Modelling Fluid Flow in Zone 2 of an Open Horseshoe Channel with Lateral Inflow Channels

Abstract

Flooding has been a problem for a long time, especially after heavy rains. Channels constructed to lessen floods are used to move water into rivers, lakes, and the ocean. Engineers have been faced with the challenge of designing drainage ditches, irrigation canals, and navigation channels while making use of hydraulic efficiency to move as much water as possible in order to generate energy. The bulk of studies on open channels have focused on rectangular, parabolic, trapezoidal, and circular channels, leaving a knowledge gap that has to be filled. Less research has been done on channels that resemble horseshoes and have lateral inflows.

Modeling a homogeneous flow in Zone 2 with a horseshoe-shaped cross-section and lateral inflows is the goal of this study. The study's goal was to ascertain how changes in the lateral inflow channels' area and an increase in the lateral inflows' length in zone 2 impact the main channel flow velocity.

The physical circumstances of the flow issue were applied to the conservation equations in order to get the governing equations. These equations were solved using the finite difference approximation method because of its accuracy, stability, and convergence. The investigation's results were displayed graphically.

The investigation found that when the cross sectional area reduces, the main channel's velocity increases. In the end, as the area of the lateral inputs reduces, so does the main channel's velocity. Flood mitigation and water collection for irrigation push the boundaries of science, technology, and engineering by requiring innovative solutions and climate change-tolerant infrastructure that increase crop yield and resilience.

Keywords: Fluid, Open channel, Lateral inflow, Horseshoe Channel, laminar flow, Flow Area.

Nomenclature:

yHeight

rRadius

Q Discharge

A Area

m Number of Lateral uniform Inflows

qVelocity of the channel

θ Angle

T Top width

U uniform Inflow

1.0 introduction

1.1 Background information

In years 2023 and 2024 Kenya experienced a heavy rainfall. In the month of April 2024, heavy rainfall was again experienced in Nairobi, Kisumu and Kajiado counties. The rainfall was too heavy that bridges, vehicles and peoples were swept awayas rivers flooded. Also more than two thousand peoples were displaced from their homes and more than hundred peoples killed.Water would flow from high places towards lowlands where the soil got saturated resulting into excess water remaining stagnant. Such a rainfall becomes a calamity as it is unexpected and the right measures have not been taken.

Sometimes when the rainfall just goes above the normal, many places becomeflooded, houses and other structures get destroyed and such floods become a healthhazard to people. This is because handling such unexpected amounts of water is achallenge even to engineers in Kenya. Designing channels that would control suchan environmental disaster is very important. Open channels have been designed. These channels have been of different cross-sectionssuch as trapezoidal, rectangular,triangular and circular. The fact that the flood problem stillpersists, there is need to come up with a hydraulically efficient channel, that is, a channel that would carry maximum discharge at a given slope, area of flow and roughness coefficient. Such a channel would be used to drive out excess water from the affected areas.

1.2 Literature review

Tsombe et al., (2011) modelled a fluid flow within open channels featuring a circular cross-section. The research findings indicate that an increase in flow depth leads to a decrease in fluid velocity. In addition, the flow velocity rises with a rise in the channel slope. The flow velocity similarly decreases as the channel's radius increases. The data also indicated that a decrease in the channel's slope results in a corresponding decrease in flow velocity, as the two variables are directly correlated. Furthermore, as the flow velocity increases with depth from the channel bottom to the free stream, the maximum velocity for a fixed flow area occurs immediately below the free surface. The flow equations in this work were solved using the finite difference approach; however, the finite element method is required for more precise findings.

Thiong'o (2013) focused on open rectangular and triangular channel flows.Determining the hydraulic efficiency of the open rectangular and triangular channels was the goal. The conservation of mass and momentum laws lead to non-linear partial differential equations. The finite difference method was used since analytical solutions to these problems are not possible. The flow's velocity and depth are two essential factors in determining discharge. Research has been conducted on the effects of changing various parameters on velocity. The relationship between fluid velocity and depth has also been explored. When solving the equations in this study, the finite element method can produce more accurate results than the finite difference method.

Kwanza et al., 2007 study examined the impacts of channel discharge, width, and slope on both trapezoidal and rectangular channels. The findings proved that trapezoidal open channel flows are more hydraulically efficient than rectangular cross-sectional open channels. The analysis indicates that a rise in the specified parameters leads to a rise in the flow volume. Since the study did not address these effects on the main flow's velocity, more research is required to determine how lateral input affects the main flow.

