Electromagnetic Radiation for Nonlinear Dynamics of Two-Neuron Based Memristive Hopfield Neural Network with Synaptic Crosstalk

Abstract : Electromagnetic radiation (EMR) is a well-known method for studying the behavior of the nervous system.In this research, Hopfield Neural Network (HNN) of two neurons has been investigated based on the interaction of synapses with hyperbolic memristor and EMR effects.By adjusting the interference networks and the weight of synapses, the ability of neurons can be controlled.Imitation of synapse interference between two neurons with mutual parameters of weight and memory, as well as the effect of EMR on the chaotic dynamics of neuronase, complex dynamic behaviors, transient disturbance behavior, phase portraits, chaos phenomena and branching diagrams in emerging neural networks.are analyzed.The dynamic behavior of HNN can be controlled by changing the EMR value of the damaged neuron.The proposed model is simulated by Pspice.

1. Introduction

Processing in the brain, the transfer of information from one neuron to another is done by the intermediate synapse (1). Neurons in the nervous system communicate with each other through electrochemical signals (2-4). External stimulation and uncertainty of information transmission cause nonlinear and complex dynamic behavior in the neural network of the brain (5).Neural networks and artificial neurons are inspired by the structure and function of the brain system(6). In the process of researching the neural network, researchers investigated the dynamic behavior of the nervous system of the brain and chaos mechanisms with different methods(7-14).In the memristor, by adjusting the voltage, it is possible to simulate the flexibility of the synapse weights, and by changing the voltage value, the strength of the nerve connection can be shown.

Memory neural networks have attracted the attention of many researchers(15-17). It has been used to improve the quality of images and solve the problem of the order of designed images(16). Human emotions are simulated through a memristor-based Hopfield neural network(17).By changing the weight of neural synapses, the features of chaotic attractors, Lyapunov power, complex bifurcation diagrams, fuzzy portraits reflect the dynamic behavior of parameters, after nonlinear dynamics are used to investigate the behavior of HNN(18). In the article (19), instead of the hyperbolic function, a new function called RELU is used as an activation function and a three-neuron has been investigated.Hopfield introduced Hopfield's artificial neural network (20), which can simulate complex brain dynamics such as chaos. HNN is widely used in image processing, associative memory, combinatorial optimization of images, etc. (21-25). One of the branches of physics, the field of dynamics, has been transformed into three areas. By using computer technology, researchers have been able to easily research nonlinear systems, so research on the dynamics of neural networks has increased.By using mathematical modeling and data analysis, its application has been developed in the development of neuroscience theories and the design of artificial intelligence systems and brain function (26-29).In recent years, researchers have investigated the importance of higher order interactions in the dynamics of neural networks in collective dynamics (30).In the article (31), various types of dynamic processes, diffusion processes, synchronization phenomena, consensus formation, and intellectual evolution have been investigated.The concept of attractors is a main element in understanding the nonlinearity of the network to research the nonlinear dynamics of HNN. The attractor can be simple or very complex, and the complexity creates chaos theory. There are two categories of attractors in investigating chaotic systems: trivial attractors. and strange attractions (32-36).Insignificant gravity has a simple form in a chaotic system, which is a point, a straight line, or a simple periodic circuit (37).The dynamic behavior of insignificant attractors is predictable and simple, but it plays an important role in chaotic systems. Strange or individual attractors have non-periodic dynamic behaviors and very complex structures (38).To encode the image, a multivariate weighted chaotic system such as Lorenz, Multivariate Weighted Chaotic Systems (MWCS) based on non-polynomial functions was used(39).Non-linear properties in HNN are due to the synapse weights, the changes in the synapse weights are similar to the synapse changes in the brain system (40-42). Adding some external stimuli to the neuron leads to an unexpected change in the neuron, so the external stimuli directly affect the behavior of the neuron and the state of the entire neural network (43-46).Hindmarsh-Rose neuron model, which is a combination of magnetic flux and memristive current as external stimulus in 2016LV et al. f introduced later research about Electro Magnetic Radiation (EMR) is an external stimulus that activates neural output through the interaction of electricity and magnetism. And the research on EMR external stimuli on the neural network for nonlinear network dynamics started. The HNN based on three neurons was developed by Wan et al. who introduced electromagnetic induction or bias current to two neurons under the influence of an EMR and found that either the power change Coupled memristor neural network system shows complex dynamic behaviors (47-48).Valin and his colleagues constructed a neural network of three neurons under EMR and discovered that a neural network with periodic attractors or applying EMR on a neuron produces irregular attractors, and the neural network creates multi-exeral attractors (49-50).It is important to study about the nonlinear dynamic behavior of neural networks of the brain and external stimuli on them (51-53).

In this article in the second part Hyperbolic memristor synapse HNN, In the third part Analyzing the dynamic behavior of HNN, In the fourth sectionCircuit simulation with pspiceAnd In the fifth section, we will also talk about CONCLUSION.

2- Hyperbolic memristor synapse HNN

2-1 HNN synapse memristor

The simulation of the memristor with the activation function of the inverse hyperbolic tangent model to simulate the synapse weight of the neurons is based on the following formula:

$$i = w(x)v = [a - b\ tanh(x)]v \quad (1)$$

Where I, v, x respectively represent the current, voltage and state variables of the memristor defined as follows:

$$i = w(x)v = \left(\frac{-1}{Ra} - \frac{1}{Rb}\tanh(x)\right)v \quad (2)$$

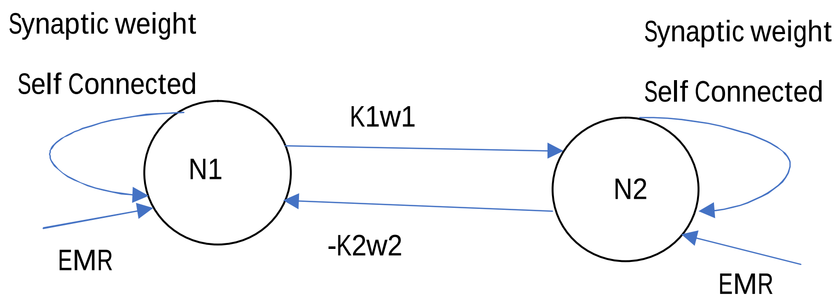

Fig 1. a: two neurons with overlapping synapses, b: single EMR, c: double EMR

According to Figure 1, the synapse interference between two neurons can be expressed as follows:

$$w_1 = a_1 - b_1 \tanh(x_1) + c_1 \tanh(x_4)$$
$$w_2 = a_2 - b_2 \tanh(x_4) + c_2 \tanh(x_3) \quad (3)$$

in which:

$$a_1 = \frac{R}{R_{a1}}, b_1 = \frac{R}{R_{b1}}, c_1 = \frac{gR^2}{R_{b1}R_{c1}}, a_2 = \frac{R}{R_{a2}}, b_2 = \frac{R}{R_{b2}}, c_2 = \frac{gR^2}{R_{b2}R_{c2}} \quad (4)$$

a1, b1, a2, b2 Memristor parameters and c1, c2 are interference power parameters.

2-2 HNN model based on two neurons:

The formula of an HNN for two neurons is as follows:

$$c_1 \frac{dx_i}{dt} = -\frac{x_i}{R_i} \sum_{i=1}^{n} w_{ij} \tanh(x_i) + I_i \quad (5)$$

where ci , Ri , xi represents, membrane capacity, membrane resistance and voltage.and tanh(xi) is the activation function of the neuron, and W is the synapse weight between neurons i,j, and Ii is the bias current. In this research, the memristor model based on EMR simulation has been used as an external stimulus for two neurons with intermediate synapses, and the mathematical model features of the memristor model are as follows:

$$i = w(\varphi)v$$
$$\frac{d\varphi}{dt} = g(\varphi, v)$$
$$w(\varphi) = p(\alpha + \beta\varphi^2)$$
$$g(\varphi, v) = \mu v + \varepsilon\varphi \qquad (6)$$

i is current, v is voltage, φ is magnetic flux and W(φ) is memory conductivity.$\alpha, \beta, \varphi, \varepsilon$ are the adjustable parameters of the EMR model In this article, the HNN model with two neurons and synapse weight values are set to appropriate weights, which are first checked without the intervention of EMR. In the following equation, a sinusoidal excitation is used for the memristor model:

$$v = A\sin(f * t)$$
$$I = p(\alpha + 3\beta\varphi^2)v$$
$$\frac{d\varphi}{dt} = \mu v \qquad (7)$$

where A, F are the amplitude and frequency of the sinusoidal stimulus, respectively, when the parameters $\alpha = \beta = \mu = 1, \varphi = 2$ and A=1 are fixed and F is set to the values of 1, 3,9 and 12 respectively, in Figure 2 it can be seen that the equation(7) Based on these values, it has three outputs of this type of memristor, each frequency forms a hysteresis loop and passes through the origin, and becomes linear as the output frequency increases.

