Assessing Performance of Estimation Techniques in Time Series Analysis when Trend-Cycle Component is Linear

Abstract: Two decomposition techniques are Buys-Ballot and least square techniques are presented in this study. The two important patterns that may be discussed are trend and seasonality and two competing models are additive and multiplicative models. The trend-cycle component is linear. The emphasis is to assess the performance of Buys-Ballot estimates and least square estimates using accuracy measures (Mean Error (ME), Mean Square Error (MSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). Results show that the two estimation techniques are very good in estimating the linear trend parameters and seasonal effects when the model for decomposition is additive. It differs for multiplicative model.

Keywords

Additive Model, Multiplicative Model, Linear Trend Curve, Buys-Ballot Technique, Least Square Technique, Accuracy Measures

1.0 Introduction

Descriptive time series involves the separation of an observed time series into components consisting of trend, the seasonal, cyclical and irregular components. This method includes the examination of trend, seasonality, cycles, turning points, changes in level, trend and scale and so on that may influence the series. This is an important preliminary to modelling, when it has to be decided whether and how to seasonally adjust, to transform, and to deal with outliers and whether to fit a model. In the examination of trend, seasonality and cycles, a time series is often described as having trends, seasonal effects, cyclic pattern and irregular or random component Iwueze and Nwogu [1]. In time series analysis, it is assumed that the data consist of a systematic pattern and random error. The systematic pattern is made up of the trend, seasonal, cyclical and irregular components. These four components may or may not coexist in real life data.

Additionally, in identifying the pattern, the two important goals of the series are better achieved if and only if the correct model is used. The specific functional relationship between these components can assume different forms. Time series models often used for decomposition are ;

Additive Model: $X_t = T_t + S_t + C_t + I_t$ (1)

Multiplicative Model: $X_t = T_t \times S_t \times C_t \times I_t$ (2)

Mixed Model: $X_t = T_t \times S_t \times C_t + I_t$ (3)

For short term period in which cyclical and trend components are jointly combined Chatfield [2] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \tag{4}$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \tag{5}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \tag{6}$$

Using equations (4) or (5) or (6) we can estimate the three components of our model and therefore decompose the series into its component parts. A summary of the traditional method of decomposition of the time series will be presented in section 1.1

1.1 Traditional Method of Decomposition

The major task of the analyst dealing with the series for descriptive purposes is to segregate each component in so far as this is possible. By isolating individual components, the impact of each may be assessed Chatfield [2]. Either of the models (4) or (5) or (6) may be employed to effect the decomposition. The first step will normally be to estimate and then to eliminate trend-cycle (M_t) for each time period from the original data either by subtraction or division. The resulting time series after elimination the trend-cycle (M_t) is the de-trended series and expresses the effects of the season and irregular components. The de-trended series is expressed mathematically as:

$$X_t - \hat{M}_t \tag{7}$$

for the additive model or

$$X_t / \hat{M}_t \tag{8}$$

for the multiplicative model or

$$X_t / \hat{M}_t \tag{9}$$

for the mixed model

The seasonal effect is obtained by estimating the average of the de-trended series at each season. The de-trended, de-seasonalized series is obtained as

$$X_t - \hat{M}_t - \hat{S}_t \tag{10}$$

for the additive model,

$$X_t / \left(\hat{M}_t \hat{S}_t\right) \tag{11}$$

for the multiplicative model,

$$X_t / \left(\hat{M}_t \hat{S}_t\right) \tag{12}$$

for the mixed model.

The least square technique is known to be associated with difficulties in computations and at same time does not give an insight for choosing appropriate model in time series decomposition and detection of presence of seasonal indices in time series data

However, by arranging a time series of length n into m rows and s column Iwueze et.al [3] demonstrated that the row, column and overall means and variances of the Buys-Ballot table can be used to (1) determine the appropriate model for decomposition of any study series (2) choose the appropriate transformation (3) obtain the estimates of trend parameter (4) estimate seasonal indices of the entire series (5) detect the presence of seasonal indices. Particularly, for a seasonal time series data, the rows are periods/years while the columns are seasons. This two-dimensional arrangement of a series is known as the Buys-Ballot table.

