

# A note on the risk model with dependence and capital injections

## Abstract

The note considers a risk model with dependence and capital injections, where the dependence structure is modeled by a Farlie-Gumbel-Morgenstern copula. In the risk model, the initial surplus starts from a level  $u \geq h$ , where  $h > 0$  is a fix constant. The author derives an expression for the Laplace transform of the Gerber-Shiu function. In particular, an explicit formula for the Gerber-Shiu function is obtained when the initial surplus is  $h$ .

Keywords: Gerber-Shiu function; Dependence; Capital injections; Laplace transform

## 1 Introduction

Nie et al. (2011, 2015) introduced a risk model with capital injections. In the model the insurer's initial

surplus starts from a level  $u \geq h$ , ( $h > 0$  is a fixed constant). In any case, if the surplus drops from above  $h$  to between 0 and  $h$ , the capital injection will restore the surplus level to  $h$ . If the surplus drops from a level above  $h$  to below 0, the ruin will occur. Dickson, D. C., Qazvini, M. (2016), studied a risk model with capital injections and obtained an expression for its Laplace transform and Gerber-Shiu function itself when the initial surplus is  $h$ , providing an effective method for studing ruin related quantities in finite time. Zhao et al. (2017) studied the optimal periodic dividend and capital injection problem for the spectral positive Lévy processes, and maximized the total value of the expected discounted dividend and penalty discounted capital injections until the time of ruin. They found that the optimal return function can be expressed as a scaling function. Xu et al. (2018) considered a capital injection strategy based on the periodic implementation of claims in the classical Poisson risk model, and derived an explicit expression for the discount density of surplus levels after a certain number of claims before the ruin occurred. They also found an explicit Laplace transform expression for the time of ruin. Yu et al. (2020) considered a classical risk model with a periodic capital injection strategy and a barrier dividend strategy, and derived boundary conditions for the Gerber-Shiu function, the expected discounted capital injection function and the expected discounted dividend function by assuming that the observation interval and claim amount are exponentially distributed, respectively.

In ruin theory, the classical compound Poisson risk model with independence between the claim size and interclaim time. In the study of the classical compound Poisson risk model, it is assumed that the claim sizes and the interclaim times are mutually independent. Although this hypothesis is applicable to certain practical situations and simplifies the study of the calculation of the amount of ruin of interest, it has been proved to be inappropriate and restrictive in other practical contexts.

For

example, in modeling damages due to natural catastrophic events the intensity of the catastrophe and the time elapsed, because the last catastrophe are expected to be dependent. See e.g. Boudreault (2003) and Nikoloulopoulos, A. K., Karlis, D. (2008) for an application of this type of dependence structure in an earthquake contex. Recently, many authors have paid lots of attention to the risk model with dependence between interclaim times and claim sizes. Cossette et al. (2010), considered the classical compound Poisson risk model with dependence structure based on a Farlie-Gumbel-Morgennstern (FGM) copula, and evaluated the defective renewal equation for the Gerber-Shiu function. Zhang, Z., Yang, H. (2011) studied the Gerber-Shiu function for a perturbed by diffusion compound Poisson risk model with dependence structure between the claim size and interclaim time by FGM copula, and show that the Gerber-Shiu function satisfy some defective renewal equations. Shi et al. (2013) consider the compound Poisson risk model with a threshold dividend strategy and dependence structure modeled by a FGM copula, and derive explicit formulas for Gerber-shiu functions and expected discounted divided payments.

In the note, we study risk model with capital injections and dependence between the claim sizes

and interclaim times, based on a FGM copula. We derive an expression for the Laplace transform of the Gerber–Shiu function, and the corresponding result in [(3)] is generalized by this note.

## 2 Model description

Consider insurer's surplus process at time  $t$  defined as  $\{U(t), t \geq 0\}$ , with

$$U(t) = u + pt - \sum_{i=1}^{N(t)} Z_i \quad (2.1)$$

where  $u$  is the initial surplus,  $p > 0$  is the premium rate which is assumed to be a positive constant.

$$N(t) = \sum_{i=1}^t Z_i$$

is a compound Poisson process and is the total claim amount process.  $\{N(t)\}_{t \geq 0}$  is a Poisson process with Poisson parameter  $\lambda$  and  $\{Z_i\}_{i=1}^{\infty}$

are assumed to form a sequence of independent

identically distributed (i. i. d.) random variables (r. v.), where  $Z_i$  represents the amount of the  $i$ th claim.

