Detection of the Presence of Seasonal Effect for some Selected Trending Curves and Choice of Model in Time Series Data

Abstract: This study discusses detection of the presence of seasonal effect and choice of model in time series data. This study considers the nature of linear, quadratic and exponential trending curves and model structure. The test is constructed in the row and overall variances of the Buys-Ballot table. Empirical example is used to determine the appropriate model for decomposition of the study data.

Keywords: Decomposition Model, Row Variance, Overall Variance, Buys-Ballot table, Trending Curves, Seasonal Indices, Choice of Model

1 INTRODUCTION

A time series data is simply define as collection of observation made sequentially in time. Examples occur in a variety of fields, ranging from economics to engineering and methods of analysing time series constitute areas of statistics. Time series forecasting is the use of a model to forecast or predict future events based on known past events.

Methods used in descriptive time series analysis are: descriptive methods, time domain methods and frequency domain methods. Frequency domain methods centre on spectral analysis and recently wavelet analysis

Two important goals in descriptive time series analysis are mainly identifying the nature of the phenomenon which can be represented by sequence of observations and predicting future values of the time series variable. Identification of the pattern and choice of model in time series data is vital to facilitate forecasting. Hence, these two major goals in time series decomposition require that the pattern of observed time series data is identified and described. The objective of time series decomposition is to separate the four time series components available in the series. That is, to de-compose an observed time series

$(X_t, t=1,2,\ldots,n)$ into components, representing the trend (T_t), the seasonal (S_t), cyclical (C_t) and irregular (u_t) Kendal and Ord [1], Chatfield [2].

The models most commonly used to describe time series decomposition are the

Additive Model:

$$X_t = T_t + S_t + C_t + u_t \tag{1}$$

Multiplicative Model:

$$X_t = T_t \times S_t \times C_t \times u_t \tag{2}$$

and Mixed Model

$$X_t = T_t \times S_t \times C_t + u_t \tag{3}$$

for short series, the trend component is jointly estimated into the cyclical Chatfield [2] and the observed time series $(X_t, t=1,2,\ldots,n)$ can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the irregular component (u_t). Hence, the models are

Additive Model:

$$X_t = M_t + S_t + u_t \tag{4}$$

Multiplicative Model:

$$X_t = M_t \times S_t \times u_t \tag{5}$$

and Mixed Model

$$X_t = M_t \times S_t + u_t. \tag{6}$$

It is always assumed that the seasonal effect, when it exists, has period *s*, that is, it repeats after *s* time periods.

$$S_{t+s} = S_t, \text{ for all } t \tag{7}$$

For additive model, it is assumed that the sum of the seasonal components over a complete period is zero, ie ,

$$\sum_{j=1}^{s} S_{t+j} = 0. \qquad (8)$$

Also, for multiplicative and mixed models, it is convenient to assume that the sum of the seasonal components over a complete period is *s*.

$$\sum_{j=1}^{s} S_{t+j} = s. \qquad (9)$$

It is also assumed that the error term u_t is the Gaussian $N(0, \sigma_1^2)$ white noise for additive and mixed models, while for multiplicative model, u_t is the Gaussian $N(1, \sigma_2^2)$ white noise and that $Cov(u_t, u_{t+k}) = 0, \forall k \neq 0$

lt is assumed that (i) the appropriate model for decomposition is known; (ii) the study series satisfied the assumptions of the models. However, one of the greatest problems identified in the use of descriptive method of time series analysis is choice of appropriate model for decomposition of any study data. That is when to use any of the additive, multiplicative and mixed models for analysis is uncertain. And it is clear that; use of wrong model will definitely lead to erroneous estimates of the components. The emphasis of this paper is to identification of seasonality in time series data and choice of appropriate model for decomposition of study data.

In some time series data, the presence of a seasonal effect in a series is quiet obvious and the seasonal periods are easy to find (example, 4 for quarterly data, 12 for monthly data , etc). Seasonality in time series data can be observed as a pattern that repeats every k elements. Some graphical methods are used to detect the presence of seasonal effect in time series are: (1) a run sequence plot (Chambers et al [3]). (2) a seasonal sub-series plot [2]; (3) multiple box plots [1]; (4) the autocorrelation plot [3]. Davey and Flores [4] added statistical tests for seasonality. Kendall and Ord [1] studied test of seasonality in time series analysis. Chatfield [2] presented the use of Buys-Ballot table for detecting the presence of trend and seasonal component in time series.