For details of this Buys-Ballot technique see Iwueze *et.al* [3], Nwogu *et.al* [4], Dozie and Uwaezuoke [5], Dozie and Ibebuogu [6], Dozie *et.al* [7], Dozie and Ijomah [8], Dozie and Nwanya [9], Dozie [10], Dozie and Uwaezuoke [11], Dozie and Ihekuna [12], Dozie and Ibebuogu [13], Akpanta and Iwueze [14]. Dozie and Ihekuna [15], Dozie [16] and Dozie and Uwaezuoke [17]

The Buys-Ballot method proposed by Iwueze and Nwogu [1] for short period series in which trend-cycle components are jointly combined; the tests developed in this study are based on this assumption. In their results, on the basis of which the proposition for choice of appropriate model was made, they showed that, for the selected trending curves, the seasonal variances depend only on the trend parameters for the additive model and both trend parameters and seasonal indices for the multiplicative model. Since the Buys-Ballot estimates of trend parameters and seasonal indices can be obtained from least squares technique, it has become necessary to assess the performance of Buys-Ballot technique to that of least square technique

2.0 Methodology

2.1 Existing Methods of Comparing Buys-Ballot Technique and Least square Technique.

The comparison of Buys-Ballot estimates and least square estimates are done directly using accuracy measures. Some of existing techniques include the Mean Error (ME), Mean Square Error (MSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE).

The comparison is usually based on the following summary statistics

1 Mean Error (ME) $= \frac{1}{W}\sum_{i=1}^{W} e_i$

2 Mean Square Error (MSE) $= \frac{1}{W}\sum_{i=1}^{W} e_i^2$

3 Mean Absolute Error (MAE) $= \frac{1}{W}\sum_{i=1}^{W} |e_i|$

4 Mean Absolute Percentage Error (MAPE) $= 100\left(\frac{1}{W}\sum_{i=1}^{W} \frac{|e_i|}{\phi_i}\right)$

where $\phi_i, i = 1, 2, \ldots W$. These measures of accuracy may be defined in terms deviations of the parameters estimates from the corresponding actual values applied in the stimulations.

$e_i = \phi_i - \hat{\phi}_i$ is defined as the error made in estimating $\phi_i, i = 1, 2, \ldots W$.

In these definitions, the comparison of parameters estimates is done using the actual and estimated values of the parameters.

The summary of the Buys-Ballot estimates of trend parameters and seasonal indices obtained by Iwueze and Nwogu [1] are listed in Table 1 Linear trend curve

Table 1: Buys-Ballot Estimates of Parameters of the Linear Trend and Seasonal Indices

Parameter	Additive Model	Multiplicative Model
a	$a^* + b^*\left(\frac{s-1}{2}\right)$	$a^* + b^*\left(s - \frac{C_1}{s}\right)$
b	$\frac{b^*}{s}$	$\frac{b^*}{s}$
s_j	$\bar{X}_{.j} - \left(a^* + \frac{b^*}{2}(n-s+2j)\right)$	$\hat{s}_j = \frac{\hat{\sigma}}{b^*\sqrt{\frac{n(n+s)}{12}}}$

Source: Iwueze and Nwogu (2014)

We note the following from Table 1

1 a^*, b^* and c^* are estimates obtained from the regression equations of row averages on row number.

2 Additive and Multiplicative models give different estimates

3 $C_1 = \sum_{j=1}^{s} j s_j$

3.0 Simulation examples

The purpose of this section is to present empirical examples to illustrate the application of the decomposition methods.

