Application of Univariate Time Series Models for Forecasting Area, Production, and Productivity of Aman Rice in Jalpaiguri

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ABSTRACT

Agricultural production is reliant on modern technology and historical information to enhance the present outcomes and ensure future sustainability. In this study, the area, production, and productivity of Aman rice in Jalpaiguri district using data from 1977-2022 is modelled by two popular time series modelling techniques i.e., the Autoregressive Integrated Moving Average (ARIMA) method and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) method. A comparison of the models based on the lowest values of Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and Mean Absolute Error (MAE) revealed that ARIMA models performed better in the training period on all three series. However, in the test period, the GARCH models on area and production performed better while the ARIMA model performed betteron productivity. The best-fitted models selected were AR (1) - GARCH (1,1)on Aman rice area, AR (1) - GARCH (1,1) onAman rice production and ARIMA (0,1,1) onAman rice productivity. Using the chosen models, forecasts are produced for thesubsequenttenyears.

Keywords: ARIMA, GARCH, ACF, PACF, Forecasting, Area, Production, Productivity

1. INTRODUCTION

In agriculture, time series forecasting is a pathway to assist the decision-making process. For agricultural management, policy-making, and planning purposes, getting estimates of future values is beneficial. By harnessing the power of data analytics using historical data to get advanced estimates, sustainability can be ensured in the face of uncertainty. Time-series modelling is a productive and reliable tool for forecasting future outcomes. Time-series models take historical information to describe the underlying time-lagged relationships in the variables under study to produce accurate forecasts.

Two of the most popular time series forecasting techniques used are the ARIMA and GARCH modelling techniques. The Autoregressive Integrated Moving Average (ARIMA) model fitting is a linear modelling technique to ascertain the stochastic process involved in a univariate series. ARIMA models are populardue to their simplicity and availability in multiple softwares [1]. The ARIMA technique is an approach to model a time series variable with cyclical variation, irregular fluctuations, and trend components. Thistechniqueassumes that the residuals variance is constant.However, in real datasets, changes in variance have been observedwith trends modelled by non-linear processes. Often, the underlying pattern isinadequately explained bya linear modelasthe error variance is non-constant over time. In that case, the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model can capture the conditional variance orvolatility present in the dataset. In India, ARIMA and GARCH models have been used in agriculture to study crop prices[2,3],area, production and yield [4, 5].

Rice is grown extensively in West Bengal making it the largest producer of India. Around 5.29 % of the rice grown in West Bengal is exported (Source: IBEF,2022). In West

Bengal, rice is grown primarily in three seasons, viz., Aus, Aman, and Boro. Of the three seasons, the area under Aman rice cultivation is the largest and so, the production is maximum. In Terai Zone,heavy rainfall favours the growth of Aman rice. Jalpaiguriaccounts for 2.7 % of the area under Aman rice cultivation and 2.5 % of Aman rice production in West Bengal (Source: Directorate of Agriculture, Govt. Of W.B., 2021). There is an emerging need to forecast the area, production, and productivity of Aman rice to help to formulate policy to narrow the gap between supply and demand. An efficient and reliable method to produce forecasts is the time series approach.This study aims to model the area, production, and productivity of Aman rice using the univariate ARIMA and GARCH modelsfor generating forecasts in the Jalpaiguri district of West Bengal.

2. MATERIAL AND METHODS

2.1 STUDY AREA

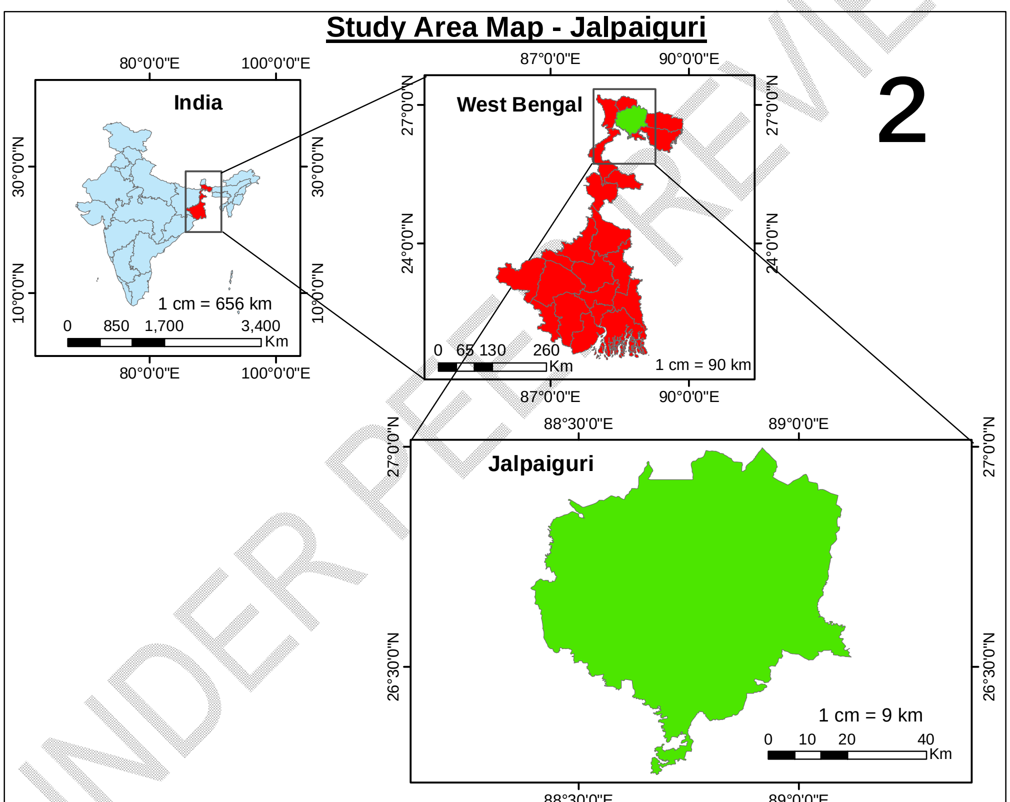


Fig. 1 Geographical map of Jalpaiguri district

Jalpaiguri is an Indian district in the state of West Bengal. As illustrated in Fig.1, it is situated in the northern portion of West Bengal at latitudes 26° 16' to 27° 0' North and longitudes 88° 4' to 89° 53' East. The gross cropped area of Jalpaiguri is 329544 ha of which Aman rice is grown in 111502 ha (Source: Directorate of Agriculture, Govt. of W.B., 2021).

2.2 DATA COLLECTION

Annual data from 1977 to 2022 on area ('000 ha), production ('000 tons), productivity (kg/ha) of Aman rice of Jalpaiguri district were collected from Statistical Abstract, Govt. of W.B. from 1977-2014 and Directorate of Agriculture, Govt. of W.B. from 2015-2022. The dataset is divided into the training dataset consisting of the first 80% i.e. 1977 to 2013 for building the model while the rest 20% i.e., 2014 to 2022 is used for approving the selected model. The analysis was carried out in R Studio IDE.

