Original Research Article

MATHEMATICAL MODELING OF BIRD HARVESTING IN INTENSIVE POULTRY SYSTEM

ABSTRACT:
This work, presents a formulation of mathematical model of bird harvesting in an intensive poultry system, under the assumption that under a favourable environmental atmosphere and good management system, the birds have logistic growth. The model is analysed using methods from dynamical system theory and theory of calculus. It was established that the system has two steady state, the two equilibrium state are both locally asymptotically stable. The first one is stable if there is a bound on the harvest rate of the birds, which is proportional to the growth rate of the birds. The second equilibrium state is locally asymptotically stable (LAS) if $K < \frac{r(\mathrm{C} + \gamma)}{\rho}$ that is if the carrying capacity is less than the ratio of the sum of $Per\ unit\ cost\ of\ birds$ and $Per\ unit\ tax\ on\ the\ bird$ to that of $Per\ unit\ price\ of\ the\ birds$. Further analysis indicates that the limiting population of bird, that is the maximum population of birds that the available resources in the system can sustain and also ensures harvesting profitability is given as $\frac{K\left(r - \frac{K\rho\,(r-\mu) - r(\mathrm{C}+\gamma)}{K\rho} - \mu\right)}{r}$

This occurs at the point

$$(\bar{t}, \bar{B}) = \left(\left(\frac{1}{\left(r - \beta\left(\frac{K\rho\,(r-\mu) - r(\mathrm{C}+\gamma)}{K\rho}\right) - \mu\right)} \mathrm{Ln}\left(\frac{\frac{K\left(r - \frac{K\rho\,(r-\mu) - r(\mathrm{C}+\gamma)}{K\rho} - \mu\right)}{r} - \mathrm{B}_0}{\mathrm{B}_0}\right), \left(\frac{K\left(r - \frac{K\rho\,(r-\mu) - r(\mathrm{C}+\gamma)}{K\rho} - \mu\right)}{2r}\right)\right)\right)$$

Keywords: Profitability, Harvesting, Stability, Poultry, Birds

1Introduction

Poultry refers to birds that are kept or raised for either for private or commercial purposes and generally includes birds such as Chicken, Turkey, Duck, Goose, Quail, Pheasant, Pigeon, Guinea Fowl, Pea fowl, Ostrich, e.t.c [1]

Poultry farming is one of the smallest livestock investment that an individual or household can easily undertake. Income generation is the primary goal of this poultry keeping. Eggs can provide a regular income albeit small, while the sales of birds provide a more flexible source of income as required. [2]

A study of income generation in poultry farming in East Kakimantan, Indonesea showed that family poultry accounted for about 53 percent of total income and was used for food, school fees and unexpected expenses such as medicine [3]. Poultry are kept under a wide range of conditions, which can be classified into one of four broad production system such as free-range extensive system, Backyard extensive system, Semi-intensive system and Intensive system. The intensive systems are used by medium or large-scale commercial enterprises and also at the household level. In this system, birds are fully confined either in houses or cages.

[4] presented a mathematical modelling of the mortality rate of broilers chicken using a standard S-I-R model. Analysis of the modelshowed that poultry industry is one of the most successful agricultural

venture in any nation's economy. They established that for successful rearing of broiler chicken, good welfare conditions must be maintained to avoid extreme case of the effective mortality rate being large. Also good management programmes must be developed in broiler production to combat early mortality. It has been established by many researchers [10, 11 12,13,19]that effective control ,reduction and prevention of infection in poultry can enhance better production and profitability in poultry.

[7] stated that harvesting models based on ordinary differential equations are commonly used in the fishery industry and wildlife management to model the evolution of a population depleted by harvest mortality. He presented a project consisting of a series of scenarios based on fishery harvesting models to teach the application of theoretical concepts learned in a differential equations course to scenarios encountered in real fisheries. In this projects a thorough understanding of simplifying assumptions inherent in various models was required, as well as a qualitative analysis of phase portraits, bifurcations, and stability of steady states. Parameters were estimated and equations were solved both analytically and numerically. Students learn to respond to a professional request from a fishery in the form of a scientific report, which requires organizing and communicating assumptions, models, solution methods, results, and a final recommendation with clarity and professionalism.