3.1 Simulations Results using the Additive Model

The simulated series used consists 100 data set of 120 observations each simulated from

$$X_t = (a+bt)+S_t+e_t,\ with\ a=1,\ b=0.02,\quad e_t \sim N(0,\ 1).\ with\ S_1=0.10\ S_2=-0.89,\ S_3=-0.15,$$

$$S_4=-1.22,\ S_5=-0.09,\ S_6=-0.97,\ S_7=-2.14,\ S_8=-0.73\ S_9=0.64\ S_{10}=1.96,\ S_{11}=2.34,\ S_{12}=1.16$$

3.2 Simulations Results using the Multiplicative Model

The simulated series used consists 100 data set of 120 observations each simulated from

$$X_t = (a + bt) \times S_t \times e_t, \; with\, a = 1, \; b = 0.02, \quad e_t \sim N(1, \sigma = 1.0). \; with\, S_1 = 0.98, \; S_2 = 0.80, \; S_3 = 0.88,$$

$$S_4 = 1.04, \; S_5 = 0.96, \; S_6 = 1.22, \; S_7 = 1.27, \; S_8 = 1.32 \; S_9 = 0.96 \; S_{10} = 0.80, \; S_{11} = 0.88, \; S_{12} = 0.89$$

Using the MINITAB 17.0 version. The computational procedure for the Buys-Ballot estimates of trend parameters and seasonal indices are done using the expression in Table 1. For each of the estimation methods under consideration, the estimates of the error means and standard deviations are calculated from the residuals from the fitted models. The deviations of these estimates from the parameters used in the simulations are given in Appendix A for the additive model and Appendix B for the multiplicative model, while the summary statistics for the 100 simulations are given in Table 2 for the additive model and Table 3 for the multiplicative model.

Table 2: Summary statistics for additive model (a = 1 and b = 0.02)

Series	ME		MAE		MSE	
	LSE	BBE	LSE	BBE	LSE	BBE
1	-0.007	-0.007	0.222	0.228	0.041	0.043
2	0.003	0.002	0.203	0.204	0.078	0.080
3	0.000	0.001	0.219	0.221	0.011	0.011
4	0.001	0.002	0.176	0.179	0.096	0.094
5	0.007	0.006	0.337	0.335	0.038	0.044
6	0.033	0.031	0.143	0.152	0.049	0.054
7	0.000	0.002	0.167	0.171	0.076	0.079
8	0.000	0.000	0.143	0.140	0.097	0.095
9	0.012	0.011	0.171	0.169	0.046	0.054
10	-0.011	-0.010	0.132	0.129	0.076	0.084
11	0.009	0.007	0.228	0.223	0.023	0.021
12	0.006	0.006	0.176	0.180	0.086	0.094
13	0.001	0.001	0.181	0.181	0.097	0.094
14	0.023	0.023	0.145	0.149	0.074	0.076
15	0.000	0.001	0.133	0.129	0.038	0.042
16	0.001	0.002	0.127	0.125	0.076	0.081
17	-0.005	-0.006	0.188	0.191	0.032	0.029
18	0.012	0.012	0.564	0.564	0.056	0.063

19	0.000	0.000	0.919	0.922	0.012	0.011
20	0.000	0.001	0.675	0.677	0.131	0.128

Table 2: Summary statistics for additive model (a = 1 and b = 0.02)

Series	ME		MAE		MSE	
	LSE	BBE	LSE	BBE	LSE	BBE
21	0.009	0.007	0.223	0.228	0.043	0.040
22	0.002	0.002	0.765	0.769	0.091	0.097
23	0.000	0.000	0.819	0.821	0.019	0.018
24	0.003	0.001	0.176	0.179	0.016	0.014
25	0.008	0.005	0.337	0.335	0.039	0.044
26	0.030	0.029	0.443	0.452	0.051	0.054
27	0.000	0.002	0.160	0.173	0.079	0.079
28	0.000	0.000	0.183	0.180	0.097	0.095
29	0.012	0.011	0.171	0.169	0.040	0.044
30	-0.011	-0.010	0.132	0.129	0.076	0.084
31	0.009	0.007	0.228	0.223	0.023	0.021
32	0.006	0.006	0.176	0.180	0.089	0.094
33	0.001	0.001	0.181	0.181	0.097	0.098
34	0.023	0.020	0.145	0.149	0.074	0.071
35	0.000	0.000	0.133	0.129	0.038	0.040
36	0.001	0.002	0.127	0.125	0.076	0.080
37	-0.005	-0.006	0.188	0.191	0.092	0.095
38	0.012	0.014	0.566	0.566	0.086	0.082
39	0.000	0.000	0.910	0.922	0.018	0.020
40	0.000	0.000	0.675	0.677	0.139	0.135

