Comparative Analysis of Stochastics Approaches in Forecasting Nigeria's key Macroeconomic Indicators

Abstract:

The Nigerian economy faces significant volatility in key macroeconomic variables, posing challenges to economic stability and growth. This study compares the performance of ARIMA, GARCH, and VAR models in forecasting GDP, exchange rates, interest rates, inflation, and unemployment, using annual data from 1981-2024. Results show that while ARIMA and GARCH models capture certain dynamics, the VAR model consistently delivers the highest forecast accuracy across all variables. These findings offer valuable insights for policymakers seeking data-driven strategies to stabilize the economy and manage macroeconomic uncertainty.

Key word: Stochastic modeling, Vector Autoregression (VAR), Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Autoregressive Integrated Moving Average (ARIMA),

1. Introduction

Macroeconomic stability is essential economic growth, national wealth and development [1,2]. Policies based on evidence-based predictions help to achieve sustainable growth; however; however, most economies globally continue to grapple with decades of instability across economic indicators [3,4]. The Nigerian economy, like many developing economies, is characterized by fluctuations in key macroeconomic variables such as GDP, inflation, interest rates, unemployment, and exchange rates. These fluctuations create challenges for policymakers in forecasting economic conditions and making informed decisions [5,6]. Given the complex volatile and non-linear nature of macroeconomic interactions, it is crucial investigate and adopt robust models to better understand these dynamics [7].

The development and application of econometrical and statistical models are crucial for understanding economic behaviours and making informed policy decisions. These models facilitate the analysis of interdependences, predict complex behaviour and patterns within the macroeconomic ecosystem, contributing to economic stability and prosperity [8]. Models serve as foundation of assessing economic relationships, allowing for evaluation of models based on data collections. Integrating statistitcal-econmetric approaches enhances decision-making process. Thereby ensuring models reflect the characteristics of economic phenomenon.

Stochastic time series modeling approaches like the Autoregressive Integrated Moving Average (ARIMA), Generalized Autoregressive Conditional Heteroskedasticity (GARCH), and Vector Autoregression (VAR) have been widely used to analyze the patterns and predict the future behaviours of such economic indicators [9,10,11,12,13]. They provide powerful tool for capturing the inherent uncertainties and volatility in macroeconomic variables. Each of these models has unique strengths in capturing different aspect of the times series data, ARIMA is well-suited for capturing linear trends and cyclical patterns, while GARCH is effective in modeling volatility, especially in financial time series. VAR, on the other hand, offers a comprehensive approach by analyzing the interrelationships among multiple variables, allowing for better policy simulation and economic forecasting, but share commonalities in determining their predictive abilities on variables [14].

Over the years, economic forecasting often fails short in capturing the complexity and interrelationship of these variables, resulting to inadequate responses to economic shocks [15]. Moreover, existing studies may not sufficiently address the stochastic nature of the Nigerian economy, which is influenced by a variety of external and internal factors, including global economic conditions, political instability and local policy decisions [16].

The study aims to apply and compare ARIMA, GARCH, and VAR models to key macroeconomic variables in Nigeria to identify the most efficient for predictive purposes. By doing so, it seeks to provide valuable insight for policymakers in their effort to enhances economic stability and implement data driven macroeconomic interventions.

The primary objectives of the research are:

i. To investigate the behaviour of key macromonomer variables using ARIMA, GARCH, and VAR models.

ii. To compare the efficiency of the models under different conditions and identify the best the best-suited model for forecasting in the Nigerian context.

This study aims to fill the gap by employing some stochastic time series modeling processes ARIMA, GARCH, and VAR to model key macroeconomic variables in Nigeria. By comparing the effectiveness of the model under varying conditions in predicting economic bahaviours, this study will provide valuable insight into their applicability and reliability in the Nigerian context. This contributes to the ongoing discourse on macroeconomic modeling in the Nigeria. It addresses current challenges within the system by offering empirical analysis of these models and providing policy with evidence-based recommendations to manage macroeconomic uncertainties.

