Original Research Article

Dynamic Modeling of Money Supply and GDP: A State-Space Approach

ABSTRACT

Using data from 2008 to 2023, this study investigates the dynamic relationship between money supply (M2) and GDP through a state-space modeling approach. Addressing the question of how effectively money supply impacts GDP, the study is grounded in the theoretical framework that monetary policy influences economic output, a concept drawn from classical and Keynesian economic theories. The data, sourced from official economic reports, highlights significant variability in both M2 and GDP, with GDP exhibiting slightly lower volatility. The distributional characteristics of the variables show a slight positive skew and a flatter-than-normal distribution, suggesting potential deviations from typical economic trends.

The state-space model captures the relationship between M2 and GDP, indicating that money supply is an important explanatory factor for GDP. While the model's observation equation shows a low and statistically significant standard deviation of the error term—suggesting that it explains variations in GDP effectively—the state transition equation reveals a mostly deterministic evolution of unobserved state variables, though this result lacks full statistical significance and warrants cautious interpretation. The log-likelihood value supports a fair model fit, yet additional macroeconomic variables or alternate model structures may enhance the analysis.

The findings affirm the critical role of money supply in economic growth, consistent with economic theory. However, future research could explore more complex dynamics, including non-linear relationships and the effects of external shocks, to refine the understanding of the M2-GDP nexus. This study contributes empirical insights using a state-space framework, offering valuable implications for monetary policy and macroeconomic forecasting.

Keywords: Money Supply, Economic Growth, State-Space Model, Time-Varying Parameters, Nigeria, Monetary Policy

1. INTRODUCTION

The relationship between the money supply and economic growth has long been emphasised in macroeconomic theory and policy (Awwad, 2021). The significance of controlling this money supply to affect economic performance has long been emphasised by schools such as classical and monetarist. But contemporary perspectives, particularly in developing countries, indicate that this relationship is more nuanced and subject to alter over time (Choudhury, 2021).

An unusual setting for examining this relationship is Nigeria. Nigeria's economy has changed significantly during the past 20 years. A number of factors have contributed, including changes in monetary policy, remittances, and foreign aid (Williams, 2020). However, despite these adjustments, there is a dearth of comprehensive empirical research on how changes in the money supply affect Nigeria's economic expansion. This dearth of study is particularly noticeable when viewed from the viewpoint of a dynamic system(Koirala, 2013; Garieli et al., 2018; Pecchi, 1993).

Although the relationship between money supply and economic growth has been the subject of numerous studies, there is a dearth of research that particularly addresses Nigeria. Furthermore, there hasn't been much work done with approaches that include relationships that change over time(Zhang et al., 2023; Andreasen et al., 2016). A fixed link throughout time is often assumed Aby conventional econometric models. Regretfully, this may not adequately convey the intricacies of a quickly changing economy. Consequently, a dynamic parameter state-space model was employed in this study [9,10]. This model aids in evaluating how the money supply's impact on Nigeria's economic growth changes over time.

1.2 Research Objectives

The purpose of this study is to:

1. Make a model that illustrates how Nigeria's money supply and growth are related. This will employ a state-space methodology.
2. Examine the relationship's stability throughout time and identify significant turning points.

1.3 Significance of the Study

The results of this study will contribute to a better knowledge of Nigerian financial systems. For individuals who create policies, this can provide insightful information. The research emphasises the necessity of flexible monetary policies by concentrating on how the money supply and economic development evolve over time. These regulations can more effectively address the particular difficulties that a transitioning economy faces.

2. LITERATURE REVIEW

2.1 Theoretical Framework

Many macroeconomic theories have examined the relationship between the money supply and economic growth. The state-space models, endogenous growth theory, and the quantity theory of money were used in this investigation.

According to Fisher and Friedman's classical economic theory, price and economic output are directly and proportionately impacted by changes in the money supply (Backhouse, 2020). Strict control over the money supply is encouraged by monetarist policies in order to preserve steady economic growth.

Romer and Lucas' endogenous growth models suggest that the money supply can affect long-term economic growth through productivity, capital accumulation, and technological innovation (Menesha and Singh, 2023). According to the models, the money supply influences investments in technology and human capital, which in turn affects economic development indirectly.

A framework for depicting time-varying relationships is offered by the state-space approach (Camarero, Sapena, & Tamarit, 2020). First created for engineering, it was eventually extended to economics. State-space models allow for dynamic parameters that can alter over time, in contrast to classic econometric models that assume constant parameters. Because of this, they are especially useful for studying economies in transition, like Nigeria's.