Table 2: Summary statistics for additive model (a = 1 and b = 0.02)

Series	ME		MAE		MSE	
	LSE	BBE	LSE	BBE	LSE	BBE
41	0.000	0.000	0.233	0.231	0.040	0.043
42	0.000	0.003	0.760	0.761	0.098	0.097
43	0.001	0.000	0.820	0.821	0.011	0.011
44	0.002	0.001	0.176	0.179	0.086	0.084
45	0.006	0.007	0.339	0.335	0.039	0.044
46	0.000	0.004	0.444	0.451	0.051	0.054
47	0.000	0.002	0.160	0.173	0.079	0.079
48	0.000	0.000	0.183	0.180	0.097	0.095
49	0.010	0.011	0.171	0.169	0.040	0.044
50	-0.010	-0.010	0.132	0.129	0.076	0.084
51	0.000	0.004	0.228	0.223	0.023	0.021
52	0.006	0.006	0.176	0.180	0.086	0.094

53	0.001	0.001	0.181	0.181	0.097	0.094
54	0.023	0.020	0.145	0.149	0.074	0.076
55	0.000	0.000	0.133	0.129	0.038	0.042
56	0.001	0.002	0.127	0.125	0.076	0.081
57	-0.005	-0.006	0.188	0.191	0.092	0.099
58	0.012	0.014	0.561	0.560	0.086	0.083
59	0.008	0.005	0.910	0.920	0.018	0.016
60	0.009	0.005	0.212	0.332	0.139	0.129

Table 2: Summary statistics for additive model (a = 1 and b = 0.02)

Series	ME		MAE		MSE	
	LSE	BBE	LSE	BBE	LSE	BBE
61	0.006	0.009	0.633	0.731	0.050	0.043
62	0.008	0.005	0.768	0.761	0.078	0.057
63	0.001	0.000	0.819	0.821	0.011	0.011
64	0.003	0.001	0.170	0.170	0.085	0.084
65	0.005	0.007	0.339	0.335	0.040	0.044
66	0.001	0.004	0.440	0.452	0.051	0.054
67	0.001	0.002	0.160	0.173	0.079	0.079
68	0.000	0.000	0.183	0.180	0.097	0.095
69	0.011	0.011	0.171	0.169	0.040	0.044
70	0.010	0.010	0.132	0.129	0.076	0.084
71	0.000	0.004	0.228	0.223	0.023	0.021
72	-0.006	-0.006	0.176	0.180	0.086	0.094
73	0.008	0.005	0.181	0.181	0.091	0.094
74	0.023	0.020	0.145	0.149	0.075	0.078
75	0.004	0.000	0.133	0.129	0.038	0.042
76	0.004	0.002	0.129	0.125	0.076	0.081
77	-0.005	-0.006	0.188	0.191	0.095	0.097
78	0.011	0.010	0.559	0.560	0.081	0.083
79	0.008	0.002	0.912	0.920	0.016	0.018
80	0.009	0.009	0.218	0.332	0.179	0.169

Table 2: Summary statistics for additive model (a = 1 and b = 0.02)

Series	ME		MAE		MSE	
	LSE	BBE	LSE	BBE	LSE	BBE
81	0.008	0.007	0.701	0.731	0.055	0.043
82	0.004	0.000	0.760	0.761	0.034	0.032
83	-0.004	-0.004	0.815	0.821	0.011	0.014
84	0.001	0.004	0.173	0.170	0.083	0.084
85	0.005	0.007	0.339	0.335	0.041	0.044
86	0.002	0.004	0.442	0.452	0.053	0.054