Let  $g(x)$  be the probability density function of  $Z_i$ , and the cumulative distribution function (c. d. f.) of  $Z_i$

is  $G(x)$ , with  $G(0) = 0$ , and  $G = 1 - \overline{G}$ . The interclaim times  $\{V_i, i \geq 1\}$  with  $V_i$  the time between the

$(i-1)$ th and the  $i$ th claim, are i. i. d with common p. d. f.  $p(t) = \lambda e^{-\lambda t}$ . Obviously  $(Z_i, V_i), i \geq 1$ , are i. i. d. random vectors. Motivated by Cossette et al. (2010) and Denuit et al. (2005), we use the FGM copula to define the joint distribution of  $(Z, V)$  and the claim size and the interclaim time is dependent.

The FGM copula is given by

$$C_{FGM}$$

$$c(u_1, u_2) = u_1 u_2 + \theta u_1 u_2 (1 - u_1)(1 - u_2)$$

$\theta \in [-1, 1]$ ,  $u_1, u_2 \in [0, 1]$ , we assume that, the joint p. d. f. of  $\{(Z_i, V_i), i \geq 1\}$  is defined by

$$g(x, t) = g(x) \lambda e^{-\lambda t} + \theta l(x) (2\lambda e^{-2\lambda t} - \lambda e^{-\lambda t})$$

$x, t \geq 0$ , where  $l(x) = (1 - 2G(x))g(x)$ ,  $x \geq 0$  with  $\hat{l}(s)$  is the Laplace transform of  $l(x)$ .

Let  $\tau = \inf \{t : U(t) < 0 | U(0) = u\}$  be the time of ruin with  $\tau = \infty$  if  $U(t) > 0$  for all  $t > 0$  (i. e.

ruin does not occur) and  $\varphi(u) = \Pr(\tau < \infty | U(0) = u)$  be the ultimate ruin probability. To guarantee

that ruin will not occur, the premium loading factor is given by  $p > \lambda E(Z_i)$ .

The note is interested in the risk model with dependence and capital injection. Further, the initial surplus starts from  $u \geq h > 0$ , when the surplus process falls between 0 and  $h$ , a capital injection causes the surplus to start from  $k$ . When the surplus falls below 0, ruin occurs. Therefore, we use the same notation as for the classical risk model, with a subscript  $h$ .

## 3 Analysis of the Gerber–Shiu function

In this section, we will investigate the Gerber–Shiu function of the risk process. In Dickson et al. [(3)],

the authors introduce Gerber–Shiu function about  $\tau_h$ ,  $N_{\tau_h}$ , and the penalty function  $\omega(x, y)$ , when

$x \geq h$  and  $y > 0$ , as follows

$$m(u) = E$$

$$h$$

$$r_{N_{\tau_h}} \exp(-\delta \tau_h) \omega(U(\tau_h -$$

$$h), |U(\tau_h)| |I(\tau_h < \infty) | U(0) = u$$

i

$$(3.1)$$

where  $\delta \geq 0$ ,  $0 < r \leq 1$  as well as  $u > h$ ,  $U(r-h)$  is the surplus prior to ruin. In Landriault et al. [(6)], authors explain  $\delta$  as the parameter of a Laplace transform and  $r$  as the parameter of a probability generating function. Especially,  $m(u) = 0$ , when  $0 \leq u < h$ .