A seasonal time series with *m* rows and *s* columns, *m* represents the period while *s* represents the seasons. This two-dimensional arrangement of a series is called the Buys-Ballot table For details of this method, see Iwueze *et.al* [5], Nwogu *et.al* [6], Dozie and Uwaezuoke [7], Dozie and Ibebuogu [8], Dozie *et.al* [9], Dozie and Ijomah [10], Dozie and Nwanya [11], Dozie [12], Dozie and Uwaezuoke [13], Dozie and Ihekuna [14], Dozie and Ibebuogu [15], Akpanta and Iwueze [16]. Dozie and Ihekuna [17], Dozie [18] and Dozie and Uwaezuoke [19]

Chatfield [2] proposed the need of the Buys-Ballot table for inspecting time series data for the presence of trend and seasonal component. In addition to the inspection for the presence of trend and seasonal component Iwueze and Nwogu [20] and Iwueze and Ohakwe [21] proposed a Buys-Ballot estimation procedure for the estimation of trend parameters and seasonal components. The Buys-Ballot method suggested by Iwueze and Nwogu [20] for short period series in which trend-cycle components are jointly combined; the tests developed in this study are based on this assumption. In their results, on the basis of which the proposition for choice of appropriate model was made, they showed that, for the selected trending curves, the seasonal variances depend only on the trend parameters for the additive model and both trend parameters and seasonal indices for the multiplicative model.

For additive model and all trending curves studied, the overall variances are functions of both trend parameters and seasonal component. Hence, in this study, the test for detection of the presence and absence of seasonal effect in a time series data is constructed for the additive decomposition model only.

2 Methodology

The summary of the Buys-Ballot estimates of the row and overall variances with error variances with linear trend component obtained by Dozie and Uwaezuoke [7] given in equations (10) and those of quadratic and exponential trend components by Iwueze and Nwogu [20] given in equations (10), (11), and (12) for additive model.

2.1 Linear Trend $(a+bt)$

$$\sigma_{i.}^2 = b^2 s\left(\frac{s+1}{2}\right) + \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \sigma_1^2 \qquad (10)$$

$$\sigma_x^2 = \frac{b^2 n(n+1)}{12} + \frac{m}{n-1}\sum_{j=1}^{s} S_j^2 + \frac{2bm}{n-1}\sum_{j=1}^{s} jS_j + \sigma_1^2 \qquad (11)$$

2.2 Quadratic Trend $(a+bt+ct^2)$

$$\sigma_{i.}^2 = \left\{ \begin{array}{l} \frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2 - 30(s-1)bc + 15b^2\right\} + \\ \frac{1}{s-1}\left\{\sum_{j=1}^{s} S_j^2 + 2[b-2cs]Z_1 + 2cZ_2\right\} \\ + \left\{\frac{s^2(s+1)}{3}\left[bc - c^2(s-1) + \frac{4csZ_1}{s-1}\right]\right\} i + \left[\frac{s^2(s+1)c^2}{3}\right] i^2 \end{array} \right\} \qquad (12)$$

$$\sigma_{..}^2 = \frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\ +\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\} \tag{13}$$

$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s} S_j^2+2\left[b+c(n-s)\right]Z_1+2cZ_2\right\}$$

Where $Z_1=\sum_{j=1}^{s} jS_j,\ Z_2=\sum_{j=1}^{s} jS_j$

2.3 Exponential Trend (bu^{ct})

$$\sigma_{i.}^2 = b^2u^{2c[(i-1)s+1]}\left[\left(\frac{1-u^{2cs}}{1-u^{2c}}\right)-\frac{1}{s}\left(\frac{1-u^{cs}}{1-u^{c}}\right)\right]+\sum_{j=1}^{s} S_j^2+2bu^{c(i-1)s}\sum_{j=1}^{s} u^{cj}S_j \tag{14}$$

$$\sigma_{..}^2 = \frac{b^2u^{2c}}{n-1}\left[\left(\frac{1-u^{2cn}}{1-u^{2c}}\right)-\frac{1}{n}\left(\frac{1-u^{cn}}{1-u^{c}}\right)^2\right]+\frac{m}{m-1}\sum_{j=1}^{s} S_j^2+\frac{2b}{n-1}\sum_{j=1}^{s} u^{cj}S_j \tag{15}$$

2.4 Levene's Test for Constant Variance

The Levene's test statistic for the null hypothesis

$$H_0:\sigma_i^2=\sigma_j^2$$

$H_1:\sigma_i^2\neq\sigma_j^2$ *for at least one* $i\neq j$ is defined as

$$W=\frac{(N-K)\sum_{i=1}^{k} N_i\left(\bar{z}_{i.}-\bar{z}_{..}\right)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{N_i}\left(z_{ij}-\bar{z}_{i.}\right)^2} \tag{16}$$

where k is the number of different groups, N_i *is the number of cases* in the *ith group*, Y_{ij} *is the value of the jth* observation in the *ith group*.

z_{ij} may be defined as deviation of y_{ij} from the mean $(\bar{y}_{i})$ or from the median $(\bar{y}_{i})$. That is

$$z_{ij} = y_{ij} - \bar{y}_{i} \ \ or \ y_{ij} - \bar{y}_{i} \tag{17}$$

$\bar{z}_{i.} = \frac{1}{N_i}\sum_{j=1}^{N_i} z_{ij}$ is the mean of the z_{ij} for group i (18)

$\bar{z}_{..} = \frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{N_i} z_{ij}$ is mean of all z_{ij}. (19)