87	0.000	0.002	0.160	0.173	0.080	0.079
88	0.000	0.000	0.183	0.180	0.096	0.095
89	0.010	0.014	0.171	0.169	0.043	0.044
90	0.012	0.010	0.130	0.129	0.076	0.084
91	0.000	0.000	0.224	0.223	0.020	0.021
92	-0.008	-0.006	0.176	0.180	0.086	0.094
93	0.005	0.005	0.181	0.181	0.093	0.094
94	0.023	0.020	0.145	0.149	0.078	0.078
95	0.003	0.001	0.133	0.129	0.038	0.042
96	0.002	0.002	0.129	0.125	0.071	0.081
97	-0.005	-0.006	0.188	0.191	0.096	0.097
98	0.011	0.010	0.562	0.560	0.093	0.083
99	0.008	0.006	0.915	0.920	0.016	0.018
100	0.002	0.002	0.217	0.332	0.179	0.169

Table: 3 Summary Statistics for Multiplicative Model (a = 1 and b = 0.02)

S/N	ME		MAE		MSE		MAPE	
	LSE	BBE	LSE	BBE	LSE	BBE	LSE	BBE
1	0.000	0.000	-0.001	0.006	0.000	0.000	0.876	5.097
2	0.000	0.000	0.001	0.004	0.000	0.000	0.442	3.098
3	-0.001	-0.001	0.000	0.007	0.001	0.001	0.127	9.908
4	0.001	0.001	0.001	0.007	0.001	0.001	0.312	2.223
5	0.000	0.000	0.001	0.003	0.000	0.000	0.112	6.098
6	0.000	0.000	0.000	0.004	0.000	0.000	0.879	3.876
7	0.000	0.000	0.000	0.003	0.000	0.000	0.112	2.112
8	0.001	0.001	0.000	0.007	0.001	0.001	0.123	3.337
9	0.000	0.000	0.001	0.003	0.000	0.000	0.765	3.998
10	0.000	0.000	0.001	0.007	0.000	0.000	1.112	2.112
11	0.000	0.000	0.001	0.007	0.000	0.000	1.002	2.119
12	0.001	-0.001	0.000	0.004	0.001	0.001	0.987	2.002
13	0.000	0.000	0.000	0.005	0.000	0.000	0.132	1.654
14	0.000	0.000	-0.001	0.003	0.000	0.000	0.654	2.098
15	0.000	0.000	0.001	0.004	0.000	0.000	0.765	1.008
16	0.000	0.000	0.000	0.004	0.000	0.000	0.998	3.098
17	0.000	0.001	-0.001	0.005	0.000	0.001	0.654	4.098
18	0.000	0.000	-0.001	0.005	0.000	0.000	1.987	3.987
19	0.000	0.001	0.000	0.004	0.000	0.001	0.876	3.112
20	0.000	0.000	-0.001	0.007	0.000	0.000	0.987	1.321

Table: 3 Summary Statistics for Multiplicative Model (a = 1 and b = 0.02)