We need to introduce the well-known Dickon-Hipp operator [(4)]  $T_s$  of a real-valued integrable function  $g$  defined as

$$T_s g(x) = \int_x^\infty e^{-s(y-x)} g(y) dy$$

**Theorem 1.** The Gerber-Shiu penalty function  $m(u)$  satisfies

$$m(h) = \frac{r\lambda}{\rho} [(\rho_2 - 2\lambda + \delta) T_{\rho_2} \beta_1(h) - (\rho_1 - 2\lambda + \delta) T_{\rho_1} \beta_1(h)] + r\lambda \theta \frac{r\lambda}{\rho} [(\rho_2 - \delta) T_{\rho_2} \beta_2(h) - (\rho_1 - \delta) T_{\rho_1} \beta_2(h)] - (\rho_2 - \rho_1) - r\lambda \frac{r\lambda}{\rho} [(\rho_2 - 2\lambda + \delta) \eta_2(h) - (\rho_1 - 2\lambda + \delta) \eta_1(h)] - r\lambda \theta \frac{r\lambda}{\rho} [(\rho_2 - \delta) \xi_2(h) - (\rho_1 - \delta) \xi_1(h)] \quad (3.2)$$

where,

$$\begin{aligned} \gamma_1(u) &= \int_0^{u-h} m(u-x)g(x) dx + \int_{u-h}^u m(h)g(x) dx + \beta_1(u) \\ \beta_1(u) &= \int_u^\infty \omega(u, x-u)g(x) dx \\ \gamma_2(u) &= \int_0^{u-h} m(u-x)l(x) dx + \int_{u-h}^u \end{aligned}$$

$$m(h)l(x)dx + \beta_2(u)$$

$$\beta_2(u) =$$

$$\int_u^\infty$$

$$u$$

$$\omega(u, x - u)l(x) dx$$

$$\eta_1(h) =$$

$$\int_h^\infty$$

$$h$$

$$\exp(-\rho_1(u - h)) (\bar{G}(u - h) - \bar{G}(u)) du$$

$$\eta_2(h) =$$

$$\int_h^\infty$$

$$h$$

$$\exp(-\rho_2(u - h)) (\bar{G}(u - h) - \bar{G}(u)) du$$

$$\xi_1(h) =$$

$$\int_h^\infty$$

$$h$$

$$\exp(-\rho_1(u - h)) (\bar{L}(u - h) - \bar{L}(u)) du$$

$$\xi_2(h) =$$

$$\int_h^\infty$$

$$h$$

$$\exp(-\rho_2(u - h)) (\bar{L}(u - h) - \bar{L}(u)) du$$

Proof. By conditioning on the time and amount of the first claim we can obtain for  $u \geq h$ ,

$$m(h) = E[r_1 \exp(-\delta t) m(u + pt - x) | X_1 = x, V_1 = t] + E[r_1 \exp(-\delta t) m(h) | X_1 = x, V_1 = t]$$

$$+ E[r_1 \exp(-\delta T_h) \omega(U(T_h -$$

$$h), |U(T_h) |) I(T_h < \infty) | U(0) = u] | X_1 = x, V_1 = t]$$

$$=$$

$$\int_0^\infty$$

$$0$$

$$\int_0^{u+pt-h}$$

$$\exp(-\delta t) r m(u + pt - x) g(x, t) dx +$$

$$\int_{u+pt-h}^{u+pt}$$

$$u+pt-h$$

$$\exp(-\delta t) r m(h) g(x, t) dx$$

$$+$$

$$\int_{u+pt}^\infty$$

$$u+pt$$

$$r \exp(-\delta t) \omega(U(T_h -$$

$$h), |U(T_h) |) g(x, t) dx$$

$$=$$

$$\int_0^\infty$$

$$dt$$

$$=$$

$$r \lambda$$

$$\int_0^\infty$$

$$u$$

$$\exp(-(\lambda + \delta)$$

$$(t - u)$$

$$r$$

$$dt$$

$$=$$

$$\int_0^{\tau-h} m(\tau-x)g(x)dx + \int_{\tau-h}^{\tau} m(h)g(x)dx + \int_{\tau}^{\infty} f(x)\omega(\tau, x-\tau)dx$$

-  
dτ

+  
rλθ

p  
 $\int_u^{\infty}$

$$(2\exp(-(\lambda + \delta)^{\tau-u}$$

$$) - \exp(-(\lambda + \delta)^{\tau-u}$$

τ - u

p  
)

