Efficiency Analysis of Public and Private Sector Banks in India using Slack-Based Measure of Efficiency

ABSTRACT

In this article, a consistent decision making model criterion is identified based on coefficient of determination in Data Envelopment Analysis (DEA). The Slack-Based Measure (SBM) of efficiency and non-radial model identified better decision-making units from the variables selected for the analysis. Both public and private sector banks play a crucial role in the Indian financial system, which is indicative of the country's progress. The nation's economy is guided by the potential of the financial system. In contemporary society, the institutional banking sector makes a notable and significant contribution to economic progress. The DEA has been used in this study to assess the effectiveness of banks in the public and private sectors during the fiscal year from 2015-16 to 2018-19. The basic CCR (Charnes, Cooper, and Rhode) model is commonly used to evaluate the effectiveness of DMUs (Decision Making Units) based on recognised characteristics. A scalar measure of efficiency known as the SBM addresses the input surplus and output shortage of certain DMUs. According to the analysis's findings, the best performing PSBs (public sector banks) are CBI, UBI, Punjab & Sind, and IDBI, whereas the best performing Pvt.SBs (private sector banks) are Axis, Yes, ICICI, and South Indian banks in terms of efficiency scores.

Keywords: DEA, CCR, BCC, SBM, Unit invariant

1. INTRODUCTION

The Data Envelopment Analysis (DEA) is an area of Operations Research (OR) that evaluates the efficacy of found DMUs using mathematical programming. With this technique, the effective DMUs for the supplied variables are found. The efficient DMUs are presented along the border line of the production possibility set and contrasted with other inefficient DMUs from a convex set [1]. The main objective of the DEA is to assess the relative efficacy of the DMU using linear programming, with a range of zero (the worst) to one (the best).

One of the foundational DEA models, the CCR, was first suggested in 1978. This method compares the DMU's many outputs and input variables to the sum of all the available DMUs to assess the efficiency [7]. The CCR model seeks to maximise outputs with the least amount of observable input values while still satisfying at least the specified output levels. 20 banks were selected for this study from the PSBs group, and 21 banks were selected from the Pvt.SBs group based on 5 input variables and 6 outcome variables. The output in this paper was produced using the DEA Solver Software. The essential input and output elements that are employed [3, 4, and 5]:

Inputs

- Borrowing
- Number of Employees
- Capital

- Fixed Assets
- Total liabilities

Outputs

- Loans & Advances
- Investment
- Deposits
- Advances
- Operating Expenses
- NPA's

The Constant Returns-to-Scale (CRS) is depicts the proportionate change in the input and output on frontier line. The BCC (Banker, Charnes and Cooper) method proposed by Banker is an extension of the CCR methodology (1984) [2]. This approach states that virtual weights of the input and output variables are used to calculate the DMU's efficiency at the specified variables. The Variable Returns Scale is the primary goal of the BCC technique (VRS). With this method, the efficiency of DMUs on the frontier line may change depending on the circumstance or remain constant. Compared to the CCR approach, the BCC approach can assist in identifying a bigger number of efficient DMUs from the production possibility set [6]. The optimum solution of BCC_0 is represented by ${\theta_B}^*$, λ^*, $s^{-*}s^{+*}$ and where $s^{-*},\ and\ s^{+*}$ represents input excesses and output shortfalls, respectively.

Although the nature of BCC approach cause the non-zero slacks are not taken into account while evaluating the radial (proportional) efficiency of the set DMUs using the CCR model. Because the non-zero slacks are not included, it highlights the flaw. Tone (1997, 2001) presented the SBM of efficiency model to address the non-zero slack insufficiency [23]. To assess efficiency based on slacks, the SBM efficiency is introduced.