2. Literatures review

Stochastic times series modeling is increasingly used in macroeconomics to understand the behaviours, complexity, volatility and non-linear interaction inherent inn economic systems. By incorporating random shocks, these models provide a realistic representation of how unexpected events affect key economic variables like GDP and inflation. This approach helps policymakers understand risks better and craft more effective responses [10,2].

The study by Mohammed, examines the relationship between structure and behaviors in a macroeconomic model [17]. [18] investigated the interactions between the oil market and the foreign exchange market using multivariate stochastic volatility (MSV) and multivariate GARCH (MGARCH) models, aiming to extract information from both markets for improved volatility forecasting. The study by [19] investigates the application of ARIMA and GARCH models to predict and analyze the fluctuations in the USD/EUR exchange rate over the next 53 weeks, using historical data from 2013 to 2023. The GARCH (1,1) model effectively analyzes volatility in finance, while the ARIMA model is not suitable for forecasting exchange rate fluctuation. Policymakers must prioritize addressing high inflation rates exchange rate and interest rate. Such rates can negatively impact purchasing power, external debt, fiscal deficit, exchange rates, interest rates, and investment [20,21]. Additionally, inflation models have been used to forecast crude oil reserves and production capacity in Nigeria [22].

Nigeria scholars often employ stochastic modeling to simulate the behavior of various macroeconomic variables, including GDP, inflation, interest rates, exchange rates, and unemployment, aiding policymakers in decision-making [23]. Sovilj *et al.* argue that dynamic stochastic general equilibrium (DSGE) models have limitations in modeling and explaining real-world phenomena, particularly in relation to the recent (2007-2009) global financial crisis [24]. Stochastics time-series modeling provides an important tool for better understanding economic variables and their analysing complex economic system by incorporating randomness and probability distribution into the model to better capture behaviors of economic variables and their interactions [25,26]. Moreover, some models often struggle to fully encapsulate the inherent randomness and external shocks affecting macroeconomic variables, leading to potential inaccuracies in predictions [19].

The application of stochastic time series modeling process helps researchers and policymakers better understand the potential outcomes of different economic policies under various scenarios, enabling the identification of effective policy interventions [27]. Stochastics modeling processes is widely applied to analyse system volatility of and random system shocks [28,29].

3. Methodology

The study applied a quantitative research design, applying a stochastic time-series approach to model and evaluate the predictive power of key macroeconomic variables. It analysed annul time series data from 1981 to 2023 on GDP, Exchange Rate (EXR), Interest rate (IR), Inflation rate (IFL), and Unemployment rate (UEMPL) as the key macroeconomic variables. The dataset was sourced from the CBN, NBS, and World Bank databases. To analyse the data, the researchers employed the R statistical software, specifically using the RStudio environment and EVIEW.

These software packages were chosen for their robust statistical and econometric capabilities, allowing for comprehensive modeling and analysis.

3.1 Model Specification

3.1.1 ARIMA MODEL

The ARIMA (AutoRegressive Integrated Moving Average) model is a robust time series forecasting method that effectively captures trends and patterns in non-stationary data. The ARIMA model was initially introduced in 1976 by George Box and Gwilym Jenkins, is characterized by linearity, combines Autoregressive (AR) and Moving Average (MA) components, and is known for its highly accurate short-term forecasting precision [12]. Its components autoregression, differencing, and moving averages work together to provide accurate predictions across various fields, including economics and finance. The general form of the ARIMA model (p, d, q) (P, D, Q)^L.

