Comparative Study of Elementary School Mathematics Textbooks in China, Japan, South Korea, Singapore, America, Germany: A Case Study on "Fraction Division"

Abstract

This study selects 15 elementary school mathematics textbooks from China, Japan, South Korea, Singapore, America, Germany as research objects (see Table 1). Taking "fraction division" as an example, the study compares the operational meaning models and arithmetic explanation processes of "fraction division." The results indicate that there are significant differences in the meaning models and arithmetic to fraction division among the different textbook versions. This paper aims to clarify the characteristics and similarities of these differences through comparison, combine national conditions of various countries, seek common ground while reserving differences, so as to find a teaching method suitable for Chinese elementary school mathematics "fractional division" ,provide elementary school mathematics teachers with a teaching path of fractional division based on national conditions and absorbing the advantages of different countries, and provide theoretical support for better implementation of curriculum standards and textbooks.

Keywords: *Elementary school mathematics; Fraction division; Textbook comparison; Division models; Procedural explanation*

1Introduction

According to the report of the International Association for the Evaluation of Educational Achievement, China, Japan, South Korea, Singapore, Germany and America have achieved relatively good results in international mathematical achievement comparison tests such as TIMISS and PISA. The reason is that elementary school students' understanding of mathematics basically comes from classroom teaching and textbooks, and the differences in textbooks may partly explain the differences in students' test performance. The textbooks of six countries can concretely present the arrangement and development of curriculum content, carry curriculum culture and transmit curriculum ideas.

As an important part of "number and algebra", fractional division is the basis for students to expand the number system. It is of great significance for students to understand decimals, ratios, proportions, percentages, trigonometric functions, derivatives, grasp the essence of mathematics, and cultivate students' number sense. Therefore, the study of fractional division in textbooks is indispensable.

In the latest round of mathematics curriculum reforms, understanding algorithm is an important goal, which is more abstract and complex for elementary school students at this stage, but understanding algorithm can help students truly understand algorithms. Many students have many

Comment [IPAS1]: The manuscript shows an effort in comparing elementary school mathematics textbooks from various countries on the topic of fraction division, however, there are several areas that could be improved. Firstly, please strengthen the introduction by providing more detailed background information on the significance of fraction division in elementary mathematics education and its broader implications. Additionally, please write a clearer explanation of the criteria used to select the textbooks for comparison and the specific methods employed for analysis for the research design section. Furthermore, please write more integration on the suggestions provided for improving teaching methods with the comparative analysis to provide a deeper understanding of the differences observed in the textbooks. Moreover, the conclusion could be expanded to discuss the implications of the findings in a broader...

Comment [IPAS2]: Please clarify the study's scope and significance by writing a clear overview of the research objectives, methods, and key findings. Moreover, please provide a concise overview of the mathematics education implications arising from the variations in fraction division representation and arithmetic across textbooks published in different nations. Lastly, please provide a clear statement of the study's contributions to the field, such as offering insights into teaching methods for fraction division.

Comment [IPAS3]: Please give a more detailed summary of the study's specific goals and objectives. Furthermore, please offer a more analysis of the present research on fraction division education, emphasizing any significant gaps or points of disagreement.

difficulties in understanding algorithm, often only remember the algorithm, through rote memorization, step-by-step procedural operations to master mathematical knowledge, improve mathematical performance. Therefore, the conflict between the importance of fractional division and the difficulty of understanding fractional algorithm makes fractional division a hot topic in mathematics education.[1]

Based on the considerations outlined above, this paper, based on JiaSuijun's reasoning model [2] andMa Liping's model describing the meaning of fractional division operation[3], grounded in the two perspectives of the models of the significance of fraction division, the revealing of fraction division theory, compares and analyzes models of fraction division Meaning, the revealing process of fraction division theory of describing fraction division operation in the textbooks of six countries. Therefore, it provides some relevant suggestions for elementary school teachers on how to learn fractional division.

2 Research Design

2.1 Research Objects

Based on the widespread influence and usage, the author selected 15 elementary school mathematics textbooks from different publishers around the world as research objects (see Table 1). These textbooks are widely used in the educational systems of China, Japan, South Korea, Singapore, America, and Germany.

