(λ, α)-interpolative Kannan Type Contractions on Super Metric Spaces

ABSTRCT: In the present research paper, Kannan, Reich and Dass-Gupta type contractions are defined and discussed in the framework of super metric space. Further, some fixed-point results are proved using the notion of interpolation.

Keywords: fixed point; iterative methods; interpolative contraction, rational contraction, super metric space.

2010 Mathematics Subject Classification: 47H10, 54H25

1. Introduction

One of the appealing areas of nonlinear functional analysis is metric fixed-point theory. In light of Banach's pioneering fixed-point theorem, several findings and publications on the topic have been made during the past 100 years. Essentially, there are two widely accepted theories about how to advance the metric fixed point: the first is changing (weakening) the constraints on the contraction mapping, and the second is altering the abstract structure. Metric spaces have already seen several generalizations and extensions. These include the quasi-metric space, the b-metric space, the symmetric space, the fuzzy metric space, the dislocated metric space, the partial metric space, the 2-metric space, the modular metric space, the cone metric space, the ultra-metric space, and a variety of other combinations of these.

It is important to note that the fixed-point theory is very practical and helpful in finding solutions to numerous issues in a variety of industries. As a result, this topic has been the subject of extensive research, the findings of which have been disseminated in the form of articles and books. These discoveries highlight the fact that the fixed-point theory is congested and constrained. As an illustration, most of the results for cone metric spaces are equivalent to the comparable results when standard metric space is used. Regarding the G-metric space, the same result may be drawn. The Banach contraction principle was then widely generalized in the literature (see [1--18]). Both pure and applied mathematics make extensive use of it. Kannan [14] defined a new variation of this theory in 1968 and eliminated the continuity condition from it. Kannan fixed-point theorem is the first significant variant of the outstanding result of Banach on the metric fixed-point theory. Kannan's theorem has been generalized in different ways. In the present note, we zoom in on one of the recent generalizations that was proposed by Karapınar [4] as interpolative Kannan-type contraction. It was indicated in [4] that each interpolative Kannan-type contraction in a complete metric space admits a fixed point. Erdal Karapinar and Andreea Fulga [5] introduced super-metric space. We were able to derive some fixed-point theorems in this structure, and we believe that this method could assist in alleviating the congestion and squeezing problems noted before.

We prove some fixed-point theorems for interpolative contraction and interpolative rational contraction in super metric space. Our findings extend the contractions of the metric space to a super metric space by Kannan's contraction, Riech's contraction and Dass-Gupta's rational contraction.

2. Preliminaries

We begin this section with the definition of the super metric.

***Definition* 2.1** (see [5]) Let $\mathfrak{D}$ is a non-empty set. We say that a function $\eta: \mathfrak{D} \times \mathfrak{D} \to [0, +\infty)$ is a super metric if it satisfies the following axioms:

(s1). $\forall\, \sigma, \varsigma \in \mathfrak{D}$, if $\eta(\sigma, \varsigma) = 0$, then $\sigma = \varsigma$.

(s2). $\forall\, \sigma, \varsigma \in \mathfrak{D}, \eta(\sigma, \varsigma) = \eta(\varsigma, \sigma)$.

(s3). *There exists* $s \geq 1$ *such that for every* $\varsigma \in \mathfrak{D}$, *there exist distinct sequences* $\{\sigma_n\}, \{\varsigma_n\} \subset \mathfrak{D}$, *with* $\eta(\sigma_n, \varsigma_n) \to 0$ *when* $n \to \infty$, *such that*

$$\limsup_{n\to\infty} \eta(\varsigma_n, \varsigma) \leq s \limsup_{n\to\infty} \eta(\sigma_n, \varsigma)$$

The tripled $(\mathfrak{D}, \eta, s)$ is called a super metric space.

Definition 2.2 (see [5]) *On a super metric space* $(\mathfrak{D}, \eta, s)$, *a sequence* $\{\sigma_n\}$:

(i). *converges to* σ *in* $\mathfrak{D}$ *if and only if* $\lim_{n\to\infty} \eta(\sigma_n, \sigma) = 0$

(ii). *is a Cauchy sequence in* $\mathfrak{D}$ *if and only if* $\lim_{n\to\infty} \sup\{\eta(\sigma_n, \sigma_p) : p > n\} = 0$.