Thiong'o *et al* (2011) investigated fluid flow in an open rectangular and triangular channel. The findings showed that rectangular cross-section open channels are more hydraulically efficient than triangular cross-section channels. Further investigation showed that for both rectangular and triangle channels, an increase in the channel's energy coefficient, top width, and slope results in an increase in flow velocity. Additionally, the flow velocity rises with depth, peaking somewhat below the free surface. The velocity profiles for both types of channels show that the rectangular channel flows water more quickly than an open triangular channel at the same depth and width.

Macharia *et al.,(*2014) studied the flow of fluids in an open rectangular channel with lateral inflow channels and discovered that increasing the channel's lateral inflow angle does not increase the velocity of the fluid in the core channel. As the cross-sectional area of the lateral inflow increases, the main channel's flow speed decreases. When the lateral inflow channel length rises, the flow velocity in both channels drops, while the flow velocity in the open rectangular channel increases as the lateral inflow channel velocity increases.

Marangu*et al.,* (2016) developed a model of open channel fluid flow with trapezoidal cross-section and a segment base. The study discovered that increasing the flow's cross-section area results in a decrease in flow velocity. In addition, an increase in the roughness coefficient, the cross-section, and the flow channel radius all result in a decrease in flow velocity. The results of the investigation show that the flow velocity decreases as the segment's circle radius increases. The results showed that increases in the cross-sectional area, channel radius, and depth of flow causes a proportional drop in fluid velocity.

Ojiambo et al.,(2014), developed a Mathematical model of fluid flow in an open channel with a circular cross-section. The results of the study demonstrated that, for a static area of flow, maximum velocity is obtained immediately below the free surface and that flow velocity increases as flow depth increases from the lowest part of the channel to the free stream. The results also showed that flow velocity increases with reducing the cross-sectional area and flow depth of the channel. The flow increases as the lateral inflow rate per unit length of the channel decreases. A research with lateral input is necessary because this one focused on a flow without it.

Jomba*et al.,* (2015) investigated a mathematical model of fluid flow in an open channel with a Horseshoe cross-section. The investigation's findings revealed that, for a fixed flow area, the depth increases with increasing flow velocity in the direction of the free stream. Moreover, it was demonstrated that increased hydraulic radii and roughness coefficients result in increased shear stresses, which lower velocity. Because of the direct relationship between flow velocity and channel slope, a decrease in the channel's slope results in a proportional drop in flow velocity. As the flow's cross-sectional area grows, the flow velocity falls. Because the horseshoe cross section without lateral input was the focus of this research project, a corresponding investigation with lateral inflow is necessary.

Omari *et al.,* (2018) Modelled circular closed channels for sewer lines.The findings showed that the sewer depth falls as the cross-sectional sewer flow area increases. When the friction slope is lowered, sewer flow velocity has been seen to rise. Additionally, it was found that the sewer velocity increased as the tunnel's angle of slope increased. Similar research on lateral inflow sewer lines is necessary because the current study focused on circular closed channels for lateral inflow sewer lines.

Karimi (2018) studied flow in an open rectangular channel with a lateral inflow channel.The results showed that, at zero degrees of the lateral rectangular channel, the results were in agreement with those of previous studies. It was discovered that while the velocity of this channel reduces as the velocity of the main channel increases, the velocity of the main channel increases when the area and length of the lateral inflow channel increases.

Rotich et al., (2021) investigated open channel flows with a parabolic cross-section.The results demonstrated that higher channel slope and energy coefficients result in higher flow velocities. Conversely, a reduction in top width results in a rise in velocity. The impact of lateral inflow on the channel's velocity has not been examined in this study. In order to solve the governing equations and provide more precise answers, the finite element method must also be used.

Several studies have been carried out over the past 20 years on open channels with a varying cross-sectional area, very few have been done on horseshoe channels with lateral inflow. Even though it is clear from the previously discussed study investigations that finite difference was used to solve the equations, finite element analysis is necessary for more accurate outcomes. Since many earlier studies have not sufficiently addressed the effect of lateral input on the channel's velocity, more research is required. The goal of this research is to build a canal that can carry the maximum flow from flooded areas into agricultural land. Flooding is still a problem in modern times.