2.3 ANALYTICAL TECHNIQUES

2.3.1 Trend Analysis

A time series displaying a clear pattern, either upward or downward, is called a trend. The nonparametricMann–Kendall (MK) test[6,7] and Sen's slope Test [8] were employed for trend analysis. The MK test is a statistical test employed for quantifying the significance of trends and Sen'sslope for measuring the magnitude of trend [9].The τ of the MK test is the coefficient of correlation ranging between ±1 [10] with positive values indicating positive trend and negative values indicate decreasing trend. The Sen's slope assumes a linear trend in a time series represented by β. Positive β values indicate upward trend while negative values indicate downward trend [11].

2.3.2 ARIMA

The ARIMA [12] model predicts future values based on a linear function of past values of the time series and its error values. The Moving Average (MA) and Autoregressive (AR) terms make up the ARIMA model. The Autoregressive terms are time-lagged values of that variable with its immediate past value. The Moving average terms, on the other hand, are time-lagged errors resulting from previously made estimates. An important criterion for ARIMA models is that the series should be stationary i.e. the autocorrelation, mean and variance should be unchanging across time. If a series is nonstationary, it implies the presence of a seasonal, cyclical, or trend component. To make it stationary, the series should be differenced *d* times. This combination of the AR, MA, and differenced terms is called the ARIMA model. The ARIMA (p, d, q) model is expressed as follows as:

$$\kappa(\beta)(1-\beta)^d z_t = \lambda(\beta)\,\omega_t$$

Where, $\kappa(\beta) = \left(1-\kappa_1\beta-\kappa_2\beta^2-\cdots-\kappa_p\beta^p\right), \lambda(\beta) = \left(1-\lambda_1\beta-\lambda_2\beta^2-\cdots-\lambda_q\beta^q\right)$; β is known as the backshift operator; $\beta z_t = z_{t-1}$; $z_t$is the t^{th} time series value; ω_t is the t^{th} white noise assumed to be independently distributed; d is the order of differencing; p is the AR order; q is the MA order; κ's are the AR coefficients and λ's are the MA coefficients.

The Box Jenkins procedure of ARIMA modelling consists of the following stages: Model Identification, Parameter Estimation, and Diagnostic Checking.

2.3.2.1 Model Identification

The identification stage is a stage of determining the orders of the AR, differencing, and MA process i.e. p, d, and q respectively. For an ARIMA model, a time series should be stationary. Non-stationary data is made stationary by differencing the series or sometimes, by transforming the data using logarithmic transformations [13].A differenced series is obtained by subtracting values from the current period from its last. The Augmented Dickey-Fuller (ADF) Test is employed to test the stationarity of a series. The results of the ADF test determine the order of *d*. To determine the *p* and *q* orders, one must visualize the ACF and PACF plots. The lag at which the PACF plot has a significant spike and the lag at which the

ACF plot has a significant spike, determines the order of the AR and MA process, respectively. The tentative models are examinedand the model with the smallest Information Criteria values is selected.

2.3.2.2 Parameter Estimation

When the series is stationary, i.e. d = 0, then, a constant term is included in the model, and when d = 1, a constant is included if it improves the AICc value. For d > 1, the constant term is omitted as higher-order trends are harmful to forecasts [14]. After the model is identified, the parameter coefficients of the AR and MA order are estimated using the maximum likelihood function [12].The model chosen for the training set is assessed using the accuracy metrics of RMSE, MAE, and MAPE on the test set.

2.3.2.3 Diagnostic checking

In this stage,residuals are screened for their departure from white noise. White noise is characterized by residuals that are independently and normally distributed. The Normality assumption is visualized by the histogram and checked using the Shapiro Wilk's statistic. The residual ACF plot and Ljung Box test are employed to check the independence of residuals.

2.3.3GARCH

The GARCH[15] model is a generalization of the ARCH [16] model. It is used for modelling a series possessing conditional heterocedasticity or non-constant variance. Totest the presence of ARCH effects, the ARCH L-M Test is used.The ARCH (q) process for the residual series ε_t is defined by the conditional distribution of $\varepsilon_t|\psi_{t-1}$, where ψ_{t-1} is the information about volatility upto$t-1$.Here, $\varepsilon_t|\psi_{t-1}$~N(0, v_t)

$$\varepsilon_t = \omega_t\sqrt{v_t} \; and \; v_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i {\varepsilon_{t-i}}^2$$

Where, ω_t is white noise and ω_t~i. i. d. (0,1), α_0 > 0, $\alpha_i \geq 0, \sum_{i=1}^{q} \alpha_i < 1 \forall\, i$ to ensure positive and unconditional variance of the stationary ε_t series.It means that ε_t is serially uncorrelated with zero mean and changing conditional variance of v_t over time.

The unconditional ACF of squared residuals of the ARCH model, if it exists declines rapidly, excepting the case when the maximum lag is large. While the unconditional ACF of squared residuals of the GARCH model decays slowly which gives a more parsimonious model of the conditional variance [1].The conditional variance of GARCHmodel represents an ARMA process [17] which is a linear function of its lagged terms as well and is given as

$$v_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i {\varepsilon_{t-i}}^2 + \sum_{j=1}^{p} \beta_j \, v_{t-j}$$

Where, $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \forall\, i, j$ to ensure positive conditional variance.${\varepsilon_{t-i}}^2$is the squared residual of *t-i* period from the mean equation and v_{t-j} is forecast variance of *t-j*period.

Such models are called GARCH (*p, q*) where *p*denotes the GARCH order and*q*denotes the ARCH order.The model parameters are estimated by the method of maximum likelihood estimation.Here, ($\alpha_i + \beta_j$) is the measure of volatility persistence. This sum values closer to 1 indicate longer duration of persistence [2].

2.4 Test Statistics

1. Augmented Dickey Fuller Test - The ADF test[18] checks the stationarity of a series. In simple words, including the lagged values of the variable to the existing model, and continuing this procedure till where the autocorrelation is eliminated, the model can be illustrated as:

$$\Delta z_t = \alpha + \beta t + \Psi z_{t-1} + \varphi_1 \Delta z_{t-1} + \cdots + \varphi_p \Delta z_{t-p} + \varepsilon_t$$

Where, α is a constant, β is the coefficient of time trend, $\Delta z_t = z_t - z_{t-1}$, $\sum_{i=1}^{p} \varphi_i \Delta z_{t-i}$ are lag terms up to order p; Ψ is the coefficient of interestand ε_t is the error term

The null hypothesis is $\Psi = 0$, i.e., the presence of unit root indicating that the series is non-stationary. The alternate hypothesis is that the time series is stationary.

$$DF_T = \frac{\widehat{\Psi}}{SE(\widehat{\Psi})}$$

If DF_T calculated <the critical value, the null hypothesis is rejected. If it is not stationary, then we travel further and test the series at first and second difference sequentially.

2. Wilk's Shapiro Test- The Wilk's Shapiro test [19] is a goodness-of-fit test to check the departure from normality. The null hypothesis is the time series $y_1, y_2, \ldots, y_t$is normally distributed. The alternative hypothesis is the series is not normally distributed.

$$W = \frac{\left(\sum_{i=1}^{t} k_i y_{(i)}\right)^2}{\sum_{i=1}^{t} (y_i - \bar{y})^2}$$

Where, k_i are coefficients generated from variance, covariance and mean of the sample order statistics; $y_{(i)}$ is the i^{th}order statistic and $\sum_{i=1}^{t} (y_i - \bar{y})^2$ is the error sum of squares.