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The effectiveness Efficiency is assessed using SBM, which is affected by objective criteria and invariant to the unit of measurement employed for various DMU-related variables [11]. SBM is a more effective strategy than earlier models since it focuses specifically on the variable gaps. SBM's primary goal is to transform an "inefficient" DMU into a "efficient" DMU by focusing on the DMU reference set. Consequently, the choice to assess efficiency based on its reference set shouldn't be impacted or influenced by extreme values or the entire data set [15]. The primary characteristic of SBM is described by

1. Unit Invariant.
2. Monotone decrease [19].

Tone (2001) proposed radial model are employed in DEA [20]. In CCR, the radial model is utilised which implies that changes in the inputs or outputs of DMUs are the subject of this model are proportional. It displays the CCR model efficiency measures are in proportional [16]. However, no inputs or outputs in real-world business situations react proportionally [17]. The radial model's failure to account for slacks while presenting efficiency score is another flaw. When judging efficiency, we have a lot of non-radial slack in other circumstances [1, 13, 18].

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When we use the efficiency score as the only index to gauge how well DMUs work, the radial model could lead us to mislead. The non-radial SBM efficiency disregards the assumption and deals directly with slacks in place of the radial method [14]. The Slack Based Measure of Efficiency in DEA analysis were used by Chunhua Chen, Haohua Liu, Lijun Tang, and Jianwei Ren [8] to investigate China's regional transportation systems (RTSs) taking transportation accidents into consideration. The findings assist decision-makers in analysing the advantages of efficient RTSs and enhancing the performance of ineffective RTSs.

The productive effectiveness of paper chemical mills was studied in [22] by Dong Guo and Zheng-Qun Cai. In this research factors with respect to input and output are incorporated into the objective function used in this study's SBM of super-efficiency to measure DMUs. They successfully combined the SBM and S-SBM models, producing scaling factors for input savings and output surplus. Through the use of two numerical examples, this study reveals and demonstrates the decision variable that affects the efficiency score for a certain DMU.

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The extension of super efficiency in DEA with respect to the additive model with integer-valued by Yu S. and Hsu C. in [9]. In this study, the two models have been compared and the model was implemented to data set of 13 bus firms operating in the Taichung municipal bus system. The two models' efficiency scores did not significantly differ, according to the data.

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2. THEORY AND METHODOLOGY

The phrases "productivity" and "technical efficiency" are employed in DEA in such a manner that the product appears to be turning its inputs into outputs because the goal of production is to turn inputs into valuable outcomes (outputs). A function is created by the production technology using input and output variables.

$$O = f(I)$$

2.1 CCR Measure

The basic methodology propounded by CCR (1981) measuring quantitative measure of efficiency of the identified DMUs for the analysis with multiple input and output variables.

In this analysis for quantitative measurement N firms has been taken from the production possibility set, and each yields the r outputs $(O_{1m}O_{2m} \dots \dots \dots O$ from the k inputs $(I_{1m}I_{2m} \dots \dots \dots I_{km})$ [10]. From the N firms the average productivity of m firms is given as follows

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$$\mathrm{P_m} = \frac{\sum_{\mathrm{r=1}}^{\mathrm{s}} \mathrm{b_{rm}O_{rm}}}{\sum_{\mathrm{i=1}}^{\mathrm{k}} \mathrm{a_{im}I_{im}}} \quad (1)$$

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From the envelopment analysis it implies that the quantitative measure efficiency should not be exceeding the unity. In this scenario the output from the function of productivity described as follows.

$$\mathrm{P_m} = \frac{\sum_{\mathrm{r=1}}^{\mathrm{s}} \mathrm{b_{rm}O_{rm}}}{\sum_{\mathrm{i=1}}^{\mathrm{k}} \mathrm{a_{im}I_{im}}} \leq 1 \quad (\mathrm{r} = 1, 2, \dots.\mathrm{s}) \quad (2)$$

$$\mathrm{a_{im}} \geq 0;\ \mathrm{b_{rm}} \geq 0; \quad (\mathrm{i} = 1, 2, \dots \dots..\mathrm{k})$$

Where,

$\mathrm{I_{im}} \rightarrow$ i^{th} input of the m^{th} firm (DMU)

a_{im}→ is the weight of that input
O_{rm} →rth output of the mth firm (DMU)
b_{rm}→ is the weight of that output