Table 1 Research objects

Nation	Version abbreviation	Textbook version	Publishing House
China	RJ	People's Education Press Grade 6 Volume 1[4]	People's Education Press
America	CM	California Mathematics Grade6[5]	The McGraw-Hill companies Glencoe
	M5	My Math Grade5[6]	The McGraw-Hill companies Education
	HM	Go Math Grade6[7]	Houghton Mifflin Harcourt
Singapore	TM	Targeting Mathematics 6A[8]	Star Publishing Pte Ltd
	MC	My Pals Are Here! Math 6A 3rd Edition[9]	Marshall Cavendish Education
Japan	Math 634	Math 6(Math 634)[10]	School Book Press
	Math 631	New Math 6(Math 631) [11]	Tokyo BooksPress
	Math 638	Math 6(Math 638)[12]	Qilin GuanPress
	Math 633	New Math 6(Math 633)[13]	Dai Nippon Book Press
	Math 636	elementary school Math6 (Math 636) [14]	Education Press

South Korea	TC	Math6-1[15]	Genius Education Press
Germany	WE	Mathematik 6 (Hauptschule Bayern) [16]	Westermann
	NA	LernstufenMathematick 6 (NeueAusgabe) [17]	Cornelsen
	MB	LernstufenMathematick 6 (Mittelschule Bayern) [18]	Cornelsen

2.2 Methodology

Comment [IPAS4]: Please provide more detailed criteria for the textbook selection process. Additionally, please further elaborate on how utilization of established frameworks by JiaSuijun and Ma Liping were applied in the analysis. Moreover, please include information on the specific data collection and analysis methods employed, such as content analysis or qualitative coding procedures.

2.2.1 Representation Models of Fraction Division Meaning

Ma Liping's research indicates that the meaning of fraction division operations can be revealed through three models: the inclusion model, the part-whole model, and the product-and-factor model[3]. Among them, the part-whole model includes a special type where the divisor is an integer. The meaning of fraction division in such special cases is similar to the meaning of integer division.[2] Many researchers separate this special case from the part-whole model, often referring to it as the equal sharing model. In this study, the 15 elementary school mathematics textbooks from various publishers around the world generally follow these four types to reveal the meaning of fraction division.

(1) ***Inclusion Model***: Given a fixed total quantity of objects, known quantities for each share, the task is to determine the number of shares contained in the total quantity. For example, the

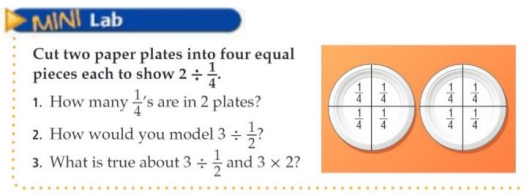


Figure 1Inclusion Model

context of the American CM textbookis illustrated in Figure 1[5].

(2) ***Equal Sharing Model***: With a fixed total quantity of objects, the task is to divide the objects equally into a certain number of shares and determine the quantity of objects in each share. For example, the context of the German WE textbook is illustrated in Figure 2[16].

Alexander, Christine and Daniel share $\frac{6}{8}$ of the pizza equally.

What portion of the pizza will each get?

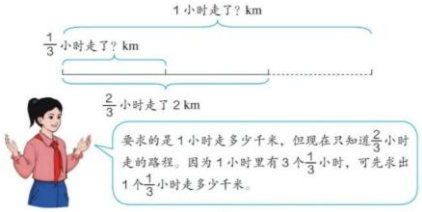
$$\frac{6}{8} \div 3 = \frac{2}{8}$$

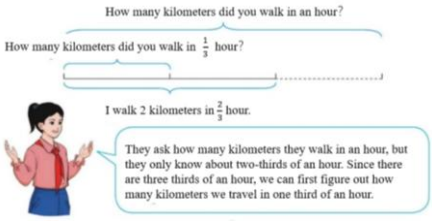
Read as six eighths divided by three equals two eighths, and each person gets two eighths of a pizza. Divide every six in the numerator by the divisor of three.

