MHD Flow from a Vertical Porous Plate in presence of Heat Source

ABSTRACT

Aim: With a heat source present, this analytical study aims to examine the thermal radiation affects the unsteady MHD free convective mass transfer flow through a semi-infinite vertical porous plate with a changing suction velocity.

Study design: On this plate, a uniform transverse magnetic field is applied perpendicularly. The non-dimensional governing equations are resolved by applying a simple perturbation technique.

Place and Duration of Study: Department of Mathematics,The Assam Royal Global University, Guwahati, Assam between January 2024 to August 2024

Methodology: We allow an infinite vertical porous plate to be passed by a viscous, incompressible fluid that is optically thin. The X-axis is introduced along the infinite vertical plate, the Y-axis is normal to the plate, and the Z-axis is along the width of the plate in a Cartesian coordinate system (x',y',z'). At the beginning, the fluid and plate were both at the same temperature (T'_∞) and concentration (C'_∞) throughout. The concentration level at the plate increased to C'_w at time $t'>0$, and the plate temperature was abruptly increased to T'_w. The plate is subjected to a normal magnetic field that is consistent. The semi-infinite plane surface assumptions cause all the flow variables, with the exception of pressure, to be functions of only y' and t'.

Results: When the Grashof number Gr rises, the fluid velocity increases, but when the Hartmann number M rises, it decreases. In the presence of a heat source S, the fluid temperature drops as the radiation parameter Q and Prandtl number Pr grow. When the Soret number (Sr) increases, the species concentration at the boundary layer increases, but when the chemical reaction parameter (Kr) increases, it drops.

Conclusion: From the work we have concluded that in presence of heat source, the fluid temperature drops with the increament of Prandtl number and Radiation parameter.

Keywords: Magnetohydrodynamics, thermal radiation, free convection, porosity

1. INTRODUCTION: The study of electrically conducting fluid dynamics in the presence of magnetic fields is known as magnetohydrodynamics. In 1970, Hannes Alfvén won the Nobel Prize in Physics for founding the science of MHD. The basic idea of MHD is that a flowing conductive fluid can induce current in it, which in turn applies forces to the fluid and modifies the magnetic field. Numerous scientific domains have found use for MHD, which has motivated numerous scientists to pursue research on it. Geophysics, plasma physics, and astrophysics subjects like magneto-convection and MHD turbulence are among the subjects covered in MHD. It is important for biomedical science and biomedical engineering as well.Natural convection is a heat-transfer process where fluid motion is not produced by Soret effect or thermal diffusion is the process of transfer of mass due to combined effect of concentration and temperature gradient. Many industrial and environmental processes such as in evaporation from large open water reservoirs, heating and cooling chambers etc.

The degree to which a material blocks light from flowing through it is known as its optical thickness. The dimensions and physical characteristics of the material determine its optical thickness. It is a dimensionless quantity represented by the symbol τ. The gas is considered optically thick if $\tau >> 1$, and optically thin if $\tau << 1$. A gray gas is one whose optical thickness is unaffected by the electromagnetic radiation's wave number. If not, the gas is referred to as non-gray gas. Non-gray gases are frequently encountered in the environment.

Survey of literature is an essential ingredient of research work. Everyone has to rely on it to understand and develop the concept to analyse the subject of research. Bejan et al. [1], Ahmed and Choudhury [8], Sattar [10] and many others have studied MHD free convective heat and mass transfer flow in a porous medium. Mythere and Balamurugan [3] have considered MHD free convective flow with thermal diffusion due to importance of Soret effect in fluid motion.Coogley et al. [2] have researched onDifferential approximation for radiative heat transfer in a Gray gas near equilibrium. Pal and Talukdar [5] have analysed MHD convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction by perturbation technique. Similarly, Cookey et al. [4], Ibrahim et al. [6] and many other scientists have analysed the radiation effect of MHD flows. Balamurugan et al. [7] have researched on unsteady MHD free convective flow with time dependent suction and chemical reaction in a slip flow regime.N. Ahmed [9] analysed the effect of chemical reaction on a transient MHD flow past a suddenly started infinite vertical plate with thermal diffusion and radiation.