The above average productivity of fractional programming transformed into a linear programming problem of the identified DMUs for the analysis, and this function leads to the input minimization of a linear programming is given as follows

$$\theta(CCR)=\mathrm{Min}\ (\theta) \quad (3)$$

$$\text{Subject to} \quad \sum_{j=1}^{m} \lambda_j I_{ij} \leq \theta I_{ij} \quad (i = 1, 2, \ldots\ldots, k)$$

$$\sum_{j=1}^{m} \lambda_j O_{rj} \geq O_{rj} \quad (r = 1, 2, 3, \ldots, s)$$

$$\lambda_j \geq 0 \quad (j = 1, 2, 3, \ldots, m)$$

Where,

$I_{ij} \rightarrow i^{th}$ input of the j^{th} DMU

$O_{rj} \rightarrow \mathrm{r}^{\mathrm{th}}$ output of the j^{th} DMU

λ_j→ Non-negative vector [λ=$(\lambda_1, \ldots\ldots, \lambda_n)^T$]

According to the duality the objective function a linear programming are equal

$$\mathrm{Max} \sum_{r=1}^{s} b_r O_{rj} = Min\ \lambda \quad (4)$$

The output-oriented DEA target is to maximise the output without a significant level of observed input values, whereas the input-oriented DEA objective is to minimise the input that at least satisfies the specified output level [12]. The mathematical approach of CCR follows that of CRS, which means that it identifies a DMU that is an efficient at increase (decrease) of weighted output and input quantities are proportional[24].

Definition 1: The solution of CCR- efficient if it satisfies

(i) Optimum value $\theta^* = 1$
(ii) $(s^{-*} = 0, \quad s^{+*} = 0)$ i.e., all the slacks are zero otherwise CCR-inefficient.

2.2 BCC Measure

The Banker, charnes, Cooper (BCC) in the year (1984) was extended an approach of CCR is called BCC. The objective of this approach is it can identify a DMU is efficient at increase, decrease, and constant returns-to-scale from the given possibility set on frontier line [3, 4, 5]. The convexity condition of BCC model is

$$\sum_{j=1}^{n} \lambda_j = 1, \ \ \lambda_j \geq 0 \text{ in its constraints.}$$

Mathematical expansion of input oriented BCC model is given by

$$\mathrm{Min}(\theta) = \theta^* \quad (5)$$

$$\text{Subject to } \theta^* I_0 - a\lambda \geq 0$$

$$\mathrm{b}\lambda \geq O_0$$

$$e\lambda = 1$$

$$\lambda \geq 0$$

Where,

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I_0 is the input of DMU_0

a is the input weight of DMU_0

O_0 is the output of DMU_0

b is the output weight of DMU_0

θ^* is a scalar.

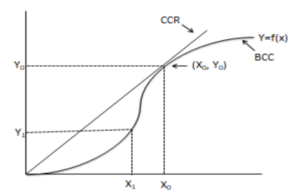

Fig.1. Projection Frontier of CCR, BCC

The above figure represents the production frontier of CCR and BCC efficiency projection of input, outputs and which exhibits to identify the efficient DMU(s) from the given set of DMUs [4, 21]. From the production frontiers, any DMU falling on the CCR frontier line consider efficient, and similarly from the BCC frontier line is in the same direction as identifying efficient DMU(s) [3, 5].

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Definition 2: (BCC_0) of an optimum solution is efficient if it as no slack ($s^{-*} = 0,\ s^{+*} = 0$) and $\theta^* = 1$.

The drawback of CCR, BCC approach is input excesses and output shortfalls not under control that lead to the non-zero slacks[25]. In this case the slack-based measures of efficiency take pioneer role to minimize the non-zero slack deficiency which was propounded by Tone [23].