$\int_0^{\tau-h}$

$$m(\tau-x)l(x)dx + \int_{\tau-h}^{\tau} m(h)l(x)dx$$

$$+ \int_{\tau}^{\infty} l(x)\omega(\tau, x-\tau)dx$$

+  
 $\int_{\tau}^{\infty}$

$$l(x)\omega(\tau, x-\tau)dx$$

-  
dτ

=  
rλ

p  
 $\int_u^{\infty}$

$$\exp(-(\lambda + \delta)^{\tau-u}$$

$$)\psi_1(\tau)d\tau +$$

rλθ

p  
 $\int_u^{\infty}$

u

$$\begin{aligned}
& (2\exp^{-(2\lambda + \delta)}) \\
& \tau - u \\
& \rho \\
& ) - \exp^{-(\lambda + \delta)} \\
& \tau - u \\
& \rho \\
& )) \gamma_2(\tau) d\tau \\
& (3.3)
\end{aligned}$$

Using the operator  $T_s$  we can obtain

$$\begin{aligned}
& m(u) = \\
& r\lambda \\
& \rho \\
& T_{\lambda+\delta} \\
& \rho \\
& \gamma_1(u) + \\
& r\lambda\theta \\
& \rho \\
& - \\
& 2T_{2\lambda+\delta} \\
& \rho \\
& \gamma_2(u) - T_{\lambda+\delta} \\
& \rho \\
& \gamma_2(u) \\
& - \\
& (3.4)
\end{aligned}$$

Noting  $m(u) = 0$ , when  $0 \leq u < h$ , then

$$\begin{aligned}
& T_s m(h) = \\
& \int_h^\infty \\
& \exp(-s(x-h)) m(x) dx = e^{sh} \hat{m}(s)
\end{aligned}$$

therefore,  $T_s \gamma_1(h) = e^{sh} \hat{\gamma}_1(s)$ ,  $T_s \gamma_2(h) = e^{sh} \hat{\gamma}_2(s)$ . By the Dickson-Hipp operator, we have

$$\begin{aligned}
& T_s m(h) = \\
& r\lambda \\
& \rho \\
& T_s T_{\lambda+\delta} \\
& \rho \\
& \gamma_1(h) + \\
& r\lambda\theta \\
& \rho \\
& (2T_s T_{2\lambda+\delta} \\
& \rho \\
& \gamma_2(h) - T_s T_{\lambda+\delta} \\
& \rho \\
& \gamma_2(p)) \\
& = \\
& r\lambda
\end{aligned}$$

$$\begin{aligned}
& \rho \\
& \times \\
& T_{\lambda+\delta} \\
& \rho \\
& - T_s \\
& (s - \lambda + \delta \\
& \rho) \\
& \gamma_1(\mathbf{h}) + \\
& r\lambda\theta \\
& \rho
\end{aligned}$$

$$\begin{aligned}
& 2 \times \\
& T_{2\lambda+\delta} \\
& \rho \\
& - T_s \\
& (s - 2\lambda + \delta \\
& \rho) \\
& \gamma_2(\mathbf{h}) - \\
& T_{\lambda+\delta} \\
& \rho \\
& - T_s \\
& (s - \lambda + \delta \\
& \rho) \\
& \gamma_2(\mathbf{h}) \\
& ! \\
& (3.5)
\end{aligned}$$

Multiplying (3.5) by  $(s - \lambda + \delta$

$$\begin{aligned}
& \rho) (s - 2\lambda + \delta \\
& \rho), \text{ we have} \\
& (s - \\
& \lambda + \delta \\
& \rho \\
& ) (s - \\
& 2\lambda + \delta \\
& \rho \\
& ) T_s m_{r,\delta}(\mathbf{h}) = \\
& r\lambda \\
& \rho \\
& (s - \\
& 2\lambda + \delta \\
& \rho \\
& ) (T_{\lambda+\delta} \\
& \rho \\
& - T_s) \gamma_1(\mathbf{h}) \\
& + 2
\end{aligned}$$