2. MATHEMATICAL FORMULATION: Allow an infinite vertical porous plate to be passed by a viscous, incompressible fluid that is optically thin. The X-axis is introduced along the infinite vertical plate, the Y-axis is normal to the plate, and the Z-axis is along the width of the plate in a Cartesian coordinate system (x', y', z'). At the beginning, the fluid and plate were both at the same temperature (T'_∞) and concentration (C'_∞) throughout. The concentration level at the plate increased to C'_w at time $t'>0$, and the plate temperature was abruptly increased to T'_w. The plate is subjected to a normal magnetic field that is consistent. The semi-infinite plane surface assumptions cause all the flow variables, with the exception of pressure, to be functions of only y' and t'.

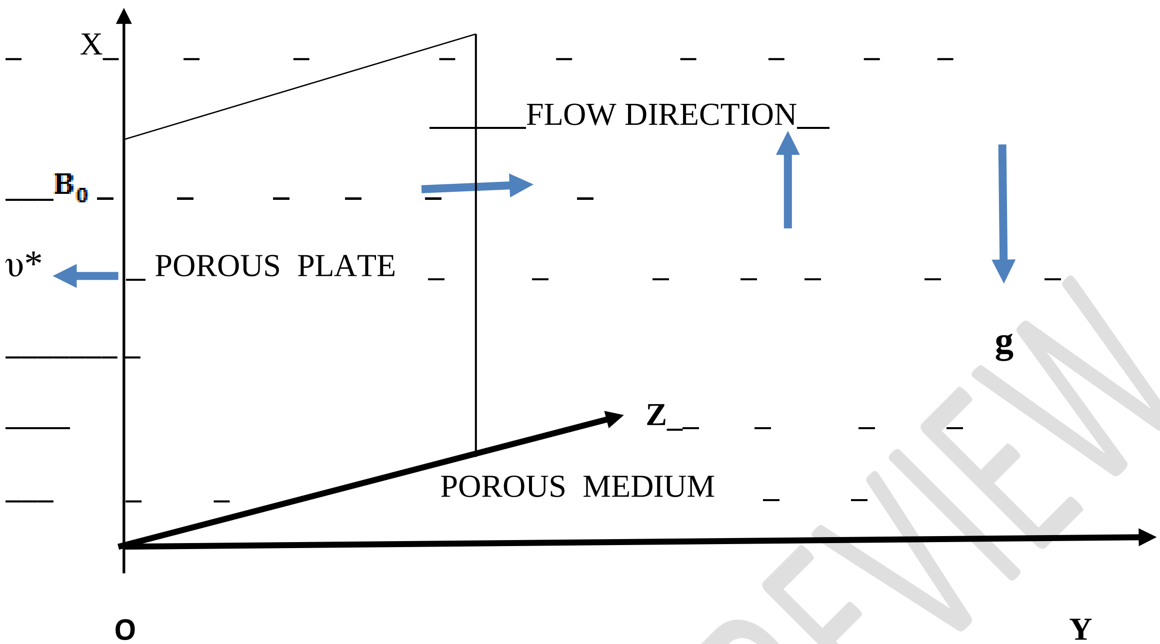

Figure 1. Physical Configuration

Governing equations

Continuity equation-

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum equation-

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{k'}u' - \frac{\sigma B_0^2}{\rho}u' \tag{2}$$

Energy equation-

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K_1}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} + S^*(T - T_\infty) \tag{3}$$

Species continuity equation-

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} + D_1\frac{\partial^2 T'}{\partial y'^2} - K_1(C' - C'_\infty) \tag{4}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} t > 0, u' = 0, T' = T'_\infty + (T'_w - T'_\infty)\left(1 + \varepsilon e^{i\omega' t'}\right), C' = C'_w \text{ at } y' = 0 \\ \\ u' \to 0, T' \to T'_\infty, C' \to C' \text{ at } y' \to \infty \end{array}\right\} \quad (5)$$

with usual meaning of the symbols.

According to Coogley's model, the radiative heat flux in optically thin non-gray gas near equilibrium is specified by

$$\frac{\partial q_r}{\partial y'} = 4I(T' - T'_\infty), \text{ where } I = \int_0^\infty (K_\lambda)_w \left(\frac{\partial e_{\lambda b}}{\partial T'}\right)_w d\lambda \quad (6)$$

Using (6) in (3) we get,

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K_1}{\rho C_P}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_P} 4I(T' - T'_\infty) \quad (3.1)$$

It is seen from equation (1) that v′ is a constant. Assuming the suction velocity to be oscillatory we have

$$v' = -v_0\left(1 + \varepsilon A e^{i\omega t}\right) \quad (7)$$

where ε, A are small such that $\varepsilon A << 1$.