The test statistic W approximately follows the F-distribution with $k-1 \ and \ N-K$ degree of freedom. To suit the Buys-Ballot procedure, the levene’s test statistic is modified with

$N = ms, \ k = s \ N_i = m$ as

$$W = \frac{(ms-s)}{s-1}\left[\frac{\sum_{j=1}^{s} m(\bar{z}_{i} - \bar{z}_{..})^2}{\sum_{j=1}^{s}\sum_{i=1}^{m}\left(z_{ij} - \bar{z}_{.j}\right)^2}\right] \tag{20}$$

$$= \frac{s(m-1)}{s-1}\left[\frac{m\sum_{i=1}^{s}\left(\bar{z}_{i} - \bar{z}_{..}\right)^2}{\sum_{i=1}^{s}\sum_{j=1}^{m}\left(z_{ij} - \bar{z}_{.j}\right)^2}\right] \tag{21}$$

2.5 Chi-Square Test

To choose between mixed and multiplicative models, Nwogu, et al. [6] and Dozie, et al. [9] conducted Chi-Square test in seasonal variances of Buy-Ballot table for mixed model Hence, test null hypothesis is:

$$H_0: \ \sigma_j^2 = \sigma_{zj}^2$$

and the suitable model is mixed

$H_1: \sigma_j^2 \neq \sigma_{zj}^2$

and the suitable model is not mixed

$\sigma_j^2 = (j = 1,\ 2,...s)$ *is the true variance of the jth season.*

$$\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (22)$$

and σ_1^2 *is the error variance assumed to be equal to* 1

Therefore, the statistic is $\chi_c^2 = \dfrac{(m-1)\sigma_j^2}{\sigma_{zj}^2}$ (23)

follows the chi-square distribution with $m-1$ degree of freedom, m is the number of observations in each column and s is the seasonal lag.

The interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ *contains the statistic* (23) with $100(1-\alpha)$% degree of confidence.

2.6 Test for seasonality in time series for some selected trending curves

Matched pairs $(A_i, B_i),\ i = 1,2,...n$, define $w_i = A_i - B_i$ are applied to the row and overall variances of the Buys-Ballot. For detection of the presence and absence of seasonal effect, we let A_i represents the row and overall variances in the presence of seasonal effect and B_i represents row and overall variances in the absence of seasonal effect (Nwogu *et al.* [22])

For instance, (1) For the linear trend, in the presence of seasonal effect, the row variance is

$$A_i(L) = \sigma_{i.}^2(L) = b^2 s\left(\frac{s+1}{2}\right) + \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \sigma_1^2 \qquad (24)$$

When there is seasonal indices, $S_j = 0 \ \forall j = 1,2,...,s,$ therefore,

$$w_i = A_i(L) - B_i(L) = \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \qquad (25)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

and when there is no seasonal indices, $S_j = 0 \forall j = 1, 2, ..., s,$ hence,

$$B_i(L) = b^2 s\left(\frac{s+1}{2}\right) + \sigma_1^2 \qquad (26)$$

3.4 The overall variance is obtained as

$$A_i(L) = \sigma_x^2(L) = \frac{b^2 n(n+1)}{12} + \frac{m}{n-1}\sum_{j=1}^{s} S_j^2 + \frac{2bm}{n-1}\sum_{j=1}^{s} jS_j + \sigma_1^2 \qquad (27)$$

When there is seasonal indices, $S_j = 0 \forall j = 1, 2, ..., s,$ therefore

$$w_i(L) = A_i(L) - B_i(L) = + \frac{m}{n-1}\sum_{j=1}^{s} S_j^2 + \frac{2bm}{n-1}\sum_{j=1}^{s} jS_j$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

and when there is no seasonal indices, $S_j = 0 \forall j = 1, 2, ..., s,$ thus

$$B_i(L) = \frac{b^2 n(n+1)}{12} + \sigma_1^2 \qquad (28)$$

3.5 For Quadratic trend, in the presence of seasonal effect, the row variance is obtained as

$$A_i(Q)=\sigma_{i.}^2(Q)=\left\{\begin{array}{l}\frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right\}+\\ \frac{1}{s-1}\left\{\sum_{j=1}^{s}S_j^2+2[b-2cs]Z_1+2cZ_2\right\}\\ +\left\{\frac{s^2(s+1)}{3}\left[bc-c^2(s-1)+\frac{4csZ_1}{s-1}\right]\right\}i+\left[\frac{s^2(s+1)c^2}{3}\right]i^2\end{array}\right\} \quad (29)$$

When there is seasonal indices, $S_j=0\forall j=1,2,...,s,\ \ Z_1=Z_2=\sum_{j=1}^{s}S_j^2=0.$ thus

$$w_i(Q)=A_i(Q)-B_i(Q)\left\{\begin{array}{l}\frac{1}{s-1}\left\{\sum_{j=1}^{s}S_j^2+2[b-2cs]Z_1+2cZ_2\right\}\\ +\left\{\left[+\frac{4csZ_1}{s-1}\right]\right\}i\end{array}\right\} \quad (30)$$