ARMA(p,q) Process

If we let $\epsilon_1, \epsilon_2, \epsilon_3, \ldots be\ a\ WhiteNoise(0, \theta_{\epsilon}^2) Process.$ It is defined that

$Y_1, Y_2, Y_2, \ldots is\ an\ ARM(p, q)$ process if for some constant parameters

$\mu, \phi_{1,\ldots,}\phi_p, \theta_1, \theta_2, \ldots, \theta_q \epsilon \mathbb{R}$

$$Y_t - \mu = \phi_1(Y_{t-1} - \mu) + \cdots + \phi_1\left(Y_{t-p} - \mu\right) + \epsilon_t - \theta_1\epsilon_t - \cdots - \theta_q\epsilon_{t-q} \qquad (3.1)$$

ARIMA (p, d, q)

A time series X_t is said to be an autoregressive integrated moving average process if $\Delta^d X_t$ Is an ARIMA (p, d, q) process:

i. Difference $X_t d$ times to achieve stationarity
ii. Model $Y_t = \Delta^d X_t$ as an ARMA (p, q) process
iii. Integrate $Y_t d$times to create a model for X_t

$$Y_t = \frac{\theta_Q \beta \Theta_Q(\beta^l)}{\phi_p(\beta)\Theta_p(\beta)(1-\beta)^d(\beta)(1-\beta^L)^D} a_t \qquad (3.2)$$

Where:

$\beta\ is\ the\ backshift\ operator\left(\beta_{yt} = y_{t-1}\right) Autorgressive\ polynomial$ (3.3)

$\phi_p(\beta) = 1 - \phi_1\beta - \phi_2\beta^2 - \cdots - \phi_p\beta^p$ is the coefficient of the non-seasonal AR component with degree of p

$\theta_q(\beta) = 1 + \theta_1\beta + \theta_2\beta^2 + \cdots + \theta_p\beta^q$ is the coefficient of the non-seasonal MA component with degree of q

$\phi_p(\beta^L) = 1 - \phi_1\beta^L - \phi_2\beta^{2L} - \cdots - \phi_p\beta^{pL}$ is the coefficient of the seasonal AR component with degree of p

$\Theta_Q(\beta) = (1 - \Theta_1\beta^L - \Theta_2\beta^{2L} - \cdots - \Theta_p\beta^{QL})$ is the coefficient of the seasonal with a component with degree of Q

$(1 - \beta)^d$ is the difference for the season order L with degree D, a_t is the residual values at time t that satisfy the white noise assumption

t=1, 2,….n, with n being the number of observation[12]

3.1.2 GARCH (Generalized Autoregressive Conditional Heteroskedasticity Model)

The GARCH model is use to model and forecast volatility, the model is specify by (p,q) where p is the order of the GARCH terms and q is the order of the ARCH term. The GARCH model is a extension of the ARCH(q) model in which the p lags of the past conditional variance were added to the equation. The model allows for both Autoregressive and moving average in the heteroscedastic variance [10]. The GARCH (p,q) model is given as:

Let $\epsilon_t \sim WhiteNoise(0,1)$. Let the process ∂_t is a generalized Auto-Regressive Heteroscedasticity p, q or GARCH(p,q) Process if;

$\boldsymbol{y_t} = \mu + \epsilon_t$

$\partial_t = \sigma_t \epsilon_t$ (3.4)

$\boldsymbol{\sigma^2}_{\boldsymbol{t}} = \alpha_0 + \sum_{i-1}^{q} \alpha_i \partial^2{}_{i-1} + \sum_{j-1}^{p} \boldsymbol{\beta}_j \sigma^2{}_{t-j}$ (3.5)

Where: $\alpha_0, \alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p \geq 0, \&$

$\sigma_t = \sqrt{\alpha_0 + \sum_{i-=1}^{q} \alpha_j \partial_{t-1}^2 + \sum_{i-=1}^{p} \beta_j \sigma_{t-1}^2}$ (3.6)

Is the conditional Standard deviation of ∂_tgiven past values

$\partial_{t-1}, \ldots, \partial_{t-q}, \sigma_{t-1}, \ldots . \sigma_{t-p}$

Square both Side by (3.6)

$\partial_t^2 = (\alpha_0 + \sum_{i-=1}^{q} \alpha_j \partial_{t-1}^2 + \sum_{i-=1}^{p} \beta_j \sigma_{t-1}^2) \in_t^2$ (3.7)

$\sigma_t = \sqrt{\alpha_0 + \sum_{i-=1}^{q} \alpha_j \partial_{t-1}^2 + \sum_{i=1}^{p} \beta_j \sigma_{t-1}^2}$ (3.8)