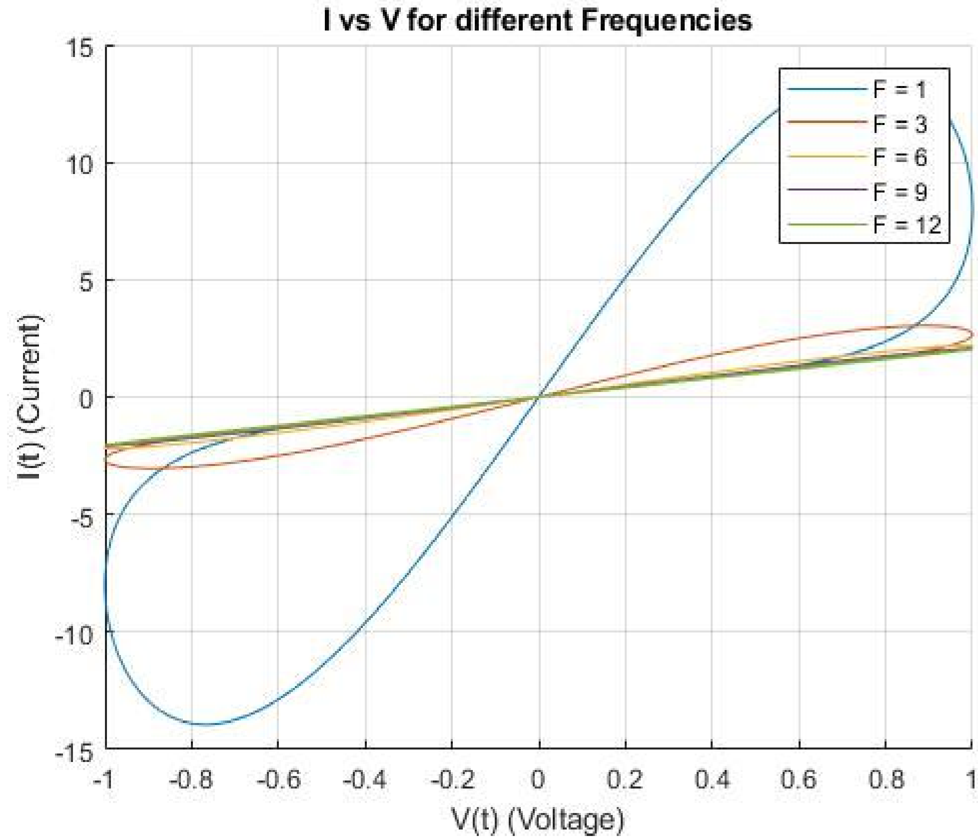

Fig 2. Memristor model with fixed values of A=1,F=1;F=3;F=6;F=9;F=12, output of hysteresis loops.

Using equation 5.6, HNN model with overlapping synapse and EMR bone according to Figure 1, part a is as follows:

$$\dot{x_1} = -x_1 - 1.5\tanh(x_1) + \tanh(x_2) - 1.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$\dot{x_2} = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$\dot{x_3} = -x_3 - .5\tanh(x_1)$$

$$\dot{x_4} = -x_4 - \tanh(x_2)(8)$$

x_i(i=1,2) and $\varphi$is the state variable of the system.

In the second case, using a single EMR according to Figure 1, part b:

$$\dot{x_1} = -x_1 - 1.5\tanh(x_1) + \tanh(x_2) - 1.5\tanh(x_3) - 1.5\tanh(x_4)$$

$+\varphi(\alpha + 3\beta\varphi^2)x_1$

$$\dot{x_2} = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$\dot{x_3} = -x_3 - .5\tanh(x_1) + 1.4\tanh(x_2) + \tanh(x_3) + 1.5\ \tanh(x_4)$$

$$\dot{x_4} = -\ x_4 + 5\tanh(x_1) - 1.5\tanh(x_2) - 2.5\tanh(x_3) + 3\ \tanh(x_4)$$

$$\varphi = \mu\, x_1 \quad (9)$$

In the third case, using two EMRs according to Figure 1, part C:

$$\dot{x_1} = -x_1 - 1.5\tanh(x_1) + \tanh(x_2) - 1.5\tanh(x_3) - 1.5\tanh(x_4) + \varphi(\alpha + 3\beta\varphi^2)x_1$$

$$\dot{x_2} = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$\dot{x_3} = -x_3 - .5\tanh(x_1) + 1.4\tanh(x_2) + \tanh(x_3) + 1.5\ \tanh(x_4)$$

$$\dot{x_4} = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$\varphi_1 = \mu_1 x_1$$

$$\varphi_2 = \mu_2 x_2 \quad (10)$$

x_i(i=1,2) and $\varphi_1, \varphi_2$is the state variable of the system.

2-3 Stability analysis with mathematical and visual model:

To analyze the stability of all three cases, the first case of two neurons with overlapping synapses without EMR, the second case of two neurons with overlapping synapses and single EMR, the third case of two neurons with overlapping synapses and with two EMRs have been analyzed by mathematical and visual analysis methods for In the first case, we set the mathematical equation 8 equal to 0.

$$0 = -x_1 - 1.5\tanh(x_1) + \tanh(x_2) - 1.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$0 = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$0 = -x_3 - .5\tanh(x_1)$$

$$0 = -x_4 - \tanh(x_2)(11)$$

From equation 11, the implied function of equation 12 can be obtained by placing $x_3 = .5\tanh(x_1)$and $x_4 = \tanh(x_2)$ instead of x_1 and x instead of x_2 , y in equation 11, and based on this equation, the figure 3 equilibrium point for the first case is determined.

$$H_{1(x,y)} = -x - 1.5\tanh(x) + \tanh(y) - 1.5\tanh\left(.5\tanh(x) - 1.5\tanh(y)\right)$$

$$H_{2(x,y)} = -y - 2.2\tanh(x) - .5\tanh(y) + \tanh(y) + 2.5\tanh(.5\tanh(y)) - 1.5\tanh(tanh(y)) \qquad (12)$$

