UNDER PEER REVIEW

A note on the risk model with dependence and capital injections

Abstract

The author considers a risk model with capital injections and a dependence structure modeled by a Farlie-Gumbel-Morgenstern copula. In this risk model, the surplus starts at a level $u \geq k$, where $k > 0$ is a fixed constant. We derive an expression for the Laplace transform of the Gerber-Shiu discounted penalty function. In particular, an explicit formula for the Gerber-Shiu function is obtained when the initial surplus is k.

Keywords: Gerber-Shiu function; Dependence; Capital injections; Laplace transform

1 Introduction

The risk model with capital injections was introduced by Nie et al.(2011) and Nie et al.(2015). In this model the insurer's surplus starts at a level $u \geq k$, where $k > 0$ is a fixed constant. On any occasion that the surplus falls between the levels 0 and k from above k, a capital injection restores the surplus level to k. If the surplus falls below 0 from a level above k, ruin occurs. In Dickson, D. C., Qazvini, M. (2016), the authors consider the risk model with capital injections and obtain an expression for its Laplace transform and for the Gerber-Shiu function itself when the initial surplus is k, and provide an effective way of studing ruin related quantities in finite time. Zhao et al. (2017) investigate an optimal periodic dividend and capital injection problem for spectrally positive L$\acute{e}$vy processes, and maximize the total value of the expected discounted dividends and the penalized discounted capital injections until the time of ruin and find that the optimal return function can be expressed in terms of the scale functions. Xu et al.(2018) consider a capital injection strategy which is periodically implemented based on the number of claims in the classical Poisson risk model, and derive an explicit expression for the discounted density of the surplus level after a certain number of claims if ruin has not yet occurred and an expression for the Laplace transform of the time to ruin is also explicitly found. Yu et al.(2020) consider the classical risk model with a periodic capital injection strategy and a barrier dividend strategy, and derive the boundary conditions satisfied by the Gerber-Shiu function, the expected discounted capital injection function and the expected discounted dividend function by assuming that the observation interval and claim amount are exponentially distributed, respectively.

In ruin theory, the classical compound Poisson risk model is based on the assumption of independence between the claim amount random variable X_j and the interclaim time r.v. W_j. This assumption is appropriate in certain practical circumstances and has the advantage of simplifying the computation of ruin quantitie of interest. However, such a hypothesis can be restrictive in other practical contexts. For example, in modeling natural catastrophic events, we can expect that, on the occurrence of a catastrophe, the total claim amount (or the intensity of the catastrophe) and the time elapsed since the previous catastrophe are dependent. See e.g. Boudreault(2003) and Nikoloulopoulos, A. K., Karlis, D. (2008) for an application of this type of dependence structure in an earthquake contex. In recent years, the risk model with dependence structure between inter-arrival times and claim sizes has got more and more attention since the independence of them is not well applicable from the pratical point of veiw. Cossette et al.(2010), propose a dependence structure between the claim amounts and the inter-arrival times which is introduced through a Farlie-Gumbel-Morgennstern(FGM) copula, where the defective renewal equation for the Gerber-Shiu function is obtained and solved. Zhang, Z., Yang, H. (2011) construct the bivariate cumulative distribution function of the claim size and interclaim time by FGM copula, and show that the Gerber-Shiu function satisfy some defective renewal equations. Shi et al.(2013) consider the compound Poisson risk model with a threshold dividend strategy and dependence structure modeled by a FGM copula, and derive explicit formulas for Gerber-shiu functions and expected discounted divided payments.

In this note, we consider the risk model with capital injections and dependence between claim amounts and inter-arrival times modeled by a FGM copula. We obtain an expression for the Laplace transform of the Gerber-Shiu function, and the corresponding result in [(3)] is generalized by this note.

2 Model description

Consider insurer's surplus process at time t defined as $\{U(t), t \geq 0\}$, with

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i \tag{2.1}$$

where u is the initial surplus, $c > 0$ is the rate of premium income per unit time (assumed to be received continuously). Further $\sum_{i=1}^{N(t)} X_i$ is a compound Poisson process where $\{N(t)\}_{t \geq 0}$ is a Poisson process with Poisson parameter λ and $\{X_i\}_{i=1}^{\infty}$ is a sequence of independent and identically distributed random variables, where X_i represents the amount of the ith claim. Let $F = 1 - \bar{F}$ be the distribution function of X_i, with $F(0) = 0$, and density function f. Define $\{W_j, j \geq 1\}$ to be a sequence of inter-arrival times of the Poisson process. It is known that $W_j, j \geq 1$, are independent an identically distributed with common pdf $p(t) = \lambda e^{-\lambda t}$. Obviously $(X_j, W_j), j \geq 1$, are independent an identically distributed random vectors. Further, the premium loading factor is given by $c > \lambda E(X_1)$.