The non-dimensional quantities for making the governing equations in dimensionless form are-

$$y = \frac{v_0 y'}{\upsilon}, u = \frac{u'}{v_0}, t = \frac{t' v_0^2}{4\upsilon}, T = \frac{T' - T'_\infty}{T'_W - T'_\infty}, C = \frac{C' - C'_\infty}{C'_W - C'_\infty}, Sc = \frac{\upsilon}{D}, v = \frac{v'}{v_0}, Gr = \frac{g\beta\upsilon(T'_W - T'_\infty)}{v_0^3},$$

$$Gc = \frac{g\beta^*\upsilon(C'_W - C'_\infty)}{v_0^3}, \omega = \frac{4\upsilon\omega'}{v_0^2}, M = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2}, K = \frac{K' v_0^2}{\upsilon^2}, Sr = \frac{D_1(T'_W - T'_\infty)}{\upsilon(C'_W - C'_\infty)}, Pr = \frac{\mu C_p}{K_1},$$

$$Q = \frac{4I\upsilon}{\rho C_p v_0^2}, \; K_r = \frac{\upsilon K_1}{v_0^2}, \; S = \frac{S^* \upsilon^2}{K_T v_0^2}$$

Using the above non-dimensional quantities in the equations (2), (3.1) and (4), the governing equation reduces to

$$\frac{1}{4}\left(\frac{\partial u}{\partial t}\right) - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = GrT + GcC + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \quad (8)$$

$$\frac{1}{4}\left(\frac{\partial T}{\partial t}\right) - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} - QT + \frac{S}{Pr}T \quad (9)$$

$$\frac{1}{4}\left(\frac{\partial C}{\partial t}\right) - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + Sr\frac{\partial^2 T}{\partial y^2} - KrC \quad (10)$$

The corresponding boundary conditions are-

$$\left.\begin{array}{l} t > 0, u = 0, T = T_W = 1 + \varepsilon e^{i\omega t}, C = 1 \text{ at } y = 0 \\ u \to 0, T \to 0, C \to 0 \text{ at } y \to \infty \end{array}\right\} \quad (11)$$

3. METHOD OF SOLUTION

The above system of partial differential equation is reduced to ordinary differential equation by taking the velocity, temperature and concentration respectively as-

$$\left.\begin{array}{l} u(y,t) = u_0(y) + \varepsilon e^{i\omega t}u_1(y) + O(\varepsilon^2) \\ T(y,t) = T_0(y) + \varepsilon e^{i\omega t}T_1(y) + O(\varepsilon^2) \\ C(y,t) = C_0(y) + \varepsilon e^{i\omega t}C_1(y) + O(\varepsilon^2) \end{array}\right\} \quad (12)$$

Substituting (12) in the equations (8) to (10) and equating the harmonic and non-harmonic terms, and also neglecting the higher order terms of ε, we get

$$u_0'' + u_0' - \left(M + \frac{1}{K}\right)u_0 = -GrT_0 - GcC_0 \quad (13)$$

$$u_1'' + u_1' - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)u_1 = -Au_0' - GrT_1 - GcC_1 \quad (14)$$

$$T_0'' + Pr\,T_0' - Pr\,Q\,T_0 + ST_0 = 0 \quad (15)$$

$$T_1'' + Pr\,T_1' - \left(Q + \frac{i\omega}{4}\right) Pr\,T_1 + ST_1 = -PrAT_0' \tag{16}$$

$$C_0'' + ScC_0' - ScKrC_0 = -ScSrT_0'' \tag{17}$$

$$C_1'' + ScC_1' - \left(Kr + \frac{i\omega}{4}\right) ScC_1 = -AScC_0' - SrScT_1'' \tag{18}$$

And the corresponding boundary conditions are-

$$\left.\begin{aligned} &t > 0, u_0 = u_1 = 0, T_0 = T_1 = 1, C_0 = 1, C_1 = 0 \text{ at } y = 0 \\ &u_0 = u_1 \to 0, T_0 = T_1 \to 0, C_0 = C_1 \to 0 \text{ as } y \to \infty \end{aligned}\right\} \tag{19}$$