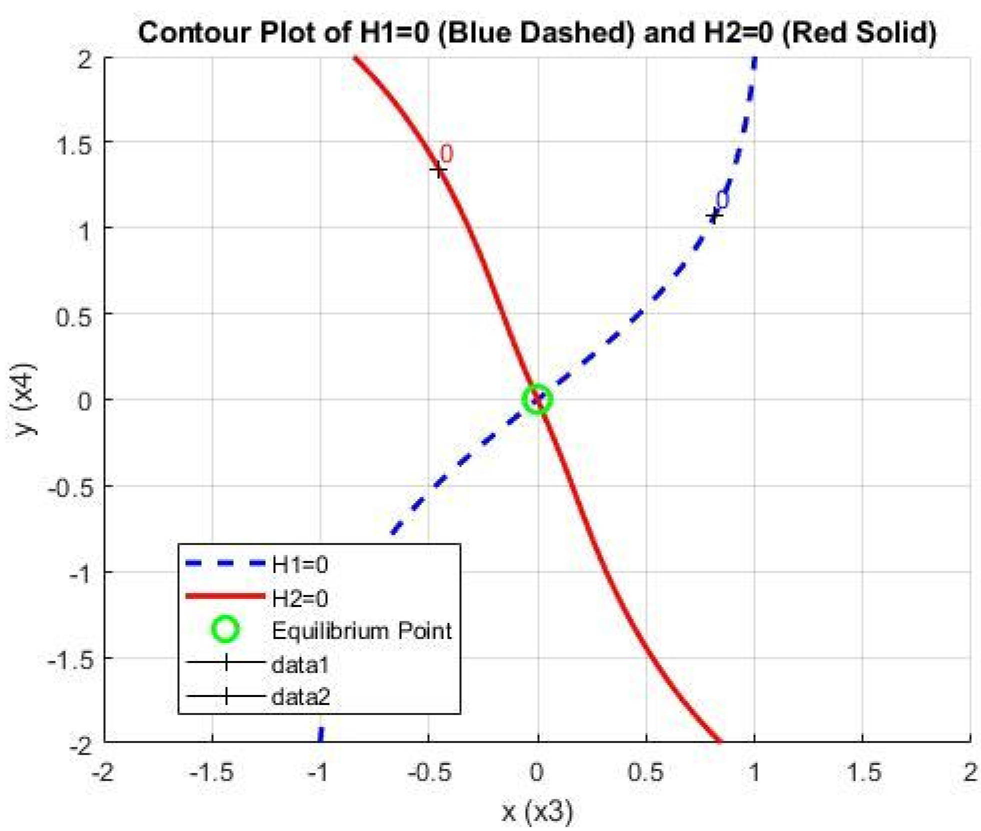

Fig 3 Fixed point for the two-neuron case with synapse interference without ENR

For the second case, we set the mathematical equation 9 equal to 0.

$$0 = -x_1 - 1.5\tanh(x_1) + \tanh(x_2) - 1.5\tanh(x_3) - 1.5\tanh(x_4) + \varphi(\alpha + 3\beta\varphi^2)x_1$$

$$0 = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$0 = -x_3 - .5\tanh(x_1) + 1.4\tanh(x_2) + \tanh(x_3) + 1.5\tanh(x_4)$$

$$0 = -x_4 + 5\tanh(x_1) - 1.5\tanh(x_2) - 2.5\tanh(x_3) + 3\tanh(x_4)$$

$$0 = \mu x_1 \qquad (13)$$

According to the section $0 = \mu x_1$ and because $\mu$the intensity coefficient is related to EMR, so x_1 = 0 and its value is non-zero, so equation 12 becomes equation 13.

$$0 = -x_3 - 1.4\tanh(1.75\tanh(x_3) - 2.25\tanh(x_4)) + \tanh(x_3) + 1.5\tanh(x_4)$$

$$0 = -x_4 + 1.5\tanh(1.75\tanh(x_3) - 2.25\tanh(x_4)) - 2.5\tanh(x_3) + 3\tanh(x_4) \qquad (14)$$

From equation 13, by setting x_3 equal to x and x_4 equal to y, the implied function of equation 15 can be obtained, and based on this equation, Figure 4 and the equilibrium point for the second case are determined.

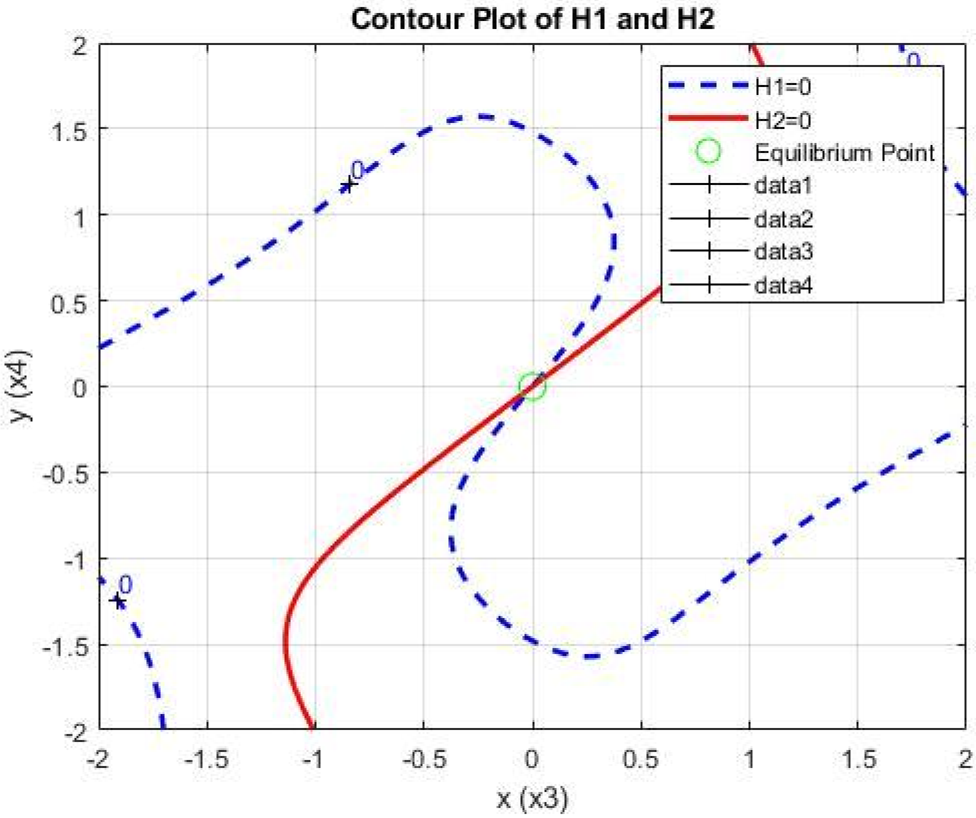

Fig 4. Fixed point for two neurons with synapse interference and single EMR

$$H_{1(x,y)} = -x + 1.4\tanh(1.75\tanh(x)) - 2.25\tanh(y) + \tanh(x) + 1.5\tanh(y))$$

$$H_{2(x,y)} = -y - 1.5\tanh(1.75\tanh(x) - 2.25\tanh(y)) - 2.5\tanh(x) + 3\tanh(tanh(y)) \qquad (15)$$

For the third case, we set the mathematical equation 10 equal to 0.

$$0 = -x_1 - 1.5\tanh(x_1) + \tanh(x_2) - 1.5\tanh(x_3) - 1.5\tanh(x_4) + \varphi(\alpha + 3\beta\varphi^2)x_1$$

$$0 = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$0 = -x_3 - .5\tanh(x_1) + 1.4\tanh(x_2) + \tanh(x_3) + 1.5\tanh(x_4)$$

$$0 = -x_2 - 2.2\tanh(x_1) - .5\tanh(x_2) + \tanh(x_2) + 2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$0 = \mu_1 x_1$$

$$0 = \mu_2 x_2 \qquad (16)$$

According to the section $0 = \mu_1 x_1$ and $0 = \mu_2 x_2$, because μ_1 and $\mu_2$are intensity coefficients related to EMR, it is non-zero.Therefore, $x_1 = 0$ and $x_2 = 0$, so equation 16 becomes equation 17.

$$0 = -1.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$0 = -2.5\tanh(x_3) - 1.5\tanh(x_4)$$

$$0 = -x_3 + \tanh(x_3) + 1.5\tanh(x_4)$$

$$0 = -x_4 - 2.5\tanh(x_3) - 3\tanh(x_4) \qquad (17)$$

From equation 17, by settingx_3 equal to x and $x_4$equal to y, the implicit function of equation 18 can be obtained, and based on this equation, Figure 5 and the equilibrium point of the third case are determined.