Which is zero under null hypothesis ($H_o:S_j=0$)

and when there is no seasonal indices, $S_j=0\forall j=1,2,...,s,\ \ Z_1=Z_2=\sum_{j=1}^{s}S_j^2=0.$ thus

$$B_i(Q)=\left\{\begin{array}{l}\frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right\}+\\ +\left\{\frac{s^2(s+1)}{3}\left[bc-c^2(s-1)\right]i\right\}+\left[\frac{s^2(s+1)c^2}{3}\right]i^2\end{array}\right\} \quad (31)$$

3.6 The overall variance is obtained as

$$A_i(Q)=\sigma_{i.}^2(Q)=\frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\+\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\} \quad \textbf{(32)}$$

$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s}S_j^2+2[b+c(n-s)]Z_1+2cZ_2\right\}$$

When there is seasonal indices, $S_j=0\,\forall j=1,2,...,s,\;\; Z_1=Z_2=\sum_{j=1}^{s}S_j^2=0.$ thus

$$w_i(Q)=A_i(Q)-B_i(Q)=\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s}S_j^2+2[b+c(n-s)]Z_1+2cZ_2\right\} \quad (33)$$

Which is zero under null hypothesis ($H_o:S_j=0$)

and when there is no seasonal indices, $S_j=0\,\forall j=1,2,...,s,\;\; Z_1=Z_2=\sum_{j=1}^{s}S_j^2=0.$ thus

$$B_i(Q)=\frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\+\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\} \quad (34)$$

$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+$$

3.7 For exponential trend, in the presence of seasonal effect, the row variance is obtained as

$$X_i(E)=\sigma_{i.}^2(E)=b^2u^{2c[(i-1)s+1]}\left[\left(\frac{1-u^{2cs}}{1-u^{2c}}\right)-\frac{1}{s}\left(\frac{1-u^{cs}}{1-u^{c}}\right)\right]+\sum_{j=1}^{s}S_j^2+2bu^{c(i-1)s}\sum_{j=1}^{s}u^{cj}S_j \quad (35)$$

When there is seasonal indices, $S_j = 0 \forall j = 1,2,...,s,$ therefore

$$w_i(E) = A_i(E) - B_i(E) = \sum_{j=1}^{s} S_j^2 + 2bu^{c(i-1)s} \sum_{j=1}^{s} u^{cj} S_j \qquad (36)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

and when there is no seasonal indices, $S_j = 0 \forall j = 1,2,...,s,$ therefore

$$Y_i(E) = b^2 u^{2c[(i-1)s+1]} \left[\left(\frac{1-u^{2cs}}{1-u^{2c}} \right) - \frac{1}{s} \left(\frac{1-u^{cs}}{1-u^{c}} \right) \right] \qquad (37)$$

3.8 The overall variance is obtained as

$$A_i(E) = \sigma_{..}^2(E) = \frac{b^2 u^{2c}}{n-1} \left[\left(\frac{1-u^{2cn}}{1-u^{2c}} \right) - \frac{1}{n} \left(\frac{1-u^{cn}}{1-u^{c}} \right)^2 \right] + \frac{m}{m-1} \sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1} \sum_{j=1}^{s} u^{cj} S_j \qquad (38)$$

When there is seasonal indices, $S_j = 0 \forall j = 1,2,...,s,$ therefore

$$w_i = A_i(E) - B_i(E) = \frac{m}{m-1} \sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1} \sum_{j=1}^{s} u^{cj} S_j \qquad (39)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

and when there is no seasonal indices, $S_j = 0 \forall j = 1,2,...,s,$ therefore

$$B_i(E) = \frac{b^2 u^{2c}}{n-1} \left[\left(\frac{1-u^{2cn}}{1-u^{2c}} \right) - \frac{1}{n} \left(\frac{1-u^{cn}}{1-u^{c}} \right)^2 \right] \qquad (40)$$

Table 1: Estimates in the Presence of Seasonal Indices for Row Variance

Linear Trend $(a+bt)$	$\frac{2b}{s-1} \sum_{j=1}^{s} j S_j + \frac{1}{s-1} \sum_{j=1}^{s} S_j^2 +$

Quadratic Trend $(a+bt+ct^2)$	$\left\{ \begin{array}{l} \frac{1}{s-1}\left\{ \sum_{j=1}^{s} S_j^2 + 2[b-2cs]Z_1 + 2cZ_2 \right\} \\ + \left\{ \left[+ \frac{4csZ_1}{s-1} \right] \right\} i \end{array} \right\}$
Exponential Trend (bu^t)	$\sum_{j=1}^{s} S_j^2 + 2bu^{(i-1)s} \sum_{j=1}^{s} u^j S_j$