The null hypothesis is rejected when W statistic> the selected critical value.

3. Ljung-Box Test- The Ljung Box test [20] is a lack-of-fit test. It is employed to test the absence of serial autocorrelation up to lag *k*. The null hypothesis states that data is distributed independently. The alternate hypothesis is the data are not independently distributed.

$$Q = t\,(t+2) \sum_{k=1}^{h} (t-k)^{-1}\, \hat{\rho}_k^2 \,,$$

Where, h is the total number of lags being tested, t is the sample size,$\hat{\rho}_k$ is the sample autocorrelation at lag k, and. Q is distributed asymptotically as $\chi^2_{(h)}$ with h degrees of freedom.

The null hypothesis of independence is rejected when the value of Q is greater than the selected critical value of the χ^2 distribution with h degrees of freedom.

5. ARCH L-M Test- Engle's [16] ARCH test is a Lagrange Multiplier test to evaluate the presence of conditional heteroscedasticity in the squared residuals. The null hypothesis is $\alpha_j = 0$, $j = 1, 2, \ldots, m$ and the alternative hypothesis is at least one of the α_j coefficients is significant which is tested by the F-test indicating the presence of autocorrelation in the squared residuals of the regression:

$$\varepsilon_t^{\,2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\,2} + \ldots + \alpha_m \varepsilon_{t-m}^{\,2} + \omega_t$$

Where, $\varepsilon_t^{\,2}$is the squared residuals, $t = m + 1, \ldots n$,$\omega_t$is the white noise term, m is the number of lags being tested, n is the total sample size.

The null hypothesis is rejected when the value of F statistic is larger than the selected critical value of the χ^2 distribution with mdegrees of freedom.

Table 1: Model Evaluation Metrics

Statistics	Formula
Akaike Information Criteria	$2f - 2\,ln(\hat{L})$
Bayesian Information Criteria	$ln\,(T)f - 2\ ln\ (\hat{L})$
Akaike Information Criteria with correction	$AIC + \frac{2f^2 + 2f}{T - f - 1}$
Mean Absolute Percent Error	$100 \times T^{-1} \times \sum_{t=1}^{T} \lvert\widehat{\omega_t}\rvert / z_t$
Mean Absolute Error	$T^{-1} \times \sum_{t=1}^{T} \lvert\widehat{\omega_t}\rvert$
Root Mean Square Error	$(T^{-1} \sum_{t=1}^{T} \widehat{\omega_t}^2)^{1/2}$

Where, T = no. of observations; f = no. of parameters; $\hat{L}$ = maximized value of maximum likelihood function of the estimated model, $\widehat{\omega_t} = z_t - p_t$, $\widehat{\omega_t}$ = estimated white noise at time 't', $\widehat{\omega_t}^2$= squared value of estimated white noise at time 't', z_t = original value at time 't', p_t =predicted value at time 't'.

3. RESULTS AND DISCUSSION

3.1 Data Description

Descriptive statistics for Aman rice data between 1977-2022 are presented in Table 2.The area has decreased from 212.00 '000 ha to 111.34 '000 ha, production has increased from 224.30 '000 tonnes to 299.23 '000 tonnes, and productivity has increased from 1058 kg/ha to 2687 kg/ha during the period. Theaverage yearly area of Aman rice is 175.44 '000 ha, production is 269.59 '000 tonnes, and productivity is 1624.28kg/ha. Kurtosis values (0.84) of area indicates leptokurtic nature, 0.50 of production indicates mesokurtic nature, and -1.00 of productivity indicates platykurtic distribution. Negative skewness value (-1.33) of area implies that area decreased continuously and positive skewness values 0.762 and 0.626 indicates increase in the Aman rice production and productivity respectively.Variability is explained by the CV and SD. The CVs of area, production and productivity are18.60%, 25.28%, 37.08%, respectively and SDsare32.63,68.15, 602.35, respectively.

Table 2 Descriptive statistics of Aman rice area, production, and productivity in Jalpaiguri

	Maximum	Minimum	Mean	S.D.	C.V. (%)	Kurtosis	Skewness
Area	220.70	99.74	175.44	32.63	18.60	0.84	-1.33
Production	433.90	138.40	269.59	68.15	25.28	0.50	0.76
Productivity	2794.00	751.00	1624.28	602.35	37.08	-1.00	0.63

3.2 Trend analysis

The Mann–Kendall testand Sen's slope test (Table 3) was used for trend analysis.The results of the MK-Test indicated that area showed a significant negative trend while both production and productivity showed a significant positive trend and the same was established by the Sen's slope method at 0.1 % significance level.Sen'sslope for area, production, and productivity were -1.24 ‘000 ha, 3.37 ‘000 tonnes, and 42.02kg/ha. The existence of trend as observed in the study encourages us to adopt time series models for forecasting.

Table 3 Mann Kendall test and Sen’s Slope Test of area, production, and productivity of Aman rice in Jalpaiguri

Parameters	**Mann Kendall Test**					**Sen’s Slope Test**	
	Kendall’s τ	**p-value**	**MK test statistic (S)**	**Z**	**p-value**	**Sen’s Slope estimator (β)**	**p-value**
Area	-0.59***	0.00	-614.00***	-5.80	0.00	-1.24***	0.00
Production	0.60***	0.00	617.00***	5.83	0.00	3.37***	0.00
Productivity	0.82***	0.00	842.00***	7.96	0.00	42.02***	0.00

3.3 ARIMA

The primary step in ARIMA modelling is to determine the stationarity of the dataset. For this purpose, the ADF test was employed. $P > 0.05$ of ADF statistic (Table 3) of the original series for Aman rice area, production and productivity depict that the series are non-stationary. So, the three series were differenced to obtain a stationary series. As shown in Table 3, differencing operation of order $d = 2$ resulted in a stationary dataset for Aman rice area and production, while productivity was transformed into a stationary series when $d = 1$.Once the stationarity condition is met, the AR and MA orders (*p* and *q*) are to be determined from the ACF and PACF plots of the differenced series shown in Fig 2. Since the ACF and PACF plots of Aman rice area is significant at lag 1 each, the ARIMA model combinations for the area of Aman rice are of the order $p = 0, 1$, and $q = 0, 1$. For production, the ACF plot is significant at lag 1 and PACF is significant at lags 1, 2, and 3.Hence, the model combinations might be in the following order:$p = 0, 1, 2, 3$ and $q = 0, 1$. The ACF and PACF plots of productivity are significant at lag 1 each. Thus, the model combinations can be of the order $p = 0, 1$and $q = 0, 1$. To be able to determine the model that performs best, the models are fitted to the training set and evaluated using the lowest values of AICc, AIC, and BIC. From the order analysis, ARIMA (1,2,1) for Aman area, ARIMA (3,2,0) for production, and ARIMA (0,1,1) with a constant for productivity were selected in Jalpaiguri district which are presented in Table 4

Table 4 :ADF Stationary test of the time series variables for different differencing orders

Time series	$d = 0$		$d = 1$		$d = 2$	
	ADF statistic	**P value**	**ADF statistic**	**P value**	**ADF statistic**	**P value**
Area	-1.84	0.64	-3.23	0.10	-4.87	0.01
Production	-1.85	0.63	-2.98	0.19	-5.32	0.01
Productivity	-1.99	0.58	-3.66	0.04	-	-

