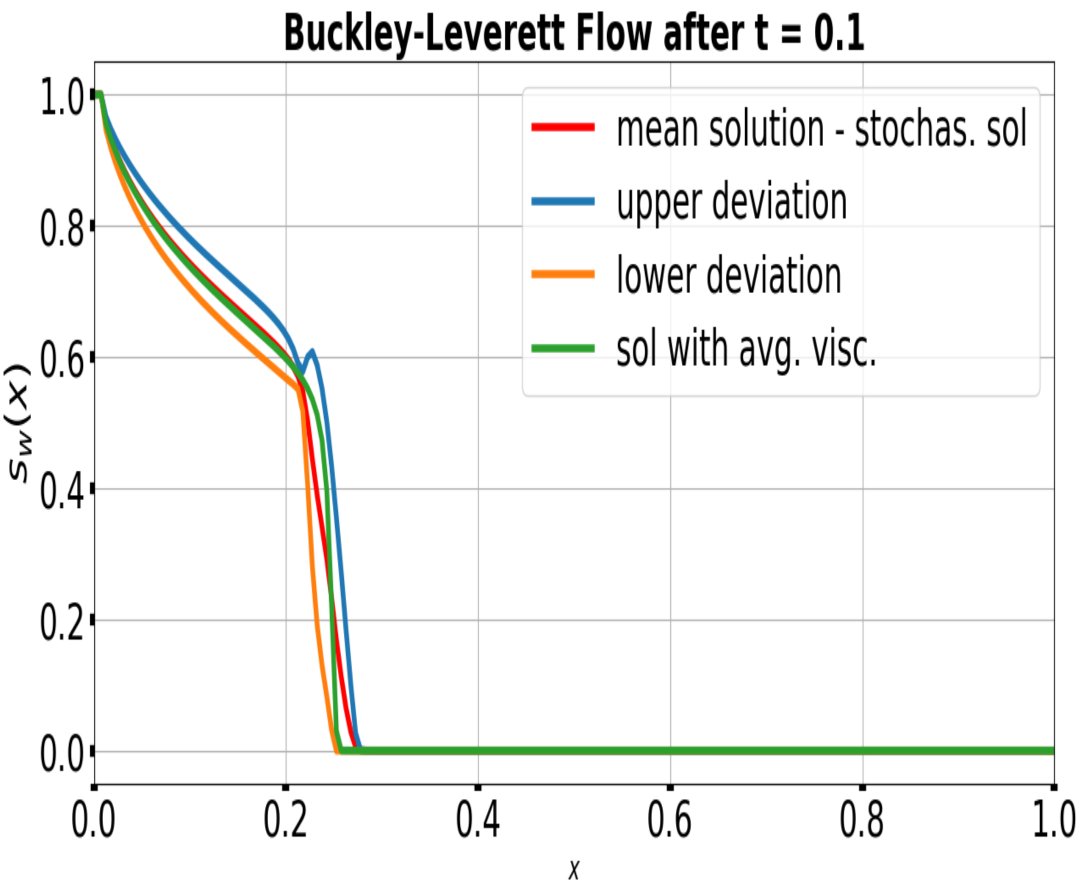

Figure 4: Stochastic solution's saturation distribution after 0.1 time units

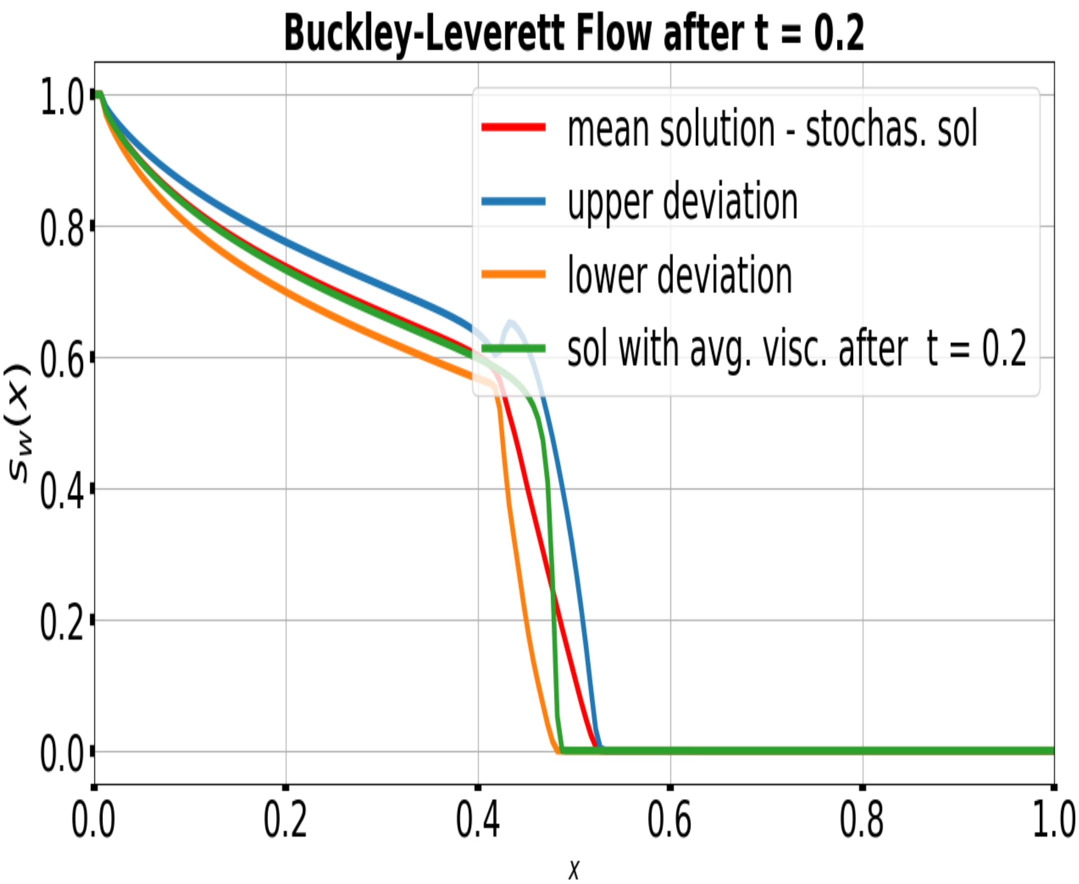

Figure 5: Stochastic solution's saturation distribution after 0.2 time units

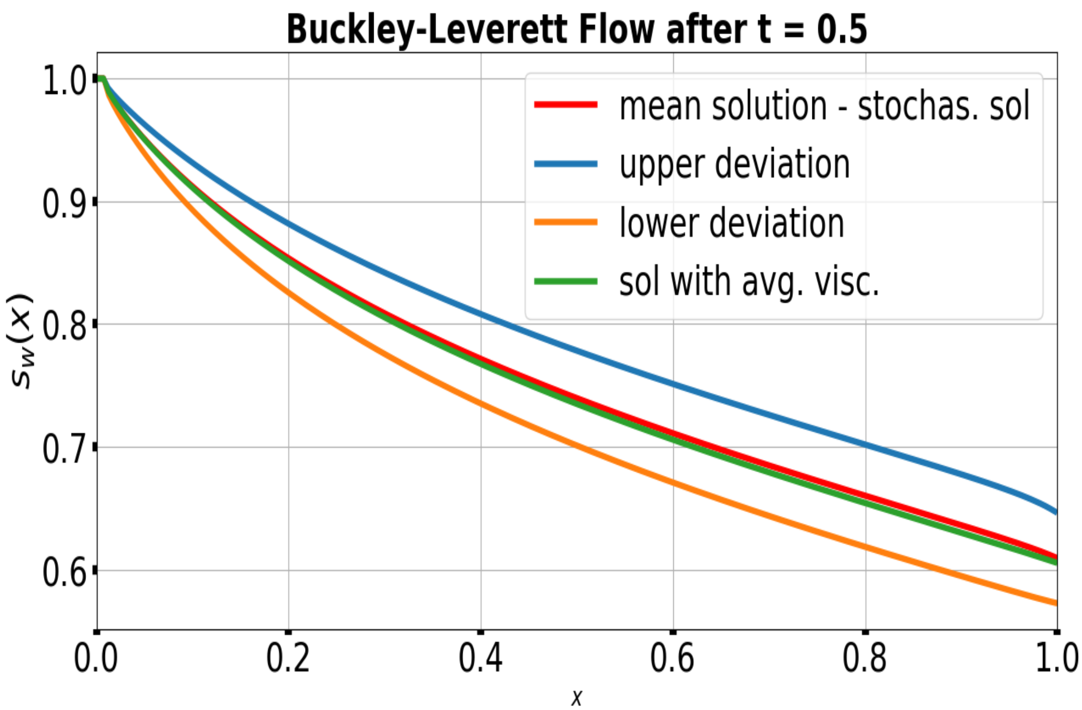

Figure 6: Stochastic solution's saturation distribution after 0.5 time units

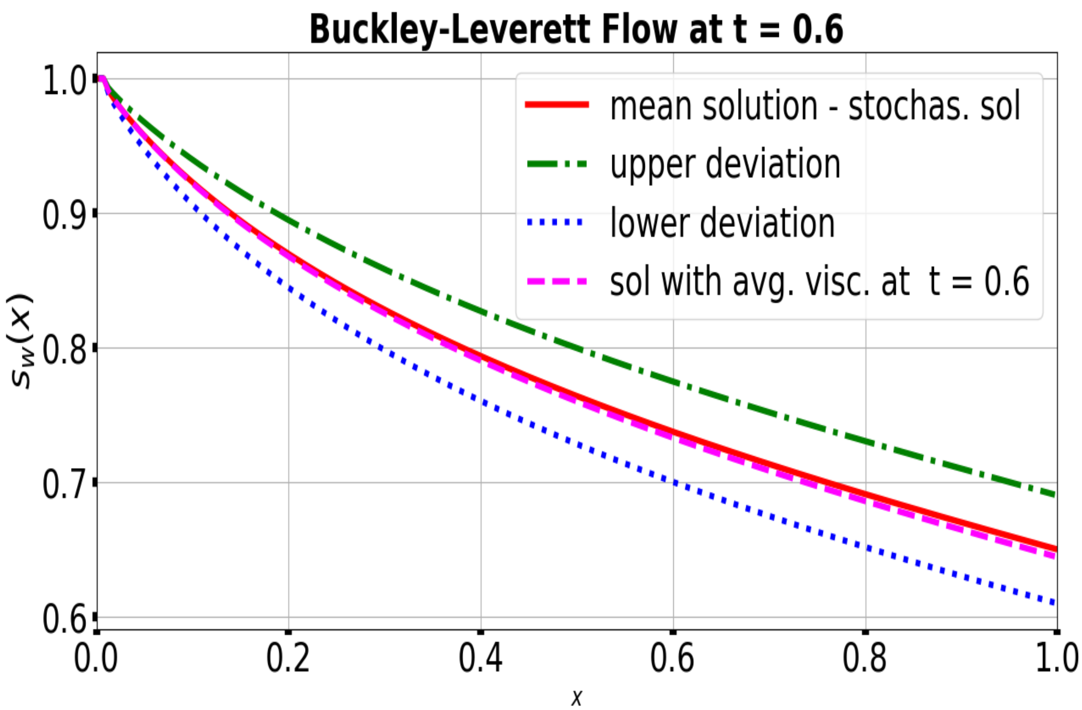

Figure 7: Stochastic solution's saturation distribution after 0.6 time units

At the left boundary, saturation of water is 100% that is at ($x = 0$) while saturation of oil in the reservoir is also 100%. As water is injected into the reservoir at a given constant velocity, it displaces the oil gradually which reduces the saturation of oil in the reservoir. As time increases, we experience an increase in water saturation in the reservoir as the saturation of oil continues to decrease. From our results, figure 1 to figure 3 show the solution of the deterministic model as time increases. we that at after 0.1 seconds water has flooded the reservoir up to a distance of 0.26 unit distance with a saturation of about 0.6%. We note here that the deterministic solution gives an exact value of the saturation at any given distance and time. Our solution also follows the physics of the water flooding process which explains the correctness of our method for the Buckley Leverett equation.