S/N	ME		MAE		MSE		MAPE	
	LSE	BBE	LSE	BBE	LSE	BBE	LSE	BBE

21	0.001	0.001	-0.001	0.004	0.001	0.001	1.870	2.097
22	0.000	0.000	0.001	0.009	0.000	0.000	0.442	3.998
23	0.001	-0.001	0.000	0.003	0.001	0.001	0.327	9.908
24	0.001	0.001	0.001	0.004	0.001	0.001	0.812	2.223
25	0.000	0.000	0.001	0.003	0.000	0.000	0.312	2.098
26	0.000	0.000	0.000	0.007	0.000	0.000	0.879	1.876
27	0.000	0.000	0.000	0.009	0.000	0.000	0.112	1.112
28	0.001	0.001	0.000	0.002	0.001	0.001	0.123	4.330
29	0.000	0.000	0.001	0.003	0.000	0.000	0.765	3.998
30	0.000	0.000	0.001	0.007	0.000	0.000	1.112	2.112
31	0.000	0.000	0.001	0.007	0.000	0.000	1.002	1.110
32	0.000	0.001	0.000	0.004	0.000	0.001	0.987	2.009
33	0.000	0.000	0.000	0.005	0.000	0.000	0.132	2.654
34	0.000	0.000	-0.001	0.003	0.000	0.000	0.654	1.098
35	0.000	0.000	0.001	0.004	0.000	0.000	0.765	1.008
36	0.000	0.000	0.000	0.004	0.000	0.000	0.187	3.098
37	0.000	0.001	-0.001	0.005	0.000	0.001	1.654	4.098
38	0.000	0.000	-0.001	0.005	0.000	0.000	1.432	3.987
39	0.000	0.001	0.000	0.008	0.000	0.001	0.231	3.112
40	0.001	0.000	-0.001	0.002	0.001	0.000	1.543	5.321

Table: 3 Summary Statistics for Multiplicative Model (a = 1 and b = 0.02)

S/N	ME		MAE		MSE		MAPE	
	LSE	BBE	LSE	BBE	LSE	BBE	LSE	BBE
41	0.000	0.000	0.001	0.003	0.000	0.000	1.234	2.009
42	0.001	0.000	0.000	0.004	0.001	0.000	0.564	6.009
43	0.001	-0.001	0.000	0.004	0.001	0.001	0.543	3.007
44	0.001	0.001	0.001	0.002	0.001	0.001	0.321	2.987
45	0.000	0.000	0.001	0.002	0.000	0.000	0.809	8.098
46	0.000	0.000	0.000	0.006	0.000	0.000	0.123	1.129
47	0.000	0.000	0.000	0.008	0.000	0.000	0.118	1.112
48	0.000	0.000	0.000	0.001	0.000	0.000	0.129	4.330
49	0.000	0.000	0.001	0.007	0.000	0.000	0.766	3.998
50	0.000	0.000	0.001	0.003	0.000	0.000	1.119	2.112
51	0.000	0.000	0.001	0.005	0.000	0.000	1.112	1.110
52	0.000	0.001	0.000	0.002	0.000	0.001	0.132	2.009
53	0.000	0.000	0.000	0.003	0.000	0.000	0.654	2.654
54	0.000	0.000	-0.001	0.003	0.000	0.000	0.654	1.098
55	0.000	0.000	0.001	0.005	0.000	0.000	0.765	1.008
56	0.000	0.000	0.000	0.002	0.000	0.000	0.187	3.098
57	0.000	0.001	0.001	0.003	0.000	0.001	1.654	4.098
58	0.000	0.000	0.001	0.005	0.000	0.000	1.432	3.987
59	0.000	0.000	0.000	0.008	0.000	0.000	0.765	3.987

60	0.001	0.000	-0.001	0.002	0.001	0.000	0.543	3.321

Table: 3 Summary Statistics for Multiplicative Model (a = 1 and b = 0.02)