2.3 The Computation Procedure of SBM

Let the indices corresponding input, output variables of DMUs a = (x_{ij}) and b = y_{ij} respectively. Here (a, b) > 0

The production possibility set in DEA analysis S is given as follows

$$S = \{(I, O) / \ I \geq a\lambda, \quad O \leq b\lambda, \ \lambda \geq 0\} \qquad (6)$$

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Here λ is a non-negative vector

The expression of a certain DMU is consider (I_0, O_0) as

$$\mathrm{I}_0 = \mathrm{a}\lambda + \mathrm{s}^- \qquad (7)$$
$$\mathrm{O}_0 = \mathrm{b}\lambda - \mathrm{s}^+$$

Using slack and surplus behaviour the index χ is given as follows

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$$\chi = \frac{1 - \frac{1}{k}\sum_{i=1}^{k} s_i^- \ / \ I_{i0}}{1 + \frac{1}{m}\sum_{i=1}^{m} s_i^+ / \ O_{io}} \qquad (8)$$

Unit invariant and Monotone property satisfies the above function and the range of the function in SBM is

$$0 < \chi < 1$$

The fractional programming of DEA in λ, s^-, s^+ given by

$$Min\,\chi = \frac{1-\frac{1}{k}\sum_{i=1}^{k} s_i^- \,/\, I_{i0}}{1+\frac{1}{m}\sum_{i=1}^{m} s_i^+ / \,O_{io}} \tag{9}$$

Subject to $I_0 = a\lambda + s^-$

$$O_0 = b\lambda - s^+$$

With $\lambda \geq 0,\quad s^- \geq 0$ and $s^+ \geq 0.$

Input surplus and output shortages s^-, s^+ respectively are the constraints in DEA.

Definition 3: A SBM is efficient from a DMU (I_0, O_0) if its $\chi^* = 1$ and s^{-*}=0 and s^{+*} = 0 i.e,. in the optimum solution there exist no input excess and output shortfall [18]. For an SBM-inefficient, the DMU (I_0, O_0) can be expressed as follows

$$I_0 = a\lambda + s^- \tag{10}$$
$$O_0 = b\lambda - s^+$$

In the above expansion, the DMU (I_0, O_0) can expect improvement, which leads to becoming an efficient DMU by decreasing the input excesses and output shortfalls [17].

2.4 SBM CCR Measure

The mathematical expansion of slack-based measure of efficiency under CCR model is given as follows

$$\text{SBM (CCR) Min } \theta \tag{11}$$

Subject to $\theta I_0 = I_\mu + Z^-$

$$O_0 = O_\mu - Z^+$$
$$\mu \geq 0, Z^{-1} \geq 0, Z^+ \geq 0.$$

The optimum solution of (CCR) is ($\theta^*, \mu^*, Z^{-*}, Z^{+*}$) obtained by

$$I_0 = a\mu^* + Z^{-*} + (1-\theta^*)I_0$$
$$O_0 = b\mu^* - Z^{+*} \tag{12}$$

Thus, (λ, Z^-, Z^+) is feasible for (SBM) and the objective value can be expressed [20] as follows

$$\chi = \left[\frac{\theta^* - \frac{1}{k}\sum_{i=1}^{k} Z_i^{-*} \,/\, I_{io}}{1+\frac{1}{m}\sum_{i=1}^{m} Z_i^{+*} / \,O_{io}}\right] \tag{13}$$

Where $\chi$the SBM coefficient which is is determines by the coefficient matrix of the model.

Theorem: Tone (1997) a DMU (I_0, O_0) is CCR-efficient if and only if it is SBM-efficient.

Definition-1: For an optimal solution of linear programing if $\theta = 1\ and\ (Z^- = s^-, Z^+ = s^+) \neq (0,0)$ then (CCR) is inefficient [15].

Definition-2: an optimum solution of CCR is efficient if $\theta = 1$ and $(Z^- = s^-, Z^+ = s^+) = (0,0)$.

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Commented [DS213]: Theorem 1

Commented [DS214]: Definition I

Commented [DS215]: Definition II

Definition-3: For an optimum solution of $\theta < 1$. Here, (I_0, O_o) is CCR-inefficient [22].