Figure 4 to figure 6 show the results for the stochastic Buckley Leverett equation for the given data presented in table 1. We used random numbers generated from the uniform distribution to capture the uncertainties from the random viscose parameter giving rise to input uncertainty and a further random response. We take the mean and variance as statistics of the collection of random response following the Monte Carlo method. Our plots show the mean distribution at various times and the deviations from the mean to obtain the region of possible solutions due to uncertainty from the viscosity of oil. From our results, we can see after 0.1 seconds a mean water saturation of 0.6% with a deviation of ±0.05% thus, we can expect a saturation of 0.58% as well as saturation of 0.64% after 0.1 seconds. As time increases, we also see an increase in the range of water saturation between the upper and lower deviations from the mean distribution.

The exact solution from the deterministic model and the mean solution from the stochastic model when compared have only little difference over all the time considered. This may be be as a result of the randomness of the viscose parameter used in the stochastic model. While the deterministic result gives an exact value for the problem, the stochastic result gives an average value and a range of expected values around the average. This is a major advantage of considering a stochastic model because in real life we can never be too certain with events. The stochastic model gives us probabilities of outcomes so that we can have a range of possibilities of results in our experiments, we can be properly guided by the statistics of our random responses.

4 Conclusion

In this research, uncertainty quantification of the Buckley Leverett model has been carried out using data from properties of the bonny light crude oil and its reservoir, specifically Nembe well 01 in the southern part of Nigeria. We have used the viscosity of the bonny light crude oil as the source of uncertainty in the Buckley Leverett equation to observe its effect on water saturation during secondary oil recovery process.

We were able to use the finite volume method which was implemented with the FV solver for a deterministic solution through which we propagated uncertainty into the model and used the Monte Carlo method to quantify the uncertainties. Our solutions, mean and variance of water saturation, were presented graphically and showed the effect of randomness of the oil viscosity on the saturation profile of water in the subsurface reservoir during water flooding process. This result can be used by petroleum engineers to observe fluctuations in production rates due to randomness of oil viscosity, since viscosity aids the speed of fluid flow.

We have been able to apply the theories of uncertainty quantification to real time data in this research. The research is the first to use the FV solver for the purpose of quantifying uncertainty with Monte Carlo method. Further studies can be done by increasing the number of random parameters in the governing equations as well as the increase in the dimension of the flux function for more rigorous mathematics. Data from other production wells can also be used for the purpose of results comparison.

References

[1] Se E Buckley and MCi Leverett. Mechanism of fluid displacement in sands. *Transactions of the AIME*, 146(01):107–116, 1942.

[2] Jincal Chang and Yanis C Yortsos. Effect of capillary heterogeneity on buckley-leverett displacement. *SPE reservoir engineering*, 7(02):285–293, 1992.

[3] Nikolai Chemetov and Wladimir Neves. The generalized buckley–leverett system: solvability. *Archive for Rational Mechanics and Analysis*, 208:1–24, 2013.

[4] Adrian Chong, Weili Xu, and Khee Poh Lam. Uncertainty analysis in building energy simulation-a practical approach. In *14th International IBPSA Conference, edited by J. Mathur and V Garg*, pages 2796–2803, 2015.

[5] Robert Connolly, Chris Stivers, and Licheng Sun. Stock market uncertainty and the stock-bond return relation. *Journal of Financial and Quantitative Analysis*, 40(1):161–194, 2005.