$$H_{1(x,y)} = -1.5\tanh(x) - 1.5\tanh(y)$$

$$H_{2(x,y)} = -2.5\tanh(x) - 1.5\tanh(y)$$

$$H_{3(x,y)} = -x + \tanh(x) + 1.5\tanh(y)$$

$$H_{4(x,y)} = -y - 2.5\tanh(x) - 3\tanh(y) \qquad (18)$$

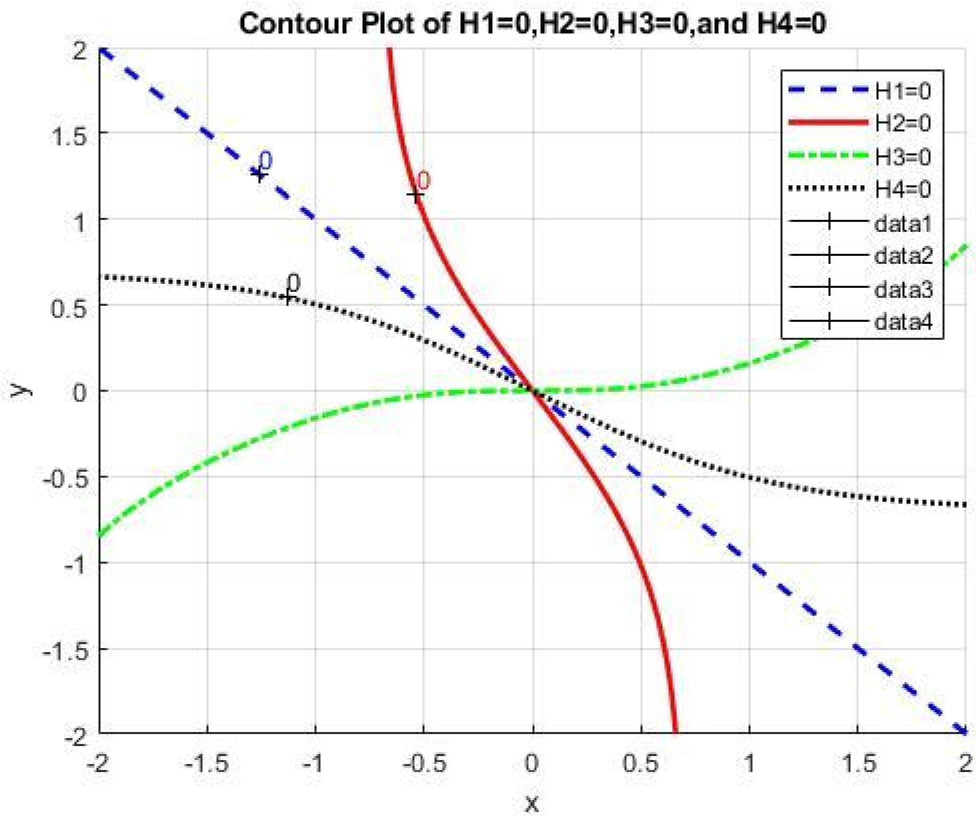

Fig 5 .Fixed point for two neurons with synapse interference and two EMRs

In Figure 4, you can see that the system is at the equilibrium point when all four points $x_1, x_2, , x_3, x_4$ are equal to zero and constant, and according to equation 13, the value of μ_1 can have an arbitrary value, and there can be an infinite number of points, and the system is still in the stateEquilibrium remains, i.e. (0,0,0,0, p), so p can change in the scope of its definition, which creates a one-dimensional path in a 5-dimensional space.In the nonlinear system, the equilibrium point is the point that does not change state in the system. In Figure 5, the values of μ_1, μ_2 are arbitrary values, so they form a 2-dimensional space and a plane, and any point on this surface can be an equilibrium state in the 6-dimensional space, because μ_1, μ_2 can take different values, so there are many equilibrium states.

3- Analyzing the dynamic behavior of HNN

3-1 Analyzing the dynamic behavior of HNN by Lyapunov power

Lyapunov power is used to describe the chaotic behavior of nonlinear dynamic systems.In this section, the analysis of the dynamic behaviors of the HNN model without EMR in Figure 6 and with a single EMR in Figure 7 and with two EMRs in Figure 8 with different values has been investigated.

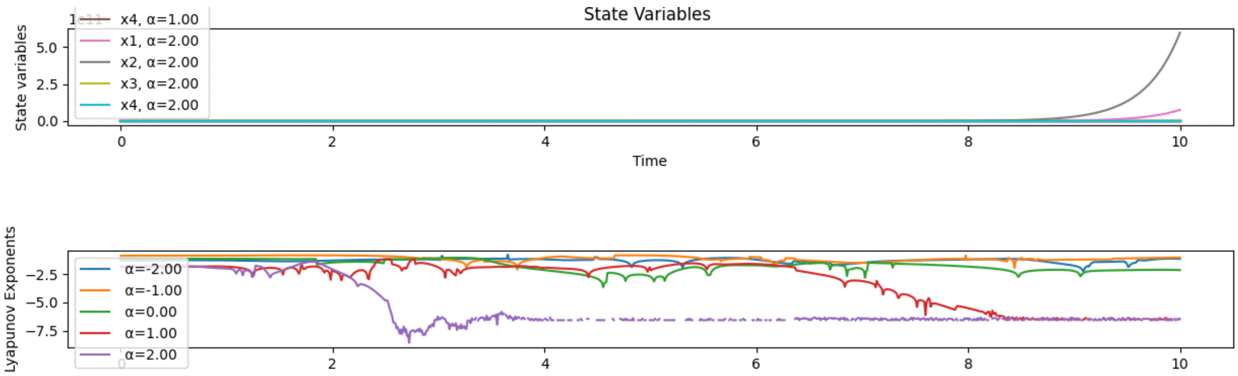

Fig 6 .Lyapunov power of HNN without EMR

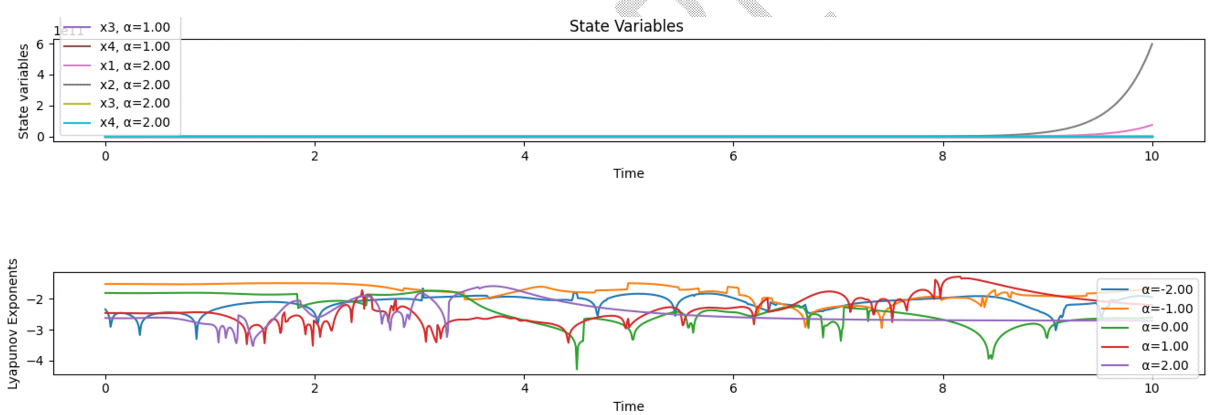

Fig 7 .Lyapunov power of HNN in single EMR mode

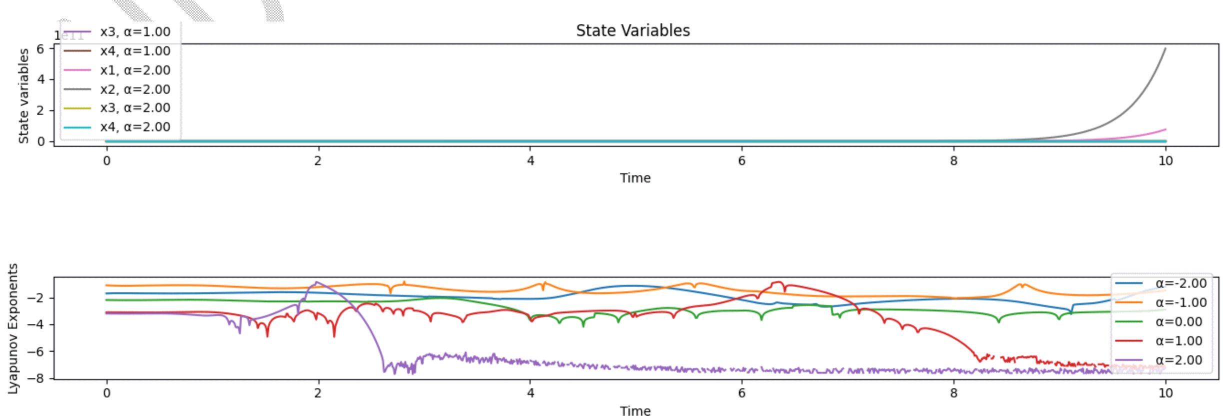

Fig 8 .Lyapunov power of HNN in the case of two EMRs

The parameter a of the memristor model is considered as a variable parameter and has been checked with different values.The dynamic behavior of HNN with Lyapunov power shows that according to Figure 6, the Lyapunov power of HNN in the state without EMR is -2.3 and Figure 7 the Lyapunov power of HNN in the single EMR state is -2.8 and Figure 8 shows that the Lyapunov power of HNNIn the case of two EMRs at -3.1, the system has an obvious chaos phenomenon.