2.2 Empirical Studies

Zhang et al. (2023) looked into how the offshore Renminbi market and capital flows were affected by Chinese monetary policy. They looked into the dynamic relationships between monetary policy, the Renminbi market, and capital flows using state space models. The study discovered strong links between policy changes and capital movements, demonstrating the consistency and efficacy of monetary policy in changing circumstances in offshore markets. The results emphasise the significance of taking into account both domestic and international issues when evaluating the impact of monetary policy.

Awe and Adepoju's (2020) study used discounted evolution variance to enhance the estimation of time-varying parameters in dynamic state space models. To facilitate more accurate modelling of dynamic systems with increasing uncertainty, they developed a Bayesian framework that includes discounted evolution variance. Even with discounted evolution variance, the Bayesian estimating approach produces reliable results that improve state space model predictions and shed light on time-varying systems.

Bhatta et al. (2021) used state space models to simulate several policy scenarios in order to investigate the policy implications of the impossible trinity, often known as the trilemma, in a small open economy. To comprehend trade-offs and policy choices within the context of the trilemma, the authors employed state space models. The simulation's results provide important insights for those making decisions about exchange rate management, monetary policy, and capital mobility. They also emphasised the difficulties small open economies face in striking a balance between desired economic outcomes and unrealistic trinity limits.

Using the Kalman filter technique, Arsad, Baharudin, and Rahman (2021) assessed the dynamic link between stock prices and macroeconomic parameters. Time-varying parameter estimate is made possible by this method, which also captures time-varying relationships. The findings provide light on how macroeconomic shifts affect the stock market's dynamic behaviour by demonstrating notable fluctuations in the correlation between stock prices and macroeconomic variables over time.

Using state space models, Bhatta, Nepal, Harvie, and Jayanthakumaran (2022) examined the uncovered interest parity (UIP) condition in a small open economy. In order to understand how UIP may be utilised in various economic situations, the authors employed state space models to take time-varying elements and structural changes in the economy into consideration. The results confirmed the UIP situation in the economy under study and emphasised the significance of time-varying economic determinants.

Hussain (2024) looked on anomalies in the conditional multifactor assets pricing model using a state space framework. Significant departures from the anticipated price behaviour were found during the analysis, indicating that conventional models might need to be modified to take these changes into consideration. The results of the study offered helpful information on asset pricing strategies.

To assess Cambodia's economic situation and identify the key determinants of economic success, Sa (2020) used a dynamic factor model. The model provided a thorough evaluation of Cambodia's economic status and highlighted important elements and patterns influencing its performance by combining data from multiple economic variables.

Using a nonlinear causality technique, Eltejaei and Montazeri Shoorekchali (2021) investigated the nonlinear relationship between inflation and money growth in Turkey. The results imply that conventional linear models were unable to adequately represent the complexities of this relationship and that an understanding of nonlinear causation is crucial for comprehending the economic environment.

Svanholm (2021) used a state-space approach to examine modifications to the modified Taylor Rule and Hungary's monetary policy framework. The approach effectively conveys the changing nature of monetary policy. The results shed light on how Hungary's monetary policy has evolved and how well adjusted policy directives have worked.

Yakubu and Suleiman (n.d.) used Bayesian VAR models and structural time series to compare how monetary policy tackled Nigeria's persistent inflation. The analysis revealed variations in how each model depicts the monetary policy response, which provided insight into the efficacy of different modelling techniques.

Wang et al.'s (2022) study looked at how China's monetary policy affected post-crisis financial stability. To evaluate its impacts, dynamic impact analysis is performed. The results gave guidance for developing post-crisis remedies and emphasised the crucial role that monetary policy plays in preserving financial stability.

In order to evaluate policy choices under different levels of uncertainty, Ajisafe et al. (2022) used a Markov-switching dynamic technique to identify the best fiscal and monetary policies for Nigeria. The results gave Nigerian authorities practical advice on how to carry out efficient monetary and fiscal policies.

Razmi, Moghadam, and Behname's (2021) study looked at how Iran's monetary policies affected the country's ability to produce renewable energy over the long run. A dynamic framework was employed to capture these impacts. The findings emphasised the necessity for focused policy interventions since they showed that monetary policy has a variety of effects on the production of renewable energy.

State space models were employed by Ahadiat et al. (2023) to aid in strategic mining decision-making. It examines the effects of economic issues on stock prices and offers data on these effects, which helps with strategic decision-making.