Figure 2Equal Sharing Model

(3) ***Part-Whole Model***: The quantity of objects as a part of the whole is known, and the task is to determine the quantity of objects as the whole. It is a hybrid model relative to the equal sharing and inclusion models. For example, the contexts of the Chinese People's Education

RJ





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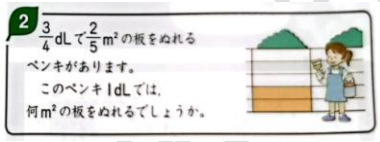

If $\frac{3}{4}$dL paint can paint $\frac{2}{5}$ square meters of wall, how many square meters of wall can be painted with 1dL paint?

Figure 3Part-Whole Model

Press[4] and Japanese 636 textbooks[14]are illustrated in Figure 3.

(4) ***Product-and-Factor Model***: The common representation of the product-and-factor model is "Area = Length × Width," where area corresponds to the product, and length or width corresponds to the other two factors. For example, the context of the Singapore MC textbook [9]is illustrated in Figure 4.

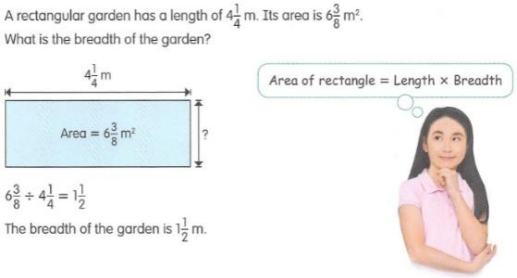

Figure 4Product-and-Factor Model

2.2.2 Revelation Methods of Arithmetic Reasoning in Fraction Division

Intuitive geometry and deductive reasoning are the main ways to reveal the computational theory in elementary school (a new perspective of procedural knowledge curriculum design: the integration of computational theory and the integration of computational law)[19]. In the fraction division operation, there are four main ways to reveal the theory: intuitive geometry; Intuitive geometry combined with multiplication; Deductive reasoning based on intuitive geometry; Deductive reasoning supplemented by intuitive geometry; Deductive reasoning.[2]

(1) ***The intuitive geometry***

According to the meaning of fraction division, the results of fraction division problems are obtained directly by means of geometric figures or other intuitive geometrys. For example, in the South Korean TC textbook[15], 5/6÷1/6=5 is directly presented by dividing the number line. (See Figure 5)

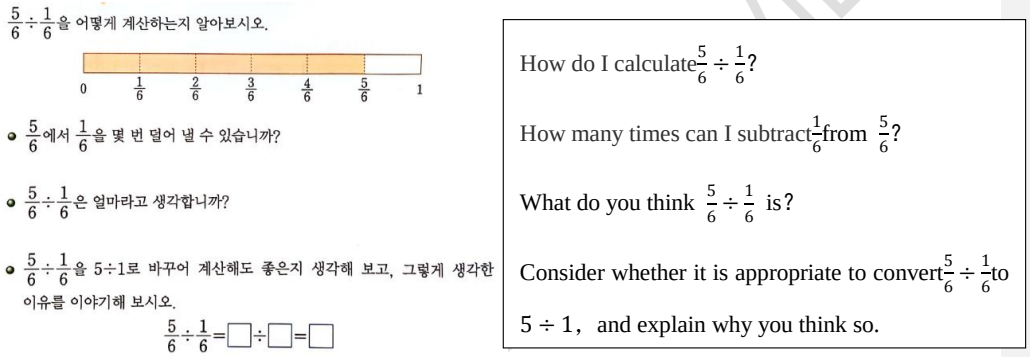

Figure 5The example of The intuitive geometry

(2) ***Intuitive geometry Associated with Fraction Multiplication.***

Obtain the result of fraction division through intuitive geometry, then express this result using fraction multiplication, thereby establishing the relationship between fraction division and multiplication operations[2]. For example, in the American CM textbook[5], the example of 4 ÷ 1/3 = 12 involves first obtaining the result of 4 ÷ 1/3 through intuitive geometry as 12 and then connecting it with multiplication operations to discover that 4 × 3 equals 12. By associating intuitive geometry with fraction multiplication, a general arithmetic for fraction division can be derived: dividing by a fraction is equivalent to multiplying by the reciprocal of that fraction (see Figure 6).