Proposition 2.3 (see [5]) *On a super metric space, the limit of a convergent sequence is unique.*

Definition 2.4 (see [5]) *We say that a super metric space* $(\mathfrak{D}, \eta, s)$ *is* complete *if and only if every Cauchy sequence is convergent in* $\mathfrak{D}$.

Example 2.5 (see [5]) *Let the set* $\mathfrak{D} = \mathbb{R}, s = 2$, *and* $\eta: \mathfrak{D} \times \mathfrak{D} \to [0, +\infty)$ *be an application defined as follows:*

$$\eta(\sigma, \varsigma) = (\sigma - \varsigma)^2, \text{ for } \sigma, \varsigma \in \mathbb{R}\backslash\{1\}$$

$$\eta(1, \varsigma) = \eta(\varsigma, 1) = (1 - \varsigma^3)^2, \text{ for } \varsigma \in \mathbb{R}.$$

Then, the *tripled* $(\mathfrak{D}, \eta, s)$ *forms a super metric space.*

Example 2.6 (see [5]) *Let the set* $\mathfrak{D} = [0, +\infty]$ *and* $\eta: \mathfrak{D} \times \mathfrak{D} \to [0, +\infty)$ *be a function, defined as follows:*

$\eta(\sigma,\varsigma)=\frac{|\sigma\varsigma-1|}{\sigma+\varsigma+1}$, *for* $\sigma,\varsigma\in[0,1)\cup(1,+\infty], \sigma\neq\varsigma$,

$\eta(\sigma,\varsigma)=0$, *for* $\sigma,\varsigma\in[0,+\infty), \sigma=\varsigma$,

$\eta(\sigma,1)=\eta(1,\sigma)=|\sigma-1|$, *for* $\sigma\in[0,+\infty]$.

We can easily see that η forms a super metric on $\mathfrak{D}$.

Proposition 2.7 (see [5]) *Let* $\Gamma:\mathfrak{D}\to\mathfrak{D}$ *be an asymptotically regular mapping on a complete supermetric space* $(\mathfrak{D},\eta,s)$. *Then, the Picard iteration* $\{\Gamma^n\sigma\}$ *for the initial point* $\sigma\in\mathbb{R}$ *is a convergent sequence on* $\mathfrak{D}$.

Theorem 2.8 (see [5]) Let $(\mathfrak{D},\eta,s)$ be a complete super-metric space and let $\Gamma:\mathfrak{D}\to\mathfrak{D}$ be a mapping. Suppose that $0<\alpha<1$ such that

$$\eta(\Gamma\sigma,\Gamma\varsigma)\leq\eta(\sigma,\varsigma)$$

for all $(\sigma,\varsigma)\in\mathfrak{D}$. Then Γ has a unique fixed point in $\mathfrak{D}$.

Theorem 2.9 (see [5]) *Let* $(\mathfrak{D},\eta,s)$ *be a complete super metric space and* $\Gamma:\mathfrak{D}\to\mathfrak{D}$ *be a mapping, such that there exist* $\alpha\in[0,1)$ *and that*

$$\eta(\Gamma\sigma,\Gamma\varsigma)\leq k\max\left\{\eta(\sigma,\varsigma),\frac{\eta(\sigma,\Gamma\sigma)\eta(\varsigma,\Gamma\varsigma)}{\eta(\sigma,\varsigma)+1}\right\}$$

Then, Γ *has a unique fixed point.*

3. Main Results

We start with the following definition.

Definition 3.1 Let $(\mathfrak{D},\eta,s)$ be a *super* metric space and $\Gamma:\mathfrak{D}\to\mathfrak{D}$ a self-map. Then Γ is called a (λ,α)-interpolative Kannan contraction, if there exist $\lambda\in[0,1), \alpha\in(0,1)$ such that

$$\eta(\Gamma p,\Gamma q)\leq\lambda\big(\eta(p,\Gamma p)\big)^{\alpha}\big(\eta(\mathrm{q},\Gamma q)\big)^{1-\alpha} \tag{3.1}$$

for all $p,q\in\mathfrak{D}$, with $p\neq q$.