By utilizing the perturbation technique to solve equations (13) through (18) and applying boundary conditions (19), the corresponding solutions for velocity, temperature, and concentration are

$$u = (A_6 + A_7)e^{-m_{10}y} - A_6e^{-m_2y} - A_7e^{-m_6y} + \varepsilon e^{i\omega t}[(A_8 + A_9 + A_{10} - A_{11} - A_{12})e^{-m_{12}y} - A_8e^{-m_2y} - A_9e^{-m_4y} - A_{10}e^{-m_6y} + A_{11}e^{-m_8y} + A_{12}e^{-m_{10}y}] \tag{20}$$

$$T = e^{-m_2y} + \varepsilon e^{i\omega t}[(1 - A_1)e^{-m_4y} + A_1e^{-m_2y}] \tag{21}$$

$$C = (1 + A_2)e^{-m_6y} - A_2e^{-m_2y} + \varepsilon e^{i\omega t}[-(A_3 + A_4 + A_5)e^{-m_8y} + A_3e^{-m_2y} + A_4e^{-m_4y} + A_5e^{-m_6y}] \tag{22}$$

Skin Friction-

The shear stress in the wall is quantified by the relation-

$$\tau = \left.\frac{\partial u}{\partial y}\right]_{y=0}$$

$$= -(A_6 + A_7)m_{10} + A_6m_2 + A_7m_6 + \varepsilon e^{i\omega t}[-(A_8 + A_9 + A_{10} - A_{11} - A_{12})m_{12} + A_8m_2 + A_9m_4 + A_{10}m_6 - A_{11}m_8 - A_{12}m_{10}] \tag{23}$$

Nusselt number-

The rate of heat transfer at the plate in terms of the Nusselt number based on Fourier's law of heat conduction is expressed as-

$$Nu = \frac{\partial T}{\partial y}\bigg]_{y=0}$$

$$= -m_2 + \varepsilon e^{i\omega t}[-(1 - A_1)m_4 - A_1 m_2] \quad (24)$$

Sherwood number-

The rate of concentration in terms of Sherwood number (*Sh*) is given by the relation-

$$Sh = \frac{\partial C}{\partial y}\bigg|_{y=0}$$

$$= -(1 + A_2)m_6 + A_2 m_2 + \varepsilon e^{i\omega t}[(A_3 + A_4 + A_5)m_8 - A_3 m_2 - A_4 m_4 - A_5 m_6] \quad (25)$$

where,

$$m_2 = \frac{Pr + \sqrt{Pr^2 - 4(S - QPr)}}{2},$$

$$m_4 = \frac{Pr + \sqrt{Pr^2 - 4\{S - \left(Q + \frac{i\omega}{4}\right)Pr\}}}{2}, m_6 = \frac{Sc + \sqrt{Sc^2 + 4ScKr}}{2},$$

$$m_8 = \frac{Sc + \sqrt{Sc^2 + (4Kr + i\omega)Sc}}{2}, \qquad m_{10} = \frac{1 + \sqrt{1 + 4\left(M + \frac{1}{k}\right)}}{2}, \quad N = M + \frac{1}{k} + \frac{i\omega}{4},$$

$$m_{12} = \frac{1 + \sqrt{1 + 4N}}{2}, \quad A_1 = \frac{PrAm_2}{m_2^2 - Prm_2 - \left(Q + \frac{i\omega}{4}\right)Pr + S}, \qquad A_2 = \frac{ScSrm_2^2}{m_2^2 - Scm_2 - ScKr},$$

$$A_3 = \frac{(SrA_1 - AA_2)Scm_2}{f_1(m_2)}, A_4 = \frac{SrSc(1 - A_1)m_4}{f_1(m_4)}, \quad A_5 = \frac{AScm_6(1 + A_2)}{f_1(m_6)},$$

$$f_1(x) = x^2 - Scx - \left(Kr + \frac{i\omega}{4}\right)Sc, A_6 = \frac{Gr - GcA_2}{f_2(m_2)}, A_7 = \frac{Gc(1 + A_2)}{f_2(m_6)}, \; f_2(x) = x^2 - x - \left(M + \frac{1}{k}\right),$$

$$A_8 = \frac{AA_6m_2 + GrA_1 + GcA_3}{f_3(m_2)}, \quad A_9 = \frac{Gr(1 - A_1) + GcA_4}{f_3(m_4)}, \quad A_{10} = \frac{AA_7m_6 + GcA_5}{f_3(m_6)},$$

$$A_{11} = \frac{Gc(A_3 + A_4 + A_5)}{f_3(m_8)}, A_{12} = \frac{A(A_6 + A_7)m_{10}}{f_3(m_{10})}, \; f_3(x) = x^2 - x - N.$$