Feedback of σ_{t-1} values σ_t to have more persistent period of high/low conditional volatility

∂_t is weakly stationarity with mean 0

∂_t has zero autocorrelation

∂_t^2 has the autocorrelation of an ARMA(p, q) process

∂_t is a robust noise term with conditional heteroscedasty (3.9)

Where;$\sigma^2{}_t$ $\textit{is the conditional variance}$

ε_t $\textit{is the error term}$

z_t $\textit{is the independent identical distribution}$

$\alpha_0, \alpha_i, \beta_{0_j}$ $\textit{are parameters to be estimated}$

The parameters $\alpha_0, \alpha_1, \beta_j \geq 0, \sigma^2{}_t$ $\textit{is the conditional variance}$ α_0 $\textit{is the constante term,}$ α_0 $\textit{and}$ β_j $\textit{are the coefficient of the ARCH and GARCH term respectively,}$ $\varepsilon^2{}_{t-1}$ $\textit{and}$ $y^2{}_{t-1}$ are the square errors at lag t-1 and t-j respectively

The GARCH (p,q) with Z_i is a discrete times stochastics process defined as

$\varepsilon_t = Z_t \sigma_t$ $\textit{which is weakly stationary with}$

$\in (\varepsilon_t) = 0$ $\textit{and}$

$\mathrm{VAR}(\varepsilon_t) = \alpha_0 [1 - (\sum \alpha_i + \sum_{j=1}^{q} \beta_j)]$ (3.10)

$COV(\varepsilon_t, \varepsilon_t) - 0$ $\textit{for}$ $t \neq s,$ $\textit{if and only if}$

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$$

$(\alpha_0 > 0)$

The GARCH model conditional variance(h)/volatility at time t depend on both past values of the shocks capture by the logged square error terms $\varepsilon^2{}_{i-1}$ and the past values of itself ($\boldsymbol{\sigma^2}_{\boldsymbol{t-1}}$)

3.1.3 Vector Autoregressive (VAR) Model

The VAR model is use to determine the dynamic relationship among variables. Consider for structural model of large-scale simultaneous equation and important to make strong prediction [14]. Vector Autoregression (VAR) model is a statistical model used to capture the linear interdependencies among multiple time series. Each variable in a VAR model is modeled as a linear function of its own past values and the past values of all other variables in the system. The specification can include multiple equations, enhancing the model's capacity to capture complex dynamics among variables [30].

3.1.3.1 Specification

$GDP = GDP_t$

$Exchange\ rate = EXR_t$

$Unemployment\ rate = UNEMP_t$

$Interest\ rate = IR_t$

$Infaltion = IFL_t$

$$X_t = \alpha + \sum_{i=1}^{p} A_i\, x_{t-1} + \varepsilon_t \qquad (3.14)$$

The stochastic part x_t is assumed to be generated by VAR process of order p (VAR(p) of the form

$X_t = A_1 x_{t-1} + A_2 x_{t-2} + \cdots + A_p x_{t-p} + \epsilon_t,$ where (3.15)

3.1.3.2 Endogenous Variables:

The vector X_t contains the time series data for the k endogenous variables. For example, in a VAR model with GDP, Exchange Rate, Interest Rate, Inflation, and Unemployment, $X_t$would be:

$$X_t = \begin{pmatrix} GDP_t \\ EXR_t \\ UNEMP_t \\ IR_t \\ IFL_t \end{pmatrix} \ is\ the\ vector\ of\ the\ variables \qquad (3.16)$$

$A_1, A_2, \ldots, A_3$ are the matrix of the coefficient that will be estimated

$$\boldsymbol{\epsilon_t} = \begin{pmatrix} \epsilon_{GDP} \\ \epsilon_{EXR} \\ \epsilon_{UNEMP} \\ \epsilon_{IR} \\ \epsilon_{IFL} \end{pmatrix} \ is\ the\ vector\ of\ the\ error\ terms\ that\ are\ assume\ to\ be\ white\ noise$$