3-2 Analyzing the dynamic behavior of HNN by Phase portrai

Fuzzy portrait is to record the path of the system with a shape that reveals the performance of the system. Then the fuzzy portrait is a graphical representation of the state trajectories of the dynamical system in the state space. Each point in this space represents the instantaneous state of the system, and the lines or curves represent the time evolution of these points. These portraits help us understand the behavior of the dynamical system, including fixed points, limit cycles, and attractors. And attraction is called a set of points in the state space towards which the dynamic system tends. In other words, if a system starts near this set of points, it will gravitate towards it. Attractors can take many forms such as Fixed Points, Limit Cycles, or more complex structures such as Strange Attractors. Transient chaos refers to a state in a dynamical system where the system is constrained for a limited period of time. It exhibits chaotic behavior, but eventually settles into an orderly behavior (usually gravity). This phenomenon is observed in many systems, where the system has an unpredictable and chaotic behavior at first, but over time it reaches a more stable pattern. Chaos in dynamic systems refers to a state in which the behavior of the system is very sensitive to It is the basic condition. Small changes in initial conditions can lead to large differences in system behavior. Chaotic systems have unpredictable and complex behavior, even if the governing equations are fully determinable. Strange gravity is a special type of gravity found in chaotic systems. Unlike simple attractions such as fixed points or limit cycles, these attractions have a complex and fractal structure. This means that they are infinitely detailed in dimensions, and the paths of the system around them can be very complex and unpredictable. In dynamic systems with fuzzy portraits, all these things can be understood with images, and in general, they help to understand the complex and unpredictable behavior of HNN dynamic systems. Chaos theory shows that even simple systems can have very complex behaviors that are sensitive to initial conditions.Dynamic behavior of the system for three states with initial values of $\alpha = 1, \beta = -1, \rho = 2, \mu_1 = \mu_2 = 1$ has been checked.Figure 9 for the HNN mode with synapse interference without EMR, where the phase portrait represents the periodic mode.

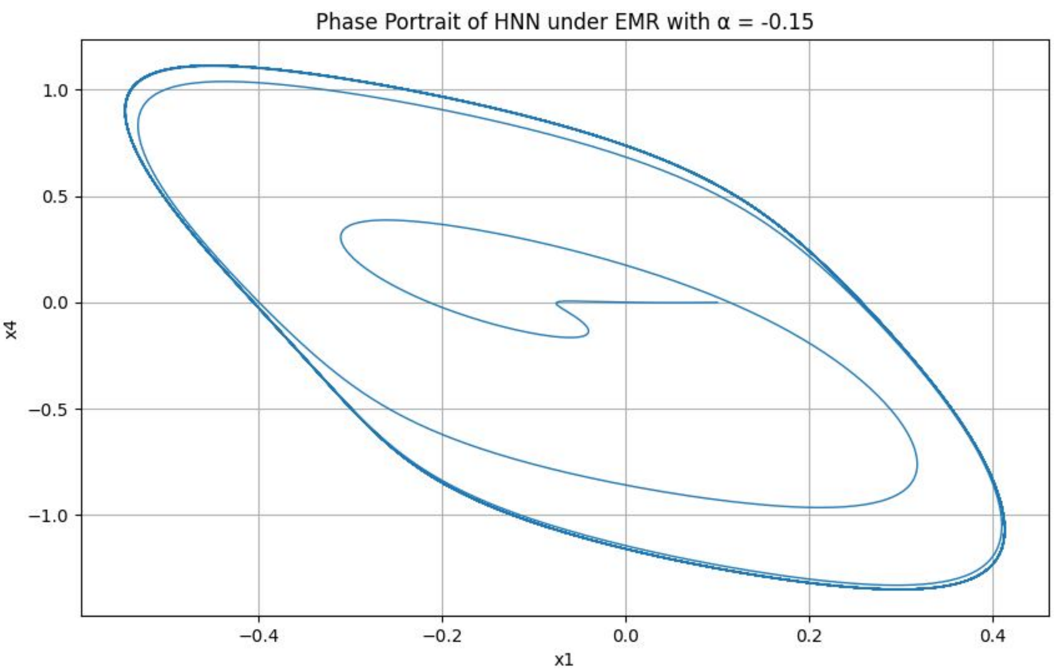

Fig 9. HNN with synapse interference without EMR

Figure 10 for the state of the neural network with synapse interference with single EMR, where the fuzzy portrait shows the state of transient chaos.

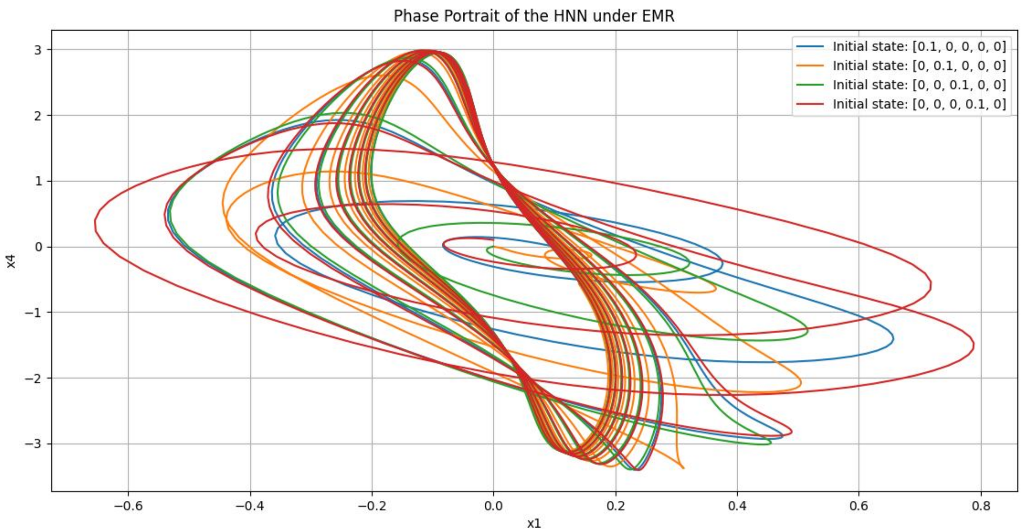

Fig. 10. Neural network with synapse interference with single EMR

Figure 11 for the state of the neural network with synapse interference with two EMRs, where the fuzzy portrait represents the state of chaos.