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2.5. Non-Radial SBM

The super-efficiency (non-radial) measure to evaluate the efficient DMU is SBM-efficiency. This approach used to measure the efficiency by minimizing the slacks of identified DMUs and produce the efficiency score more than one or less [7]. To adopt the super efficiency model to input (output) orientation, the linear programming of CRS is given by [26]

$$[\text{Super SBM-I-C}] \quad \delta_I^* = \min 1 + \frac{1}{m}\sum_{i=1}^{m} \emptyset_i \qquad (14)$$
$$\text{Sub to c} \quad \sum_{j=1,\neq 0}^{n} I_{ij}\lambda_j - \frac{1}{2} I_{i0}\emptyset_i \leq I_{i0} \qquad (i = 1, 2, \ldots \ldots \ldots, m)$$
$$\sum_{j=1,\neq 0}^{n} O_{rj}\lambda_j \geq O_{r0} \qquad (r = 1, 2, \ldots \ldots \ldots . s)$$
$$\emptyset_i \geq 0, \; \lambda_j \geq 0$$

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3.0 Empirical Study

The primary information of descriptive statistics is related to the input and output variables. The efficiency computed in this study is relative in nature. The banking performance is relatively not assessed absolutely but is compared with the best in the industry i.e., benchmark to improve the banks in the industry. From the data, efficiency can be determined by comparing the relative sizes of various efficiency measures. The table 1 in appendix represents the descriptive statistics of the sample of 41 public and private sector banks exhibit the standard deviation (SD) is low at the input variable "Number of Employees" and high at "Fixed Assets".

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3.1. Correlation matrix of identified DMUs

The correlation matrix consisting of 10 variables and corresponding 41 decision-making units exhibit the results. The correlation matrix was used to identify the strong and weak correlation between the identified variables, and this significant relationship between the variables is useful for further analysis of banking data concerning the identified variables. The correlation matrix of this analysis represents the correlation coefficients between several variables related to public and private sector banks. From the identified variables the correlation between "Net Income & Operating Expenses" is 0.98, which indicates that they are strongly positively correlated i.e., increasing in Net Income of a bank leads to the increasing Operating Expenses of a bank for maintenance of employees and other expenses. The correlation between "Investment and Net Profit" is -0.13, which indicates a low degree of negative correlation.

Table.2 in Appendix represents the efficiency scores of slack-based measures from the CRS approach. The SBM approach directly deals with slacks of the input, output DMUs and it follows Constant returns to scale. In the appendix table, the first 20 DMUs belong to PSBs and the rest of the DMUs are Pvt.SBs. All the DMUs are efficient in the fiscal year 2015–16 except Vijaya bank (DMU 20) from the PSBs, but in the subsequent year, the results are unstable. Concerning the Pvt.SBs 12 DMUs are efficient out of 21 DMUs from the production possibility set. The worst performance from the Pvt.SBS is IndusInd bank (DMU 29). In this DMU except for Advances, Loans & Advances in all other variables should improve its performance to become an efficient DMU as projection score is concerned.

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From the result, the average efficiency score in 2015–16 was found to be 94%. A financial improvement is observed in the average efficiency score of 2016–17 compared with the previous year. This year, except Canara, CBI, and Dena bank all other DMUs are efficient from the PSBs.

From 2015–16 to 2017–18 efficiency score is improved due to the implementation of new reforms in the banking industry. The average 2% efficiency score is improved from the last fiscal year. The efficiency score in the fiscal year 2017–18 is improved from the last two subsequent years and the average efficiency score is 0.961. In the fiscal year, 2018–19 the performance of DMUs comparatively less than the previous three years and there is a 13% or less efficiency score on average. In this year, the worst performance from all DMUs is DCB Bank with a score of 0.29 (DMU 24).