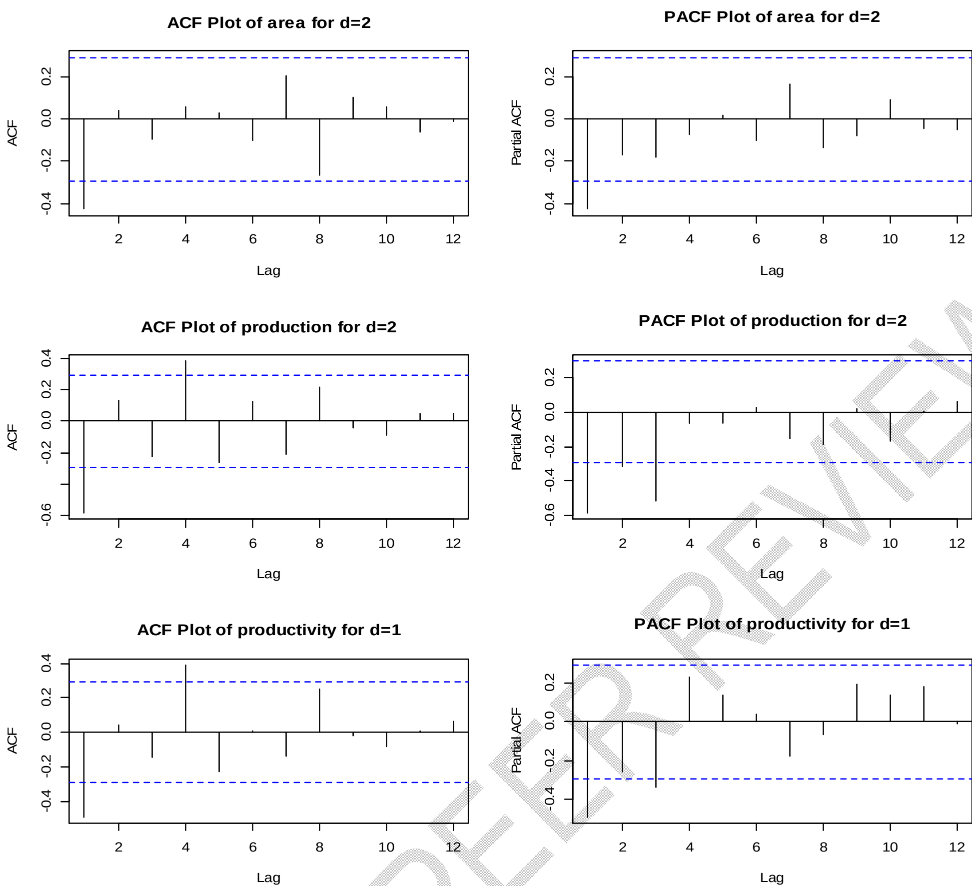


Fig. 2: ACF plot and PACF plots of the differenced series of Aman rice area, production and productivity in Jalpaiguri

Table 5: Selection criteria for the plausible model identification in Jalpaiguri

Aman Rice Area			
ARIMA (p,d,q) Model	**AICc**	**AIC**	**BIC**
ARIMA (1,2,1)	**289.30**	**288.53**	**293.20**
AR (m) - GARCH (p,q) model	**AIC**	**BIC**	**LLH**
AR (1) - GARCH (1,1)	**285.40**	**293.46**	-137.70
Aman Rice Production			
ARIMA(p,d,q) Model	**AICc**	**AIC**	**BIC**
ARIMA (3,2,0)	**356.85**	**355.52**	**361.74**
AR (m) - GARCH (p,q) model	**AIC**	**BIC**	**LLH**
AR (1) - GARCH (1,1)	**388.98**	**397.04**	-189.49
Aman Rice Productivity			
ARIMA (p,d,q) Model	**With constant**		

	AICc	AIC	BIC
ARIMA (0,1,1)	**479.95**	**479.20**	**483.95**
AR (m) - GARCH (p,q) model	**AIC**	**BIC**	**LLH**
AR (1) - GARCH (1,1)	**510.77**	**518.83**	-250.39

The parameters of the selected models are tested for their statistical significance which is given in Table 5. The AR and MA components of all the series are negative and statistically significant at 0.1% level of significance implying that previous period observations as well as previous period shocks have a crucial role in describing Jalpaiguri's area, production, and productivity.Then, the residuals from the models are analyzed for independence and normality to ascertain that they are white noise. There is no serial autocorrelation in the residuals (Fig. 3) and the Q-statistic's p-value is P>0.05, meaning that every series is independent (Table 6). To detect the presence of normality, the Shapiro Wilk's Test presented in Table 6 and the histogram (Fig. 3) explain thatthe residuals for area are non-normal (*P<0.05*) while production and productivity are normal *(P>0.05)*. Ultimately, ARIMA (1,2,1) for area, ARIMA (3,2,0) for production, and ARIMA (0,1,1) with constantfor productivity are selected as best fitted models.

Table 6: Estimated parameters of Aman rice Area, Production, and Productivity for the chosen models

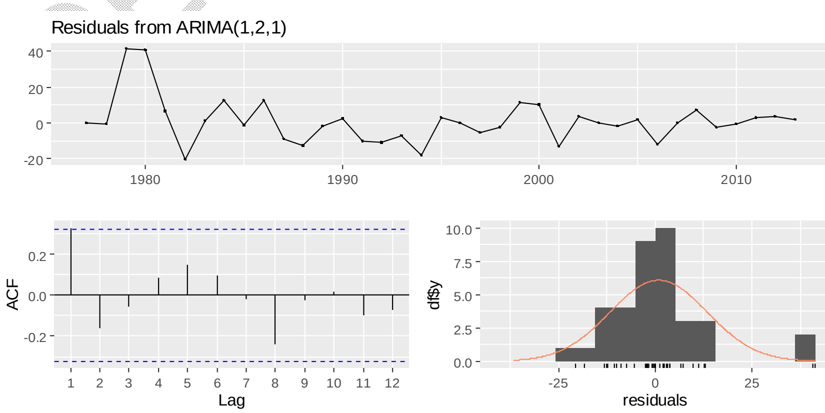
Area					
ARIMA (1,2,1)	**Parameters**	**Coefficient**	**Std. error**	**z- statistic**	**P value**
	AR1	-0.61**	0.19	-3.22	0.00
	MA1	-0.999***	0.09	-11.55	0.00
AR (1)-GARCH (1,1)	**Mean Equation**				
	Parameters	**Coefficient**	**Std. error**	**t statistic**	**P value**
	Constant	180.17***	2.28	78.98	0.00
	AR1	0.54***	0.15	3.67	0.00
	Variance Equation				
	Parameters	**Coefficient**	**Std. error**	**t statistic**	**P value**
	Constant	0.00	0.01	0.00	1.00
	α_1	0.16	0.16	0.99	0.32
	β_1	0.76***	0.14	5.32	0.00
	$(\alpha_1 + \beta_1)$	0.92			
Production					
ARIMA (3,2,0)	**Parameters**	**Coefficient**	**Std. error**	**z- statistic**	**P value**
	AR 1	-1.23***	0.12	-10.41	0.00
	AR 2	-1.13***	0.15	-7.66	0.00
	AR 3	-0.74***	0.11	-6.72	0.00
AR (1) - GARCH (1,1)	**Mean Equation**				
	Parameters	**Coefficient**	**Std. error**	**t statistic**	**P value**
	Constant	255.81***	37.87	6.76	0.00
	AR1	0.87***	0.14	6.31	0.00
	Variance Equation				