3. MATERIAL AND METHODS

3.1 Data Collection

For Nigeria, this analysis makes use of yearly data from 2000 to 2023. The information comprises:

Money Supply (M2): This refers to the total amount of money in circulation as well as different kinds of deposits.

GDP: The gross domestic product, adjusted for inflation, is calculated using constant prices.

Data was sourced from the Central Bank of Nigeria and the World Bank's World Development Indicators.

3.2 Model Specification

A state-space model with dynamic parameters is used to depict the relationship between money supply and economic growth over time. The model forms are presented in (1) and (2) as follows:

$$y_t = \beta_t x_t + \epsilon_t \quad (1)$$

$$\beta_t = \beta_{t-1} + \eta_t \quad (2)$$

where:

y_t is the GDP growth rate at time t,

x_t is the growth rate of money supply at time t,

β_t represents the time-varying parameter, and

ϵ_t and η_t are error terms.

The Kalman filter, a recursive technique that yields estimates of the unobserved time-varying parameter, β_t, is used to estimate the model.

3.3 Estimation Procedures

Maximum Likelihood Estimation (MLE) is used to estimate the state-space model's parameters. The statistical program R incorporates the Kalman filter, enabling effective estimate of the time-varying coefficients.

4. RESULTS AND DISCUSSION

4.1 Descriptive Statistics

Table 1: Summary Statistics, using the observations 2008 – 2023

Variable	M2	GDP
Mean	2.90E+09	6.47E+09
Median	2.54E+09	6.47E+09
Minimum	1.39E+09	4.58E+09
Maximum	5.49E+09	8.53E+09

Std. Dev.	1.23E+09	1.25E+09
C.V.	0.42378	0.19278
Skewness	0.77215	0.092921
Ex. kurtosis	-0.48503	-1.1906
5% Perc.	undefined	undefined
95% Perc.	undefined	undefined
IQ range	1.88E+09	2.07E+09

The distribution and characteristics of both economic variables over the observed period are summarised in Table 1, which displays the Money Supply (M2) and GDP summary data from 2008 to 2023.

The average GDP for M2 over this time period is roughly 6.47 billion, while the average money supply is roughly 2.90 billion. The distribution for M2 appears to be slightly tilted to the right, as indicated by the median being marginally lower than the mean. The median for GDP is quite near to the mean, indicating a distribution that is reasonably symmetrical. While GDP has fluctuated between 4.58 billion and 8.53 billion, M2 has fluctuated between 1.39 billion and 5.49 billion. The range shows how much these variables have changed over time. M2 and GDP both exhibit comparable degrees of variability around their respective averages, with GDP displaying somewhat greater variance. M2's C.V. is higher. compared to GDP, indicating that M2 is more erratic in relation to its mean value. GDP has very low skewness, suggesting near symmetry, whereas M2 has substantial positive skewness, suggesting a right-skewed distribution. While both GDP and M2 show negative kurtosis, GDP's flatness is more noticeable. The central 50% of GDP data has a broader dispersion than M2, as evidenced by GDP's marginally bigger IQR than M2.

According to the data, GDP is less volatile than its mean (as shown by the C.V.) but has a larger average value than M2. Both M2 and GDP have reasonably symmetric distributions, with M2 exhibiting a little greater positive skewness. Particularly for GDP, the lower kurtosis values imply that both distributions are less peaked and have thinner tails than a normal distribution.