From the PSBs, Allahabad, Andhra, BOB, Corporation, India, Punjab & Sind, PNB, SBI, Syndicate, UCO, UBI, United BI are efficient banks from the fiscal year 2015–16 to 2018–19. Similarly, the performance from the Pvt.SBs, City (DMU 23), HDFC (DMU 27), Jammu & Kashmir (DMU 31), Karur Vysya (DMU 33), YES (DMU 38), and Bandhan (DMU 39) banks are efficient. On average the performance of PSBs and Pvt.SBs is declined in the fiscal year 2018–19.

3.2. Peer group of PSBs and Pvt.SBs using SBM CRS Approach

The peer group (reference set) in DEA is useful for evaluating the most efficient DMUs and which is the benchmark for the inefficient ones whose efficiency score is less. Once they attain the performance of efficient DMUs, inefficient DMUs can improve the efficiency score. A DMU having the highest peer score classifies it as the most referred DMU (bank) to the other inefficient DMUs.

Table 1: Peer score of SBM CRS approach

S. No	DMU (Bank)	Peer Score
1	Indian	8
2	Bank of Baroda	7
3	P&SB	7
4	CBI	7
5	HDFC	7
6	Axis	6
7	Kotak Mahindra	4

The technical efficiency benchmark (peers) for all public and private sector banks is non-radial SBM under the CRS method. The peer score represents the weights to construct a linear combination of the efficient banks to represent an inefficient one. From the peer counts of efficient banks, Indian (DMU 10) is used more than Bank of Baroda (DMU 3), P&SB (DMU 13), CBI (DMU 7), HDFC (DMU 27), Axis (DMU 21), and Kotak Mahindra (DMU 34) as a peer. So, using SBM under CRS approach, the DMU 10 better performed than other efficient DMUs 3, 13, 7, 27, 21, and 34. Hence, DMU 10 is the most efficient and referred DMU for other DMUs.

Table 2. The descriptive statistics of SBM CRS approach

**	2015-16	2016-17	2017-18	2018-19
Average	0.9439	0.9565	0.961	0.8266
Max	1	1	1	1
Min	0.557	0.5839	0.6605	0.2604
St. Dev	0.1226	0.1075	0.0936	0.2697

From the above descriptive statistics, the result of the efficiency score in the fiscal year 2017–18 is higher than other years on average. The minimum efficiency score of 0.2604 is obtained in 2018–19. The variability in efficiency score found in 2017–18 was comparatively less and higher in the fiscal year 2018–19. On average, the performance of banks in 2017–18 is better than in other years. The variability in the year expected in 2017–18 is comparatively less and the average efficiency score is more.

From the above,the performance of DMUs can conclude that the performance of DMUs is variated from one another fiscal year, but the average efficiency score between the years is not significantly different. This can be proved statistically by Kruskal–Wallis H-test.

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3.3 Tests for Significance of Efficiency Scores

Non-parametric study (Charnes, Cooper, Rhodes (1978) does not need the requirement of any specific functional to proceed to define the efficient frontier or envelopment surface. The non-parametric technique permits several alternate formulations. A model definitely suggested by Groskopf & Valdmanis (1987), Brockett and Golany (1996), and Dasgupta, Sarkis & Talluri (1999) is Kruskal Wallis rank test. To test the significance among the efficiency in different years, the best non-parametric approach is Kruskal–Wallis H-test. Using super efficiency SBM under the CRS model efficiency scores were calculated for the years 2015–16 to 2018–19 and it is found that the average score of efficiency does not significantly differ. This can be proved by statistically using Kruskal Wallis H- test

Hypothesis:

Null Hypothesis Ho: The efficiency scores for the given period is not significantly differ.
Alternative Hypothesis H_1: The efficiency scores for the given period is significantly differ.

Test Statistic: $H = \frac{12}{N(N+1)} * \sum \frac{{T_i}^2}{n_i} - 3(N+1)$

Where, N → Total number of observations

T_i→ Sum of the ranks of i^{th} sample

n_i→ Number of observations from i^{th} sample

$H = \frac{12}{163(163+1)} *(\frac{3374^2}{40} + \frac{3560^2}{41} + \frac{3579.5^2}{41} + \frac{2852.5^2}{41}) - 3(163+1)$

H = 3.889

Decision: Since P-Value (.2738) > $\propto = .05$, do not reject H_0.