Where $Z_1 = \sum_{j=1}^{s} jS_j, \; Z_2 = \sum_{j=1}^{s} jS_j$

Table 2: Estimates in the Presence of Seasonal Indices for Overall Variance

Linear Trend $(a+bt)$	$\frac{m}{n-1}\sum_{j=1}^{s} S_j^2 + \frac{2bm}{n-1}\sum_{j=1}^{s} jS_j$
Quadratic Trend $(a+bt+ct^2)$	$\frac{n}{s(n-1)}\left\{ \sum_{j=1}^{s} S_j^2 + 2[b+c(n-s)]Z_1 + 2cZ_2 \right\}$
Exponential Trend (bu^t)	$\frac{m}{m-1}\sum_{j=1}^{s} S_j^2 + \frac{2b}{n-1}\sum_{j=1}^{s} u^j S_j$

Where $Z_1 = \sum_{j=1}^{s} jS_j, \; Z_2 = \sum_{j=1}^{s} jS_j$

Table 3: Estimates in the Absence of Seasonal Indices for Row Variance

Linear Trend $(a+bt)$	$b^2 s\left(\frac{s+1}{2}\right) + \sigma_1^2$
Quadratic Trend $(a+bt+ct^2)$	$\left\{ \begin{array}{l} \frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2 - 30(s-1)bc + 15b^2\right\} + \\ + \left\{ \frac{s^2(s+1)}{3}\left[bc - c^2(s-1)\right] i \right\} + \left[\frac{s^2(s+1)c^2}{3} \right] i^2 \end{array} \right\}$
Exponential Trend (bu^t)	$\frac{b^2u^{2c}}{n-1}\left[\left(\frac{1-u^{2cn}}{1-u^{2c}}\right) - \frac{1}{n}\left(\frac{1-u^{cn}}{1-u^{c}}\right)^2 \right]$

Table 4: Estimates in the Absence of Seasonal Indices for Overall Variance

Linear Trend $(a+bt)$	$\frac{b^2 n(n+1)}{12}+\sigma_1^2$
Quadratic Trend $(a+bt+ct^2)$	$\frac{nc^2}{n-1}\left\{\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}+\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\right\}+\frac{bcn(n+1)^2}{6}+\frac{b^2 n(n+1)}{12}+$
Exponential Trend (bu^{ct})	$\frac{b^2u^{2c}}{n-1}\left[\left(\frac{1-u^{2cn}}{1-u^{2c}}\right)-\frac{1}{n}\left(\frac{1-u^{cn}}{1-u^{c}}\right)^2\right]$

The presence of seasonal indices for row and overall variances are given Tables 1 and 2 and that of absence seasonal indices are listed in tables 3 and 4 respectively. It is clear from Tables 1 and 2 that the row and overall variances detect presence of seasonal indices in time series data. Hence, it is important to isolate the trend before constructing test for presence of seasonal indices. It is important to note that Tables 1 and 2 are functions seasonal indices only when trend is removed.

3.2.3 Choice of Appropriate Model

The real life example is based on monthly data on number of church marriages collected from Assumpta Cathedral Owerri, Imo State, Nigeria for a period of 2012 to 2023 given in Appendix A.

To choose the appropriate model for decomposition of the study data. The first step is to check if the time series data is additive using the test statistic given in equation (18). The null hypothesis that the data admits additive model is reject if the calculated value (W) is greater than the critical value, for which $\alpha=0.05$ level of significance and $m-1=11$ degree of freedom equal to 1.82 or do not reject H_0 otherwise. The test statistic (W) is greater, indicating the decomposition model is not additive. Having done that, we are now to choose between mixed and multiplicative models using the chi-square test proposed by Nwogu, *et al*, [6] and Dozie *et al*, [7]. Hence, the null hypothesis is not mixed model, if the test statistic stated in equation (20) lies outside the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ which for $\alpha = 0.05$ level of significance and $m-1=11$ degrees of freedom, equals (3.8, 21.9) or do not reject H_0 otherwise, and from (20) the calculated values, χ^2_{cal} given in Table 6 were obtained. When compared with the critical values (3.8 and 21.9), all the calculated values lie outside the interval, indicating that the data does not admit mixed model.

However, the choice of model for decomposition of the study series may be affected by violation of the underlying assumptions, hence, there is need for transformation to meet the assumptions in the distribution. When the column variances of the transformed data in Table 7 are subjected to test for constant variance, the test statistic (0.78) is less than the tabulated (1.82) at $\alpha = 0.05$ level of significance and $m-1=11$ degrees of freedom. The result shows that, the transformed data is additive model.

Table 5: Deviations of the Observed Values from Means ($Z_{ij} = \left|y_{ij} - \bar{y}_{,j}\right|$)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{Z}_{i.}$	$\sigma_{i.}$
2012	6.42	4.75	8.17	13.08	2.17	3.75	3.67	3.58	0.50	10.67	5.25	12.67	74.67	6.22	4.09
2013	6.42	1.75	1.83	2.08	5.17	0.75	0.67	3.58	0.50	2.33	6.75	6.67	38.50	3.21	2.42
2014	5.58	1.75	0.83	2.92	0.83	0.75	2.33	6.42	4.50	4.33	1.75	4.67	36.67	3.06	1.98
2015	3.42	0.25	4.83	8.08	3.83	1.25	1.33	4.42	3.50	5.33	2.25	2.33	40.83	3.40	2.14
2016	5.42	1.25	0.17	12.92	0.83	5.25	2.67	1.58	25.50	9.33	9.25	20.33	94.50	7.88	8.13
2017	2.58	0.75	1.17	16.92	5.17	6.25	0.33	12.58	5.50	16.67	16.25	3.33	87.50	7.29	6.50
2018	3.58	5.25	0.83	2.92	10.17	2.25	0.33	0.42	0.50	5.33	11.25	8.33	51.17	4.26	3.87
2019	7.58	5.25	1.17	4.92	3.17	2.75	0.33	1.58	2.50	1.33	5.75	5.33	41.67	3.47	2.25