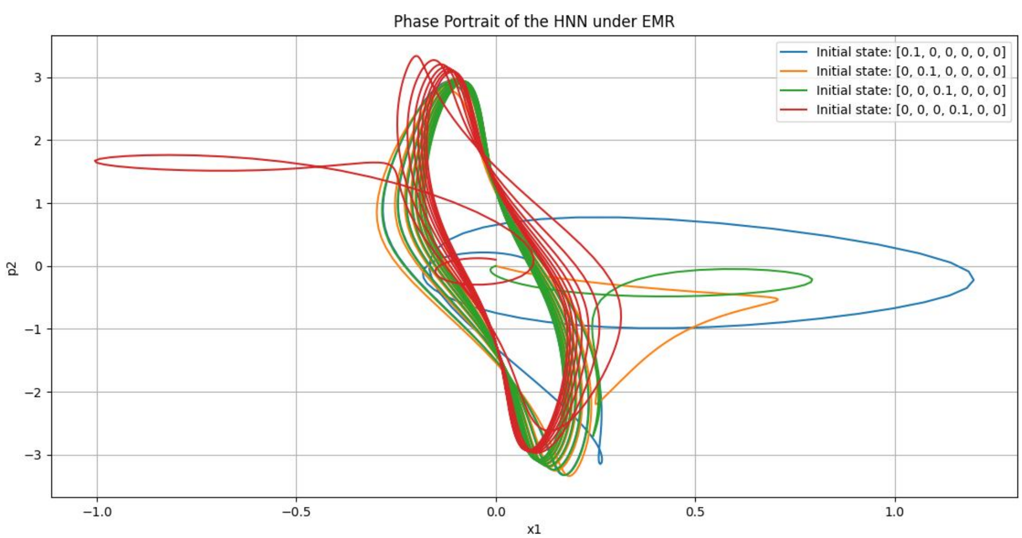

Fig 11. Neural network with synapse interference with two EMRs

If we change the value of $\alpha = -1$in Figure 11 to the value of $\alpha = -1.4$, the chaotic state of the tidal system will become a transient chaotic state as shown in Figure 12. Therefore, changing the initial value of the values directly affects the system state.

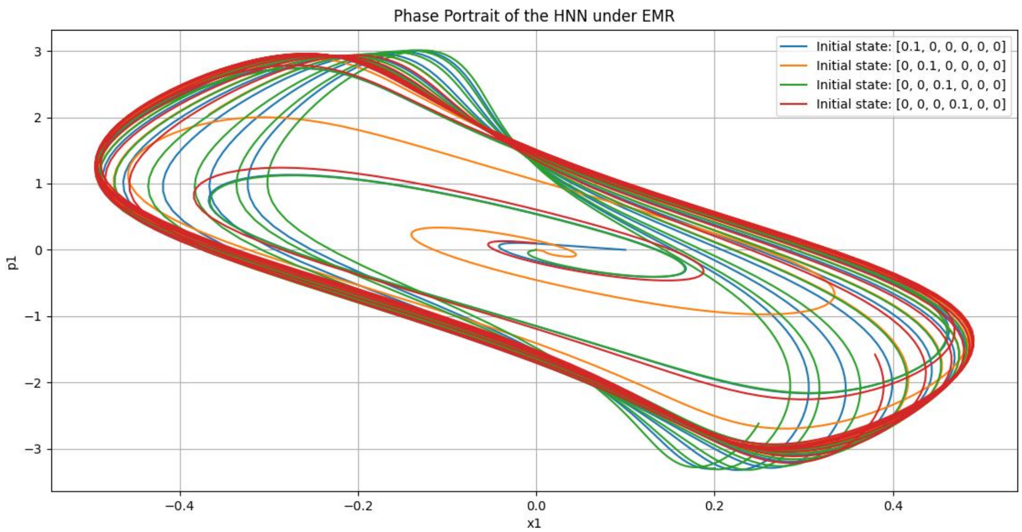

Fig 12. Neural network with synapse interference and with two EMRs and $\alpha = -1.4$

4- Circuit simulation with pspice

To design the HNN circuit based on mathematical equations and based on two neurons, and without EMR in Figure 13 and the EMR circuit itself is also seen in Figure 15.The HNN circuit based on two neurons and overlapping synapses with single EMR is shown in Figure 16 and the HNN circuit based on two neurons and overlapping synapses with two EMRs is shown in Figure 18.

4-1 Simulation of neural network circuit with synapse interference without EMR with pspice

It includes tanh circuit and synapse weight and the overall circuit is specified in diagram 13

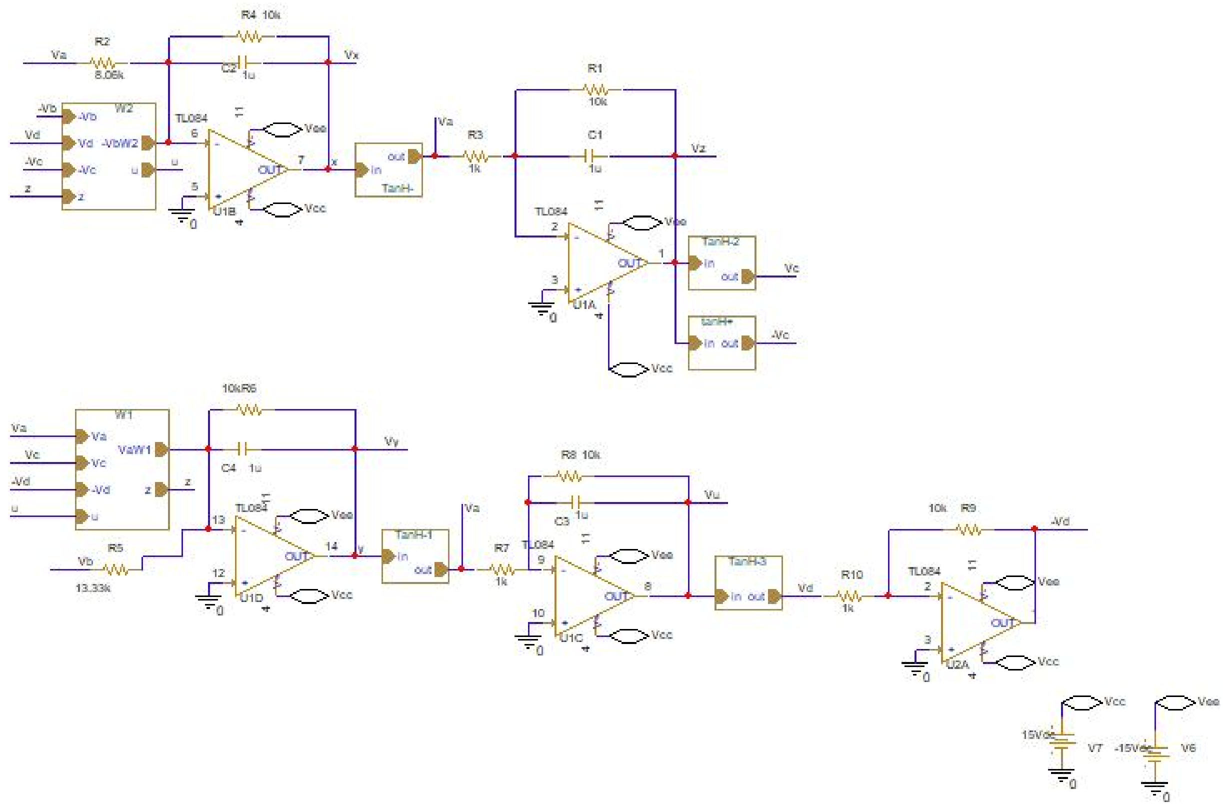

Fig 13.Memristive HNN circuit with synaptic interference and no EMR

All values and components of the circuit are specified in the figure.After designing the circuit and implementing the output of the circuit in Figure 13, the output of this dynamic system is periodic.which you can see in diagram 14.

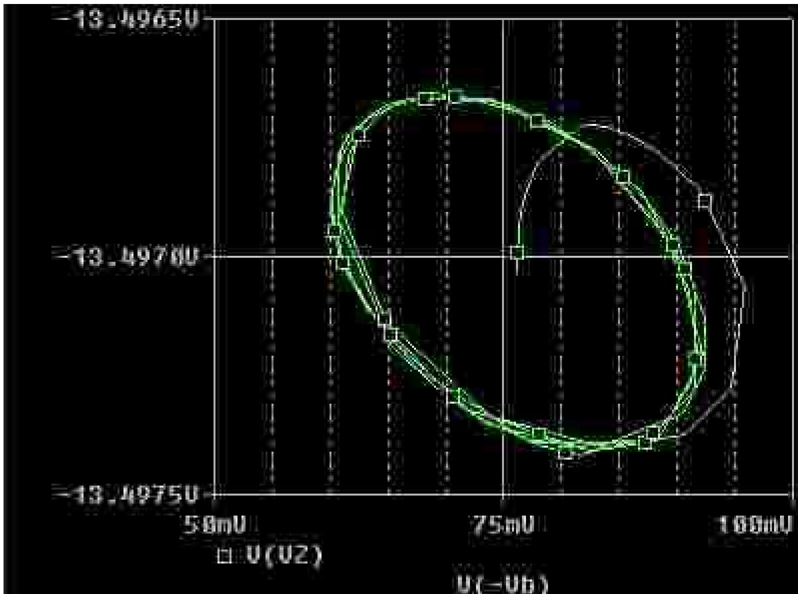

Fig 14 HNN circuit with synapse interference without EMR

4-2 Simulating neural network circuit with synapse interference with single EMRwith pspice

The EMR circuit is shown in Figure 15, and the HNN circuit with overlapping synapses and an EMR is designed in Figure 16.