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \\ X_{5t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} X_{1,t-1} \\ X_{2,t-1} \\ X_{3,t-1} \\ X_{4,t-1} \\ X_{5,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{bmatrix} \qquad (3.17)$$

Individual Equations:

$GDP_i = a_{11}GDP_{t-1} + a_{12}EXR_{t-2} + a_{13}UNEMP_{t-3} + a_{14}IR_{t-4} + a_{15}IFR_{t-5} + \epsilon_{GDP_i}$

$EXR_i = a_{21}GDP_{t-1} + a_{22}EXR_{t-2} + a_{23}UNEMP_{t-3} + a_{24}IR_{t-4} + a_{25}IFR_{t-5} + \epsilon_{EXR_i}$

$UNEMP_i = a_{31}GDP_{t-1} + a_{32}EXR_{t-2} + a_{33}UNEMP_{t-3} + a_{34}IR_{t-4} + a_{35}IFR_{t-5} + \epsilon_{UNEMP_i}$

$IR_i = a_{41}GDP_{t-1} + a_{42}EXR_{t-2} + a_{43}UNEMP_{t-3} + a_{44}IR_{t-4} + a_{45}IFR_{t-5} + \epsilon_{IR_i}$

$IFR_i = a_{51}GDP_{t-1} + a_{52}EXR_{t-2} + a_{53}UNEMP_{t-3} + a_{54}IR_{t-4} + a_{55}IFR_{t-5} + \epsilon_{IFR_i}$

$where\ a_{ij}$ are the coefficient estimate

3.1.4 Model Evaluation Criteria

Forecasting the performance of various forecasting model is essential in selecting best accuracy model, the Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean absolute percentage error. Reliability testing of the models is crucial to assess and validate their performance within the system. This process ensures that the model metrics, which are integral to determining the accuracy of the series ratios, reflect the true performance of the forecast ratios based on the model itself. To evaluate and validate the models used in this study, error metrics are employed, including Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Thiel's

Inequality Coefficient. RMSE is widely recognized as a robust metric for measuring a model's error in predicting quantitative data, as it calculates the standard deviation of the mean residual and then takes the square root of that mean. In contrast, MAE offers a simpler measure of forecast accuracy by using the absolute residual values. Both metrics provide insights into the model's performance, with RMSE focusing on the variance of errors and MAE on the average magnitude of errors. [31].

The forecast evaluation metrics used in this study are mean absolute error (MAE) is defined as:

$$MEA = \frac{1}{n}\sum_{t=1}^{n}[\, {r^2}_t \; - {\sigma^2}_t] \qquad (3.19)$$

The Root Mean Square Forecast Error (RMSE) is defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t-1}^{n}({r^2}_t - {\sigma^2}_t)} \qquad (3.20)$$

The r^2 *is the realised or actual variance and* σ^2, is the square root of the conditional forecast variance and n is the number of fitted parameter [32]

and the Mean Absolute Percentage Error is defined as

$$MAP = = \frac{1}{n}\sum_{t=1}^{n} \left| \frac{(r_t - \sigma_t)}{r_t} \right| \qquad (3.21)$$

*whe*re the actual and predicted values for corresponding t values are denoted by r_t *and* σ_t respectively.

3.1.5 Thiel's Inequality Coefficient

The Theil Inequality Coefficient (U) is a measure of the accuracy of a forecasting model. It compares the forecasted values to the actual values, where a value of 0 indicates a perfect forecast and values closer to 1 indicate worse performance.

The model specification is given as:

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t-1}^{n}({r^2}_t - {\sigma^2}_t)}}{\sqrt{\frac{1}{n}\sum_{t-1}^{n}{r^2}_t} + \sqrt{\frac{1}{n}\sum_{t-1}^{n}{\sigma^2}_t}} \qquad (3.22)$$

Where:

- ${r^2}_t$is the forecasted value at time t
- ${\sigma^2}_t$ is the actual value at time t
- n is the number of observations
- The numerator $\sqrt{\frac{1}{n}\sum_{t-1}^{n}({r^2}_t - {\sigma^2}_t)}$ represents the Root Mean Squared Error (RMSE) between the forecasted and actual values.
- The denominator is the sum of the root mean squares of the forecasted and actual values, providing a normalization factor to ensure that the coefficient is between 0 and 1.