2020	1.58	1.25	3.17	10.08	1.83	3.25	0.67	0.58	3.50	10.67	0.25	7.67	44.50	3.71	3.70
2021	1.58	2.25	0.17	3.08	7.83	4.75	3.33	2.42	5.50	13.33	10.75	0.33	55.33	4.61	4.12
2022	3.58	4.75	0.83	2.92	5.83	2.75	1.67	5.42	4.50	9.33	9.75	2.33	53.67	4.47	2.80
2023	4.42	1.75	4.83	7.08	4.83	2.75	1.33	4.42	4.50	12.67	9.75	10.67	69.00	5.75	3.58
Total	**52.17**	**31.00**	**28.00**	**87.00**	**51.67**	**36.50**	**18.67**	**47.00**	**61.00**	**101.33**	**89.00**	**84.67**	688.00		
$\bar{Z}_{.j}$	**4.35**	**2.58**	**2.33**	**7.25**	**4.31**	**3.04**	**1.56**	**3.92**	**5.08**	**8.44**	**7.42**	**7.06**		4.78	
$\sigma_{.j}$	**1.96**	**1.86**	**2.46**	**4.99**	**2.82**	**1.74**	**1.19**	**3.31**	**6.69**	**4.72**	**4.63**	**5.52**			4.32

Table 6: Square of Deviations of the Observed Values from Seasonal Means

$\left(Z_{ij} - \bar{Z}_{.j}\right)^2$													
	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total
2012	4.3	4.7	34	34	4.6	0.5	4.5	0.1	21	4.9	4.7	32	149
2003	4.3	0.7	0.3	27	0.7	5.3	0.8	0.1	21	37	0.4	0.2	98
2014	1.53	0.7	2.3	18	12	5.25	0.60	6.25	0.34	16.90	32.11	5.71	102.47
2015	0.9	5.4	6.3	0.7	0.2	3.21	0.05	0.25	2.51	9.68	26.69	22.30	78.17
2016	1.1	1.8	4.7	32	12	4.88	1.23	5.44	416.84	0.79	3.36	176.30	660.63
2017	3.1	3.4	1.4	93	0.7	10.29	1.49	75.11	0.17	67.60	78.03	13.85	348.58
2018	0.6	7.1	2.3	19	34	0.63	1.49	12.25	21.01	9.68	14.69	1.63	124.46
2019	10	7.1	1.4	5.4	1.3	0.09	1.49	5.44	6.67	50.57	2.78	2.97	95.69
2020	7.6	1.8	0.7	8	6.1	0.04	0.79	11.11	2.51	4.94	51.36	0.37	95.38
2021	7.6	0.1	4.7	17	12	2.92	3.16	2.25	0.17	23.90	11.11	45.19	130.95
2022	0.6	4.7	2.3	19	2.3	0.09	0.01	2.25	0.34	0.79	5.44	22.30	59.86

2023	0.1	0.7	6.3	0.1	0.3	0.09	0.05	0.25	0.34	17.83	5.44	13.04	44.29
Total	42	38	66	274	87	33.23	15.63	120.83	492.92	244.96	236.17	335.30	**1987.05**

Table 7: Calculation of $m\left(\bar{Z}_{.j}-\bar{Z}_{..}\right)^2$ **m = 12**

$\bar{Z}_{.j}$	$\bar{Z}_{..}$	$\bar{Z}_{.j}-\bar{Z}_{..}$	$\left(\bar{Z}_{.j}-\bar{Z}_{..}\right)^2$	$12\times\left(\bar{Z}_{.j}-\bar{Z}_{..}\right)^2$
4.35	4.78	-0.43	0.18	2.22
2.58	4.78	-2.20	4.84	58.08
2.33	4.78	-2.45	6.00	72.03
7.25	4.78	2.47	6.10	73.21
4.31	4.78	-0.47	0.22	2.65
3.04	4.78	-1.74	3.03	36.33
1.56	4.78	-3.22	10.37	124.42
3.92	4.78	-0.86	0.74	8.88
5.08	4.78	0.30	0.09	1.08
8.44	4.78	3.66	13.40	160.75
7.42	4.78	2.64	6.97	83.64
7.06	4.78	2.28	5.20	62.38
				685.66