Definition 3.2 Let $(\mathfrak{D},\eta,s)$ be a *super* metric space and $\Gamma:\mathfrak{D}\to\mathfrak{D}$ a self-map. Then Γ is called a (λ,α,β)-interpolative Kannan contraction, if there exist $\lambda\in[0,1), \alpha,\beta\in(0,1)$, $\alpha+\beta<1$ such that

$$\eta(\Gamma p,\Gamma q)\leq\lambda\big(\eta(p,\Gamma p)\big)^{\alpha}\big(\eta(\mathrm{q},\Gamma q)\big)^{\beta} \tag{3.2}$$

for all $p,q\in\mathfrak{D}$, with $p\neq q$.

Definition 3.3 Let $(\mathfrak{D}, \eta, s)$ be a *super* metric space and $\Gamma: \mathfrak{D} \to \mathfrak{D}$ a self-map. Then Γ is called a $(\lambda, \alpha, \beta, \gamma)$-interpolative Reich contraction, if there exist $\lambda \in [0,1), \alpha, \beta, \gamma \in (0,1)$, $\alpha + \beta + \gamma < 1$ such that

$$\eta(\Gamma p, \Gamma q) \leq \lambda\big(\eta(p, q)\big)^{\alpha}\big(\eta(p, \Gamma p)\big)^{\beta}\big(\eta(\mathrm{q}, \Gamma q)\big)^{\gamma} \tag{3.3}$$

for all $p, q \in \mathfrak{D}$, with $p \neq q$.

Definition 3.4 Let $(\mathfrak{D}, \eta, s)$ be a *super* metric space and $\Gamma: \mathfrak{D} \to \mathfrak{D}$ a self-map. Then Γ is called a (λ, α, β)-interpolative Dass-Gupta rational contraction, if there exist $\lambda \in [0,1), \alpha, \beta \in (0,1)$, $\alpha + \beta < 1$ such that

$$\eta(\Gamma p, \Gamma q) \leq \lambda\big(\eta(p, q)\big)^{\alpha}\left(\frac{[1+\eta(p,\Gamma p)]\eta(\mathrm{q},\Gamma q)}{1+\eta(p,q)}\right)^{\beta} \tag{3.4}$$

for all $p, q \in \mathfrak{D}$, with $p \neq q$.

Our first result as follows.

Theorem 3.5 Let $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space and $\Gamma: \mathfrak{D} \to \mathfrak{D}$ be a (λ, α)-interpolative Kannan contraction. Then, Γ has a unique fixed point.

Proof Let $p_0 \in \mathfrak{D}$ and let $\Gamma p_0 = p_1$. If $p_0 = p_1$ then $p_1$is the fixed point and the proof is completed. Henceforward, assume that $p_0 \neq p_1$. Thus, $\eta(p_0, p_1) > 0$. Thus, without loss of generality, for each nonnegative integer n, we can define

$$p_{n+1} = \Gamma p_n \tag{3.5}$$

such that $p_{n+1} \neq \Gamma p_n$. So $\eta(p_n, p_{n+1}) > 0$, for all $n \in \mathbb{N}$. From inequality (3.1), we have

$$\begin{aligned}\eta(p_n, p_{n+1}) &= \eta(\Gamma p_{n-1}, \Gamma p_n)\\ &\leq \lambda\big(\eta(p_{n-1}, \Gamma p_{n-1})\big)^{\alpha}\big(\eta(p_n, \Gamma p_n)\big)^{1-\alpha}\\ &= \lambda\big(\eta(p_{n-1}, p_n)\big)^{\alpha}\big(\eta(p_n, p_{n+1})\big)^{1-\alpha}.\end{aligned}$$

Thus,

$$\big(\eta(p_n, p_{n+1})\big)^{\alpha} \leq \lambda\big(\eta(p_{n-1}, p_n)\big)^{\alpha}$$

Therefore, the above inequality gives,

$$\eta(p_n, p_{n+1}) \leq \lambda^{\frac{1}{\alpha}}\eta(p_{n-1}, p_n) \leq \lambda\eta(p_{n-1}, p_n) \tag{3.6}$$