3.4 Main Results and Discussions

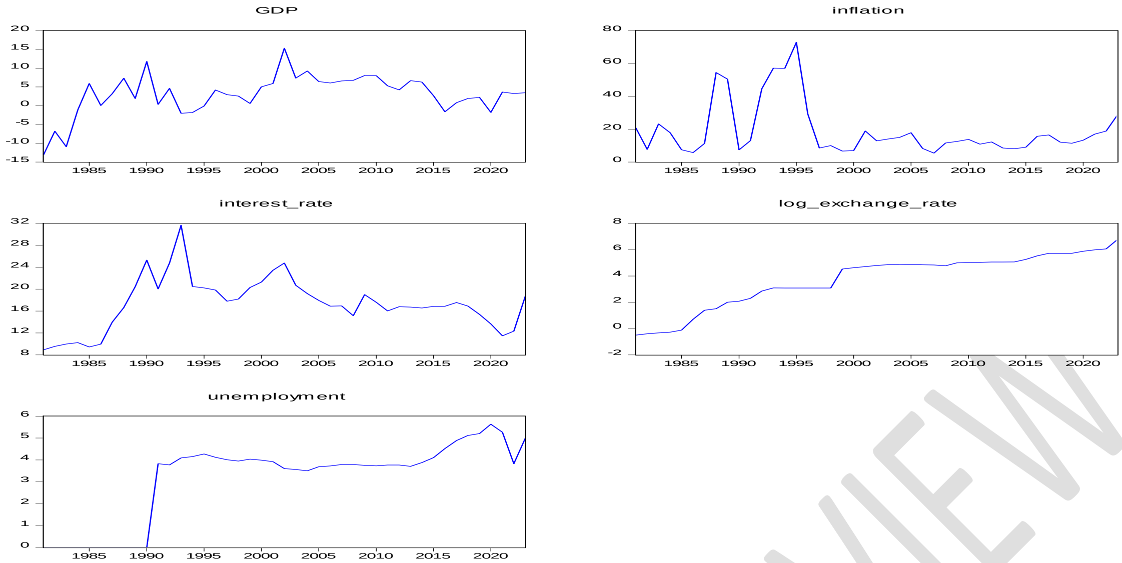

Figure 3.1: Time Series Plot of key Macroeconomic Variables Over 1981 to 2023 Period

The graph in Figure 3.1, shows the trends of key macroeconomic variables in Nigeria from 1981 to 2023, It illustrates the dynamic and volatile nature of Nigeria's key macroeconomic variables: GDP, inflation, interest rate, exchange rate, and unemployment, each exhibiting significant fluctuations and trends. The data shows high volatility and uncertainty across all macroeconomic variables.

3.4.1 Empirical Analysis of the Stochastic time series process Using ARIMA, GARCH, and VAR Models

The Autoregressive Integrated Moving Average (ARIMA) model, denoted as ARIMA (p, d, q), captures both linear and non-linear relationships among macroeconomic variables, making it a robust tool for time series analysis [33]. In this study, key macroeconomic indicators, including GDP, inflation, interest rates, unemployment, and exchange rates, were modeled using ARIMA techniques. The dataset was partitioned into training (80%) and testing (20%) sets for effective model evaluation.