S/N	ME		MAE		MSE		MAPE	
	LSE	BBE	LSE	BBE	LSE	BBE	LSE	BBE
61	0.001	0.001	0.000	0.001	0.001	0.001	0.876	8.009
62	0.000	0.000	0.000	0.001	0.000	0.000	0.321	2.009
63	0.001	0.001	0.000	0.009	0.001	0.001	0.764	2.007
64	0.001	0.001	0.001	0.001	0.001	0.001	0.898	5.987
65	0.000	0.000	0.001	0.004	0.000	0.000	0.321	2.098
66	0.000	0.000	0.000	0.003	0.000	0.000	0.221	2.129
67	0.000	0.000	0.000	0.009	0.000	0.000	0.204	2.108
68	0.000	0.000	0.000	0.009	0.000	0.000	0.298	9.987
69	0.000	0.000	0.001	0.005	0.000	0.000	0.766	3.332
70	0.000	0.000	0.001	0.005	0.000	0.000	1.109	1.876
71	0.000	0.000	0.001	0.004	0.000	0.000	1.112	3.654
72	0.000	0.001	0.000	0.005	0.000	0.001	0.132	4.987
73	0.000	0.000	0.000	0.006	0.000	0.000	0.654	4.443
74	0.000	0.000	-0.001	0.004	0.000	0.000	0.609	6.098
75	0.000	0.000	0.001	0.003	0.000	0.000	0.112	4.008
76	0.000	0.000	0.000	0.003	0.000	0.000	0.543	8.098
77	0.000	0.001	0.001	0.003	0.000	0.001	1.976	4.098
78	0.000	0.000	0.001	0.009	0.000	0.000	1.876	3.987
79	0.000	0.000	0.000	0.002	0.000	0.000	0.776	3.765
80	0.000	0.000	-0.001	0.004	0.000	0.000	0.432	3.987

Table: 3 Summary Statistics for Multiplicative Model (a = 1 and b = 0.02)

S/N	ME		MAE		MSE		MAPE	
	LSE	BBE	LSE	BBE	LSE	BBE	LSE	BBE
81	0.001	0.001	0.001	0.006	0.001	0.001	0.443	2.009
82	0.001	0.001	0.000	0.003	0.001	0.001	0.765	2.009
83	0.001	0.001	0.000	0.001	0.001	0.001	0.987	5.007
84	0.001	0.001	0.001	0.003	0.001	0.001	0.232	7.987
85	0.000	0.000	0.001	0.003	0.000	0.000	0.277	6.098
86	0.000	0.000	0.000	0.004	0.000	0.000	0.439	4.129
87	0.000	0.000	0.000	0.005	0.000	0.000	0.654	4.108
88	0.000	0.001	0.000	0.005	0.000	0.001	0.987	2.987
89	0.000	0.000	0.001	0.006	0.000	0.000	0.987	2.332
90	0.000	0.000	0.001	0.006	0.000	0.000	1.901	3.876
91	0.000	0.000	0.001	0.003	0.000	0.000	1.876	3.654
92	0.000	0.001	0.000	0.002	0.000	0.001	0.323	5.987
93	0.000	0.000	0.000	0.002	0.000	0.000	0.212	6.443

94	0.000	0.000	-0.001	0.005	0.000	0.000	0.987	3.987
95	0.000	0.000	0.001	0.002	0.000	0.000	0.119	5.436
96	0.000	0.000	0.000	0.004	0.000	0.000	0.549	3.323
97	0.001	0.001	0.001	0.002	0.001	0.001	1.098	3.332
98	0.000	0.000	0.001	0.005	0.000	0.000	1.119	5.545
99	0.000	0.000	0.000	0.004	0.000	0.000	0.987	2.987
100	0.001	0.000	-0.001	0.003	0.000	0.001	0.765	4.909

3.3 Comparison of Two Time Series Decomposition Techniques

1 The Buys-Ballot technique is computational easy to understand and easy to apply, while the least square technique is associated with computational difficulties.

2 The Buys-Ballot technique is helpful in determining the modal structure: additive or multiplicative or mixed model, while the least square techniques does not give an insight for choice of model for time series decomposition

3 The Buys-Ballot technique is useful for diagnosing the presence or absence of trend parameters and seasonal indices, while the least square technique does not

4 The Buys-Ballot technique as developed is for data that has stable seasonal pattern, while the least square technique will perform poorly in the presence of seasonal pattern that are not stable over time.