	Parameters	Coefficient	Std. error	t statistic	P value
	Constant	6.049	206.02	0.03	0.98
	α_1	0.00	0.04	0.00	1.00
	β_1	0.991***	0.12	8.03	0.00
	$(\alpha_1 + \beta_1)$	0.991			
Productivity					
ARIMA (0,1,1)	**Parameters**	**Coefficient**	**Std. error**	**z- statistic**	**P value**
	MA1	-0.62***	0.12	-5.09	0.00
	drift	34.23***	11.37	3.01	0.00
AR (1) - GARCH (1,1)	**Mean Equation**				
	Parameters	**Coefficient**	**Std. error**	**t statistic**	**P value**
	Constant	1057.91***	238.59	4.43	0.00
	AR1	1.00***	0.08	11.87	0.00
	Variance Equation				
	Parameters	**Coefficient**	**Std. error**	**t statistic**	**P value**
	Constant	168.09	9199.60	0.02	0.99
	α_1	0.00	0.04	0.00	1.00
	β_1	0.995***	0.01	73.23	0.00
	$(\alpha_1 + \beta_1)$	0.995			

#P < 0.1; P < 0.05; ** P < 0.01; *** P < 0.001*

Table 7. Tests of Normality, and Independence for the residuals of the selected models for Aman rice in Jalpaiguri

Model	**Shapiro Wilk's Test**		**Ljung- Box (Q) Test**		
	W statistic	**p value**	**Q statistic**	**df**	**p value**
Area: ARIMA (1,2,1)	0.85	0.00	7.36	5	0.20
Production: ARIMA (3,2,0)	0.96	0.22	8.14	4	0.09
Productivity: ARIMA (0,1,1)	0.99	0.94	6.82	6	0.34



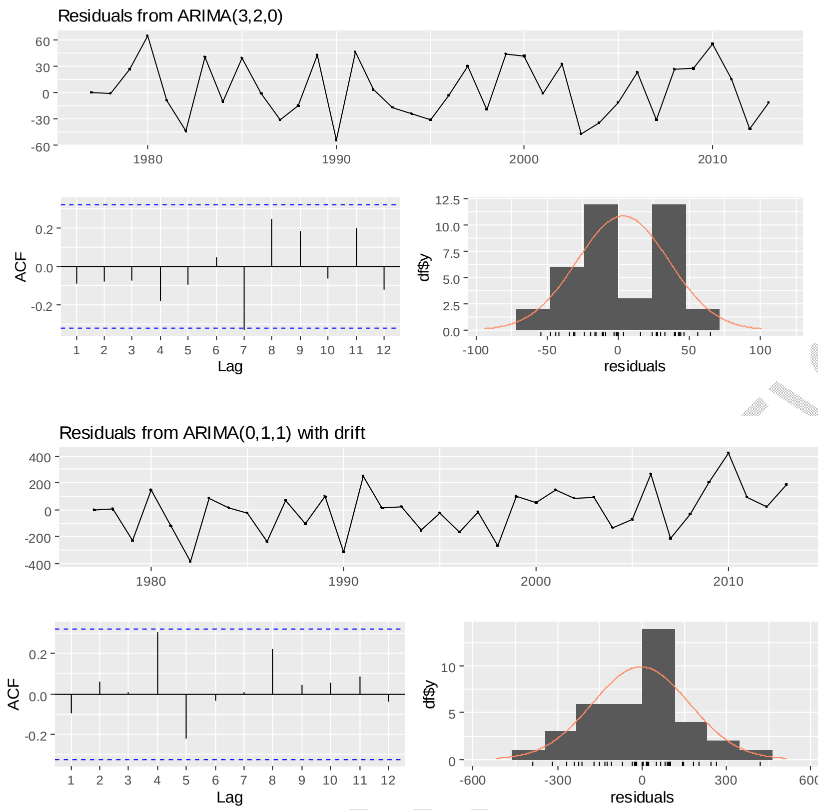


Fig. 3: Residual plots, ACF plots, and distribution of ARIMA (1,2,1), ARIMA (3,2,0), and ARIMA (0,1,1) for Aman rice area, production, and productivity in Jalpaiguri

3.4GARCH:

Like ARIMA models, GARCH models also undergo the identification, estimation and diagnostic checking procedure. The first step of GARCH modelling is to fit a suitable autoregressive (m) process by examining the PACF plot. From the PACF plots given in Fig..4, it is seen that 1st lag for area, production, and productivity have a significant spike. It can be concluded that the model for the conditional mean is AR (1). To test the heteroscedasticity of the residuals obtained from the AR (1) process, firstly, we looked at the ACFs of the squared series of area, production, and productivity and their standardized squared residuals. The ACF plots depicted dependency in both the squared series as well as squared residuals (Fig. 4). The ARCH-LM Test is tested on the squared residuals. The result of the test shows that$P$ < 0.05 (Table8) indicatingthe presence of conditional heteroscedasticity. Hence, we can proceed to build a GARCH model.

Table 8. Test of heteroscedasticity for the residuals of the AR (1) models

Model	ARCH - LM Test	
	Statistic	p-value
Area: AR1	404.89	0.00
Production: AR1	30.32	0.00
Productivity: AR1	22.57	0.00