Table 3.1: Summary of Fit ARIMA Model

Series	GDP	Exchange Rate	Interest Rate	Inflation	Unemployment
ARIMA Model	(1,1,0)	(0,1,0)	(1,0,0)	(0,0,1)	(0,1,0)
Coefficients					
Ma	-0.515	0.1734	0.803	0.738	
s.e.	0.150	0.0445	0.103	0.097	
sigma²	18.91	0.0852	10.080	0.5491	0.455
log likelihood	-94.96	-7.38	-87.02	-111.34	-33.85
AIC	193.93	18.76	180.04	229.08	69.69
BIC	196.92	22.24	184.61	239.05	71.19

The ARIMA models fitted to GDP, Exchange Rate, Interest Rate, Inflation, and Unemployment reveal varying levels of model fit. The Exchange Rate model, an ARIMA (0,1,0), demonstrates the best fit with the lowest AIC and BIC values, indicating it effectively captures the series' dynamics as a random walk. In contrast, the GDP model (ARIMA (1,1,0)) shows the poorest fit, with high residual variance and the highest AIC/BIC, suggesting it may not fully capture the complexities of GDP movements. The Interest Rate and Inflation models also fit reasonably well but exhibit some residual variability.

Table 3.2: Training Set Error Measure

Measure	GDP	Exchange Rate	Interest Rate	Inflation	Unemployment
ME	0.5480	-1.5578	0.282	-0.0011	0.114
RMSE	4.0207	0.2857	3.080	0.7303	0.664
MAE	3.0365	0.1853	2.248	0.5631	0.174
MPE	-195.23	5.1353	-1.231	-6.6500	4.168
MAPE	7.2884	9.2524	12.557	20.923	6.443
MASE	0.8728	1.0329	1.013	0.5999	0.970

The training set error measures provide valuables insight into the performances of ARIMA models, with RMSE and MAPE revealing varying predictive accuracies across the macroeconomic variables. GDP's RMSE of 4.02 indicates a moderate level of prediction error. The performances measurement errors for exchange rate, inflation and unemployment rate are also better.

Generalized Autoregressive Conditional Heteroskedasticity (GARCH)

The GARCH model was applied to capture the dynamic volatility and influence of macroeconomic variables on GDP. The GARCH model's result reveal significant coefficients that signifies relationship between GDP and other key variables in the study.

Table 3.3: GARCH Model Estimation Results for GDP Dynamics

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Inflation	-0.075415	0.032810	-2.298502	0.0215
Interest rate	0.494225	0.107573	4.594334	0.0000
Exchange rate	1.936157	0.552474	3.504523	0.0005
Unemployment	-2.198102	0.555234	-3.958877	0.0001
C	-3.490293	2.397591	-1.455750	0.1455

As seen in Table 3.3, the mean equation indicates that inflation negatively impact GDP, while interest rate and exchange rate positively influence GDP growth. Unemployment has a detrimental effect on GDP, emphasizing the importance of these variables in economic policy.

Table 3.4: Evaluation of GARCH Models for Forecasting Key Macroeconomic Variables in Nigeria

Criteria	GDP	Exchange Rate	Interest Rate	Inflation	Unemployment
Akaike	6.142	2.398	5.431	7.974	1.919
Bayesian	6.306	2.562	5.594	8.138	2.083
Shibata	6.122	2.383	5.450	7.951	1.902
Hannan-Quinn	6.203	2.459	5.491	8.035	1.979
RMSE	2.202	5.871	15.856	18.702	4.802

The GARCH model's evaluation metrics demonstrated its predictive capability for GDP, exchange rate and unemployment rate, with lower RMSE values. This indicates better accuracy compared to inflation and interest rate, which showed higher prediction errors.

3.4.2 Model Evaluation and Validation Using the VAR Model

Table 3.5: Model Evaluation

Variable	RMSE	MAE	Theil's inequality coefficient	Symmetric MAPE
GDP	1.054	0.79	0.337	67.0
Exchange Rate	0.350	0.309	0.049	5.22
Interest Rate	0.186	0.124	0.032	4.34
Inflation	0.543	0.415	0.096	14.2
Unemployment	0.321	0.217	0.110	10.1