4.0 Conclusion

This study has discussed assessing performance of two decomposition techniques when trend cycle component is linear. The two important models discussed in this study are additive and multiplicative. The emphasis is to assess the performance of Buys-Ballot estimates and least

square estimates using accuracy measures (Mean Error (ME), Mean Square Error (MSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). One hundred (100) stimulated series of 120 observations are used to illustrate the application of the decomposition techniques. Results indicate that Mean Square (MS), Mean Square Error (MSE) and Mean Absolute Error (MAE) are the same and equally good in estimating trend parameters and seasonal indices for the two estimating methods for additive model. While, the two decomposition techniques differ for multiplicative model

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2.

3.

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Appendix A: Deviations of the Buys-Ballot and Least Square Techniques of the linear tread parameters and Seasonal effects from parameter values (for additive model with $a=1$ and $b=0.02$)

S/N	Parameter	Parameter values	ME		ME	
			LSE	ERROR	BBE	ERROR
1	$\phi_1=a$	1.00	1.105	-0.109	1.187	-0.129
2	$\phi_2=b$	0.02	0.020	-0.001	0.098	-0.002
3	$\phi_3=S_1$	0.10	-0.091	-0.006	0.087	-0.080
4	$\phi_4=S_2$	-0.89	-0.032	-0.004	-0.087	-0.001
5	$\phi_5=S_3$	-0.015	-0.008	-0.001	-0.017	-0.011
6	$\phi_6=S_4$	-1.22	-2.079	-1.001	-3.005	-0.320
7	$\phi_7=S_5$	-0.09	-0.034	-0.003	-1.098	-0.004
8	$\phi_8=S_6$	-0.97	-0.659	-0.078	-0.661	-0.231
9	$\phi_9=S_7$	-2.14	-2.008	-0.077	-2.980	-0.006

10	$\phi_{10} = S_8$	-0.73	-0.505	-0.004	-0.712	-0.006
11	$\phi_{11} = S_9$	0.64	0.498	-0.005	0.611	0.548
12	$\phi_{12} = S_{10}$	1.96	1.320	-0.879	1.689	0.964
13	$\phi_{13} = S_{11}$	2.23	1.804	0.008	2.007	0.876
14	$\phi_{14} = S_{12}$	1.16	0.897	0.012	1.329	1.238
15	$\phi_{15} = \mu$	0.00	0.000	0.000	0.000	0.000
16	$\phi_{16} = \alpha$	1.00	0.798	0.237	0.779	0.237

Appendix B: Deviations of the Buys-Ballot and Least Square Techniques of the linear tread parameters and Seasonal effects from parameter values (for multiplicative model with $a=1$ and $b=0.02$)

S/N	Parameter	Parameter values	ME		ME	
			LSE	ERROR	BBE	ERROR
1	$\phi_1 = a$	1.00	1.001	-0.100	1.1003	-0.100
2	$\phi_2 = b$	0.02	0.020	-0.001	0.098	0.002
3	$\phi_3 = S_1$	0.98	0.981	-0.003	0.986	0.001
4	$\phi_4 = S_2$	0.80	0.805	0.001	0.809	0.000
5	$\phi_5 = S_3$	0.88	0.878	0.002	0.882	0.003
6	$\phi_6 = S_4$	1.04	1.004	0.000	1.041	-0.001
7	$\phi_7 = S_5$	0.98	0.960	-0.003	0.979	-0.004
8	$\phi_8 = S_6$	1.22	1.201	-0.002	1.221	-0.002
9	$\phi_9 = S_7$	1.27	1.271	0.001	1.272	0.004
10	$\phi_{10} = S_8$	1.32	1.298	-0.002	1.323	-0.001
11	$\phi_{11} = S_9$	0.96	0.951	-0.003	0.959	0.005
12	$\phi_{12} = S_{10}$	0.80	0.791	-0.004	0.801	0.001
13	$\phi_{13} = S_{11}$	0.88	0.860	0.003	0.870	0.001
14	$\phi_{14} = S_{12}$	0.89	0.889	0.002	0.891	0.004
15	$\phi_{15} = \mu$	1.00	1.000	0.000	0.000	0.000
16	$\phi_{16} = \alpha$	0.02	0.021	0.007	0.779	0.007