The time of ruin is denoted by T and is defined as $T = \inf\{t : U(t) < 0 | U(0) = u\}$ with $T = \infty$ if $U(t) > 0$ for all $t > 0$. The ultimate ruin probability is defined as $\varphi(u) = Pr(T < \infty | U(0) = u)$. In this note, we assume that the claim amounts $\{X_i\}_{i=1}^{\infty}$ and the inter-arrival times $W_j, j \geq 1$ are not independent but with a dependent structure modeled by a FGM copula. To be specific, recalling taht (e.g., Cossette et al.(2010) and Denuit et al.(2005)) the FGM copula is defined. The FGM copula is given by

$$C_\theta^{FGM}(u_1, u_2) = u_1 u_2 + \theta u_1 u_2 (1 - u_1)(1 - u_2)$$

$\theta \in [-1, 1]$, $u_1, u_2 \in [0, 1]$, we assume that, for fixed $j \geq 1$, the joint pdf of (X_j, W_j) is defined by

$$f(x, t) = f(x)\lambda e^{-\lambda t} + \theta h(x)(2\lambda e^{-2\lambda t} - \lambda e^{-\lambda t})$$

$x, t \geq 0$, where $h(x) = (1 - 2F(x))f(x), x \geq 0$ with its Laplace transform denoted by $\hat{h}(s)$. We are interested in the risk model with dependence and capital injections. Specifically, starting from initial surplus $u \geq k > 0$, if the surplus process falls between 0 and k a capital injection restores the surplus level to k, and ruin occurs only if the surplus process falls below 0 from a level above k. For the surplus process with capital injections we use the same notation as for the classical risk model, but with a subscript k, so that, for example, T_k denotes the time of ruin and $\varphi_k(u)$ denotes the ultimate ruin probability from initial surplus u.

3 Analysis of the Gerber-Shiu function

In this section, we will investigate the Gerber-Shiu function of the risk process. In Dickson et al. [(3)], the authors define Gerber-Shiu function in terms of T_k, N_{T_k}, and a general penalty function $\omega(x, y)$, defined for $x \geq k$ and $y > 0$, as

$$m_{r,\delta}(u) = E\left[r^{N_{T_k}} e^{-\delta T_k} \omega(U(T_k^-), |U(T_k)|) I(T_k < \infty) | U(0) = u\right] \tag{3.1}$$

for $\delta \geq 0$, $0 < r \leq 1$ and $u > k$, where $U(T_k^-)$ is the surplus immediately prior to ruin. As in Landriault et al. [(6)], we interpret δ as the parameter of a Laplace transform and r as the parameter of a probability generating function. Further, $m_{r,\delta}(u) = 0$, for $0 \leq u < k$.

The operator T_s introduced by Dickon and Hipp [(4)], and defined for an integrable function f as

$$T_s = \int_x^\infty e^{-s(u-x)} f(u) \mathrm{d}u$$

Theorem 1.The Laplace transform of the Gerber-Shiu function $m_{r,\delta}(u)$ satisfies

$$m_{r,\delta}(k) = \frac{\frac{r\lambda}{c}[(\rho_2 - \frac{2\lambda+\delta}{c})T_{\rho_2}\beta_1(k) - (\rho_1 - \frac{2\lambda+\delta}{c})T_{\rho_1}\beta_1(k)] + \frac{r\lambda\theta}{c}[(\rho_2 - \frac{\delta}{c})T_{\rho_2}\beta_2(k) - (\rho_1 - \frac{\delta}{c})T_{\rho_1}\beta_2(k)]}{(\rho_2 - \rho_1) - \frac{r\lambda}{c}[(\rho_2 - \frac{2\lambda+\delta}{c})\eta_2(k) - (\rho_1 - \frac{2\lambda+\delta}{c})\eta_1(k)] - \frac{r\lambda\theta}{c}[(\rho_2 - \frac{\delta}{c})\xi_2(k) - (\rho_1 - \frac{\delta}{c})\xi_1(k)]} \tag{3.2}$$