The model evaluation result (Table 3.5) shows the performance metrics for; GDP, Exchange Rate, Interest Rate, Inflation, and Unemployment. The metrics used the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Theil's Inequality Coefficient, and Symmetric Mean Absolute Percentage Error (SMAPE). The Theil static statistics predictive model for GDP shows moderate errors, with RMSE of 1.054 and an MAE of 0.79, indicating relatively low prediction accuracy. The Theil's Inequality Coefficient of 0.337, which is less than 1, affirmed that the model has a good predictive power with insignificant inaccuracies. The moderate SMAPE of 67.0 indicates that the relative error in percentage terms is considerable, pointing to significant challenges in accurately predicting GDP. The VAR model performs well in predicting the exchange rate, with a low RMSE of 0.350 and MAE of 0.309, indicating an average prediction error. The Theil's Inequality Coefficient of 0.049 indicates a higher forecasting accuracy. Moreso, the low SMPE of 5.22% further confirms the model's strong performance for exchange rate.

The prediction indices for interest rate also shows high accuracy, with a low RMSE of 0.186 and MAE of 0.124. The Thiel's Inequality Coefficient of 0.032, indicating excellent predictive power. Low SMAPE of 4.34 % affirmed the model's accuracy in forecasting interest rate. On the same scale, inflation measurement shows moderate accuracy, with an RMSE of 0.543 and MAE of 0.415. The Theil's Inequality Coefficient of 0.096 implies significant predictive power. The SMAPE of 14.2 % indicates a moderate level of relative predictive error. The model's predictive power for unemployment is fairly accurate, with a RMSE of 0.321 and MAE of 0.217, indicating average prediction error. The Theil's Inequality Coefficient of 0.110 indicates good forecasting ability, and SMAPE of 10.1 % confirmed reliable model performance.

5. Discussion of Findings

The study presents an empirical insight into the performance of stochastic time series approaches to macroeconomic variables. It presents the comparative analysis of three statistical models: ARIMA, GARCH and VAR in forecasting key macroeconomic variables in Nigeria. The ARIMA, GARCH model, which incorporates both autoregressive integrated moving average (ARIMA) and generalized autoregressive conditional heteroskedasticity (GARCH) components, generally outperforms the GARCH model in terms of in-sample forecast accuracy, as evidenced by the lower RMSE, MAE, and MAPE values. This suggests that the ARIMA component effectively captures the linear dynamics of the variables, while the GARCH component adequately models the conditional heteroskedasticity.

Moreover, the VAR model demonstrate strong performance in forecasting the studied variables, compared to the other models. It exhibits low RMSE, MAE, Theil's inequality coefficient, and SMAPE values across all variables, indicating its accuracy and reliability. While ARIMA and GARCh models are effective in capturing certain forms of volatility in the Nigeria economy, the Var model is more capable in providing accurate forecast for the studied variables in the Nigeria economy. This reinforced the study by Taiwo *et al.,* asserted that the VAR model give a better forecast of macroeconomic data in Nigeria [34]. Studies also reveal that the VAR model outperformed other traditional forecasting approaches in terms of accuracy [35,36]. This aligned with the studies by Ibrahim et al., and Yang *et al.* that VAR performance better compared to BVAR and ARIMA [37,38]. Making it suitable for forecasting time series model for policymakers making reliable forecast [39,40]. More so, it validates the model's ability to handle the interdependences between variables is a key advantage.

6. Conclusion

The findings of this study have important implications for the future study of macroeconomic variables in Nigeria. First, the comparative analysis demonstrates the importance of considering both linear and non-linear dynamics when modeling macroeconomic variables. The studied provide insight into improving economic forecasting strategies for Nigeria, which is essential for formulating economic policies. Making informs decisions that will address national challenges, better understanding and control of economic instabilities. The ARIMA-GARCH model's performance highlights the benefit of incorporating both components. The VAR model demonstrate strong performance in forecasting The key macroeconomic variables in Nigeria. It outperformed others in terms of forecast accuracy and lower error rates, indicating its predictive power and reliability. Moreso, the study underscores the challenges associated with forecasting macroeconomic variables. Hence, undertaking this predictive modeling, offers valuable insight into model accuracy and effectiveness for economic forecasting which is critical for data-driving policy making

Disclaimer (Artificial intelligence)
Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

Type of Article – Original research article

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