[6] Pulok Kanti Deb. *A finite volume approach for the numerical analysis and solution of the Buckley-Leverett equation including capillary pressure.* PhD thesis, Memorial University of Newfoundland, 2018.

[7] J Díaz, S Hernández, LE Romera, and A Baldomir. Uncertainty quantification and reliability analysis methods applied to aircraft structures. In *ICAS 2010. 27th Congress of the International Council of the Aeronautical Sciences*, 2010.

[8] Vinzenz Gregor Eck, Wouter Paulus Donders, Jacob Sturdy, Jonathan Feinberg, Tammo Delhaas, Leif Rune Hellevik, and Wouter Huberts. A guide to uncertainty quantification and sensitivity analysis for cardiovascular applications. *International journal for numerical methods in biomedical engineering*, 32(8):e02755, 2016.

[9] Fritjof Fagerlund, Fredrik Hellman, Axel Målqvist, and Auli Niemi. Multilevel monte carlo methods for computing failure probability of porous media flow systems. *Advances in Water Resources*, 94:498–509, 2016.

[10] Hermano Frid. Solution to the initial boundary-value problem for the regularized buckley-leverett system. *Acta Applicandae Mathematica*, 38:239–265, 1995.

[11] Cedric Fraces Gasmi and Hamdi Tchelepi. Uncertainty quantification for transport in porous media using parameterized physics informed neural networks. *arXiv preprint arXiv:2205.12730*, 2022.

[12] Rafael E Guzman and F John Fayers. Solutions to the three-phase buckley-leverett problem. *SPE Journal*, 2(03):301–311, 1997.

[13] Helge Holden and Nils Henrik Risebro. Stochastic properties of the scalar buckley-leverett equation. *SIAM Journal on Applied Mathematics*, 51(5):1472–1488, 1991.

[14] Fayadhoi Ibrahima, Hamdi A Tchelepi, and Daniel W Meyer. An efficient distribution method for nonlinear two-phase flow in highly heterogeneous multidimensional stochastic porous media. *Computational Geosciences*, 22:389–412, 2018.

[15] Nwaigwe C. Isobo S. and Weli A. Monte carlo analysis of uncertainty of heat distribution in a copper matetrial. *Journal of research in applied mathematics*, 9(03):147–21, 2023.

[16] Nagoor Kani Jabarullah Khan and Ahmed H Elsheikh. A machine learning based hybrid multi-fidelity multi-level monte carlo method for uncertainty quantification. *Frontiers in Environmental Science*, 7:105, 2019.

[17] Enrique Francisco Kaasschieter. Solving the buckley–leverett equation with gravity in a heterogeneous porous medium. *Computational Geosciences*, 3:23–48, 1999.

[18] Hans Petter Langtangen, Aslak Tveito, and Ragnar Winther. Instability of buckley-leverett flow in a heterogeneous medium. *Transport in Porous Media*, 9(3):165–185, 1992.

[19] Leif Larsen, Knut Kviljo, and Terje Litlehamar. Estimating skin decline and relative permeabilities from cleanup effects in well-test data with buckley-leverett methods. *SPE formation evaluation*, 5(04):360–368, 1990.

[20] Boxiao Li and Hamdi A Tchelepi. Nonlinear analysis of multiphase transport in porous media in the presence of viscous, buoyancy, and capillary forces. *Journal of Computational Physics*, 297:104–131, 2015.

[21] Richard J Licata and Piyush M Mehta. Uncertainty quantification techniques for space weather modeling: Thermospheric density application. *arXiv preprint arXiv:2201.02067*, 2022.

[22] Yingxia Liu, Gerard BM Heuvelink, Zhanguo Bai, and Ping He. Uncertainty quantification of nitrogen use efficiency prediction in china using monte carlo simulation and quantile regression forests. *Computers and Electronics in Agriculture*, 204:107533, 2023.

[23] S Mustafiz, SH Mousavizadegan, and MR Islam. Adomian decomposition of buckley-leverett equation with capillary effects. *Petroleum science and technology*, 26(15):1796–1810, 2008.