where,

$$\gamma_1(u) = \int_0^{u-k} m_{r,\delta}(u-x) f(x) \mathrm{d}x + \int_{u-k}^u m_{r,\delta}(k) f(x) \mathrm{d}x + \beta_1(u)$$

$$\beta_1(u) = \int_u^\infty \omega(u, x-u) f(x) \mathrm{d}x$$

$$\gamma_2(u) = \int_0^{u-k} m_{r,\delta}(u-x) h(x) \mathrm{d}x + \int_{u-k}^u m_{r,\delta}(k) h(x) \mathrm{d}x + \beta_2(u)$$

$$\beta_2(u) = \int_u^\infty \omega(u, x-u) h(x) \mathrm{d}x$$

$$\eta_1(k) = \int_k^\infty e^{-\rho_1(u-k)} (\bar{F}(u-k) - \bar{F}(u)) \mathrm{d}u$$

$$\eta_2(k) = \int_k^\infty e^{-\rho_2(u-k)} (\bar{F}(u-k) - \bar{F}(u)) \mathrm{d}u$$

$$\xi_1(k) = \int_k^\infty e^{-\rho_1(u-k)} (\bar{H}(u-k) - \bar{H}(u)) \mathrm{d}u$$

$$\xi_2(k) = \int_k^\infty e^{-\rho_2(u-k)} (\bar{H}(u-k) - \bar{H}(u)) \mathrm{d}u$$

Proof. Using the standard argument of conditioning on the and the amount of the first claim we obtain for $u \geq k$,

$$
\begin{aligned}
m_{r,\delta}(k) =& E[r^{N_t} e^{-\delta t} m_{r,\delta}(u+ct-x)|X_1 = x, V_1 = t] + E[r^{N_t} e^{-\delta t} m_{r,\delta}(k)|X_1 = x, V_1 = t] \\
&+ E[[r^{N_t} e^{-\delta T_k} \omega(U(T_K^-), |U(T_k)|) I(T_k < \infty)|U(0) = u]|X_1 = x, V_1 = t] \\
&= \int_0^\infty \left[\int_0^{u+ct-k} e^{-\delta t} r m_{r,\delta}(u+ct-x) f_{X,V}(x,t) \mathrm{d}x + \int_{u+ct-k}^{u+ct} e^{-\delta t} r m_{r,\delta}(k) f_{X,V}(x,t) \mathrm{d}x \right. \\
&+ \left. \int_{u+ct}^\infty r e^{-\delta t} \omega(U(T_K^-), |U(T_k)|) f_{X,V}(x,t) \mathrm{d}x \right] \mathrm{d}t \\
&= \frac{r\lambda}{c} \int_u^\infty e^{-(\lambda+\delta)\frac{\tau-u}{c}} \left[\int_0^{\tau-k} m_{r,\delta}(\tau-x) f(x) \mathrm{d}x + \int_{\tau-k}^{\tau} m_{r,\delta}(k) f(x) \mathrm{d}x + \int_\tau^\infty f(x) \omega(\tau, x-\tau) \mathrm{d}x \right] \mathrm{d}\tau \\
&+ \frac{r\lambda\theta}{c} \int_u^\infty (2e^{-(2\lambda+\delta)\frac{\tau-u}{c}} - e^{-(\lambda+\delta)\frac{\tau-u}{c}}) \left[\int_0^{\tau-k} m_{r,\delta}(\tau-x) h(x) \mathrm{d}x + \int_{\tau-k}^{\tau} m_{r,\delta}(k) h(x) \mathrm{d}x \right. \\
&+ \left. \int_\tau^\infty h(x) \omega(\tau, x-\tau) \mathrm{d}x \right] \mathrm{d}\tau \\
&= \frac{r\lambda}{c} \int_u^\infty e^{-(\lambda+\delta)\frac{\tau-u}{c}} \gamma_1(\tau) \mathrm{d}\tau + \frac{r\lambda\theta}{c} \int_u^\infty (2e^{-(2\lambda+\delta)\frac{\tau-u}{c}} - e^{-(\lambda+\delta)\frac{\tau-u}{c}}) \gamma_2(\tau) \mathrm{d}\tau
\end{aligned} \tag{3.3}
$$