So, inequality (3.6) implies that

$$\eta(p_n, p_{n+1}) \leq \lambda\eta(p_{n-1}, p_n)$$

$$\leq \lambda^2 \eta(p_{n-1}, p_n)$$

$$\vdots$$

$$\leq \lambda^n \eta(p_0, p_1) \tag{3.7}$$

Taking the limit n tends to infinity in inequality (3.7), we get

$$\lim_{n\to\infty} \eta(p_n, p_{n+1}) = 0. \tag{3.8}$$

In what follows, we want to show that the sequence $\{p_n\}$ is a Cauchy sequence. Now suppose that, $m, n \in \mathbb{N}$ with $m > n$. If $p_n = p_m$, then $\Gamma^m p_0 = \Gamma^n p_0$. This implies that $\Gamma^{m-n}(\Gamma^n p_0) = \Gamma^n p_0$. Thus, we have $\Gamma^n p_0$ is the fixed point of Γ^{m-n}. Also,

$$\Gamma\big(\Gamma^{m-n}(\Gamma^n p_0)\big) = \Gamma^{m-n}\big(\Gamma(\Gamma^n p_0)\big) = \Gamma(\Gamma^n p_0) \tag{3.9}$$

It means that, $\Gamma(\Gamma^n p_0)$ is the fixed point of Γ^{m-n}. Thus, $\Gamma(\Gamma^n p_0) = \Gamma^n p_0$. So, $\Gamma^n p_0$ is the fixed point of Γ. Now, suppose that $p_n \neq p_m$. Then from inequality (3.8) and using (s3), we get

$$\limsup_{n\to\infty} \eta(p_n, p_{n+2}) \leq s \limsup_{n\to\infty} \eta(p_{n+1}, p_{n+2}) \leq s \limsup_{n\to\infty} \{\lambda^{n+1}\eta(p_0, p_1)\} = 0. \tag{3.10}$$

Hence, $\limsup_{n\to\infty} \eta(p_n, p_{n+2}) = 0$. Similarly, we have

$$\limsup_{n\to\infty} \eta(p_n, p_{n+3}) \leq s \limsup_{n\to\infty} \eta(p_{n+2}, p_{n+3}) \leq s \limsup_{n\to\infty} \{\lambda^{n+2}\eta(p_0, p_1)\} = 0. \tag{3.11}$$

Inductively, one can conclude that $\limsup_{n\to\infty} \{\eta(p_n, p_m) : m > n\} = 0$. Thus $\{p_n\}$ is a Cauchy sequence in a complete super metric space $(\mathfrak{D}, \eta, s)$, the sequence $\{p_n\}$ converges to $p^* \in \mathfrak{D}$. We claim that p^* is the fixed point of Γ. On the contrary, assume $\eta(p^*, \Gamma p^*) > 0$. Note that

$$\eta(p_{n+1}, \Gamma p^*) = \eta(\Gamma p_n, \Gamma p^*)$$

$$\leq \lambda\big(\eta(p^*, \Gamma p^*)\big)^{\alpha}\big(\eta(p_n, \Gamma p_n)\big)^{1-\alpha}$$

$$= \lambda\big(\eta(p^*, \Gamma p^*)\big)^{\alpha}\big(\eta(p_n, p_{n+1})\big)^{1-\alpha} \to 0 \quad \text{as } n \to 0. \tag{3.12}$$

Thus, $\limsup_{n\to\infty} \eta(p_{n+1}, \Gamma p^*) = 0$. If there is $N > 0$ such that for all $n > N$ such that $p_{N+1} = p^*$, then we can conclude that $\eta(p^*, \Gamma p^*) = 0$ and so p^* is the fixed point for Γp^*. Otherwise, suppose that for all $n \in \mathbb{N}, p_n \neq p^*$. Thus, we have,

$$\eta(p^*, \Gamma p^*) \leq \limsup_{n\to\infty} \eta(p_{n+1}, \Gamma p^*) \tag{3.13}$$

and one can conclude that $\eta(p^*, \Gamma p^*) = 0$, which is a contradiction. Hence $p^* = \Gamma p^*$ is the fixed point of Γ in $\mathfrak{D}$. We shall now prove the uniqueness of the fixed point. If $q^* \in \mathfrak{D}$ is another fixed point of Γ, that is, $\Gamma q^* = q^*$, then we get

$$\eta(p^*, q^*) = \eta(\Gamma p^*, \Gamma q^*) \leq \lambda\big(\eta(p^*, \Gamma p^*)\big)^{\alpha}\big(\eta(q^*, \Gamma q^*)\big)^{1-\alpha} \leq 0$$

which is a contradiction, and hence, $p^* = q^*$.