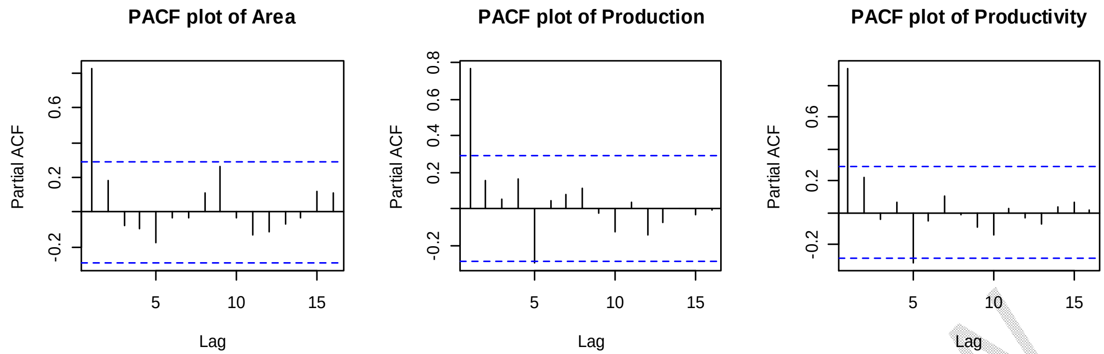


Fig. 4: PACF plot of Aman rice area, production, and productivity in Jalpaiguri

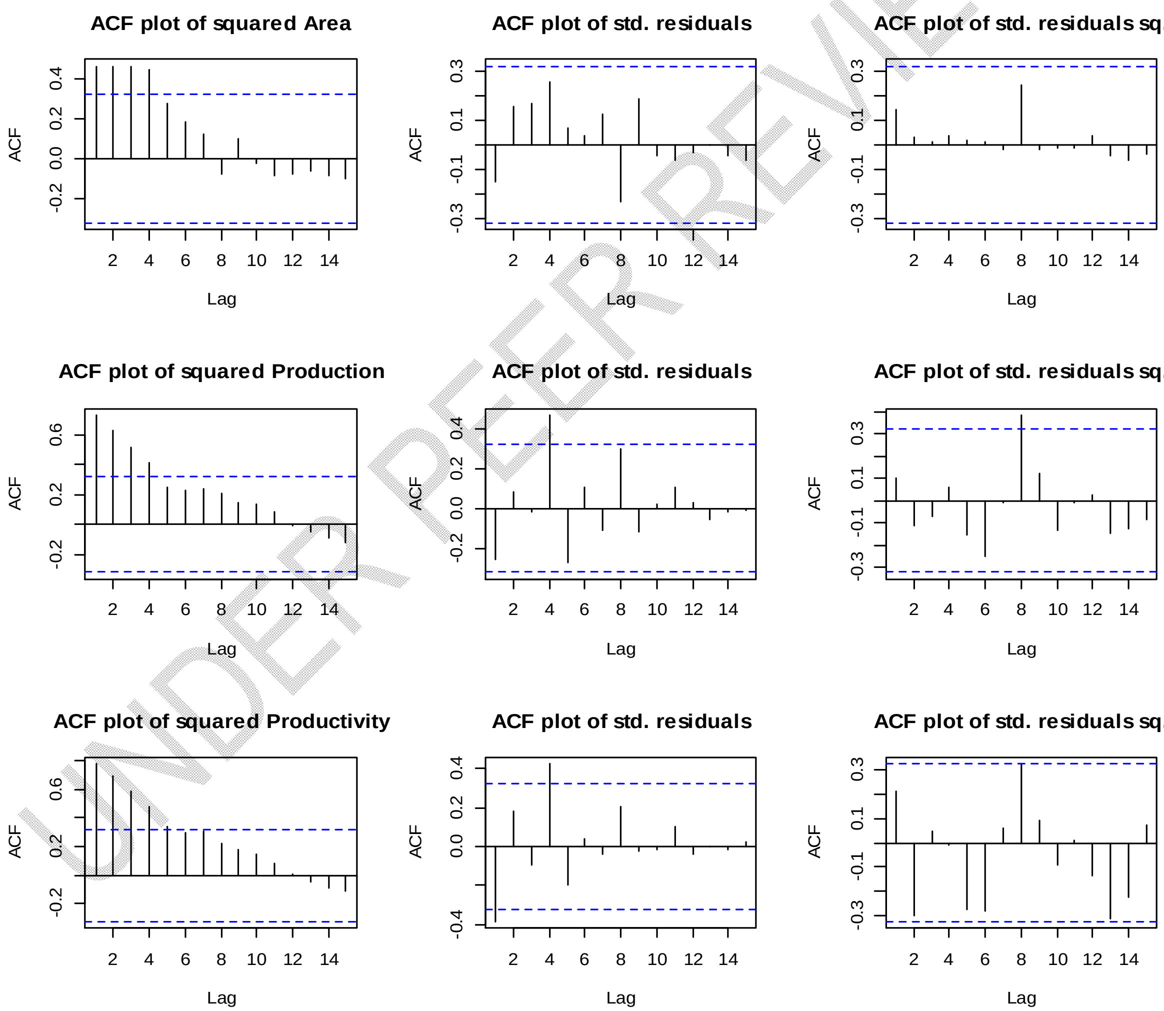


Fig. 5: ACF plot of squared series, standardized residuals, and standardized squared residuals of Aman rice area, production, and productivity in Jalpaiguri

To establish a GARCH (p,q) model, the orders of p and q must be determined. Generally, GARCH models of the order p=,1,2 and q=1,2 are formulated on the training set. The different specification models were compared and AR (1) - GARCH (1,1) was found to be the best model for area, production, and productivity based on smallest value of AIC and BIC criteria which are given in Table 8. The estimated model parameters and their statistical significance are given in Table 8. The constant term, AR (1) term of the mean equation and β_1 of the variance equation are statistically significant at a 1 % level of significance for area, production, and productivity. Significant AR (1) coefficients indicate significant conditional mean effect and β_1 coefficients indicate strong GARCH effects. However, there are no significant ARCH effects in all the three series. This implies that there is no significant error variance, however, thereis conditional variance in the lagged innovations.The sum of α and β coefficients give a measure of the volatility persistence. The results given in Table 8 indicate that there is persistent volatility in Aman rice area (0.9173), production (0.9911) and productivity (0.9954) in Jalpaiguri. It is seen that β_1 values are high indicating persistent volatility that takes longer duration to change [21].

Once the presence of volatilityis confirmed, residual diagnostics are employed to further check for a systematic pattern in the data. The standardized residuals time plot in Fig. 6show no noticeable patterns.The Ljung-Box statistic for the standardized residuals and standardized squared residuals are given in Table 9. The values indicate absence of serial autocorrelation in the standardized squared residuals for area, production and productivity at different lags (Table 9) demonstrating that it is suitable for describing the dynamic volatility. The ACF plot of the standardized residuals (Fig. 5) andsquared standardized residuals (Fig. 5) depict no departure from the model assumptions.The residuals of area are non-normal while production and productivity is normal as observed from the histogram (Fig. 6) and the p-values($P > 0.05$) of the W-S statistic (Table 9). The $P > 0.05$ of the ARCH L-M Test show no ARCH effect exists in the standardized squared residuals.

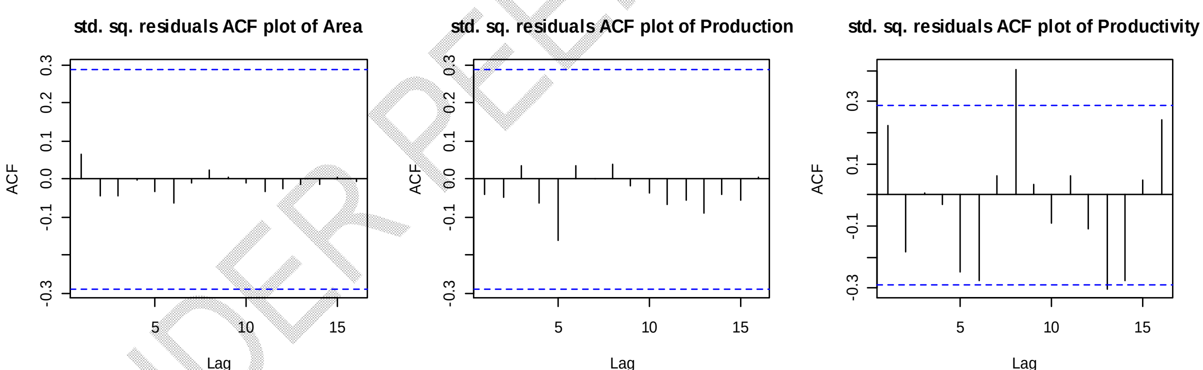


Fig. 6:ACF plots of standardized squared residuals of AR (1) - GARCH (1,1) models for Aman rice area, production, and productivity in Jalpaiguri