Using the operator T_s we can obtain

$$
m_{r,\delta}(u) = \frac{r\lambda}{c} T_{\frac{\lambda+\delta}{c}} \gamma_1(u) + \frac{r\lambda\theta}{c} \left(2T_{\frac{2\lambda+\delta}{c}} \gamma_2(u) - T_{\frac{\lambda+\delta}{c}} \gamma_2(u) \right) \tag{3.4}
$$

Noting $m_{r,\delta}(u) = 0$, when $0 \leq u < k$, then

$$
T_s m_{r,\delta}(k) = \int_k^\infty e^{-s(x-k)} m_{r,\delta}(x) \mathrm{d}x = e^{sk} \hat{m}_{r,\delta}(s)
$$

therefore, $T_s\gamma_1(k) = e^{sk}\hat{\gamma}_1(s)$, $T_s\gamma_2(k) = e^{sk}\hat{\gamma}_2(s)$. Applying the Dickson-Hipp operator to (3.4), have

$$
\begin{aligned}
T_s m_{r,\delta}(k) =& \frac{r\lambda}{c} T_s T_{\frac{\lambda+\delta}{c}} \gamma_1(k) + \frac{r\lambda\theta}{c} (2T_s T_{\frac{2\lambda+\delta}{c}} \gamma_2(k) - T_s T_{\frac{\lambda+\delta}{c}} \gamma_2(k)) \\
=& \frac{r\lambda}{c} \times \frac{T_{\frac{\lambda+\delta}{c}} - T_s}{(s - \frac{\lambda+\delta}{c})} \gamma_1(k) + \frac{r\lambda\theta}{c} \left(2 \times \frac{T_{\frac{2\lambda+\delta}{c}} - T_s}{(s - \frac{2\lambda+\delta}{c})} \gamma_2(k) - \frac{T_{\frac{\lambda+\delta}{c}} - T_s}{(s - \frac{\lambda+\delta}{c})} \gamma_2(k) \right)
\end{aligned} \tag{3.5}
$$

Multiplying (3.5) by $(s - \frac{\lambda+\delta}{c})(s - \frac{2\lambda+\delta}{c})$, we have

$$
\begin{aligned}
(s - \frac{\lambda+\delta}{c})(s - \frac{2\lambda+\delta}{c}) T_s m_{r,\delta}(k) =& \frac{r\lambda}{c} (s - \frac{2\lambda+\delta}{c})(T_{\frac{\lambda+\delta}{c}} - T_s)\gamma_1(k) \\
&+ 2\frac{r\lambda\theta}{c} (s - \frac{\lambda+\delta}{c})(T_{\frac{2\lambda+\delta}{c}} - T_s)\gamma_2(k) \\
&- \frac{r\lambda\theta}{c} (s - \frac{2\lambda+\delta}{c})(T_{\frac{\lambda+\delta}{c}} - T_s)\gamma_2(k)
\end{aligned} \tag{3.6}
$$

Multiplying (3.4) by $(s - \frac{\lambda+\delta}{c})$ and rewrite (3.5), we can obtain

$$
\begin{aligned}
(s - \frac{\lambda+\delta}{c})(s - \frac{2\lambda+\delta}{c}) T_s m_{r,\delta}(k) =& (s - \frac{2\lambda+\delta}{c}) m_{r,\delta}(k) + \frac{\lambda}{c} \times 2\frac{r\lambda\theta}{c} T_{\frac{2\lambda+\delta}{c}} \gamma_2(k) \\
&- \frac{r\lambda}{c} (s - \frac{2\lambda+\delta}{c}) T_s\gamma_1(k) - \frac{r\lambda\theta}{c} (s - \frac{\delta}{c}) T_s\gamma_2(k)
\end{aligned} \tag{3.7}
$$

$$\begin{aligned}T_s\gamma_1(k) &= \int_k^\infty e^{-s(u-k)}\gamma_1(u)\mathrm{d}u \\ &= e^{sk}\int_k^\infty e^{-su}\int_0^{u-k} f(x)m_{r,\delta}(u-x)\mathrm{d}x\mathrm{d}u + m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}\int_{u-k}^{u} f(x)\mathrm{d}x\mathrm{d}u \\ &+ \int_k^\infty e^{-s(u-k)}\beta_1(u)\mathrm{d}u \\ &= e^{sk}\hat{f}(s)\hat{m}_{r,\delta}(s) + m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}(\bar{F}(u-k)-\bar{F}(u))\mathrm{d}u + T_s\beta_1(k) \end{aligned} \tag{3.8}$$