Conclusion: Hence we may conclude that the efficiency scores for the given period is not significantly differ.

4.0 Results and Discussion of non-radial SBM CRS Approach

The slack-based measure of efficiency is a scalar measure, which directly deals with input surplus and output shortage of the identified DMUs. Andersen, and Petersen (1993) propounded a model super-efficiency used for improve the performance of an inefficient DMU(s). Under this model, the efficiency scores obtained by eliminating the data on the DMU_0 are to be evaluated from the production possibility set. The possible removal of the DMUs Andersen and Petersen (1993) measure can be regarded as deficient in its treatment of non-zero slack. To eliminate these deficiencies, a model is called non-radial super efficiency SBM under CCR used, which calculates the efficiency score of DMU units and projects the scores between 0 and more than1.

Table.3. in appendix represents efficiency scores and ranks of non-radial Super efficiency SBM under the CRS model. The results of the above efficiency from 2015–16 to 2018–19 indicate a change in efficiency scores concerning the ranks over the given financial years. In 2015–16 Pvt.SBs performed better than PSBs. This year, PNB (DMU 14), Tamil Mercantile (DMU 38), Nainital (DMU 38), Catholic Syrian (DMU 22), and CBI (DMU 8) are shown better performance from PSBs and Pvt.SBs but these performances are not constant for the given years.

In the fiscal year 2016–17, the PSB CBI (DMU 8), P&S (DMU 14), Vijaya (DMU 21) are showed their performance is better and from the Pvt.SBs Axis (DMU 22), IDBI (DMU 31), IndusInd (DMU 30) are the best performers. From the Pvt.SBs, Axis (DMU 22), Federal (DMU 27), YES (DMU 40) showed their potential performance continuously from 2015 to 2018. Looking at the results of 2018–19, ICICI, Dena, Axis, CBI, P&SB, IDBI, and Bank of Baroda are the top performers from the PSBs and Pvt.SBs in the sequence.

The performance decline banks are Allahabad (DMU 41), IDFC (DMU 41), Corporation (DMU 8), Bank of Maharashtra (DMU 5), and Lakshmi Vilas (DMU 35). These bank's performance is poor due to in proper maintenance of the banking system. From the given financial years on average banking, performance is fluctuating due to the planning and implementation of reforms in the banking system.

4.1 Peer group (Reference set) of PSBs and Pvt.SBs using convex non-radial SBM CRS Approach

The Peer group (reference set) in DEA is useful for evaluating the most efficient DMUs. A DMU having the highest peer score classifies it as the most referred DMU (Bank) to the other inefficient DMUs.

Table 3. Peer score of convex non-radial SBM CRS

S. No	DMU (Bank)	Peer Score
1	CBI	14
2	HDFC	10
3	P&SB	6
4	Indian	6
5	Kotak Mahindra	6
6	Canara	6
7	IDBI	5
8	Bank of Baroda	5

The technical efficiency benchmark (peers) for all the Public and Private sector banks is non-radial SBM under the CRS method. From the peer counts of efficient banks, CBI (DMU 7) is more used than HDFC (DMU 27), P&SB (DMU 13), Indian (DMU 10), Kotak Mahindra (DMU 34), Canara (DMU 6), Bank of Baroda (DMU 3) and IDBI (DMU 30) as a peer. So, using non-radial SBM under CRS approach the DMU 7 is benchmark than other efficient DMUs 27, 13, 10, 34, 6, 3, and 30. Hence, DMU 7 is the most efficient and referred DMU for other DMUs.