[5], presented a mathematical model of fish harvesting in a common access fishery. The model was formulated and analysed using methods from dynamical system theory. The analysis of the model showed that the system has two steady states. The first one is unstable while the second one is globally asymptotically stable if the carrying capacity of the water has a lower bound.

[9], developed mathematical model of bacteria-nutrient harvesting in a cultured environment. The model which assumes that the rate of harvesting of these bacteria is constant results in a system of first order differential equations. Analyzing the model, it was discovered that the product of the maximum nutrient uptake per cell and the number of cells produced per unit of nutrient uptake is constant ($\boldsymbol{VY} = \boldsymbol{ln2} + \boldsymbol{h}$). It was also assumed that the rate of harvesting of these bacteria varies and a corresponding model was developed. Analyzing this model using methods from dynamical systems theory, it was seen that the system has two steady states. The first steady state is unstable while the second is globally asymptotically stable if the carrying capacity of the environment has a lower bound, which is a ratio of the harvesting coefficient of the bacteria, cost per unit effort per unit price of the bacteria.

In this paper, we present a mathematical model of bird harvesting in an intensive poultry system. We assume that under a favourable environmental atmosphere and good management system the birds have logistic growth. The model is formulated and analysed using methods from dynamical system theory and theory of calculus.

2 FORMULATION OF THE MATHEMATICAL MODEL

2.1 ASSUMPTIONS

1. In the absence of epidemic outbreak, the population of birds has a logistic growth.

2. The population of birds is reduced by the number of birds harvested at time t provided the net income is positive

3. The per carpita rate of birds reproduction decreases with increase in the population of birds.

4. in the presence of favourable environmental conditions and good management system, death due to natural occurrences does not cause any damage.

5, There is no migration to or out of the poultry except for harvesting purpose.

6. Work done increases with positive revenue and decrease otherwise.

2.2 Symbols/Parameters

$B(t) \;= Population\ of\ birds\ at\ time\ t > 0$

$W(t) \;= Workdone\ in\ the\ poultry\ \ at\ time\ t > 0$

$r \;= Rate\ of\ reproduction$

$K \;= Carrying\ capacity\ of\ the\ poultry\ house$

$\beta \;= harvest\ rate\ of\ birds\ which\ depends\ on\ the\ revenue\ and\ maturity$

$\mu \;= Na\ tural\ death\ rate\ of\ birds$

$\rho = Per\ unit\ price\ of\ the\ birds$

$C = \ Per\ unit\ cost\ of\ birds$

$\gamma \;= Per\ unit\ tax\ on\ the\ birds$

2.3 The Model Equation

$$\frac{dB}{dt} \;=\; rB\left(1-\frac{B}{K}\right)-\beta WB-\mu B \ldots\ldots\ldots\ldots\ldots\ldots.. (2.1)$$

$$\frac{dW}{dt} \;=\; \alpha W(\rho B-c-\gamma) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2.2)$$

Where $R \;=\; W(\rho B-c-\ \gamma) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2.3)$

3 Analysis of the Model

Using methods from [14, 15,16,20,21,22] the system (2.1) – (2.3) is analysed as follows:

$$\frac{dB}{dt} \;=\; rB\left(1-\frac{B}{K}\right)-\beta WB-\mu B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \;(3.1)$$

$$\frac{dW}{dt} \;=\; \alpha W(\rho B-C-\gamma) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3.2)$$

At equilibrium

$$rB\left(1-\frac{B}{K}\right)-\beta WB-\mu B \;=0 \qquad \ldots\ldots\ldots\ldots.. (3.3)$$

$$\alpha W\rho B-C\alpha W-\gamma\alpha W \;=\; 0 \qquad \ldots\ldots\ldots\ldots\ldots. (3.4)$$

From (5.2)$B^0 \;=\; \frac{C\ +\gamma}{\rho}$

From (5.1)$W^0 \;=\; \frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho\beta}$

Linearizing, we obtain the Jacobian matrix thus:

$$J \;=\; \begin{pmatrix} r\left(1-\frac{B^0}{K}\right)-\ \frac{B^0}{K}-\beta W^0-\mu & -\beta B^0 \\ \alpha\rho W^0 & \alpha\rho B^0\ -C\alpha-\gamma\alpha \end{pmatrix}$$