Since $T_s m_{r,\delta}(k) = e^{sk}\hat{m}_{r,\delta}(s)$, we have

$$T_s\gamma_1(k) = T_s m_{r,\delta}(k)\hat{f}(s) + m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}(\bar{F}(u-k)-\bar{F}(u))\mathrm{d}u + T_s\beta_1(k)$$

Similaly

$$T_s\gamma_2(k) = T_s m_{r,\delta}(k)\hat{h}(s) + m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}(\bar{H}(u-k)-\bar{H}(u))\mathrm{d}u + T_s\beta_2(k)$$

Substiituting in (3.7), we can derive

$$\begin{aligned}(s-\frac{\lambda+\delta}{c})(s-\frac{2\lambda+\delta}{c})T_s m_{r,\delta}(k) =&(s-\frac{2\lambda+\delta}{c})m_{r,\delta}(k) - \frac{r\lambda}{c}(s-\frac{2\lambda+\delta}{c})\Big[T_s m_{r,\delta}(k)\hat{f}(s) \\ &+ m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}(\bar{F}(u-k)-\bar{F}(u))\mathrm{d}u + T_s\beta_1(k)\Big] \\ &- \frac{r\lambda\theta}{c}(s-\frac{\delta}{c})\Big[T_s m_{r,\delta}(k)\hat{h}(s) \\ &+ m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}(\bar{H}(u-k)-\bar{H}(u))\mathrm{d}u + T_s\beta_2(k)\Big] \\ &+ \frac{\lambda}{c}\times\frac{2r\lambda\theta}{c}\times e^{\frac{2\lambda+\delta}{c}}\hat{\gamma}_2(s)\end{aligned} \tag{3.9}$$

$$\begin{aligned}T_s m_{r,\delta}(k) =& \frac{1}{(s-\frac{\lambda+\delta}{c})(s-\frac{2\lambda+\delta}{c}) + \frac{r\lambda}{c}(s-\frac{2\lambda+\delta}{c})\hat{f}(s) + \frac{r\lambda\theta}{c}(s-\frac{\delta}{c})\hat{h}(s)} \\ &\times\Big\{(s-\frac{2\lambda+\delta}{c})m_{r,\delta}(k) - \frac{r\lambda}{c}(s-\frac{2\lambda+\delta}{c})\Big[m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}(\bar{F}(u-k)-\bar{F}(u))\mathrm{d}u + T_s\beta_1(k)\Big] \\ &-\frac{r\lambda\theta}{c}(s-\frac{\delta}{c})\Big[m_{r,\delta}(k)\int_k^\infty e^{-s(u-k)}(\bar{H}(u-k)-\bar{H}(u))\mathrm{d}u + T_s\beta_2(k)\Big] \\ &+\frac{\lambda}{c}\times\frac{2r\lambda\theta}{c}\times T_{\frac{2\lambda+\delta}{c}}\gamma_2(k)\Big\}\end{aligned} \tag{3.10}$$

Since $T_s m_{r,\delta}(k) = e^{sk}\hat{m}_{r,\delta}(s)$, we can rewrite (3.10)

$$\begin{aligned}\hat{m}_{r,\delta}(s) =& \frac{1}{(s-\frac{\lambda+\delta}{c})(s-\frac{2\lambda+\delta}{c}) + \frac{r\lambda}{c}(s-\frac{2\lambda+\delta}{c})\hat{f}(s) + \frac{r\lambda\theta}{c}(s-\frac{\delta}{c})\hat{h}(s)} \\ &\times\Big\{(s-\frac{2\lambda+\delta}{c})e^{-sk}m_{r,\delta}(k) - \frac{r\lambda}{c}(s-\frac{2\lambda+\delta}{c})\Big[m_{r,\delta}(k)\int_k^\infty e^{-su}(\bar{F}(u-k)-\bar{F}(u))\mathrm{d}u + e^{-sk}T_s\beta_1(k)\Big] \\ &-\frac{r\lambda\theta}{c}(s-\frac{\delta}{c})\Big[m_{r,\delta}(k)\int_k^\infty e^{-su}(\bar{H}(u-k)-\bar{H}(u))\mathrm{d}u + e^{-sk}T_s\beta_2(k)\Big] \\ &+\frac{\lambda}{c}\times\frac{2r\lambda\theta}{c}\times e^{-sk}T_{\frac{2\lambda+\delta}{c}}\gamma_2(k)\Big\}\end{aligned} \tag{3.11}$$