[24] Chinedu Nwaigwe. *Coupling methods for 2D/1D shallow water flow models for flood simulations.* PhD thesis, University of Warwick, 2016.

[25] Chinedu Nwaigwe and Sudi Mungkasi. Comparison of different numerical schemes for 1d conservation laws. *Journal of Interdisciplinary Mathematics*, 24(3):537–552, 2021.

[26] Tim N Palmer. Predicting uncertainty in forecasts of weather and climate. *Reports on progress in Physics*, 63(2):71, 2000.

[27] Yun Qian, Charles Jackson, Filippo Giorgi, Ben Booth, Qingyun Duan, Chris Forest, Dave Higdon, Z Jason Hou, and Gabriel Huerta. Uncertainty quantification in climate modeling and projection. *Bulletin of the American Meteorological Society*, 97(5):821–824, 2016.

[28] Talha Siddique, Md Shaad Mahmud, Amy M Keesee, Chigomezyo M Ngwira, and Hyunju Connor. A survey of uncertainty quantification in machine learning for space weather prediction. *Geosciences*, 12(1):27, 2022.

[29] S Siddiqui, PJ Hicks, and AS Grader. Verification of buckley-leverett three-phase theory using computerized tomography. *Journal of petroleum Science and Engineering*, 15(1):1–21, 1996.

[30] Christian Soize. *Uncertainty quantification: An accelerated course with advanced application in computational engineering*. Springer International publishing, Switzerland, 2017.

[31] Samuel Subbey, H Monfared, M Christie, and Malcolm Sambridge. Quantifying uncertainty in flow functions derived from scal data. *Transport in porous media*, 65(2):265–286, 2006.

[32] T. J. Sullivan. *An introduction to uncertainty quantification*. national press papers, $bms_uq_notes_00-05$, 2015.

[33] Deniz Sumnu-Dindoruk and Birol Dindoruk. Analytical solution of non-isothermal buckley-leverett flow including tracers. *SPE Reservoir Evaluation & Engineering*, 11(03):555–564, 2008.

[34] Ivan E Terez and Abbas Firoozabadi. Water injection in water-wet fractured porous media: experiments and a new model with modified buckley–leverett theory. *SPE Journal*, 4(02):134–141, 1999.

[35] Wei Tian, Yeonsook Heo, Pieter De Wilde, Zhanyong Li, Da Yan, Cheol Soo Park, Xiaohang Feng, and Godfried Augenbroe. A review of uncertainty analysis in building energy assessment. *Renewable and Sustainable Energy Reviews*, 93:285–301, 2018.

[36] Andrés Ricardo Valdez, Bernardo Martins Rocha, Grigori Chapiro, and Rodrigo Weber dos Santos. Uncertainty quantification and sensitivity analysis for relative permeability models of two-phase flow in porous media. *Journal of Petroleum Science and Engineering*, 192:107297, 2020.

[37] CJ Van Duijn, Lambertus A Peletier, and Iuliu Sorin Pop. A new class of entropy solutions of the buckley–leverett equation. *SIAM Journal on Mathematical Analysis*, 39(2):507–536, 2007.

[38] Peng Wang, Daniel M Tartakovsky, KD Jarman Jr, and Alexandre M Tartakovsky. Cdf solutions of buckley–leverett equation with uncertain parameters. *Multiscale Modeling & Simulation*, 11(1):118–133, 2013.

[39] Ying Wang and Chiu-Yen Kao. Central schemes for the modified buckley–leverett equation. *Journal of Computational Science*, 4(1-2):12–23, 2013.

[40] Ying Wang and Chiu-Yen Kao. Bounded domain problem for the modified buckley–leverett equation. *Journal of Dynamics and Differential Equations*, 26:607–629, 2014.

[41] Dongxiao Zhang and Hamdi Tchelepi. Stochastic analysis of immiscible two-phase flow in heterogeneous media. *SPE journal*, 4(04):380–388, 1999.