2.0 Geometric Properties of HorseshoeOpen Channel low for Zone 2.

The horseshoe cross section is divided into three zones of flow depth according to Merkley (2005), calculated the depths $y_2$using the following formulae.

1. Height

$$y_2 = \frac{r}{2} - r\left[1 - \left(\frac{1+\sqrt{7}}{4}\right)\right] \qquad (1)$$

2. Cross-sectional area

$$A_2 = r^2\left[c_2 + sin^{-1}\left(\frac{2y-r}{2r}\right)\right] - \left(y - \frac{r}{2}\right)\left(r - \sqrt{r^2 - \left(y - \frac{r}{2}\right)^2}\right) + (y - r)\sqrt{y(2r-y)} + r^2\left[sin^{-1}\left(\frac{y-r}{r}\right) + \frac{\pi}{2}\right] \quad \text{for } 0 \leq y \leq y_2 \qquad (2)$$

where c_1 and c_2 are

$$c_1 = 1 - \left(\frac{1+\sqrt{7}}{4}\right)$$

$$c_2 = \frac{c_1}{2}\left(1 - \sqrt{\frac{c_1^2}{4}}\right) - sin^{-1}\left(\frac{c_1}{2}\right)$$

3. Wetted perimeter

$$p_2 = 2r\left[cos^{-1}\left(\frac{r-2h}{2r}\right)\right] - cos^{-1}\left(-\frac{c_1}{2}\right) + 2rcos^{-1}\left(1 - \frac{y}{r}\right) \qquad (3)$$

for $0 \leq y \leq y_2$

4. Top width

$$T_2 = 2\sqrt{r^2 - \left(y - \frac{r}{2}\right)^2 - r} \qquad \text{for } 0 \le y \le y_2 \qquad (4)$$

3.0 Governing Equations

3.1 Continuity equation

A continuity equation is a differential equation that describes the transport of some kind of conserved quantity, in particular-mass. All examples of continuity equations express the same idea that the total amount of the conserved quantity inside any region can only change by the amount that passes in or out of the region through the boundary. This equation combines the law of mass conservation and that of the transport theorem. This equation arises from the fundamental prepositions that matter can neither be created nor be destroyed.

The St. Venant's equation of continuity governing open channel flows of arbitrary shape is given as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = u \qquad (5)$$

Let main channel have m uniform uniflows converging into it, each entering at consistent velocities and flow rates. Equation (5) is modified to equation (6)

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = m\,u \qquad (6)$$

But discharge Q = Aq (7)

Differentiating (3) partially with respect to x and substituting in (1), it becomes

$$q\frac{\partial A}{\partial x} + A\frac{\partial q}{\partial x} + \frac{\partial A}{\partial t} - mu = 0 \qquad (8)$$

The flow area is assumed to be a known function of the depth; therefore, derivatives of A in (8) may be expressed in terms of y as

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial y}\frac{\partial y}{\partial x} = T\frac{\partial y}{\partial x}$$

(9)

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial y}\frac{\partial y}{\partial t} = T\frac{\partial y}{\partial t} \qquad (9)$$

In this discussion, it is assumed that T is determined by

$$T= \frac{\partial A}{\partial y} \qquad \text{Franz (1982)} \qquad (10)$$

Substituting equations (9) and (10) in equation (8) yields:

$$qT\frac{\partial y}{\partial x} + A\frac{\partial q}{\partial x} + T\frac{\partial y}{\partial t} - mu = 0 \qquad (11)$$

Dividing (11) through by T, it becomes

$$q\frac{\partial y}{\partial x} + \frac{A}{T}\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} - \frac{mu}{T} = 0 \qquad (12)$$

Equation (12) is the general equation of continuity for open channel flows.

Substituting equations (2), (3) and (4) into (12) to obtain the specific continuity equation for zone 2.

$$q2\sqrt{r^2 - \left(y - \frac{r}{2}\right)^2} - r\frac{\partial y}{\partial x} + \left\{r^2\left[c_2 + sin^{-1}\left(\frac{2y-r}{2r}\right)\right] - \left(y - \frac{r}{2}\right)\left(r - \sqrt{r^2 - \left(y - \frac{r}{2}\right)^2}\right) + (y - r)\sqrt{y(2r - y)} + r^2\left[sin^{-1}\left(\frac{y-r}{r}\right) + \frac{\pi}{2}\right]\right\}\frac{\partial q}{\partial x} + 2\sqrt{r^2 - \left(y - \frac{r}{2}\right)^2} - r\frac{\partial y}{\partial t} - mu = 0 \qquad \text{for } 0 \le y \le y_2 \quad (13)$$