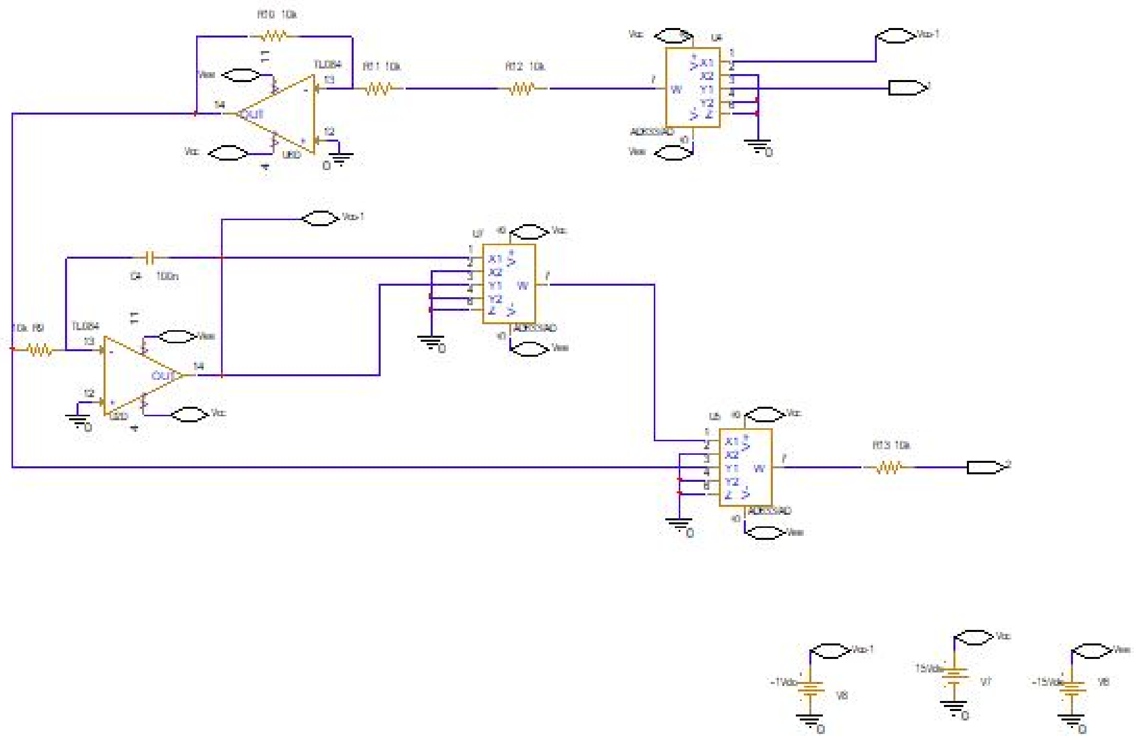

Fig 15.EMR circuit

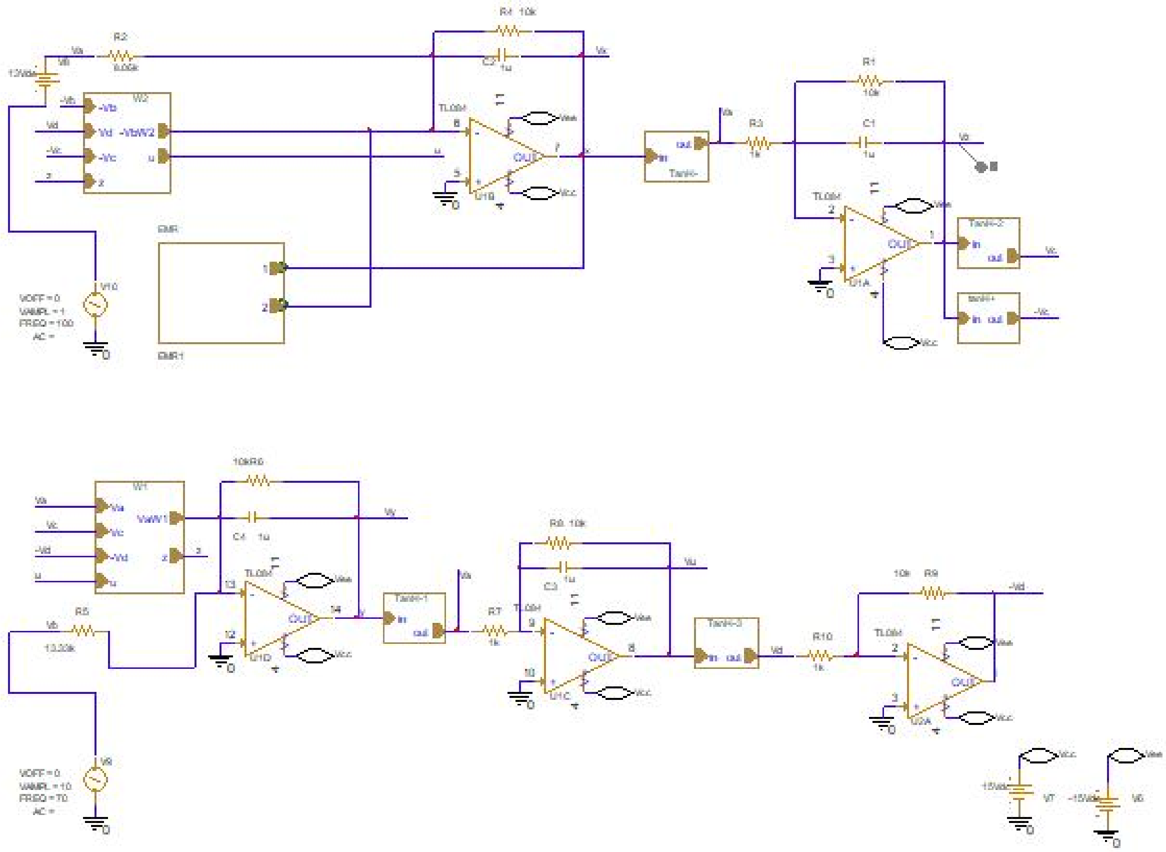

Fig 16HNN circuit with cross synapses and an EMR

After the execution of circuit 16, the output of circuit 17 is obtained, which produces a transient chaotic state.

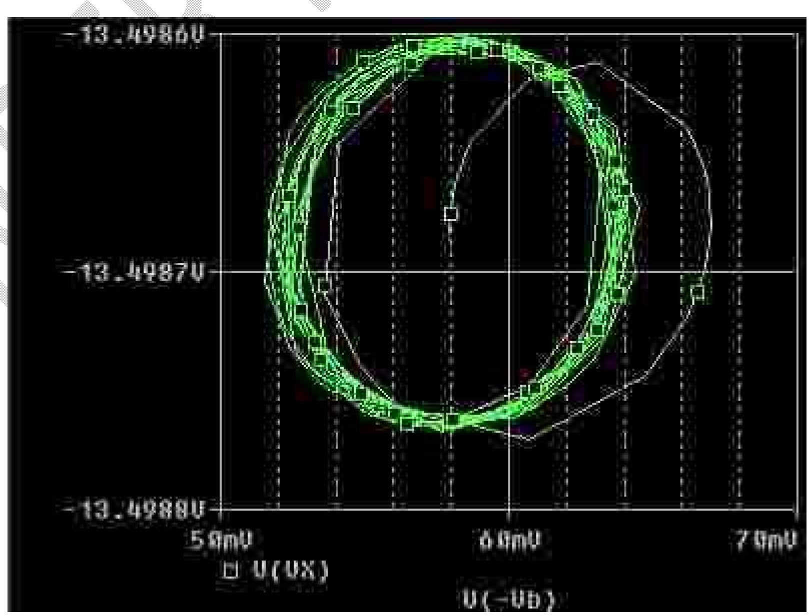

Fig 17 HNN circuit with synapse interference with single EMR

4-3 Simulation of neural network circuit with synapse interference with two EMRs with pspice

It is the same as circuit 16, only the circuit has two EMRs, which is designed in figure 18.