Theorem 3.6 Let $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space and $\Gamma: \mathfrak{D} \to \mathfrak{D}$ be a (λ, α, β)-interpolative Kannan contraction. Then Γ has a unique fixed point.

Proof Following the steps of proof of Theorem 3.5, we construct the sequence $\{p_n\}$ by iterating

$$p_{n+1} = \Gamma\, p_n, \forall\, n \in \mathbb{N},$$

where $p_0 \in \mathfrak{D}$ is arbitrary starting point. Then, by (3.2), we have

$$\begin{aligned}\eta(p_n, p_{n+1}) &= \eta(\Gamma p_{n-1}, \Gamma p_n)\\ &\leq \lambda\big(\eta(p_{n-1}, \Gamma p_{n-1})\big)^{\alpha}\big(\eta(p_n, \Gamma p_n)\big)^{\beta}\\ &= \lambda\big(\eta(p_{n-1}, p_n)\big)^{\alpha}\big(\eta(p_n, p_{n+1})\big)^{\beta}\end{aligned}$$

Since $\alpha < 1 - \beta$, the last inequality gives

$$\eta(p_n, p_{n+1})^{1-\beta} \leq \lambda\eta(p_{n-1}, p_n)^{\alpha} \leq \lambda\eta(p_{n-1}, p_n)^{1-\beta} \tag{3.14}$$

Hence

$$\eta(p_n, p_{n+1}) \leq \lambda^{\frac{1}{1-\beta}}\eta(p_{n-1}, p_n) \leq \lambda\eta(p_{n-1}, p_n)$$

and then

$$\eta(p_n, p_{n+1}) \leq \lambda^n \eta(p_0, p_1) \tag{3.15}$$

and in taking the limit from the above inequality, we get

$$\lim_{n\to\infty} \eta(p_n, p_{n+1}) = 0. \tag{3.16}$$

As already elaborated in the proof of Theorem 3.5, the classical procedure leads to the existence of a fixed-point $p^* \in X$. Now, we prove the uniqueness of p^*. If $q^* \in \mathfrak{D}$ is another fixed point of Γ, that is, $\Gamma q^* = q^*$, then from (3.2), we get

$$\eta(p^*, q^*) = \eta(\Gamma p^*, \Gamma q^*) \leq \lambda\big(\eta(p^*, \Gamma p^*)\big)^{\alpha}\big(\eta(q^*, \Gamma q^*)\big)^{\beta} \leq 0 \tag{3.17}$$

This yields that $p^* = q^*$. This completes the proof.

Theorem 3.7 Let $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space and $\Gamma: \mathfrak{D} \to \mathfrak{D}$ be a $(\lambda, \alpha, \beta, \gamma)$-interpolative Reich contraction. Then Γ has a unique fixed point.

Proof Following the steps of proof of Theorem 3.5, we construct the sequence $\{p_n\}$ by iterating

$$p_{n+1} = \Gamma\, p_n, \forall\, n \in \mathbb{N},$$

where $p_0 \in \mathfrak{D}$ is arbitrary starting point. Then, by (3.3), we have

$$\begin{aligned}\eta(p_n, p_{n+1}) &= \eta(\Gamma p_{n-1}, \Gamma p_n)\\ &\leq \lambda\big(\eta(p_{n-1}, p_n)\big)^{\alpha}\big(\eta(p_{n-1}, \Gamma p_{n-1})\big)^{\beta}\big(\eta(p_n, \Gamma p_n)\big)^{\gamma}\\ &= \lambda\big(\eta(p_{n-1}, p_n)\big)^{\alpha+\beta}\big(\eta(p_n, p_{n+1})\big)^{\gamma}\end{aligned}$$