From appendix A and Table 7

$$W=\frac{12*(12-1)(685.66)}{(12-1)(1987.05)}=\frac{90507.12}{21857.55}=4.14$$

Table 8: Seasonal effects (S_j), estimate of the column variance ($\hat{\sigma}_j^2$) and Calculated Chi-square $\left(\chi_{cal}^2\right)$

j	1	2	3	4	5	6	7	8	9	10	11	12
S_j	1.7	0.8	0.8	2.2	1.2	0.5	0.3	0.7	0.6	0.9	0.9	1.6
$\hat{\sigma}_j^2$	24.5	10.8	11.9	82.3	28.2	13.1	4.1	27.7	73.0	100	81.5	84.8
χ_{cal}^2	0.03	0.06	0.06	0.05	0.07	0.14	0.14	0.19	0.58	0.32	026	0.10

From appendix B and Table 8, $\sigma_1^2 = 1,\ b = 0.1143,\ n = 144,\ m = 12$

Therefore, from (8), $\sigma_{zj}^2 = (0.1143)^2 \times 144\left(\frac{144+12}{12}\right)S_j^2 + 1$

Table 9: Transformed series of Deviations of the Observed Values from Means ($Z_{ij} = \left| y_{ij} - \bar{y}_{,j} \right|$)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{z}_{i.}$	$\sigma_{i.}$
2012	0.64	1.08	0.81	0.67	0.20	0.56	1.42	0.37	0.19	0.57	0.32	0.40	7.22	0.60	0.36
2013	0.64	0.16	0.17	0.02	0.36	0.00	0.03	0.37	0.11	0.00	0.30	0.17	2.35	0.20	0.19
2014	0.46	0.16	0.02	0.17	0.00	0.00	0.53	0.73	0.48	0.12	0.00	0.11	2.79	0.23	0.25
2015	0.24	0.17	0.86	0.33	0.24	0.25	0.38	0.40	0.21	0.19	0.19	0.09	3.55	0.30	0.20
2016	0.49	0.31	0.12	0.48	0.00	0.62	0.72	0.22	1.32	0.53	0.46	0.47	5.74	0.48	0.34
2017	0.27	0.02	0.23	0.58	0.36	0.70	0.19	0.84	0.50	0.75	0.67	0.12	5.23	0.44	0.27
2018	0.34	0.71	0.02	0.17	0.58	0.36	0.19	0.06	0.11	0.19	0.52	0.23	3.49	0.29	0.22
2019	0.56	0.71	0.23	0.24	0.26	0.33	0.19	0.22	0.09	0.06	0.23	0.16	3.31	0.28	0.19
2020	0.20	0.31	0.44	0.45	0.07	0.46	0.03	0.14	0.21	0.57	0.10	0.21	3.19	0.27	0.18
2021	0.20	0.42	0.12	0.07	0.69	0.84	0.66	0.14	0.50	1.04	0.64	0.04	5.37	0.45	0.33
2022	0.34	1.08	0.02	0.17	0.44	0.33	0.32	0.55	0.35	0.53	0.54	0.09	4.75	0.40	0.28
2023	0.36	0.16	0.86	0.27	0.33	0.33	0.38	0.40	0.35	0.64	0.54	0.32	4.93	0.41	0.19
Total	**4.74**	**5.30**	**3.90**	**3.63**	**3.53**	**4.80**	**5.04**	**4.43**	**4.42**	**5.19**	**4.53**	**2.40**	51.91		
$\bar{z}_{.j}$	**0.40**	**0.44**	**0.32**	**0.30**	**0.29**	**0.40**	**0.42**	**0.37**	**0.37**	**0.43**	**0.38**	**0.20**		0.36	
$\sigma_{.j}$	**0.16**	**0.37**	**0.34**	**0.20**	**0.21**	**0.25**	**0.38**	**0.24**	**0.34**	**0.32**	**0.22**	**0.13**			0.27

Table 10: Transformed Series of Square of Deviations of the Observed Values from Seasonal Means

$\left(z_{ij} - \bar{z}_{.j}\right)^2$													
	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total
2012	0.06	0.41	0.24	0.14	0.01	0.02	0.99	0.00	0.03	0.02	0.00	0.04	1.96
2013	0.06	0.08	0.02	0.08	0.00	0.16	0.15	0.00	0.07	0.18	0.01	0.00	0.81
2014	0.00	0.08	0.09	0.02	0.08	0.16	0.01	0.13	0.01	0.10	0.14	0.01	0.84
2015	0.02	0.07	0.29	0.00	0.00	0.02	0.00	0.00	0.02	0.06	0.03	0.01	0.54

