Short communication

Teaching Reform and Practice of Linear Algebra Based on the "Three Transformations and Six Dimensions" Progressive Model

Abstract: Linear Algebra serves as a crucial foundational course for economics and management majors in universities, where its abstract nature and complexity often pose challenges to student learning. To address the pain points of the course, we have proposed a "Three Transformations and Six Dimensions" Progressive model. This model integrates Linear Algebra with professional education and course education, using stratified and progressive teaching strategies to simplify teaching content, guide learning methods, and achieve effective teaching evaluation. Specific measures include reconstructing course content, building knowledge maps, integrating course ideology and politics, optimizing teaching models, strengthening practical innovation, and advancing a diversified evaluation system across six dimensions. Practice has demonstrated that this model effectively enhances teaching quality and markedly boosts students' mathematical literacy, logical thinking, and application capabilities.

Keywords: Linear Algebra; Teaching Reform and Practice; Blended Learning; Knowledge Map

1 Introduction

Linear Algebra, a key branch of mathematics, has deeply penetrated various fields such as natural science, social science, and engineering technology. As mathematical problems continue to emerge in cutting-edge technology fields such as artificial intelligence, data science, 5G or 6G communications, and blockchain, the foundational role and critical basic function of Linear Algebra courses are further accentuated. However, the abstract nature and complex mathematical concepts of Linear Algebra often pose challenges to students, particularly when integrating theoretical knowledge with practical applications. Teachers and scholars worldwide have made numerous beneficial attempts to improve teaching methods in Linear Algebra, enhance teaching quality, and improve learning outcomes in [1]-[3].

Taking our university as an example, the course has a long history and has always maintained an excellent teaching tradition[4]. Having been passed down for over three decades, it has undergone continuous construction and optimization, staying true to its core while embracing innovation. It has successively been approved as a school-level excellent open course, a model course for integrating ideological and political education into the curriculum, and a provincial-level top course, gradually achieving a transition from traditional teaching to smart teaching. Below, we will discuss the reform and innovation of Linear Algebra teaching based on our university's teaching practices.

2 Teaching Innovation Approach: "Three Transformations and Six Dimensions Progressive" Model

2.1 Teaching Pain Points

Years of teaching research and student surveys have identified three main pain points in the course.

The first Teaching Pain Point: Abundance of Theory with Few Applications, Abstract Content Difficult to Grasp

Linear Algebra textbooks are rigid, focusing on theoretical systems and complex calculations, lacking practical examples and ideological and political cases. There are many concepts, rules, and computations, without considering students' learning habits, and there is insufficient connection with secondary school content and professional education.

The Second teaching pain point: Many Students, Little Interaction, Difficulty in Sustaining Learning Motivation

As a required public course, students come from diverse backgrounds with significant foundational differences. Students are accustomed to passive listening and have little active participation; as the course difficulty increases, it becomes challenging to sustain their motivation to learn.

The Third Teaching Pain Point: More Problem Solving, Less Critical Thinking, Difficulty in Enhancing Innovation Ability

There is a contradiction between the abundance of knowledge content and the limited class hours. Teachers inadvertently focus on knowledge lectures and problem-solving training, neglecting the cultivation of thinking quality and innovation ability. Students are accustomed to problem-solving training and techniques, neglecting the study of mathematical thought methods, often falling into the vicious cycle of "negligence during regular times→intensive problem-solving before exams→confusion after exams," leading to insufficient comprehensive quality and innovation application ability.

2.2 Innovation Approaches

To address the aforementioned challenges, grounded in the needs of relevant majors and adhering to the teaching philosophy of "student-centered, problem-oriented, and continuous improvement," we propose a teaching design approach based on the "Three Transformations and Six Dimensions " Progressive model.

The "Three Transformations" refer to the approaches of "simplifying complexity, guiding teaching, and grounding abstraction in reality" .

The "Six Dimensions" encompass six innovative measures: reconstructing course content, building knowledge maps, integrating course ideology and politics, optimizing teaching models, strengthening practical innovation, and conducting diversified evaluations.

The "Progressive Approach" is based on the principles of "Strengthening foundations, focusing on applications, and staying informed about the cutting edge", with an in-class progression from "foundation to improvement to expansion" and an out-of-class progression from "teaching to competition to practice."

3 Innovative Methods and Approaches

Addressing the first teaching pain point, where students find the course abstract and difficult to understand, how can we simplify the teaching content? This is addressed by the two approaches detailed in sections 3.1-3.2.

3.1 Reconstructing the Course Content

3.1.1 Aligning with professional needs

Add a practical module to the theoretical module to cultivate students' awareness and ability to apply mathematics. Build an application case library, for example, determinants and investment portfolio returns and risks, matrix multiplication and movie recommendation systems, rotation matrices and lotteries, matrix multiplication and Huawei's polar codes, inverse matrices and cryptographic encoding and decoding, matrix high power calculations and population forecasting, eigenvectors and facial recognition, etc. Use a wealth of practical cases to enliven the classroom, allowing abstract concepts and theorems to step out of textbooks, enabling students to truly experience the utility of mathematics, and enhancing students' ability to analyze and solve practical problems.