Since $\alpha + \beta < 1 - \gamma$, the last inequality gives

$$\big(\eta(p_n, p_{n+1})\big)^{1-\gamma} \leq \lambda\big(\eta(p_{n-1}, p_n)\big)^{\alpha+\beta} \leq \lambda\big(\eta(p_{n-1}, p_n)\big)^{1-\gamma} \tag{3.18}$$

Hence

$$\eta(p_n, p_{n+1}) \leq \lambda^{\frac{1}{1-\gamma}}\eta(p_{n-1}, p_n) \leq \lambda\eta(p_{n-1}, p_n)$$

and then

$$\eta(p_n, p_{n+1}) \leq \lambda^{n}\eta(p_0, p_1) \tag{3.19}$$

and in taking the limit from the above inequality, we get

$$\lim_{n\to\infty} \eta(p_n, p_{n+1}) = 0. \tag{3.20}$$

As already elaborated in the proof of Theorem 3.5, the classical procedure leads to the existence of a fixed-point $p^* \in X$. Now, we prove the uniqueness of p^*. If $q^* \in \mathfrak{D}$ is another fixed point of Γ, that is, $\Gamma q^* = q^*$, then from (3.3), we get

$$\begin{aligned}\eta(p^*, q^*) &= \eta(\Gamma p^*, \Gamma q^*)\\ &\leq \lambda\big(\,\eta(p^*, q^*)\big)^{\alpha}\big(\eta(p^*, \Gamma p^*)\big)^{\beta}\big(\eta(q^*, \Gamma q^*)\big)^{\gamma} \leq 0\end{aligned} \tag{3.21}$$

This yields that $p^* = q^*$. This completes the proof.

Theorem 3.8 Let $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space and $\Gamma: \mathfrak{D} \to \mathfrak{D}$ be a (λ, α, β)-interpolative Dass-Gupta rational contraction. Then Γ has a unique fixed point.

Proof Following the steps of proof of Theorem 3.5, we construct the sequence $\{p_n\}$ by iterating

$$p_{n+1} = \Gamma\, p_n, \forall\, n \in \mathbb{N},$$

where $p_0 \in \mathfrak{D}$ is arbitrary starting point. Then, by (3.4), we have

$$\begin{aligned}
\eta(p_n, p_{n+1}) &= \eta(\Gamma p_{n-1}, \Gamma p_n) \\
&\leq \lambda\left(\eta(p_{n-1}, p_n)\right)^{\alpha}\left(\frac{[1+\eta(p_{n-1}, \Gamma p_{n-1})]\eta(p_n, \Gamma p_n)}{1+\eta(p_{n-1}, p_n)}\right)^{\beta} \\
&\leq \lambda\left(\eta(p_{n-1}, p_n)\right)^{\alpha}\left(\frac{[1+\eta(p_{n-1}, p_n)]\eta(p_n, p_{n+1})}{1+\eta(p_{n-1}, p_n)}\right)^{\beta} \\
&= \lambda\left(\eta(p_{n-1}, p_n)\right)^{\alpha}\left(\eta(p_n, p_{n+1})\right)^{\beta}
\end{aligned}$$

Since $\alpha + \beta < 1$, the last inequality gives

$$\left(\eta(p_n, p_{n+1})\right)^{1-\beta} \leq \lambda\left(\eta(p_{n-1}, p_n)\right)^{\alpha} \leq \lambda\left(\eta(p_{n-1}, p_n)\right)^{1-\beta}$$

i.e.
$$\eta(p_n, p_{n+1}) \leq \lambda^{\frac{1}{1-\beta}}\eta(p_{n-1}, p_n) \leq \lambda\eta(p_{n-1}, p_n)$$

and then

$$\eta(p_n, p_{n+1}) \leq \lambda^n \eta(p_0, p_1) \tag{3.22}$$

As already elaborated in the proof of Theorem 3.5, the classical procedure leads to the existence of a fixed-point $p^* \in X$. Now, we prove the uniqueness of p^*. If $q^* \in \mathfrak{D}$ is another fixed point of Γ, that is, $\Gamma q^* = q^*$, then from (3.4), we get

$$\begin{aligned}
\eta(p^*, q^*) &= \eta(\Gamma p^*, \Gamma q^*) \\
&\leq \lambda\left(\eta(p^*, q^*)\right)^{\alpha}\left(\frac{[1+\eta(p^*, \Gamma p^*)]\eta(q^*, \Gamma q^*)}{1+\eta(p^*, q^*)}\right)^{\beta} \\
&= 0
\end{aligned}$$

This yields that $p^* = q^*$. This completes the proof.