3.2 Momentum equation

The equation of conservation of momentum is derived from the Newton's second law of motion, which states that the rate of change of momentum of a body is equal to the net external forces applied to the body.This external force is divided into two types of forces i.e. surface forces; static pressure, viscous stresses and body forces; gravitational force, magnetic force, centrifugal force or electric fields. The surface forces are due to the interaction between the body and the immediate contact with it and act on the bounding surfaces. Their intensities are expressed in terms of stress and defined as force per unit area. The body forces are defined as forces which act on a body from a distance and are usually expressed as force per unit mass.

The St. Venants equation of momentum for open channels of arbitrary shape is

$$\frac{\partial q}{\partial t} + \beta q \frac{\partial q}{\partial x} + g\frac{\partial y}{\partial x} = g(s_0 - s_f) - -\frac{qu}{A} \quad (14)$$

The cross-sectional area of this study is a uniform horseshoe cross-section, thus the value of momentum coefficient is equal to one. Replacing the continuity equation for unsteady flow the momentum equation becomes

$$\frac{\partial q}{\partial t} + q\frac{\partial q}{\partial x} + g\frac{\partial y}{\partial x} = g(s_0 - s_f) - \frac{qu}{A} \quad (15)$$

In computation of unsteady flow, it is usually assumed that the friction slope s_f can be estimated from either the Manning or Chezy resistance equations (chow 1973). The manning resistance equation is as follows:

$$s_f = \frac{n^2 q^2}{R^{\frac{4}{3}}} \quad (16)$$

Substituting equation (15) in equation (14) yields

$$\frac{\partial q}{\partial t} + q\frac{\partial q}{\partial x} + g\frac{\partial y}{\partial x} = g(s_o - \frac{n^2 q^2}{R^{\frac{4}{3}}}) - \frac{qu}{A} \quad (17)$$

Substituting equation (2) to get specific momentum equation for zone 2.

$$\frac{\partial q}{\partial t} + q\frac{\partial q}{\partial x} + g\frac{\partial y}{\partial x} = g(s_o - \frac{n^2 q^2}{R^{\frac{4}{3}}}) - \frac{qu}{A}$$

Where $A = r^2\left[c_2 + sin^{-1}\left(\frac{2y-r}{2r}\right)\right] - \left(y - \frac{r}{2}\right)\left(r - \sqrt{r^2 - \left(y - \frac{r}{2}\right)^2}\right) +$

$(y - r)\sqrt{y(2r-y)} + r^2\left[sin^{-1}\left(\frac{y-r}{r}\right) + \frac{\pi}{2}\right]$ (18)

Equation (18) is the specific momentum equation of an open channel with lateral inflow for zone 2.

4.0 Equations Governing the Fluid Flow in Finite Difference Form for Zone 2.

Subject to their boundary and initial conditions, the governing equations describing the unsteady, incompressible fluid flow through a horseshoe cross-section in finite difference form are solved using the finite difference approach as follows:

$$y^*_{i,j+1} = 0.5\left(y^*_{i-1,j} + y^*_{i+1,j}\right) -$$

$$\Delta t \left\{ \frac{\left\{r^2\left[c_2 + sin^{-1}\left(\frac{2y-r}{2r}\right)\right] - \left(y-\frac{r}{2}\right)\left(r - \sqrt{r^2 - \left(y-\frac{r}{2}\right)^2}\right) + (y-r)\sqrt{y(2r-y)} + r^2\left[sin^{-1}\left(\frac{y-r}{r}\right) + \frac{\pi}{2}\right]\right\}}{2L\sqrt{r^2-\left(y-\frac{r}{2}\right)^2} - r} \frac{q^*_{i+1,j} - q^*_{i-1,j}}{2\Delta x} + \right.$$

$$\left. q^*_{i,j} \frac{y^*_{i+1,j} - y^*_{i-1,j}}{2\Delta x} - \frac{u}{2V\sqrt{r^2-\left(y-\frac{r}{2}\right)^2} - r} \right\}$$

$$q^*_{i,j+1} = 0.5\left(q^*_{i-1,j} + q^*_{i+1,j}\right) - \Delta t \left\{ q^*_{i,j} \frac{q^*_{i+1,j} - q^*_{i-1,j}}{2\Delta x} + \frac{g}{Fr} \frac{y^*_{i+1,j} - y^*_{i-1,j}}{2\Delta x} + \right.$$