3.1 FIRST EQUILIBRIUM STATE

At $(B^0, W^0) = (0 , 0)$

Then

$$J_0 = \begin{pmatrix} r - \mu & 0 \\ 0 & -\text{C}\alpha - \gamma\alpha \end{pmatrix}$$

$$|J_0 - \lambda I| = \begin{vmatrix} r - \mu - \lambda & 0 \\ 0 & -\alpha(\text{C} + \gamma) - \lambda \end{vmatrix}$$

This implies that, $\lambda_1 = r - \mu$, $\lambda_2 = -\alpha(\text{C} + \gamma)$

The system is stable at $(B^0, W^0) = (0 , 0)$ if $r - \mu < 0$, $-\alpha(\text{C} + \gamma) < 0$. This implies $\text{C} + \gamma > 0$. That is, there must be a bound on the harvest rate of the the birds which is proportional to the growth rate of the birds.

3.2SECOND EQUILIBRIUM STATE

$$(B^0, W^0) = \left(\frac{\text{C} + \gamma}{\rho} , \frac{K\rho(r - \mu) - r(\text{C} + \gamma)}{K\rho\beta}\right)$$

$$J = \begin{pmatrix} r\left(1 - \frac{B^0}{K}\right) - \frac{B^0}{K} - \beta W^0 - \mu & -\beta B^0 \\ \alpha\rho W^0 & \alpha\rho B^0 - \text{C}\alpha - \gamma\alpha \end{pmatrix}$$

Substituting the endemic equilibrium point into the Jacobian, we obtain the Jacobian for the endemic equilibrium point.

$$J_E = \begin{pmatrix} \frac{-(\text{C} + \gamma)}{\rho K} & \frac{-\beta(\text{C} + \gamma)}{\rho} \\ \frac{\alpha\rho K - \alpha r(\text{C} + \gamma)}{K\beta} & 0 \end{pmatrix}$$

$$|J_E - \lambda I| = \begin{vmatrix} \frac{-(\text{C} + \gamma)}{\rho K} - \lambda & \frac{-\beta(\text{C} + \gamma)}{\rho} \\ \frac{\alpha\rho K - \alpha r(\text{C} + \gamma)}{K\beta} & 0 - \lambda \end{vmatrix}$$

This implies that, $\rho K\lambda^2 + (\text{C} + r)\lambda + \delta = 0, where\ \delta = (\text{C} + r)[\alpha\rho K - \alpha r(\text{C} + \gamma)$

Hence $\lambda_1 = \frac{-(\text{C} + \gamma)}{2\rho K} + \frac{\sqrt{(\text{C} + \gamma)^2 - 4K\rho\delta}}{2K\rho}$, $\lambda_2 = \frac{-(\text{C} + \gamma)}{2\rho K} - \frac{\sqrt{(\text{C} + \gamma)^2 - 4K\rho\delta}}{2K\rho}$

For real λ, then $(\text{C} + \gamma)^2 - 4K\rho\delta \geq 0$

This implies that $(\text{C} + \gamma)^2 \geq 4K\rho\delta \Rightarrow \text{C} + \gamma) \geq \sqrt{4K\rho\delta}$

$$\text{C} + \gamma) \geq \sqrt{4K\rho(\text{C} + r)[\alpha\rho K - \alpha r(\text{C} + \gamma)}$$

$$\Rightarrow K \geq \frac{1}{\alpha\rho}\alpha r(\text{C} + \gamma)$$

The steady state is stable only if $\lambda_1 < 0$ *and* $\lambda_2 < 0$ This is possible only if

$$\frac{-(C + \gamma)}{2\rho K} \pm \frac{\sqrt{(C + \gamma)^2 - 4K\rho\delta}}{2K\rho} < 0$$

$$\Rightarrow \frac{-(C + \gamma)}{2\rho K} < \pm \frac{\sqrt{(C + \gamma)^2 - 4K\rho\delta}}{2K\rho}$$

$$\Rightarrow -(C + \gamma) < \pm\sqrt{(C + \gamma)^2 - 4K\rho\delta}$$

Squaring both sides, we have

$$(C + \gamma)^2 = (C + \gamma)^2 - 4K\rho\delta$$

$$(C + \gamma)^2 = (C + \gamma)^2 - 4K\rho(\,(C+r)\big[\alpha\rho K - \alpha r(C + \gamma)\big)$$

$$\Rightarrow 4K\rho(\,(C+r)\big[\alpha\rho K - \alpha r(C + \gamma)\big) < 0$$

$$\Rightarrow \alpha\rho K - \alpha r(C + \gamma)$$

$$\Rightarrow \alpha\rho K < \alpha r(C + \gamma)$$

$$\Rightarrow K < \frac{r(C + \gamma)}{\rho}$$

Hence, the endemic steady state $(B^0, W^0) = \left(\frac{C + \gamma}{\rho}, \frac{K\rho(r - \mu) - r(C+\gamma)}{K\rho\beta}\right)$