$r\lambda\theta$

$\rho$

$(s -$

$\lambda + \delta$

$\rho$

$) (T_{2\lambda+\delta}$

$\rho$

$- T_s) \gamma_2(\mathbf{h})$

$-$

$r\lambda\theta$

$\rho$

$(s -$

$2\lambda + \delta$

$\rho$

$) (T_{\lambda+\delta}$

$\rho$

$- T_s) \gamma_2(\mathbf{h}) \quad (3.6)$

Multiplying (3.4) by  $(s - \lambda + \delta$

$\rho)$  and rewrite (3.5), we can obtain

$(s -$

$\lambda + \delta$

$\rho$

$) (s -$

$2\lambda + \delta$

$\rho$

$) T_s m(\mathbf{h}) = (s -$

$2\lambda + \delta$

$\rho$

$) m(\mathbf{k}) +$

$\lambda$

$\rho$

$\times 2$

$r\lambda\theta$

$\rho$

$T_{2\lambda+\delta}$

$\rho$

$\gamma_2(\mathbf{h})$

$-$

$r\lambda$

$\rho$

$(s -$

$2\lambda + \delta$

$\rho$

)  $T_s \gamma_1(h) -$

$r\lambda\theta$

$\rho$

$(s -$

$\delta$

$\rho$

)  $T_s \gamma_2(h)$  (3.7)

$T_s \gamma_1(h) =$

$\int_h^\infty$

$h$

$\exp(-s(u-h)) \gamma_1(u) du$

$= \exp(sh)$

$\int_h^\infty$

$h$

$\exp(-su)$

$\int_{u-h}^\infty$

$0$

$g(x) m(u-x) dx du + m(h)$

$\int_h^\infty$

$h$

$\exp(-s(u-h))$

$\int_u^\infty$

$u-h$

$g(x) dx du$

$+$

$\int_h^\infty$

$h$

$\exp(-s(u-h)) \beta_1(u) du$

$= \exp(sh) \hat{g}(s) \hat{m}(s) + m(h)$

$\int_h^\infty$

$h$

$\exp(-s(u-h)) (\overline{G}(u-h) - \overline{G}(u)) du + T_s \beta_1(h)$  (3.8)

Since  $T_s m(h) = e^{sh} \hat{m}(s)$ , we have

$T_s \gamma_1(h) = T_s m(h) \hat{g}(s) + m(h)$

$\int_h^\infty$

$h$

$\exp(-s(u-h)) (\overline{G}(u-h) - \overline{G}(u)) du + T_s \beta_1(h)$

Similarly

$T_s \gamma_2(h) = T_s m(h) \hat{l}(s) + m(h)$

$\int_h^\infty$

$h$

$\exp(-s(u-h)) (\overline{L}(u-h) - \overline{L}(u)) du + T_s \beta_2(h)$

Substituting in (3.7), we can derive

$(s -$

$\lambda + \delta$

$\rho$

)  $(s -$

$2\lambda + \delta$

$$\rho \int_0^{\infty} T_s m(h) = (s - 2\lambda + \delta)$$

$$\rho \int_0^{\infty} m(h) - r\lambda$$

$$\rho \int_0^{\infty} (s - 2\lambda + \delta)$$

$$\rho \int_0^{\infty} [T_s m(h) \hat{g}(s) + m(h)]$$

$$\int_h^{\infty} \exp(-s(u-h)) (\bar{G}(u-h) - \bar{G}(u)) du + T_s \beta_1(h)$$

$$= r\lambda \theta$$

$$\rho \int_0^{\infty} (s - \delta)$$

$$\rho \int_0^{\infty} T_s m(h) \hat{l}(s)$$

$$+ m(h) \int_h^{\infty} \exp(-s(u-h)) (\bar{L}(u-h) - \bar{L}(u)) du + T_s \beta_2(h)$$