$$\frac{u}{Fr\left\{r^2\left[c_2 + sin^{-1}\left(\frac{2y-r}{2r}\right)\right] - \left(y-\frac{r}{2}\right)\left(r - \sqrt{r^2 - \left(y-\frac{r}{2}\right)^2}\right) + (y-r)\sqrt{y(2r-y)} + r^2\left[sin^{-1}\left(\frac{y-r}{r}\right) + \frac{\pi}{2}\right]\right\}} q^*_{i,j} -$$

$$\left. \frac{g}{Fr}\left[S_0 - \frac{n^2 v^2}{2R^{\frac{4}{3}}} \left(q^{*2}_{i-1,j} + q^{*2}_{i+1,j}\right)\right] \right\}$$

where c_1 and c_2 are

$$c_1 = 1 - \left(\frac{1+\sqrt{7}}{4}\right)$$

$$c_2 = \frac{c_1}{2}\left(1 - \sqrt{\frac{c_1^2}{4}}\right) - sin^{-1}\left(\frac{c_1}{2}\right)$$

Subject to the condition

$H = 5.65, h = 0.5, k = 1,2,3,4,5$ *and* $6. L = 1,2$ $A = 0.32$ & $0.08, \theta =$

$30^0, 40^0, 50^0$ and 60^0, $Fr = 0.05, s_o = 0.004$; $n = 0.012$; $g = 9.8$; $v = 0.002, u =$

0.002;

5.0 Results and Discussion

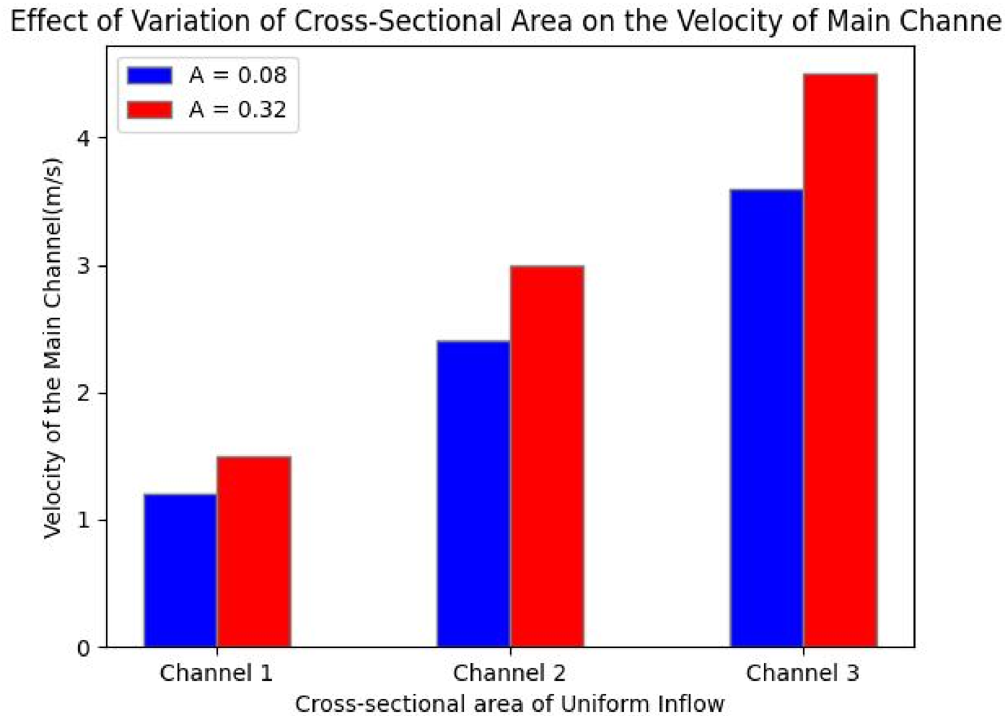

Figure 1: Effects of Cross-sectional Area on Velocity in the Main Channel at Different Angles.

Figure 1 shows how the velocity in the main channel decreases as the cross-sectional area of the lateral inflow channels decreases. In fluid dynamics, the relationship between velocity and cross-sectional area of a flow is described by the principle of conservation of mass. According to this principle, for a given flow rate (volume of fluid passing through a section per unit time) to remain constant, an increase in cross-sectional area results in a decrease in velocity, and vice versa. However, in certain scenarios such as open channel flow or natural streams, the situation can be different due to external factors like gravitational force and channel shape that influence the velocity.