Is locally asymptotically stable (LAS) if $K < \frac{r(C + \gamma)}{\rho}$ that is if the carrying capacity is less than the ratio of the sum of $Per\ unit\ cost\ of\ birds$ and $Per\ unit\ tax\ on\ the\ bird$ to that of $Per\ unit\ price\ of\ the\ birds$

3.3 LIMITING CAPACITY AND POINT OF LIMITING CAPACITY OF THE SYSTEM

Assuming the initial population of the bird is given as $B(0) = B_0$.
$W = \frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho\beta}$, $Workdone\ in\ the\ poultry$. From the theory of initial value problem of a differential equation and the methods in [8,9,10],we obtain the population of the bird to be

$$B(t) = \frac{\dfrac{K\left(r - \frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho} - \mu\right)}{r}}{1 + \left(\dfrac{K\left(r - \frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho} - \mu\right)}{r} B_0 - 1\right) e^{-\left(r - \beta\left(\frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho}\right) - \mu\right)t}}$$

By taking the limit of the bird's population as t approaches infinity, the limiting population of bird, that is the maximum population of birds that the available resources in the system can sustain and also ensures harvesting profitability is given as $\frac{K\left(r - \frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho} - \mu\right)}{r}$

This occurs at the point

$$(\bar{t}, \bar{B}) = \left(\left(\frac{1}{\left(r - \beta\left(\frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho}\right) - \mu\right)} \mathrm{Ln}\left(\frac{\frac{K\left(r - \frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho} - \mu\right)}{r} - B_0}{B_0}\right), \left(\frac{K\left(r - \frac{K\rho(r-\mu)-r(C+\gamma)}{K\rho} - \mu\right)}{2r}\right)\right)\right)$$

4CONCLUSION

The work, presents a mathematical model of bird harvesting in an intensive poultry system, under the following assumptions:In the absence of epidemic outbreak, the population of birds has a logistic growth. The population of birds is reduced by the number of birds harvested at time t provided the net income is positive. The per carpita rate of bird's reproduction decreases with increase in the population of birds. In the presence of favourable environmental conditions and good management system, death due to natural occurrences does not cause any damage.There is no migration to or out of the poultry except for harvesting purpose. Work done increases with positive revenue and decrease otherwise. The model is analysed using methods from dynamical system theory and theory of calculus. It was established that the system has two steady state, the two equilibrium state are both locally asymptotically stable. The first one is stable if there is a bound on the harvest rate of the birds, which is proportional to the growth rate of the birds. The second equilibrium state is locally asymptotically stable (LAS) if $K < \frac{r(C + \gamma)}{\rho}$ that is if the carrying capacity is less than the ratio of the sum of $Per\ unit\ cost\ of\ birds$ and $Per\ unit\ tax\ on\ the\ bird$ to that of $Per\ unit\ price\ of\ the\ birds$. Further analysis indicates that the limiting population of bird, that is the maximum population of birds that the available resources in the system can sustain and also ensures harvesting profitability is given as

$$\frac{K\left(r-\frac{K\rho\,(r-\mu)-r(C+\gamma)}{K\rho}-\mu\right)}{r}$$

This occurs at the point

$$(\bar{t},\bar{B}) = \left(\left(\frac{1}{\left(r-\beta\left(\frac{K\rho\,(r-\mu)-r(C+\gamma)}{K\rho}\right)-\mu\right)}\,\mathrm{Ln}\left(\frac{\frac{K\left(r-\frac{K\rho\,(r-\mu)-r(C+\gamma)}{K\rho}-\mu\right)}{r}-B_0}{B_0}\right),\left(\frac{K\left(r-\frac{K\rho\,(r-\mu)-r(C+\gamma)}{K\rho}-\mu\right)}{2r}\right)\right)\right)$$

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