where, the denominator of Eq. (3.11) $(s-\frac{\lambda+\delta}{c})(s-\frac{2\lambda+\delta}{c})+\frac{r\lambda}{c}(s-\frac{2\lambda+\delta}{c})\hat{f}(s)+\frac{r\lambda\theta}{c}(s-\frac{\delta}{c})\hat{h}(s)=0$ is the Lundberg's generalized equation with two different positive real roots $\rho_i,\ i=1,2$. See the analysis of the Lunderberg's generalized equation [(14)] and [(10)]. These roots must also be roots of the numerator of Eq. (3.11), given that it is analytic. Therefore we obtain $m_{r,\delta}(k)$ by the following linear systerm.

$$m_{r,\delta}(k)=\frac{\left\{\frac{r\lambda}{c}(\rho_1-\frac{2\lambda+\delta}{c})T_{\rho_1}\beta_1(k)+\frac{r\lambda\theta}{c}(\rho_1-\frac{\delta}{c})T_{\rho_1}\beta_2(k)-\frac{\lambda}{c}\times\frac{2r\lambda\theta}{c}\times T_{\frac{2\lambda+\delta}{c}}\gamma_2(k)\right\}}{\left\{(\rho_1-\frac{2\lambda+\delta}{c})-\frac{r\lambda}{c}(\rho_1-\frac{2\lambda+\delta}{c})\eta_1(k)-\frac{r\lambda\theta}{c}(\rho_1-\frac{\delta}{c})\xi_1(k)\right\}} \tag{3.12}$$

$$m_{r,\delta}(k)=\frac{\left\{\frac{r\lambda}{c}(\rho_2-\frac{2\lambda+\delta}{c})T_{\rho_2}\beta_1(k)+\frac{r\lambda\theta}{c}(\rho_1-\frac{\delta}{c})T_{\rho_2}\beta_2(k)-\frac{\lambda}{c}\times\frac{2r\lambda\theta}{c}\times T_{\frac{2\lambda+\delta}{c}}\gamma_2(k)\right\}}{\left\{(\rho_2-\frac{2\lambda+\delta}{c})-\frac{r\lambda}{c}(\rho_2-\frac{2\lambda+\delta}{c})\eta_2(k)-\frac{r\lambda\theta}{c}(\rho_2-\frac{\delta}{c})\xi_2(k)\right\}} \tag{3.13}$$

Hence,

$$m_{r,\delta}(k)=\frac{\frac{r\lambda}{c}[(\rho_2-\frac{2\lambda+\delta}{c})T_{\rho_2}\beta_1(k)-(\rho_1-\frac{2\lambda+\delta}{c})T_{\rho_1}\beta_1(k)]+\frac{r\lambda\theta}{c}[(\rho_2-\frac{\delta}{c})T_{\rho_2}\beta_2(k)-(\rho_1-\frac{\delta}{c})T_{\rho_1}\beta_2(k)]}{(\rho_2-\rho_1)-\frac{r\lambda}{c}[(\rho_2-\frac{2\lambda+\delta}{c})\eta_2(k)-(\rho_1-\frac{2\lambda+\delta}{c})\eta_1(k)]-\frac{r\lambda\theta}{c}[(\rho_2-\frac{\delta}{c})\xi_2(k)-(\rho_1-\frac{\delta}{c})\xi_1(k)]} \tag{3.14}$$

The proof is completed. □

When $\theta=0$, the claim amount r.v. X_j and the interclaim time r.v. W_j is independent,

$$f(x,t)=f(x)\lambda e^{-\lambda t}$$

then,

$$m_{r,\delta}(k)=\frac{\frac{r\lambda}{c}\int_k^\infty\int_u^\infty e^{-\rho(u-k)}f(x)\omega(u,x-u)\mathrm{d}x\mathrm{d}u}{1-\frac{r\lambda}{c}\int_k^\infty e^{-\rho(u-k)}(\bar{F}(u-k)-\bar{F}(u))\mathrm{d}u} \tag{3.15}$$

Which coincides with the result obtained by Dickson et al.(2016).

4 CONCLUSIONS

a In the section (2), a risk model with capital injections and a dependence structure modeled by a Farlie-Gumbel-Morgenstern copula is described.

b In the section (3), an explicit formula for the Gerber-Shiu function is obtained when the initial surplus is k.

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