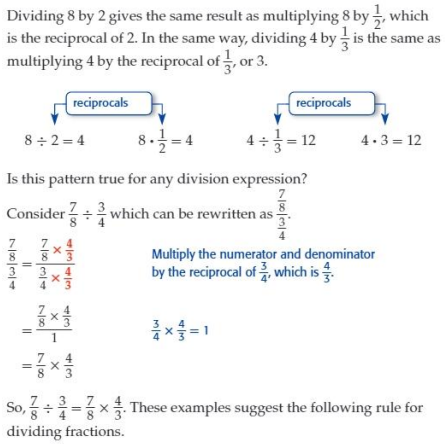

Figure 6The example ofIntuitive Geometry Associated with Fraction Multiplication

(3) ***Deductive Reasoning Based on Intuitive geometry.***

Utilize intuitive geometry to establish a mathematical conclusion, and then, starting from this conclusion, employ deductive reasoning to discover a new mathematical result.[2] For instance, in the Singapore MC textbook[9], when solving 5 ÷ 2/3, intuitive geometry reveals that there are three occurrences of 2/3 in 2. This becomes a crucial mathematical conclusion. Based on this conclusion, we can determine that 1 contains 3/2 occurrences of 2/3, and subsequently, 5 contains 5 × 3/2 occurrences of 2/3. Through deductive reasoning, a new mathematical result is obtained (see Figure 7).

This approach differs from "The intuitive geometry" and " intuitive geometry associated with fraction multiplication" in two main aspects. Firstly, "The intuitive geometry" relies solely on intuitive geometry to derive mathematical conclusions without presenting any additional explanations or reasoning. Secondly, "deductive reasoning based on intuitive geometry" primarily depends on explanations and reasoning to obtain mathematical conclusions, with intuitive geometry serving as the starting point for explanations and reasoning. [2]Secondly, "deductive reasoning based on intuitive geometry" reveals the intrinsic connection between fraction division and multiplication, answering the question of "why dividing by a fraction is equivalent to multiplying by its reciprocal." Although "Intuitive geometry associated with fraction multiplication" also establishes a connection between fraction division and multiplication, the mechanism of this "intrinsic connection" remains a black-box problem and has not been answered

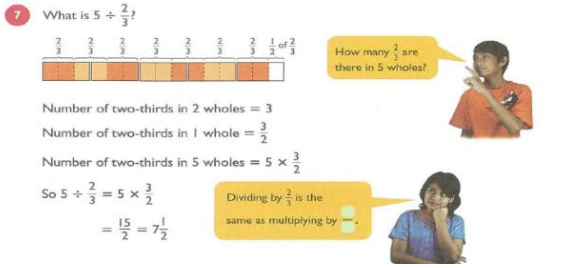

Figure 7The example ofDeductive Reasoning Based on Intuitive geometry

positively (see Figure 7).

(4) ***Deductive reasoning supplemented by intuitive geometry.***

The fraction division rule is constructed by deductive reasoning mainly in virtue of fractional meaning and fraction multiplication meaning. The intuitive geometry only plays an auxiliary role, and the extraction of intuitive geometry has little influence on the reasoning process.

The Japanese arithmetic 634 textbook[10] contains deductive reasoning supplemented by intuitive geometry (see Figure 8). The problem to be solved in Figure 8 is: 3/4 liter of paint can paint 2/5 square meters of wall surface, so how many square meters of wall surface can be painted with 1 liter of paint? According to the meaning of the score, 3/4 is 3 1/4, then the wall painted by 1/4 liter is 2/5÷3=2/5×1/3=2/15. According to the meaning of the score, 1 contains 4 1/4, then the