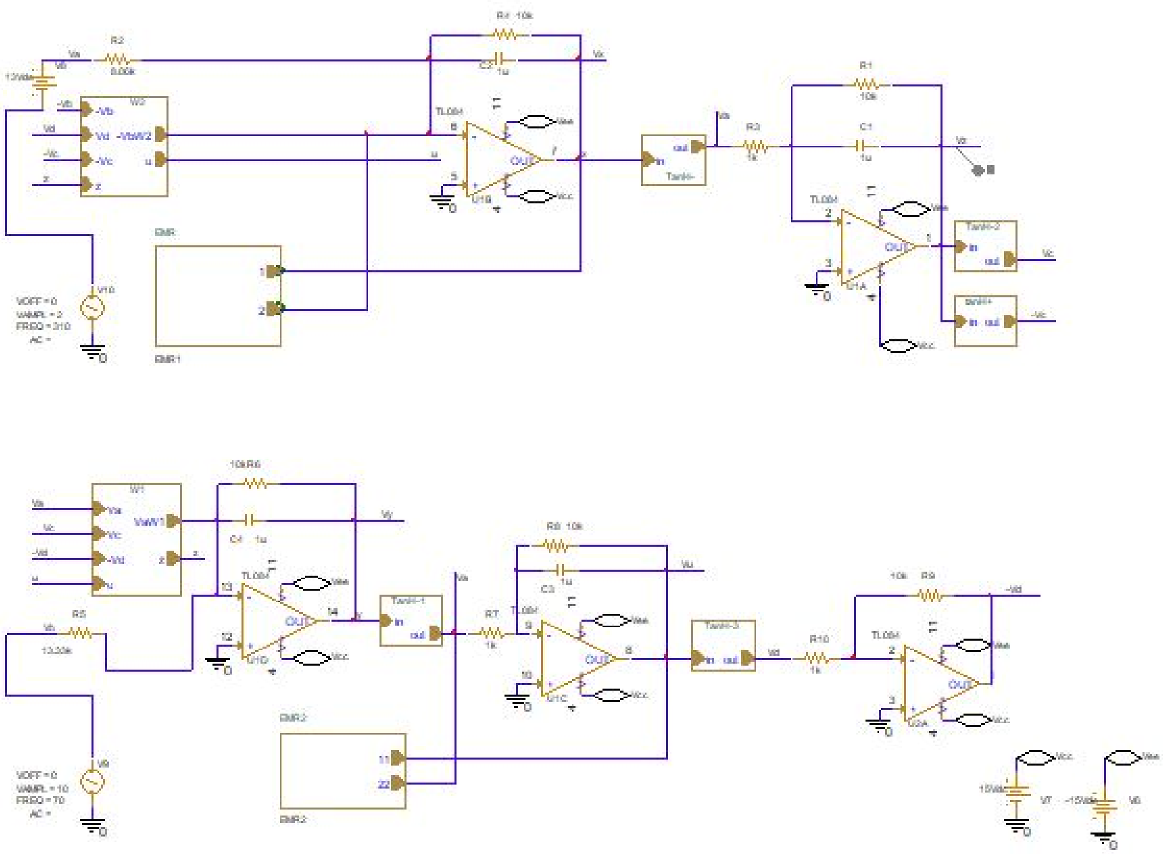

Fig18. HNN circuit with crossed synapses and two EMRs

The output of the circuit in Figure 18 is checked in two states with frequency 100 in Figure 19 and 130 in Figure 20. In Figure 19, the system has a chaotic state, but with the change of frequency in Figure 20, the system turns into a transient state of chaos, which shows that the system has a slight change.It has different modes and is sensitive to initial conditions

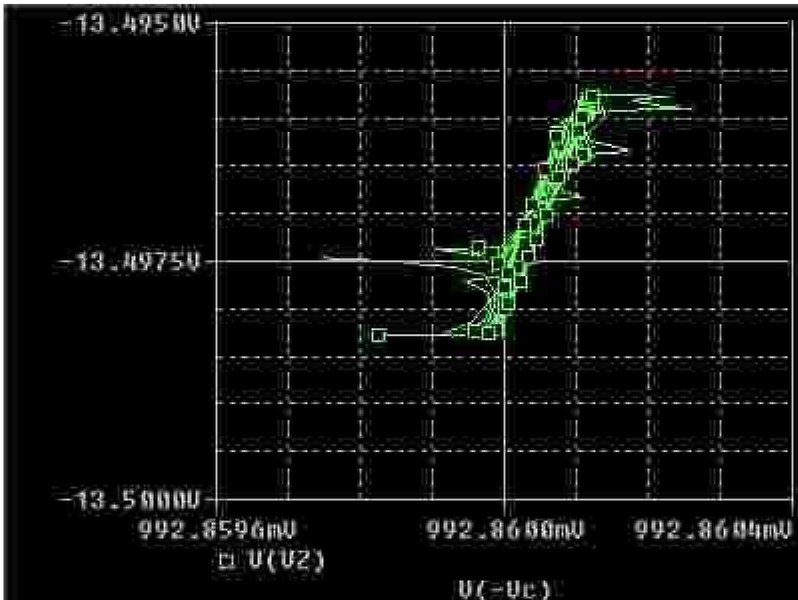

Fig 19.The output of the circuit in Figure 18 with a frequency of 100

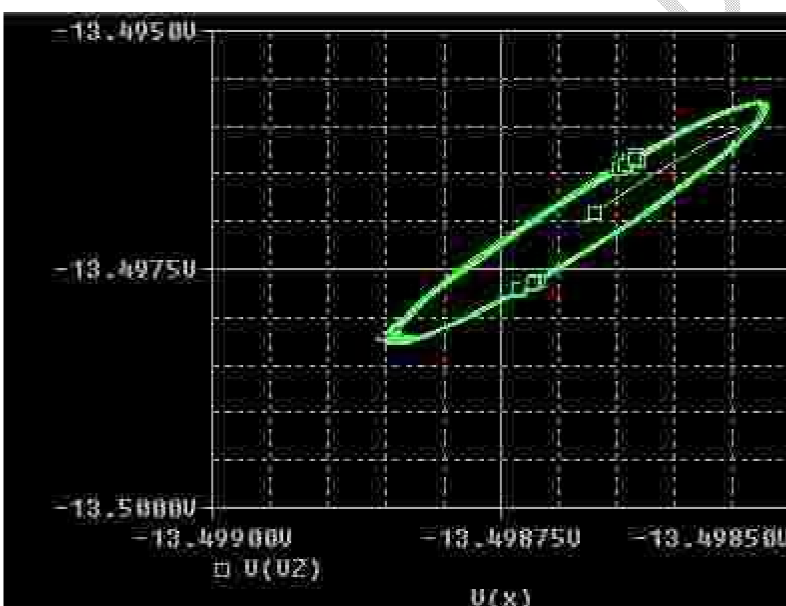

Fig 20.The output of the circuit in Figure 18 with a frequency of 130

5 CONCLUSION

In this research, the chaotic dynamics of HNN of two neurons with synapse interference were investigated for three body states of EMR, single EMR, and two EMRs.The second cases of single EMR affecting one neuron and the third case of double EMR affecting two neurons are presented respectively.Through the study, it was first found that chaotic phenomena can be observed by

Lyapunov power.By examining the phase portraits for all three states of chaos and multi-period phenomena appeared, and the properties of dual EMR suppression in the chaos of the system are also changed by changing the initial values.The results show that the external stimulus represented by EMR can affect the inherently chaotic system.It can both have more complex dynamic behavior and suppress complex chaotic behavior by changing parameters.Finally, the feasibility of this theory is confirmed by circuit and pspice experiments, and the results of this study are used in the control of chaotic phenomena.

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