4. RESULTS AND DISCUSIONS:

The physical depth of the problem is analyzed by studying graphically the effects of various parameters like Thermal Grashof number (*Gr*), Hartmann number(*M*), Radiation parameter(*Q*) etc. on velocity, temperature, concentration, Nusselt number and Sherwood number which are shown in Figure 2-10.

Figure 2 illustrates how velocity rises as Gr increases. As seen in Figure 3, fluid flow in the presence of a magnetic field generates the resistive Lorentz force, which slows the fluid's speed. As radiation parameter Q rises, Figure 4 illustrates how the temperature's magnitude decreases. Figure 5 shows that a rise in pressure causes the fluid temperature to decrease. This is because heat diffuses from the heated plate more quickly with smaller values of Pr than with higher values of Pr since smaller values of Prin improve the thermal conductivities. It is observed that the rise in temperature reduces the species concentration in the boundary layer.

Figure 8 shows that when Hartmann number M increases, the skin friction magnitude τdecreases. As Figure 9 illustrates, the rate of heat transfer Nu increases significantly with increasing radiation parameter Q. Figure 10 shows that the radiation parameter Q has an effect on the rate of mass transfer Sh at the plate.

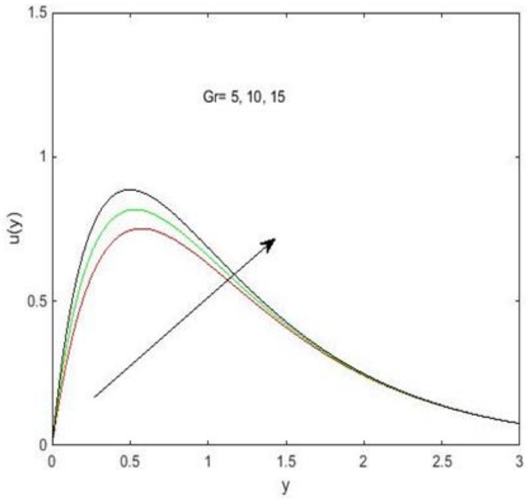

Figure 2. Velocity profile versus y for variation in Gr when Q=5, M=20, K=20, Sc=0.7, Sr=10, Gc=5, Kr=1, Pr=0.71, ε=0.01, ω=2, A=0.01, t=0.2

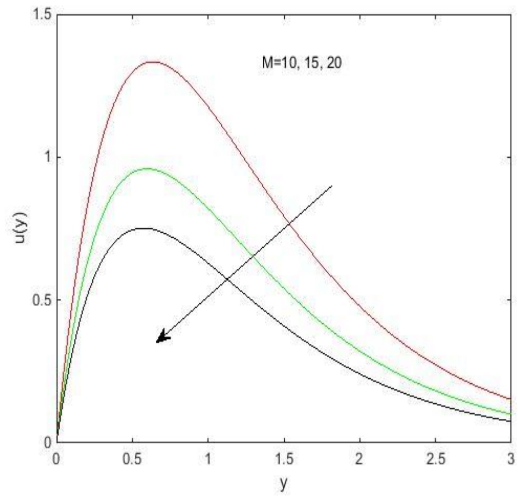

Figure 3. Velocity profile versus y for variation in M when Q=5, K=20, Sc=0.7, Sr=10, Gc=5, Kr=1, Gr=5, Pr=0.71, ε=0.01, ω=2, A=0.01, t=0.2

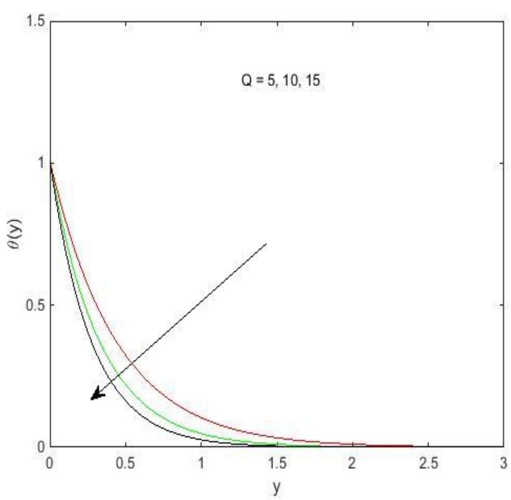

Figure 4. Temperature profile versus y for variation in Q when Pr=0.71, ε=0.01, ω=2, A=0.01, t=0.2, S=5