3.1.2 Modular design

Combining Bloom's taxonomy of educational objectives, dividing the course content into basic modules, application modules, and cutting-edge modules, allowing students to choose their learning paths based on their interests and needs. The basic module includes foundational knowledge, historical background, simple calculations, etc., for online self-study by students. The application module includes comprehensive calculations, basic modeling, theoretical applications, etc., suitable for in-class research and study. The cutting-edge module includes theoretical derivation, advanced modeling, method innovation, etc., learned through a combination of in-class intensive lectures and post-class expansion studies.

3.2 Building Knowledge Maps

The teaching team established a Linear Algebra MOOC in 2020. By 2024, utilizing AI technology from the Zhihuishu platform, an AI-powered knowledge map course for Linear Algebra was developed and made available free of charge across the country.

3.2.1 Digitization of Teaching Resources

The AI smart course breaks down teaching content into 6 modules with a total of 87 knowledge points, constructs relationships between knowledge points through a knowledge graph, and associates with 311 learning resources and over 2000 related questions.

Each knowledge point is complemented by a rich array of learning resources, including micro-courses, PPTs, test questions, advanced thinking questions, and other learning materials such as documents, journals, and books, recording the entire learning process to fully support students' self-directed learning. Provides advanced thinking questions and past postgraduate entrance exam questions in Linear Algebra, which helps to enhance students' problem-solving and thinking abilities. The introduction of Matlab mathematical experiments and Python operation examples provides students with opportunities to apply theoretical knowledge to practice, enhancing their hands-on and problem-solving skills.

Teachers can understand students' learning progress and mastery through the knowledge graph, thereby formulating more precise teaching plans and methods. With the assistance of the knowledge graph, teachers can better assess students' learning outcomes, adjust teaching strategies in a timely manner, and enhance teaching quality.

3.2.2 Personalized Learning

Through the interactive features of the knowledge map, students can browse and learn course content, and interact with the knowledge map through actions like clicking and dragging, thereby gaining a deeper understanding of the course material. Relying on the platform's powerful AI intelligent data search function, online intelligent Q&A is realized, effectively improving students' learning efficiency and interest. Additionally, the course AI assistant offers tailored learning assistance, adept at smartly suggesting learning materials and practice exercises, and mapping out personalized learning routes according to students' advancement and understanding.

In sum, the AI course reform based on the knowledge graph can achieve visualization of teaching content, digitalization of teaching resources, individualization of the teaching process, and refinement of teaching management [5].

Addressing the second teaching pain point, where students learn passively, how can we transform teaching methods from instructing to guiding? This is addressed by the two approaches detailed in sections 3.3-3.4.

3.3 Integrating Ideological and Political Education into the Course

Integrating four dimensions of educational elements: "dialectical thinking, mathematical culture, scientific spirit, and the beauty of mathematics" organically into the course teaching.

Dialectical thinking

Algebraic thought methods profoundly reflect the worldview and methodology of dialectical materialism. In teaching, guide students to explore the philosophical ideas contained in the methods and clarify the context and connotations of knowledge.

Mathematical culture

Incorporating ancient mathematical achievements from home and abroad and contemporary cutting-edge results into the classroom, encouraging students to apply their knowledge to technological development.

Scientific spirit

The history of algebra fully demonstrates the scientific spirit of mathematicians who are pioneering, innovative, and in pursuit of truth. Introduce related mathematical history when introducing theorems and concepts, promoting the spirit of craftsmanship.

Mathematical application

Introducing the application of linear algebra in high-tech, guiding students to use their knowledge to explain phenomena in life, and enhancing their understanding through practice.

3.4 Optimizing Teaching Models

Based on outcome-oriented, problem-driven, and BOPPPS teaching model[4], deeply integrating information technology and smart tools, precisely designing each teaching link, and implementing teaching objectives. Following the principle of "Strengthening foundations, focusing on applications, and staying informed about the cutting edge", with a progressive approach from "foundation to improvement to expansion."

Seamlessly combine the six components of the BOPPPS model including entry, objectives, pre-assessment, participatory learning, post-assessment, and summary, with the three phases of blended learning, which include pre-class guidance, in-class research and study , and post-class expansion.

Pre-class Guidance

Teachers post learning objectives and tasks through the platform. Students engage in self-directed learning using textbooks, literature, micro-courses, MOOCs, etc., completing self-assessments and topic discussions. Teachers provide online Q&A, collect pre-class feedback, and prepare lessons meticulously.

In-class Research and Study

Implement a flipped classroom interactive design with students taking the lead and teachers facilitating. Use forms such as group discussions, teacher's in-depth lectures, practice while teaching, and student presentations to help students clarify knowledge points and apply what they've learned to solve specific problems. The session concludes with a class summary and assignment of homework.

Post-class Expansion

Assess students' mastery through post-tests and summaries. Provide extended resources, thought-provoking questions, and homework assignments (evaluated by teachers, peers, and the system) to keep students engaged and make teaching outcomes tangible.