4. Conclusion

In this paper, using the new framework of super metric spaces, we introduced the concept of (λ, α)-interpolative Kannan contraction, $(\lambda\ \alpha, \beta)$-interpolative Kannan contraction and

$(\lambda, \alpha, \beta, \gamma)$-interpolative Riech contraction and $(\lambda\ \alpha, \beta)$-interpolative Dass-Gupta rational contraction and proved the existence of fixed points for self-mapping.

Authors' Contributions

All authors contributed equally and significantly in writing this paper. All authors read and

approved the final manuscript.

References

[1] Alqahtani, B.; Fulga, A.; Karapınar, E., Sehgal Type Contractions on b-Metric Space. Symmetry 2018, 10, 560.

[2] Alqahtani, B.; Fulga, A.; Karapinar, E.; Rakocevic, V. Contractions with rational inequalities in the extended b-metric space. J. Inequal. Appl. 2019, 2019, 220.

[3] Czerwik, S. Contraction mappings in b-metric spaces. Acta Math. Inform. Univ. Ostrav. 1993, 1, 5–11.

[4] E. Karapinar, Revisiting the Kannan Type Contractions via Interpolation, *Advances in theory of nonlinear analysis and its applications,* 2, 2(2018), 8587.

[5] Erdal Karapinar and Andreea Fulga, Contraction in Rational Forms in the Framework of Super Metric Spaces, MPDI, Mathematic, 2022, 10, 3077, pages 1-12, https://doi.org/10.3390/math10173077

[6] Huang, H.; Singh, Y.M.; Khan, M.S.; Radenovic, S. Rational type contractions in extended b-metric spaces. Symmetry 2021, 13, 614.

[7] Jleli, M.; Samet, B. A generalized metric space and related fixed-point theorems. Fixed Point Theory Appl. 2015, 2015, 61.

[8] Karapinar, E. A note on a rational form contraction with discontinuities at fixed points. Fixed Point Theory 2020, 21, 211–220.

[9] Khan, Z.A.; Ahmad, I.; Shah, K. Applications of Fixed-Point Theory to Investigate a System of Fractional Order Differential Equations. J. Funct. Spaces 2021, 2021, 1399764. [CrossRef]

[10] Mebawonduy,; A.A.; Izuchukwuz, C.; Aremux, I.K.O.; Mewomo, O.T. Some fixed-point results for a generalized TAC-SuzukiBerinde type F-contractions in b-metric spaces. Appl. Math.- Notes 2019, 19, 629–653.

[11] Rezapour, S.; Deressa, C.T.; Hussain, A.; Etemad, S.; George, R.; Ahmad, B. A Theoretical Analysis of a Fractional Multi Dimensional System of Boundary Value Problems on the Methylpropane Graph via Fixed Point Technique. Mathematics 2022, 10, 568.

[12] Roldán López de Hierro, A.F.; Shahzad, N. Fixed point theorems by combining Jleli and Samet's, and Branciari's inequalities. J. Nonlinear Sci. Appl. 2016, 9, 3822–3849.

[13] Yae Ulrich Gaba, E. Karapinar, A new approach to the interpolative contractions, *Axioms*, 8(2019), 1-4.

[14] R. Kannan: Some results on fixed-point s. Bull. Calcutta Math. Soc. **60**, 71–76 (1968).

[15] S. Reich: Kannan's fixed-point theorem. Boll. Un. Mat. Ital. (4) **4**, 1–11 (1971).

[16] B. K. Dass and S. Gupta, "An extension of Banach contraction principle through rational expressions," Indian J. Pure Appl. Math. vol. 6, 1455-1458, 1975.

[17] I. A. Bakhtin, "The contraction principle in quasi-metric spaces," *Journal of Functional Analysis*, vol. 30, pp. 26–37, 1989.

[18] S. G. Matthews, "Partial metric topology," *Annals of the New York Academy of Sciences*, vol. 728, no. 1, pp. 183–197, 1994.