2016	0.01	0.02	0.04	0.03	0.08	0.05	0.09	0.02	0.91	0.01	0.01	0.07	1.34
2017	0.01	0.18	0.01	0.08	0.00	0.09	0.05	0.22	0.02	0.10	0.08	0.01	0.85
2018	0.00	0.07	0.09	0.02	0.08	0.00	0.05	0.10	0.07	0.06	0.02	0.00	0.57
2019	0.03	0.07	0.01	0.00	0.00	0.00	0.05	0.02	0.07	0.14	0.02	0.00	0.43
2020	0.04	0.02	0.01	0.02	0.05	0.00	0.15	0.05	0.02	0.02	0.08	0.00	0.47
2021	0.04	0.00	0.04	0.05	0.16	0.20	0.06	0.05	0.02	0.37	0.07	0.03	1.08
2022	0.00	0.41	0.09	0.02	0.02	0.00	0.01	0.03	0.00	0.01	0.03	0.01	0.64
2023	0.002	0.08	0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.03	0.01	0.46
Total	0.28	1.48	1.24	0.46	0.50	0.71	1.63	0.63	1.24	1.10	0.52	0.19	**9.99**

Table 11: Calculation of $m\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ m =12

$\bar{z}_{.j}$	$\bar{z}_{..}$	$\bar{z}_{.j}-\bar{z}_{..}$	$\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$	$12\times\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$
0.4	0.36	0.040	0.002	0.019
0.44	0.36	0.080	0.006	0.077
0.32	0.36	-0.040	0.002	0.019
0.3	0.36	-0.060	0.004	0.043
0.29	0.36	-0.070	0.005	0.059
0.4	0.36	0.040	0.002	0.019
0.42	0.36	0.060	0.004	0.043
0.37	0.36	0.010	0.000	0.001
0.37	0.36	0.010	0.000	0.001
0.43	0.36	0.070	0.005	0.059
0.38	0.36	0.020	0.000	0.005
0.2	0.36	-0.160	0.026	0.307
				0.653

From Appendix B and Table 11

$$W=\frac{12*(12-1)(0.653)}{(12-1)(9.99)}=\frac{86.196}{109.89}=0.78$$

4 Concluding Remarks

This study has examined detection of presence and absence of seasonal effect and choice of appropriate model in time series data. The Buys-Ballot table is very useful for inspecting the presence or absence of trend and seasonal effect in time series data. It is also useful in determining the model structure. The emphasis of this study is to detect the presence and absence of seasonal effect and choice of model. Successful transformation is done to meet the assumptions in the distribution. Results indicate that (1) Tables 1 and 2 are dominated with the presence of seasonal effects only when the trend is removed. Hence, it is important to note, before constructing test for the presence of seasonal effect, the trend must be isolated (2) the transformed series admits additive model. This further suggests that the original data is multiplicative model. There is suggestion that choice of appropriate model may be affected by violation of underlying assumptions, hence, it is recommended that a study series should be evaluated for the assumptions of time series model before applying test for choice of model.

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Appendix A: Original series from 2012 to 2023

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
2012	16	34	21	12	23	24	14	27	13	11	39	27
2013	16	24	20	15	11	26	23	18	22	13	28	23
2014	8	18	11	14	26	14	24	18	21	19	25	10
2015	14	14	18	22	14	15	26	14	19	16	24	20
2016	17	21	20	23	16	11	27	22	14	12	5	21
2017	8	11	14	23	32	22	13	20	25	13	19	15
2018	5	21	9	10	18	15	9	20	18	10	15	16
2019	9	18	11	12	12	13	10	27	9	18	18	14
2020	6	10	10	11	18	12	11	10	11	8	11	10
2021	5	13	9	14	13	8	18	20	9	10	17	10
2022	2	4	9	10	12	3	12	21	4	10	18	12
2023	7	9	12	12	23	21	7	2	11	6	21	18

Source: Assumpta Cathedral Parish, Owerri 2012-2023

Appendix B: Transformed series from 2012 to 2023

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
2012	6.42	4.75	8.17	13.08	2.17	3.75	3.67	3.58	0.50	10.67	5.25	12.67
2013	6.42	1.75	1.83	2.08	5.17	0.75	0.67	3.58	0.50	2.33	6.75	6.67
2014	5.58	1.75	0.83	2.92	0.83	0.75	2.33	6.42	4.50	4.33	1.75	4.67
2015	3.42	0.25	4.83	8.08	3.83	1.25	1.33	4.42	3.50	5.33	2.25	2.33
2016	5.42	1.25	0.17	12.92	0.83	5.25	2.67	1.58	25.50	9.33	9.25	20.33
2017	2.58	0.75	1.17	16.92	5.17	6.25	0.33	12.58	5.50	16.67	16.25	3.33

2018	3.58	5.25	0.83	2.92	10.17	2.25	0.33	0.42	0.50	5.33	11.25	8.33
2019	7.58	5.25	1.17	4.92	3.17	2.75	0.33	1.58	2.50	1.33	5.75	5.33
2020	1.58	1.25	3.17	10.08	1.83	3.25	0.67	0.58	3.50	10.67	0.25	7.67
2021	1.58	2.25	0.17	3.08	7.83	4.75	3.33	2.42	5.50	13.33	10.75	0.33
2022	3.58	4.75	0.83	2.92	5.83	2.75	1.67	5.42	4.50	9.33	9.75	2.33
2023	4.42	1.75	4.83	7.08	4.83	2.75	1.33	4.42	4.50	12.67	9.75	10.67