Tackling the third pedagogical issue, where students are unable to apply the knowledge they've acquired, how can we transform teaching assessment into a practical exercise? This is addressed by the two approaches detailed in sections 3.5-3.6.

3.5 Strengthening Innovative Practice

Building on the in-class progression from "foundation to improvement to expansion," we establish an out-of-class progression from "teaching to competition to practice," stimulating students' innovative thinking and practical abilities through experiments, projects, and competitions.

Project-driven Approach

Introduce cutting-edge technology developments and teachers' academic research into the curriculum, allowing students to understand the application of mathematics in the digital age. Design projects related to their majors, enabling students to apply knowledge of linear algebra in practice to solve real-world problems.

Innovation Competitions

Organize innovation competitions related to linear algebra, encouraging students to participate in various subject competitions, combining learning with competition to stimulate students' innovative thinking and practical abilities.

3.6 Promoting Diversified Evaluation

Implement a diversified teaching evaluation that combines online and offline methods, as well as formative and summative assessments, to comprehensively evaluate students' regular learning performance and outcomes, stimulating their motivation to advance in learning. The course total grade is calculated as (15% for unit quizzes + 20% for written assignments + 5% for video learning + 5% for course discussions + 5% for course interactions) + 50% for the final exam, where the first five components are formative assessments and the sixth part is a summative assessment using a unified closed-book exam. Formative assessment covers the entire process from pre-class to in-class and post-class, promoting regular study habits among students. The course assessment includes a variety of forms beyond the final exam, such as 12 assignments per semester, 4 unit tests, multiple in-class exercises, course discussions, and PBL tasks, assessing not only students' problem-solving abilities on paper but also their practical innovation and communication skills.

Additionally, introduce a peer evaluation mechanism, allowing students to evaluate each other, promoting communication and learning among students.

4 Innovation Achievements and Dissemination

4.1 Enhanced Interaction and Participation, Increased Course Satisfaction

Survey questionnaires indicate that students are highly satisfied with the teaching methods, means, and diversified formative assessments of this course. Students report a strong sense of participation in the course, with diversified formative assessments creating a sense of urgency in learning; the course contains many practical examples, making mathematics classes less dry and abstract, and more interesting; self-directed learning, teamwork, and communication skills have all improved.

From learning statistics and classroom interaction, students are more actively engaged in group discussions and PBL group tasks, improving learning efficiency through questioning and reflection, and gaining a more solid grasp of knowledge.

4.2 Enhanced Mathematical Literacy and Applied Innovation Ability

Grade statistics show a significant increase in the proportion of excellent students and a marked decrease in the failure rate, indicating a substantial improvement in students' overall quality. Additionally, students can apply their knowledge to subsequent subject competitions and innovation practices, achieving numerous awards. In the past five years, under team guidance, students have won over 100 awards in national and provincial-level competitions such as the National University Student Mathematical Modeling Contest and Provincial Higher Mathematics Contest; students have led six national-level university innovation training projects and published more than 30 papers.

4.3 Achieving Mutual Progress and Extensive Teaching Demonstration

Teaching innovation has enhanced the teaching abilities and professional qualities of the faculty team. In the past five years, team teachers have led four provincial-level course projects, one provincial classroom teaching reform project, one higher education research project, and two Ministry of Education industry-university-research projects. Team teachers have won five awards in provincial teaching competitions.

The course SPOC has been running for 12 periods on the Chaoxing platform, with over 2000 students on campus completing blended learning. The MOOC and knowledge map AI course are shared nationwide on the Zhihuishu platform and have been recommended by Chaoxing platform as a "demonstration teaching package". Currently, more than 600 universities in China are using our MOOC, with a cumulative total of more than 100,000 students engaged in learning, showcasing an effective exemplary impact on curriculum development.

5 Conclusion

Through the innovation and practice of the "Three Transformations and Six Dimensions " Progressive approach, not only can the effectiveness of teaching be enhanced, but a foundation can also be laid for students' lifelong learning and the improvement of their comprehensive abilities. This teaching model is not limited to linear algebra courses; it is equally suitable for a variety of mathematics and engineering disciplines.

However, with the progress of the times, the Linear Algebra course still faces some challenges. On one hand, the rapid development of language intelligence majors (directions) both domestically and internationally has put forward new requirements for the teaching content, methods, and means of Linear Algebra. On the other hand, applying knowledge graph technology to Linear Algebra teaching also faces a series of challenges, including technical implementation challenges such as the construction, maintenance, and updating of knowledge graphs; as well as teaching implementation challenges, such as professional training for teachers, students' usage habits and acceptance levels, and the integration and optimization of educational resources.

Future research directions include developing more intelligent knowledge graph tools to support more personalized and adaptive teaching strategies; exploring interdisciplinary collaboration models to promote collaborative innovation between mathematics and information technology fields; and continuously optimizing the application model of knowledge graphs in Linear Algebra teaching through case studies and teaching practices.

In summary, facing the dual challenges of technological development and educational needs, future research will continue to promote the innovation and development of Linear Algebra teaching models.

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