In open channels or natural watercourses, as the cross-sectional area increases, the velocity can increase due to factors such as the reduction in frictional resistance and the potential energy contributing to kinetic energy. In wider sections of rivers or channels, the influence of the walls or bed on the flow decreases, reducing friction and allowing the water to move faster. Additionally, if the channel slope remains constant, the gravitational force acting on the larger volume of water can result in a higher velocity as the water accelerates to maintain the flow rate. This is often observed in natural streams where the widening of the channel and reduction in resistance lead to an increase in flow velocity, particularly in areas where the channel depth and gradient support such dynamics.

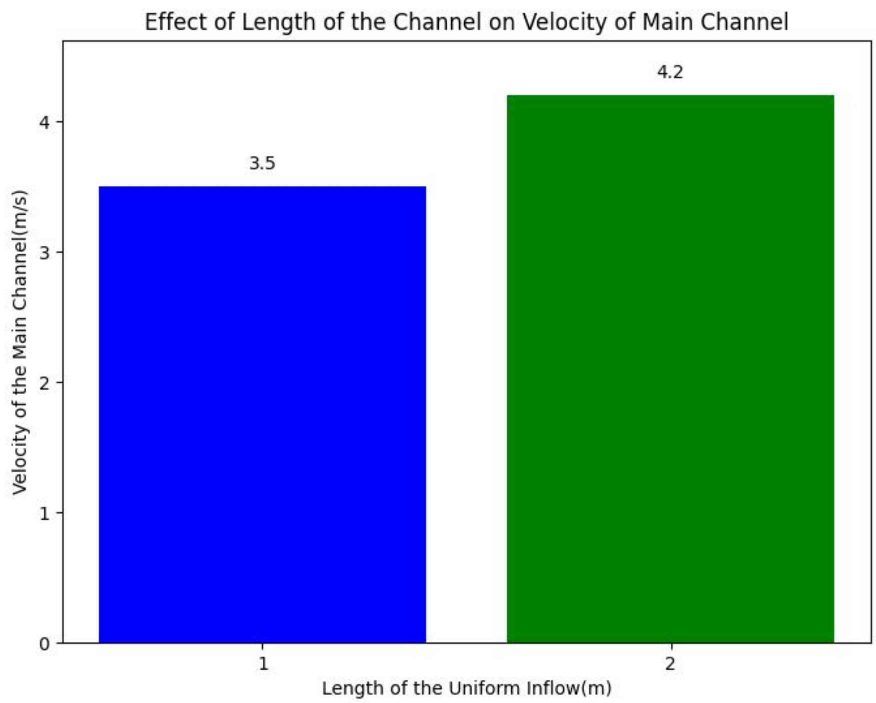

Figure2: Effect of Length of Lateral Inflow Channels on Velocity

Figure 2 shows that increase in length of lateral inflows increases the velocity in the main channel. The distance that water must travel before entering the main channel increases with the length of the lateral inflow channels. When the water reaches the main channel, its velocity may have decreased due to greater frictional losses along the lateral channels as a result of the longer trip distance. Because of this, intake from longer lateral channels adds less momentum to the flow in zone two, where the main channel has already formed, resulting in lower velocities.

Where lateral inflow channels join the main channel, their length can change the dynamics of flow convergence. Longer lateral channels have more chances to spread out their flow along the main channel, which more uniformly distributes inflow throughout the main channel's breadth. In zone two, this lessens localized disruptions and preserves a more consistent velocity profile. On the other hand, very long lateral channels may cause the momentum and velocity in the main channel to significantly decrease.

The main channel's velocity is influenced by the geometry and length of the lateral inflow channels. If longer lateral channels broaden gradually, they can minimize flow disturbances and sustain greater velocities in zone two, resulting in a more progressive transfer of flow into the main channel. On the other hand, abrupt joinings or short lateral channels might generate disruptions and lower velocities in zone two.

5.0 Knowledge gaps

It is recommended that future research should be carried out on

1. The effect of lateral outflow and outflow channels on discharge
2. The effect of two or more lateral inflow and outflow channels at various locations on discharge in the main channel
3. Flow in a trapezoidal, triangular, circular and horseshoe open channel with lateral inflow/outflow channel

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