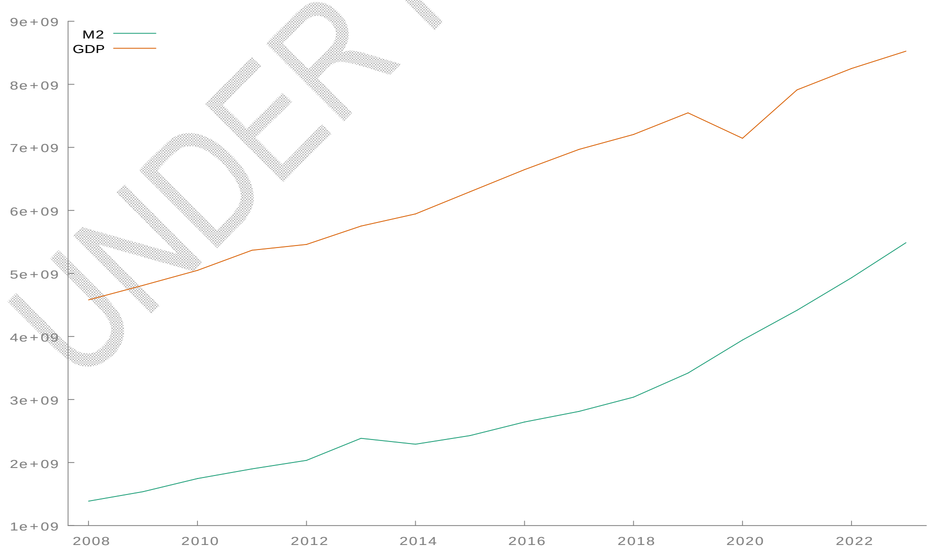

Fig. 1: Time plot for Money Supply and Gross Domestic Product, 2008-2023

Figure 1 illustrates the trend and connection between GDP (Gross Domestic Product), a more comprehensive indicator of the money supply, and M2 throughout time. Over time, M2 and GDP have both shown steady rising trends. This implies that the money supply and the economy have both been expanding. The relationship between M2 and GDP seems to be positive. GDP often rises in tandem with M2, and vice versa. This connection is frequently interpreted as an indication of economic expansion and advancement. Compared to the GDP growth rate, the M2 growth rate appears to have varied more. M2 increases more slowly than GDP during some times and more quickly during others.

4.2 State Space Modelling

Table 2: Observation equation

	Coefficient	std. error	z	p-value
stdev[1]	0.0325691	0.00615485	5.292	1.21e-07 ***

The results of the observation equation are displayed in Table 2. In a state-space model, observable data and unobserved state variables are connected by the observation equation. The calculated standard deviation of the error term in the observation equation is 0.0325691. This is the typical size of deviations from expected results caused by measurement error or noise. With a high z-statistic of 5.292 and an extraordinarily low p-value (1.21e-07), the standard deviation is clearly different from zero. The finding is highly significant at conventional levels (e.g., 1%, 5%, or even 0.1%) because to the low p-value. This demonstrates that the error term in the observation equation has a non-zero standard deviation and is highly precisely approximated.

Table 3: State transition equation

	Coefficient	std. error	z	p-value
stdev[1]	1.64459e-09	0.00651196	2.525e-07	1.0000

Log-likelihood = 21.1982

The state transition equation's results are shown in Table 3. The table gives statistical details on the coefficients related to the state transition equation, which describes the evolution of the unobserved state variables.

A highly predictable process is indicated by the extremely small coefficient value (1.64459×10^{-9}), which implies that the process noise is very near zero. The precision of the coefficient estimate is indicated by the standard error value of 0.00651196, which is 0.00651196. In this instance, the incredibly small coefficient value is accompanied by a comparatively significant standard error. This disparity suggests that there is a great deal of uncertainty surrounding the calculated coefficient, even though it is extremely small. The z-statistic is incredibly small, which is indicative of the coefficient's close to zero value. The coefficient is not substantially different from zero, according to the modest z-statistic. A p-value of 1.0000 indicates that the coefficient is not statistically significant, meaning that the null hypothesis cannot be discarded. Although this number should be interpreted in relation to other models or benchmarks, the log-likelihood value of 21.1982 in this case indicates a reasonable fit of the state-space model to the data.

The error term of the state transition equation is calculated to have a standard deviation of 1.64459e-09, which is very near to zero. This implies that there is very little random noise and that the process controlling the evolution of the state variables is nearly totally deterministic.

This coefficient is not statistically significant, though, as the p-value is 1.0000 and the z-statistic is quite small (2.525e−072.525e-072.525e−07). The model implies that there is no significant random variation in the state transition process, and this lack of relevance raises the possibility that the estimate of the near-zero standard deviation is unreliable.

Although the model's overall fit is reflected in the log-likelihood value of 21.1982, it is difficult to assess the model's adequacy on its own without comparing it to other models. Although this conclusion is not statistically significant, the results show that the state transition process is nearly completely deterministic, with little to no random noise. This could suggest that the process is predictable or that the model needs to be updated to accurately capture the state variables' dynamics.

5. CONCLUSION

Using a state-space model, this study sought to investigate the dynamic link between GDP growth and the expansion of the money supply (M2) between 2008 and 2023. The results show a number of significant revelations.