$$= \lambda$$

$$\rho \times 2r\lambda \theta$$

$$\rho \times e^{2\lambda + \delta}$$

$$\rho \hat{\gamma}_2(s) \quad (3.9)$$

$$T_s m(h) =$$

$$1 (s - \lambda + \delta)$$

$$\rho (s - 2\lambda + \delta)$$

$$\rho + r\lambda$$

$$\rho (s - 2\lambda + \delta)$$

$$\begin{aligned}
& \rho) \hat{g}(s) + r\lambda\theta \\
& \rho (s - \delta \\
& \rho) \hat{l}(s) \\
& \times \\
& - \\
& (s - \\
& 2\lambda + \delta \\
& \rho \\
& ) m(h) - \\
& r\lambda \\
& \rho \\
& (s - \\
& 2\lambda + \delta \\
& \rho \\
& ) \\
& - \\
& m(h) \\
& \int_h^\infty \\
& \exp(-s(u-h)) (\overline{G}(u-h) - \overline{G}(u)) du + T_s \beta_1(h) \\
& - \\
& - \\
& r\lambda\theta \\
& \rho \\
& (s - \\
& \delta \\
& \rho \\
& ) \\
& - \\
& m(h) \\
& \int_h^\infty \\
& \exp(-s(u-h)) (\overline{L}(u-h) - \overline{L}(u)) du + T_s \beta_2(h) \\
& - \\
& + \\
& \lambda \\
& \rho \\
& \times \\
& 2r\lambda\theta \\
& \rho \\
& \times T_{2\lambda+\delta} \\
& \rho \\
& \gamma_2(h) \\
& - \\
& (3.10)
\end{aligned}$$

Since  $T_s m(h) = e^{sh} \hat{m}(s)$ , we can rewrite (3.10)

$$\begin{aligned}
& \hat{m}(s) = \\
& \frac{1}{(s - \lambda + \delta)} \\
& \frac{\rho}{\rho} (s - 2\lambda + \delta) \\
& \frac{\rho}{\rho} + r\lambda \\
& \frac{\rho}{\rho} (s - 2\lambda + \delta) \\
& \frac{\rho}{\rho} \hat{g}(s) + r\lambda\theta \\
& \frac{\rho}{\rho} (s - \delta) \\
& \frac{\rho}{\rho} \hat{l}(s) \\
& \times \\
& - \\
& \frac{\rho}{(s - 2\lambda + \delta)} \\
& \frac{\rho}{\rho} \\
& \frac{\rho}{\rho} \exp(-sh) m(h) - \\
& \frac{\rho}{\rho} \\
& \frac{\rho}{(s - 2\lambda + \delta)} \\
& \frac{\rho}{\rho} \\
& \frac{\rho}{\rho} \\
& - \\
& \frac{\rho}{m(h)} \\
& \frac{\rho}{Z_\infty} \\
& \frac{\rho}{h} \exp(-su) (\int_h^\infty G(u-h) - \int_h^\infty G(u)) du + \exp(-sh) T_s \beta_1(h) \\
& - \\
& - \\
& \frac{\rho}{r\lambda\theta} \\
& \frac{\rho}{\rho} \\
& \frac{\rho}{(s - \delta)} \\
& \frac{\rho}{\rho} \\
& \frac{\rho}{\rho} \\
& - \\
& \frac{\rho}{m(h)} \\
& \frac{\rho}{Z_\infty} \\
& \frac{\rho}{h} \exp(-su) (\int_h^\infty L(u-h) - \int_h^\infty L(u)) du + \exp(-sh) T_s \beta_2(h) \\
& - \\
& + \\
& \lambda \\
& \frac{\rho}{\rho} \\
& \times \\
& 2r\lambda\theta \\
& \frac{\rho}{\rho}
\end{aligned}$$

$$\times \exp(-sh) T_{2\lambda+\delta}$$

$$\begin{matrix} p \\ \gamma_2(h) \end{matrix}$$

$$(3.11)$$

where, the denominator of Eq. (3.11)  $(s - \lambda + \delta$

$$p) (s - 2\lambda + \delta$$

$$p) + r\lambda$$

$$p) (s - 2\lambda + \delta$$

$$p) \hat{f}(s) + r\lambda\theta$$

$$p) (s - \delta$$

$$p) \hat{h}(s) = 0$$

is the Lundberg's generalized equation with two different positive real roots  $\rho_i$ ,  $i = 1, 2$ . See the analysis of the Lunderberg's generalized equation [(14)] and [(10)]. These roots must also be roots of the numerator of Eq. (3.11), given that it is analytic. Therefore we obtain  $m(k)$  by the following linear system.