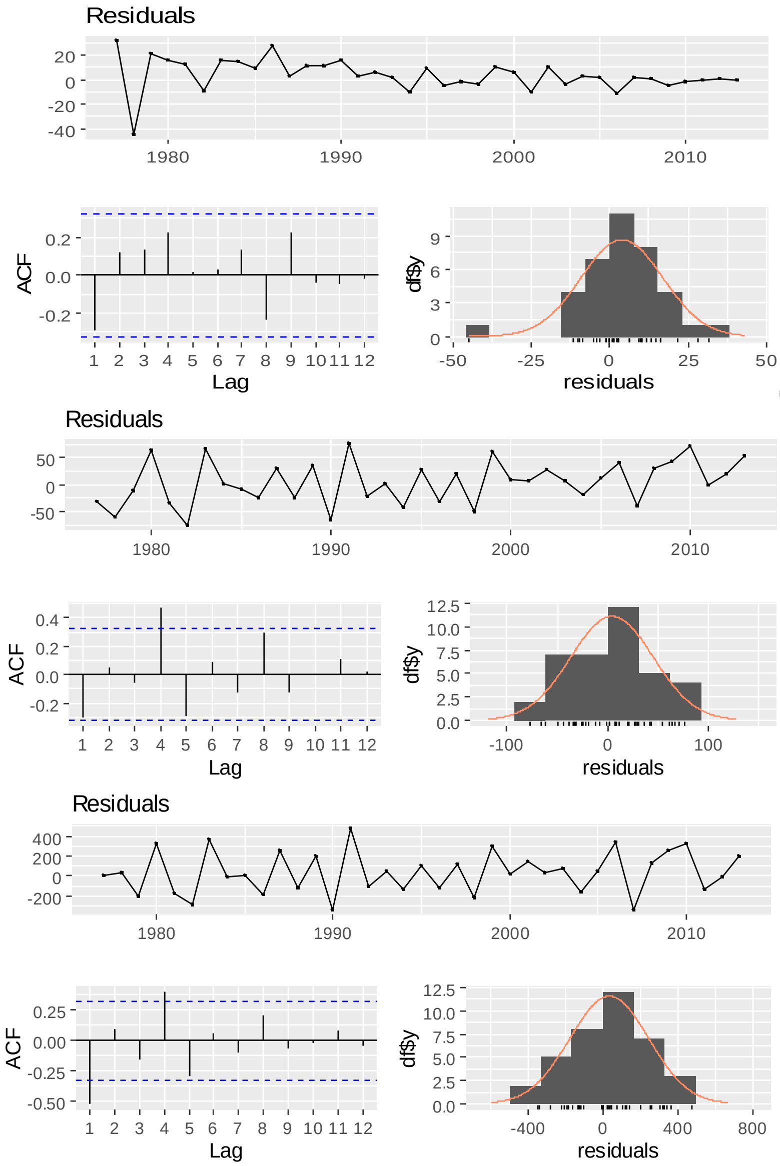


Fig. 6: Residual plots, ACF plots, and distribution of standardized residuals of AR (1) - GARCH (1,1) modelsfor Aman rice area, production, and productivity in Jalpaiguri

Table 9: Standardized residual tests for AR (1) – GARCH (1,1) models of Aman rice area, production, and productivity in Jalpaiguri

Area				
Residual Test	**Test statistic**	**Variable**	**Test value**	**p-value**
Shapiro Wilk	W	$Residual$	0.91	0.01
Ljung Box	Q (1)	$Residual$	3.05	0.08
Ljung Box	Q (2)	$Residual$	3.14	0.03
Ljung Box	Q (5)	$Residual$	4.61	0.14
Ljung Box	Q (1)	$Residual^2$	1.09	0.30

Ljung Box	Q (5)	$Residual^2$	1.45	0.75
Ljung Box	Q (9)	$Residual^2$	1.90	0.92
ARCH L-M	χ^2(3)	$Residual^2$	0.17	0.68
Production				
Residual Test	Test statistic	**Variable**	**Test value**	**p-value**
Shapiro Wilk	W	$Residual$	0.98	0.69
Ljung Box	Q (1)	$Residual$	3.49	0.06
Ljung Box	Q (2)	$Residual$	3.58	0.01
Ljung Box	Q (5)	$Residual$	8.31	0.01
Ljung Box	Q (1)	$Residual^2$	0.39	0.53
Ljung Box	Q (5)	$Residual^2$	1.51	0.74
Ljung Box	Q (9)	$Residual^2$	5.75	0.33
ARCH L-M	χ^2(3)	$Residual^2$	0.11	0.74
Productivity				
Residual Test	**Test statistic**	**Variable**	**Test value**	**p-value**
Shapiro Wilk	W	$Residual$	0.98	0.64
Ljung Box	Q (1)	$Residual$	10.68	0.00
Ljung Box	Q (2)	$Residual$	10.87	0.00
Ljung Box	Q (5)	$Residual$	15.11	0.00
Ljung Box	Q (1)	$Residual^2$	1.46	0.23
Ljung Box	Q (5)	$Residual^2$	4.41	0.21
Ljung Box	Q (9)	$Residual^2$	10.59	0.04
ARCH L-M	χ^2 (3)	$Residual^2$	0.03	0.85

3.5 Validation and Forecasting

Table 9 presents the actual versus predicted Area, production and productivity values from 2014-2022. The predicted values are close to the actual values indicating that the fitted model is appropriate. A two-way validation of the training and testing set is done to select the best model for forecasting. The selected models are compared and the model with the lowest accuracy statistics is selected (Table 10).It can be seen that ARIMA models perform best in the training series of all the three series. In the case of Aman rice area, the AR (1) - GARCH (1,1) outperformed ARIMA (1,2,1) in the test period. The performance on Aman rice production reveals that AR (1) - GARCH (1,1) is better due to lower values of evaluation metrics in the test period. From the results of the accuracy statistics of the test set, linear ARIMA (0,1,1) model performed better for Aman rice productivity due to its lowest values of all forecasting accuracy statistics. Eventually, the models that performed better during the testing period i.e., AR (1) - GARCH (1,1), AR (1) - GARCH (1,1), and ARIMA (0,1,1) models are chosen for forecasting Aman rice area, production, and productivity since they have better performance in the new dataset. The forecast values along with their 95 % confidence interval for the next ten years from 2023 to 2032 are presented in Table 10 and Fig. 7.