Both M2 and GDP exhibit notable variability over the research period, according to summary statistics. However, the coefficient of variation (C.V.) indicates that GDP growth is less variable than M2 growth. These variables' distributions have a negative excess kurtosis and are somewhat skewed to the right. This implies that they are flatter than distributions in general. The observation equation emphasises how important money supply expansion is in explaining changes in GDP.

Additionally, this equation's error term is statistically significant and has a modest standard deviation. This suggests that there is little unexplained variability in the model, which well captures the relationship between M2 and GDP growth. According to the state transition equation, the evolution of unobserved state variables—which could be a sign of underlying shocks or economic conditions—is primarily predictable. Interestingly, the transition error term's standard deviation is extremely near to zero. However, we should use caution when drawing this conclusion because this coefficient lacks statistical significance. It implies that even though our model assumes a deterministic transition, more research or better modelling may be required to capture the actual dynamics.

The log-likelihood score is 21.1982, indicating that the state-space model provides a reasonably good fit to the data. However, comparisons with other models could be useful for a more thorough assessment of its suitability.

The findings support the idea that monetary issues can affect economic performance and highlight the money supply as a major predictor of GDP growth. However, the nearly deterministic character of state transition and the sparse statistical findings highlight the possibility that other elements or alternative models may be required to adequately represent the complex relationships between money supply and GDP growth.

By examining non-linear dynamics and adding more macroeconomic variables, future studies could expand on these conclusions. Understanding the mechanisms underlying these observed associations may also be improved by using various modelling tools. Furthermore, it could be helpful to examine sub-periods or carry out robustness tests to validate these results in light of possible exogenous shocks and structural alterations during this time.

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2.

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REFERENCES

Ahadiat, A., Kesumah, F. S. D., Azhar, R., & Wisnu, F. K. (2023). Strategic decision-making on mining sector company stock prices and economic variable (state space model application). *International Journal of Energy Economics and Policy*, *13*(3), 177-184.

Ajisafe, R. A., Fasoye, K., Fatai, M. O., & AJIDE, F. M. (2022). Optimal Fiscal and Monetary Policy Under Uncertainty in Nigeria: A Markov-Switching Dynamic Approach. *Revista Romana de Economie*, *55*.

Ali, A., & Hassan, F. (2024). The Role of Economic Theory in Shaping Monetary Policy Decisions. *International Journal of Accounting, Finance, and Economic Studies*, *2*(1).

Andreasen MM, Fernández-Villaverde J, Rubio-Ramírez JF. The pruned state-space system for non-linear dsge models: Theory and empirical applications. FEDEA; 2016 Apr 21.

Arsad, Z., Baharudin, N. A., & Rahman, R. A. (2021, November). Estimating dynamic relationship between stock price and macroeconomic variables using Kalman filter approach. In *AIP Conference Proceedings* (Vol. 2423, No. 1). AIP Publishing.

Awe, O. O., & Adepoju, A. A. (2020). Bayesian estimation of time-varying parameters in dynamic state space models in the presence of discounted evolution variance. *Journal of the Nigerian Mathematical Society*, *39*(2), 183-209.

Awwad, R. F. J. (2021). THE IMPACT OF MONETARY POLICY ON ECONOMIC GROWTH IN DEVELOPING COUNTRIES FOR THE PERIOD 2010-2020 (MALAYSIA AS A MODEL). *International Journal of Research Science and Management*, *8*(8), 39-60.

Backhouse, R. (2020). The debate over Milton Friedman's theoretical framework: an economist's view. In *Economics and Language* (pp. 103-131). Routledge.

Bhatta, G. R., Nepal, R., Harvie, C., &Jayanthakumaran, K. (2022). Testing for the uncovered interest parity condition in a small open economy: A state space modelling approach. *Journal of Asian Economics*, *82*, 101525.

Bhatta, G. R., Nepal, R., Jayanthakumaran, K., & Harvie, C. (2021). Impossible trinity in a small open economy: A state-space model informed policy simulation.

Camarero, M., Sapena, J., & Tamarit, C. (2020). Modelling time-varying parameters in panel data state-space frameworks: An application to the Feldstein–Horioka puzzle. *Computational Economics*, *56*(1), 87-114.

Choudhury, M. A. (2021). Monetary and Fiscal Complementarities in Socioeconomic Sustainability. In *Monetary Policy, Islamic Finance, and Islamic Corporate Governance: An International Overview* (pp. 85-111). Emerald Publishing Limited.