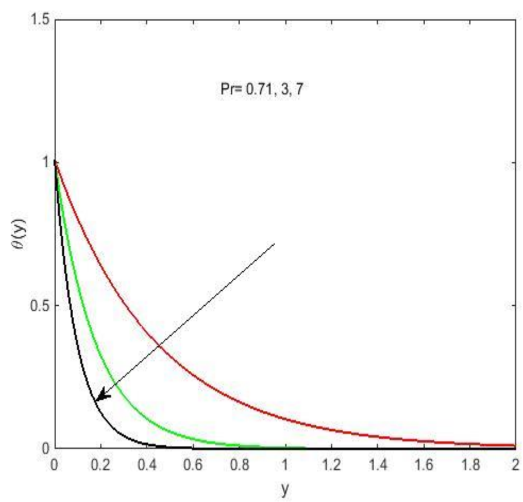

Figure 5. Temperature profile versus y for variation in Pr when Q=5, ε=0.01, ω=2, A=0.01, t=0.2, S=5

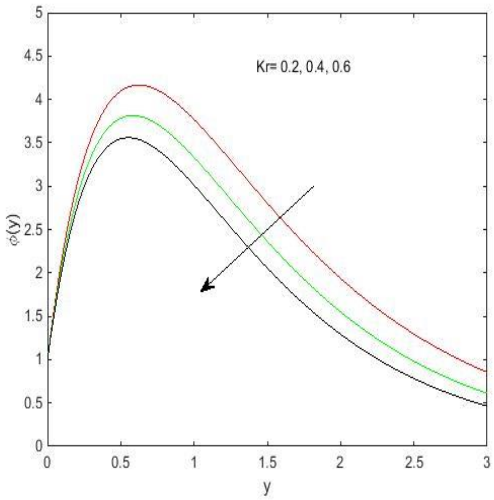

Figure 6. Concentration profile versus y for variation in Kr when Pr=0.71, Sr=10, Sc=0.7, Q=5, ε=0.01, ω=2, A=0.01, t=0.2

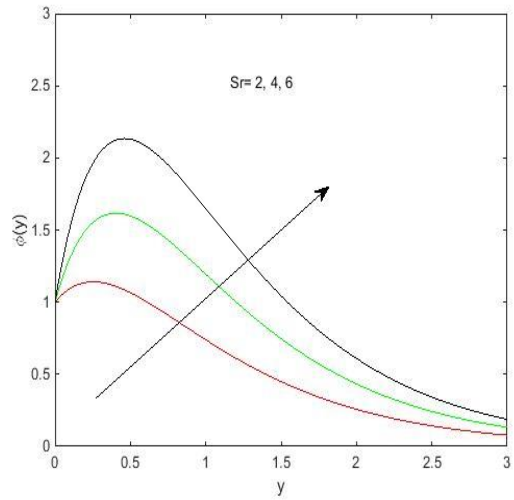

Figure 7. Concentration profile versus y for variation in Sr when Pr=0.71, Kr=1, Sc=0.7, Q=5, ε=0.01, ω=2, A=0.01, t=0.2

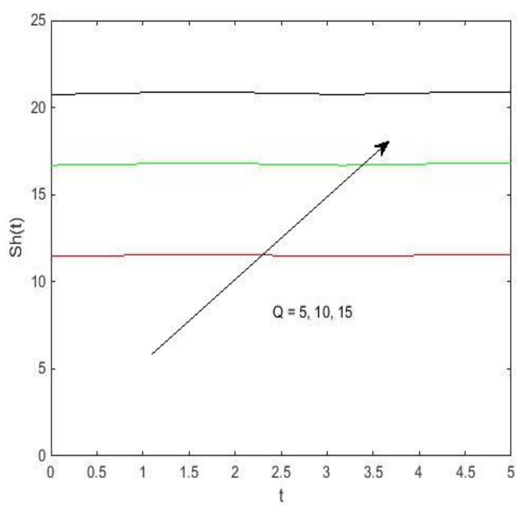

Figure 8. Skin friction versus t for variations in M when Q=5, Gc=5, Sr=10, Sc=0.7, Gr=5, K=20, Pr=0.71, Kr=1, ε=0.01, ω=2, A=0.01