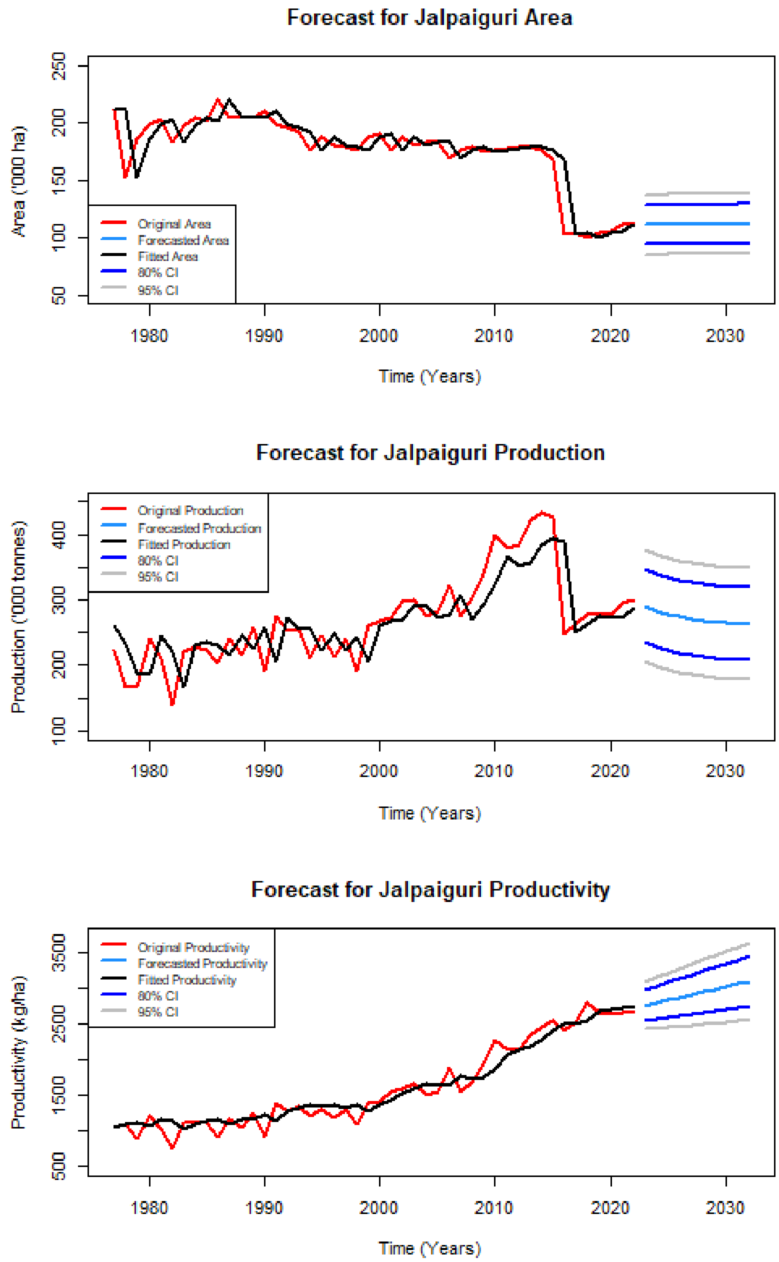


Fig. 7: Forecast plot of AR (1) - GARCH (1,1) of Aman rice Area, AR (1) - GARCH (1,1) of Production, ARIMA (0,1,1) of Productivity in Jalpaiguri

Table 10: Actual vs. predicted values for validation set

Year	Area ('000 ha)			Production ('000 tons)			Productivity (kg/ha)		
	Actual	ARIMA	AR-GARCH	Actual	ARIMA	AR-GARCH	Actual	ARIMA	AR-GARCH
2014	176.40	179.33	180.13	433.9	450.08	384.76	2460.00	2280.12	2336.00

2015	167.78	176.44	176.44	425.33	431.38	395.00	2535.00	2394.11	2460.00
2016	103.43	168.10	167.82	249.33	439.03	388.35	2411.00	2491.65	2535.00
2017	104.95	116.90	103.54	263.93	269.12	251.74	2515.00	2495.77	2411.00
2018	99.74	101.52	105.06	278.70	236.29	263.07	2794.00	2542.00	2515.00
2019	104.68	98.08	99.85	276.90	232.90	274.54	2645.00	2686.41	2794.00
2020	105.29	100.58	104.79	277.91	186.49	273.14	2640.00	2707.07	2645.00
2021	111.50	102.45	105.40	296.18	289.31	273.92	2656.00	2716.91	2640.00
2022	111.34	107.39	111.60	299.23	304.28	288.10	2687.00	2729.35	2656.00

Table 11: Model accuracy statistics for validation of Aman Rice in Jalpaiguri

	Training set			**Testing set**		
Model	**RMSE**	**MAE**	**MAPE**	**RMSE**	**MAE**	**MAPE**
Area: ARIMA (1,2,1)	**7.98**	**12.41**	**4.19**	20.22	**23.71**	19.23
Area: AR (1) - GARCH (1,1)	9.57	13.49	5.06	**17.40**	24.38	**13.81**
Production: ARIMA (3,2,0)	**27.09**	**32.14**	**10.87**	44.08	61.11	27.09
Production: AR (1) - GARCH (1,1)	33.91	40.65	14.39	**29.30**	**59.31**	**10.81**
Productivity: ARIMA (0,1,1)	**131.74**	**169.49**	**10.53**	**67.56**	**97.37**	**2.56**
Productivity: AR (1) - GARCH (1,1)	171.19	210.25	13.38	87.00	121.61	3.31

Table 12: Forecasted values of best-fitted models of Aman rice Area, Production, and Productivity in Jalpaiguri

Year	**Area ('000 ha)**			**Production ('000 tons)**			**Productivity (kg/ha)**		
	low 95	**Point forecast**	**high 95**	**low 95**	**Point forecast**	**high 95**	**low 95**	**Point forecast**	**high 95**
2023	85.26	111.44	137.62	205.21	290.47	375.74	2421.41	2749.61	3077.82
2024	85.37	111.54	137.72	198.40	283.67	368.95	2431.54	2787.74	3143.94
2025	85.47	111.64	137.81	193.11	278.40	363.68	2443.72	2825.87	3208.01
2026	85.57	111.74	137.91	189.00	274.30	359.60	2457.55	2863.99	3270.43
2027	85.67	111.84	138.01	185.81	271.12	356.43	2472.76	2902.12	3331.48
2028	85.77	111.94	138.11	183.33	268.65	353.98	2489.12	2940.25	3391.37
2029	85.87	112.04	138.21	181.41	266.74	352.07	2506.49	2978.37	3450.25
2030	85.97	112.14	138.31	179.91	265.25	350.60	2524.74	3016.50	3508.26
2031	86.07	112.24	138.41	178.74	264.10	349.45	2543.75	3054.62	3565.49
2032	86.16	112.34	138.51	177.84	263.20	348.57	2563.46	3092.75	3622.04

4. CONCLUSION

The production of Aman season rice is the highest among all the three seasons- Aus, Aman, and Boro in Jalpaiguri district. Aman rice is an indispensable crop as it is the major rice contributor providing extensive employment and income. It is imperative to study the scenario of the crop area, production, and productivity and make predictions of the future outcomes to ascertain that the demands are met for feeding the Jalpaiguri populace. The model AR (1) - GARCH (1,1) for Aman rice area, AR (1) - GARCH (1,1) for production, and ARIMA (0,1,1) for productivity is found to be a parsimonious model for Aman rice in Jalpaiguri. It is unsuitable to comment on the supremacy of either of the models since both ARIMA and GARCH models can be used for forecasting. The forecast estimates suggest that Aman rice areais increasing at a slow ratewhile production is likely to decrease and productivity is projected to increase. This research thus suggests to make policies so as to increase Aman rice area and production in Jalpaiguri for assuring nutritional security.

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