Eltejaei, E., & Montazeri Shoorekchali, J. (2021). Investigating the relationship between money growth and inflation in Turkey: A nonlinear causality approach. *Journal of Money and Economy*, *16*(3), 305-322.

Gabrieli T, Pilbeam K, Wang T. (2018). Estimation of bubble dynamics in the Chinese real estate market: a State space model. *International economics and economic policy*. 15:483-99.

Hussain, H. (2024). Conditional Multifactor Assets Pricing Model: An Empirical Analysis of Anomalies in the State Space Framework. *Audit and Accounting Review*, *4*(1), 29-53.

Jouni Helske (2017). KFAS: Exponential Family State Space Models in R. *Journal of Statistical Software, 78*(10), 1-39. doi:10.18637/jss.v078.i10.

Koirala TP. (2013). Time-varying parameters of inflation model in Nepal: State space modeling. *NRB Economic Review*. *25*(2):66-77.

Ledenyov D, Ledenyov V. (2013). On the Stratonovich-Kalman-Bucy Filtering Algorithm Application for Accurate Characterization of Financial Time Series with Use of State-Space Model by Central Banks. Available at SSRN 2594333.

Mengesha, Z. D., & Singh, L. (2023). Human capital accumulation and economic growth of Ethiopian economy. *African Journal of Science, Technology, Innovation and Development*, *15*(2), 211-226.

Pecchi L. Time series evidence on the money supply process in the USA: A new state space modelling approach. Economic modelling. 1993 Jul 1;10(3):187-200.

Razmi, S. F., Moghadam, M. H., &Behname, M. (2021). Time-varying effects of monetary policy on Iranian renewable energy generation. *Renewable Energy*, *177*, 1161-1169.

Risse M, Ohl L. (2017). Using dynamic model averaging in state space representation with dynamic Occam's window and applications to the stock and gold market. *Journal of Empirical Finance*. *1*(44), 158-76.

Sa, K. (2020). Estimating Cambodials Economic Conditions by Dynamic Factor Model. *Asian Journal of Economics and Empirical Research*, *7*(2), 268-281.

Svanholm, D. (2021). Monetary Policy Regime Shifts in Hungary: A State-Space Approach: A study of a modified Taylor Rule in Hungary with thime-varying properties of monetary policy.

Wang, H., Xu, N., Yin, H., & Ji, H. (2022). The dynamic impact of monetary policy on financial stability in China after crises. *Pacific-Basin Finance Journal*, *75*, 101855.

Williams, N. (2020). Moving beyond financial remittances: The evolution of diaspora policy in post-conflict economies. *International Small Business Journal*, *38*(1), 41-62.

Yakubu, A. L. F. A., Suleiman, S. A. A. D., & AA, A. RESPONSE OF MONETRAY POLICY TO INFLATION PERSISTENCE IN NIGERIA: A COMPARISON OF STRUCTURAL TIME SERIES MODEL AND BAYASIAN VAR MODEL.

Zhang, W., Zhang, J., & Zhang, H. (2023). The Interplay of Chinese Monetary Policy, the Offshore Renminbi Market, and Capital Flows: An Empirical Analysis via State Space Models. *Applied Mathematics and Nonlinear Sciences*, *9*(1).

APPENDIX: STATA CODE

```
import excel "C:\Users\Olugbenga\Downloads\NigeriaData.xlsx") firstrow clear
tsset Year
gen log_GDP = log(GDP)
gen log_M2 = log(M2)

* Calculate growth rates (percentage change)
gen GDP_growth = (log_GDP - log_GDP[_n-1]) / log_GDP[_n-1]
gen M2_growth = (log_M2 - log_M2[_n-1]) / log_M2[_n-1]

* Drop missing values generated by the differencing
drop if missing(GDP_growth) | missing(M2_growth)

* Define the state-space model
sspace (GDP_eq: GDP_growth = {alpha}*M2_growth, state(beta)) ///
      (state_eq: beta = L.beta, state noconstant)

* Set constraints
constraint 1 [GDP_eq]alpha = 1
constraint 2 [state_eq]L.beta = 1

* Run the model with the constraints
sspace (GDP_eq: GDP_growth = {alpha}*M2_growth, state(beta)) ///
      (state_eq: beta = L.beta, state noconstant), constraints(1/2)
estimate

display _b[beta]
predict beta_t, state
list Date beta_t

twoway (line beta_t Date), title("Time-Varying Relationship Between M2 and GDP Growth")

predict resids, resid
tslineresids, title("Residuals from State-Space Model")
```