Table 4. Descriptive statistics of non-radial SBM score from PSBs & Pvt.SBs

***	2015-16	2016-17	2017-18	2018-19
Average	1.113	0.986	1.054	0.885
SD	0.953	0.319	0.275	0.388
Maximum	6.762	1.682	1.608	1.301
Minimum	0.120	0.118	0.102	0.074

The above table represents the efficiency scores of PSBs & Pvt.SBs from the super-efficiency under Constant Returns-to-Scale. The above data used to exhibit the overall performance of DMUs in an individual year is concerned. The average score of banks from the results exhibit fluctuating trends is observed at given years. The variability within the DMUs is comparatively less in 2017–18. The maximum efficiency is credited in the fiscal year 2015–16, and it belongs to Tamilnad Mercantile (DMU 38) Bank. From the result, we may conclude that the average efficiency scores differ in different years but it is very less significantly differ on average. This result can be proved statistically using Kruskal–Wallis H-test.

4.2. Tests for Significance in Efficiency Scores

Hypothesis:
Null Hypothesis Ho: The efficiency scores for the given period is not significantly differ.
Alternative Hypothesis H_1: The efficiency scores for the given period is significantly differ.

Test Statistic: $H = \frac{12}{N(N+1)} * \sum \frac{{T_i}^2}{n_i} - 3(N+1)$

$H = \frac{12}{163(163+1)} * (\frac{3284.5^2}{40} + \frac{3277.5^2}{41} + \frac{3600^2}{41} + \frac{3204^2}{41}) - 3(163+1)$

H = .972

P-Value at $\propto = .05$ significance is .808

Decision: Since P-Value (.808) > $\propto = .05$, do not reject H_0.

Conclusion**:** Hence we may conclude that there is no significant difference in efficiency scores from 2015-16 to 2018-19.

5.0 Conclusion and Recommendation

A non-parametric mathematical model called the DEA enables us to assess the effectiveness of numerous input and output variables used in DMUs. The production possibility set's efficient DMUs are assessed using the DEA's Slack-Based Measure of Efficiency. Slacks are not taken into consideration by the CCR model, which is based on the proportional reduction of input (output) variables. The SBM model, which deals directly with input surplus and output shortage, has been employed as an alternative to CCR.

The non-radial super efficiency of the SBM model has been used in this investigation to create superior DMUs. In general, the VRS result under radial SBM is impractical. To get around this, a workable solution from the discovered DMUs is obtained using the non-radial super-efficiency of SBM of the CRS model.

The findings indicate that the mean overall or economic efficiency from the Slack-Based Measure under Constant Returns-to-scale was 94 percent in 2015–16, rose to 96 percent in 2016–17, and remained unstable in 2017–18 and 2018–19. The performance in the fiscal year 2015–16 showed that IndusInd bank (DMU 29) had the lowest efficiency score, while Vijaya bank (DMU 20) had the highest efficiency score over the course of the following years. In terms of efficiency, Canara (DMU 6), CBI (DMU 7), and Dena Bank (DMU 9) earned the worst rankings in 2016–17. When compared to prior years, efficiency declined by an average of 13% in 2018–19. The top-performing banks among the identified DMUs are Kotak Mahindra, Indian, P & SB, HDFC, Bank of Baroda, and Axis Banks.

The variability score for this year (0.275) is considerably lower, while the average efficiency score for the fiscal year 2017–18 is the second highest. Comparatively speaking, the PSBs CBI (DMU 8), Punjab & Sindh (DMU 14), and SBI (DMU 16) have better overall years' performance than the Pvt.SBs YES (DMU 40), Axis (DMU 22), IDBI (DMU 31), and South Indian Bank (DMU 38). While Pvt.SBs typically outperforms PSBs in terms of potential performance, an upward tendency is seen over the course of the study period. The average efficiency performance of decision-making units is statistically insignificant. From the results of the efficient banks, an inefficient bank(s) adopt their financial performance then there is scope to improve the performance. From the results, financial banks are encouraged to adopt this model to improve their financial performance in the listed financial parameters.

Disclaimer (Artificial Intelligence)

I hereby declare that No generative AI technologies such as Larger Language Models (ChatGPT, COPILOT etc.) and text-to-image generators have been used during the writing or editing of the manuscript.

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