$$m(h) =$$

$$n$$

$$r\lambda$$

$$p) (\rho_1 - 2\lambda + \delta$$

$$p) T_{\rho_1} \beta_1(h) + r\lambda\theta$$

$$p) (\rho_1 - \delta$$

$$p) T_{\rho_1} \beta_2(h) - \lambda$$

$$p) \times 2r\lambda\theta$$

$$p) \times T_{2\lambda+\delta}$$

$$p)$$

$$\gamma_2(h)$$

$$0$$

$$n$$

$$p) (\rho_1 - 2\lambda + \delta$$

$$p) - r\lambda$$

$$p) (\rho_1 - 2\lambda + \delta$$

$$p) \eta_1(h) - r\lambda\theta$$

$$p) (\rho_1 - \delta$$

$$p) \xi_1(h)$$

$$0 (3.12)$$

$$m(h) =$$

$$n$$

$$r\lambda$$

$$p) (\rho_2 - 2\lambda + \delta$$

$$p) T_{\rho_2} \beta_1(h) + r\lambda\theta$$

$$p) (\rho_2 - \delta$$

$$p) T_{\rho_2} \beta_2(h) - \lambda$$

$$p) \times 2r\lambda\theta$$

$$p) \times T_{2\lambda+\delta}$$

$$p)$$

$$\begin{aligned}
& \gamma_2(\mathbf{h}) \\
& 0 \\
& \mathbf{n} \\
& (\rho_2 - 2\lambda + \delta \\
& \rho) - r\lambda \\
& \rho (\rho_2 - 2\lambda + \delta \\
& \rho) \eta_2(\mathbf{h}) - r\lambda\theta \\
& \rho (\rho_2 - \delta \\
& \rho) \xi_2(\mathbf{h}) \\
& 0 \quad (3.13) \\
& \text{Hence,} \\
& \mathbf{m}(\mathbf{h}) = \\
& r\lambda \\
& \rho [(\rho_2 - 2\lambda + \delta \\
& \rho) T_{\rho_2} \beta_1(\mathbf{h}) - (\rho_1 - 2\lambda + \delta \\
& \rho) T_{\rho_1} \beta_1(\mathbf{h})] + r\lambda\theta \\
& \rho [(\rho_2 - \delta \\
& \rho) T_{\rho_2} \beta_2(\mathbf{h}) - (\rho_1 - \delta \\
& \rho) T_{\rho_1} \beta_2(\mathbf{h})] \\
& (\rho_2 - \rho_1) - r\lambda \\
& \rho [(\rho_2 - 2\lambda + \delta \\
& \rho) \eta_2(\mathbf{h}) - (\rho_1 - 2\lambda + \delta \\
& \rho) \eta_1(\mathbf{h})] - r\lambda\theta \\
& \rho [(\rho_2 - \delta \\
& \rho) \xi_2(\mathbf{h}) - (\rho_1 - \delta \\
& \rho) \xi_1(\mathbf{h})] \\
& (3.14)
\end{aligned}$$

The proof is completed.

When  $\theta = 0$ , the claim amount r.v.  $X_j$  and the interclaim time r.v.  $W_j$  is independent,

$$\mathbf{g}(\mathbf{x}, t) = \mathbf{g}(\mathbf{x}) \lambda e^{-\lambda t}$$

then,

$$\mathbf{m}(\mathbf{h}) =$$

$r\lambda$

$\rho$

$\mathbf{R}_\infty$

$\mathbf{h}$

$\mathbf{R}_\infty$

$$\int_0^u e^{-\rho(u-h)} \mathbf{g}(\mathbf{x}) \omega(u, \mathbf{x} - u) dx du$$

$$1 - r\lambda$$

$\rho$

$\mathbf{R}_\infty$

$$\int_0^h e^{-\rho(u-h)} (\overline{\mathbf{G}}(u - h) - \overline{\mathbf{G}}(u)) du$$

$$(3.15)$$

Which coincides with the result obtained by Dickson et al. (2016).

## 4 CONCLUSIONS

**a** In the section (2), a risk model with capital injections and a dependence structure modeled by a Farlie-Gumbel-Morgenstern copula is described.

**b** In the section (3), an explicit formula for the Gerber–Shiu function is obtained when the initial surplus is  $k$ .

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## Acknowledgment

The author is grateful to the anonymous referee for constructive comments which improved the quality of this note.

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