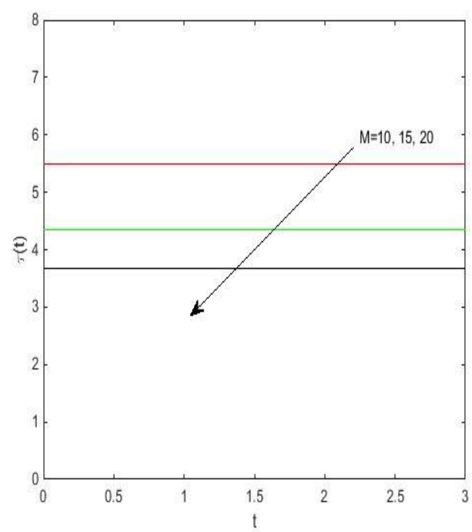

Figure 9. Nusselt number versus t for variation in Q when Pr=0.71, ε=0.01, ω=2, A=0.01

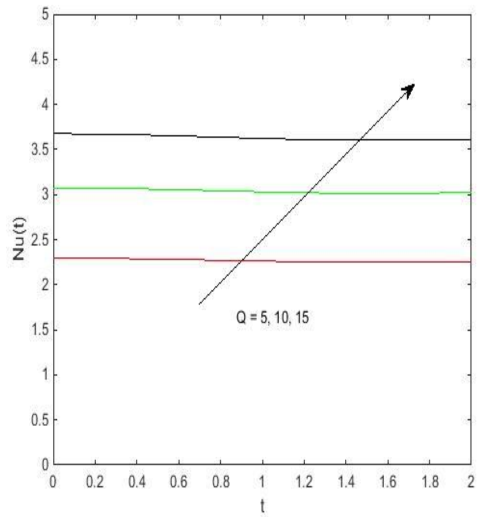

Figure 10. Sherwood number versus t for variation in Q when Sr=10, Sc=0.7, Pr=0.71, Kr=1, ε=0.01, ω=2, A=0.01

CONCLUSIONS

i. When the Grashof number Gr rises, the fluid velocity increases, but when the Hartmann number M rises, it decreases

ii. In the presence of a heat source S, the fluid temperature drops as the radiation parameter Q and Prandtl number Pr grow.

iii. When the Soret number (Sr) increases, the species concentration at the boundary layer increases, but when the chemical reaction parameter (Kr) increases, it drops.

SCOPE FOR FUTURE WORK

The project can be extended by adding some terms to the governing equations and to find its applications in scientific fields.

NOMENCLATURE

u' = Component of velocity along x-axis.

v' = Component of velocity along y-axis.

t' = Time.

t= Non-dimensional time.

ρ= Density of the fluid.

g = Acceleration due to gravity.

v = Velocity of the fluid.

v_0= Scale of the suction velocity.

σ= Electrical conductivity.

υ = Kinematic viscosity.

β = Coefficient of volume expansion of the fluid.

β^*= Coefficient of thermal expansion with concentration.

B_0 = Magnetic induction parameter.

T= Temperature of the fluid.

$T_\infty^/$= Temperature of the fluid far away from the plate.

$T_w^/$= Temperature of the plate.

C = Species Concentration of the fluid.

$C_\infty^{'}$= Species Concentration of the fluid far away from the plate.

$C_w^{'}$= Species Concentration of the fluid near the plate.

μ= Coefficient if viscosity.

C_p = Specific heat at constant pressure.

K_T= Thermal conductivity.

D= Chemical molecular diffusivity.

D_1= Thermal diffusion ratio.

M= Hartmann number.

Sr= Soret number.

Sc= Schimdt number.

Gr=Thermal Grashof number.

Gc= Solutal Grashof number.

Pr= Prandtl number.

Q= Radiation parameter.

Kr= Chemical reaction parameter.

q_r= Radiative heat flux.

K= Porosity parameter.

K_1 = First order chemical reaction rate constant

K'= Permeability of the porous media.

S^*= Coefficient of heat source.

S= Heat source parameter.

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