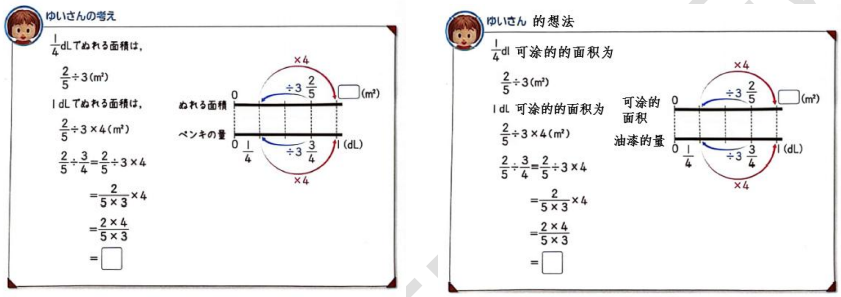

Figure 8The example ofDeductive reasoning supplemented by intuitive geometry

wall painted by 1 liter of paint is 2/15×4=8/15.

(5) ***Deductive reasoning.***

The theory of fraction division is revealed by deductive reasoning, mainly by means of invariant properties of quotient and fraction multiplication law. In this process, there is no penetration of intuitive geometry. For example, in the Japanese arithmetic 634 textbook[10], 2/5÷3/4 is used as an

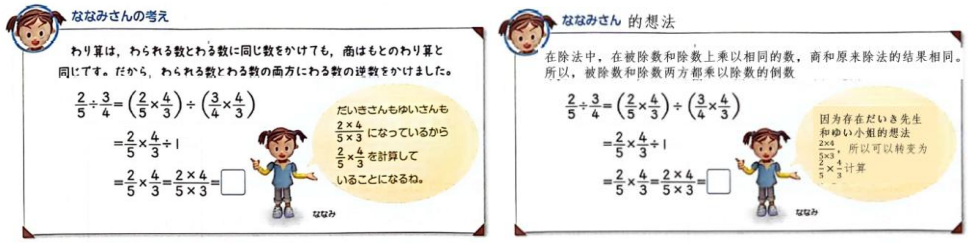

Figure 9The example ofDeductive reasoning

example to reveal the arithmetic of fraction division. (See Figure 9)

3Results

3.1 Significance of fraction division

The models of the significance of fraction division in each teaching material are illustrated in Table 2.

Table 2 Models that introduce the meaning of fractional division

Nation	Textbook version	Using models
China	People's Education Press<	Part-Whole Model
		Equal Sharing Model
America	CM	Inclusion Model
	M5	Inclusion Model
	HM	Inclusion Model
Singaporee	TM	Equal Sharing Model
		Part-Whole Model
	MC	Part-Whole Model
		Inclusion Model
		Product-and-Factor Model
		Equal Sharing Model
Japan	Math634	Part-Whole Model
	Math631	Part-Whole Model
	Math638	Part-Whole Model
	Math633	Part-Whole Model
	Math636	Part-Whole Model
Germany	WE	Equal Sharing Model
		Inclusion Model
	NA	Equal Sharing Model
		Inclusion Model
	MB	Equal Sharing Model
South Korea	TC	Inclusion Model

Fraction division is the inverse operation of fraction multiplication. The inclusion model and the equal division model are based on the concept of "a certain number of a certain quantity" in the multiplication of fraction, the former emphasizes how many parts, that is, the "a certain quantity" in the front, the latter emphasizes how many parts, that is, the "a certain number" in the back, and students can understand the rationality of the fraction division operation from the cognitive basis of the fraction multiplication.

Part-Whole Model emphasizes the concept of "unit" and focuses on students' reasoning and analysis ability of reverse thinking. Product and factor models combined with intuitive geometry make it easier for students to understand and relate to reality, and have a deeper understanding of fraction division.

The model the significance of fraction division used in the United States and South Korea is relatively simple. The inclusion model is used in the United States and South Korea, and the reason is related to the curriculum standards in the United States and South Korea. The new

curriculum standards of the two countries not only focus on letting students master the arithmetic of fraction division, but also emphasize the application of practical problems, so as to cultivate students' ability to flexibly use mathematical knowledge to solve complex problems, rather than just staying in the mechanical calculation level.

Japanese curriculum standards focus on students' better understanding of the concept and operation methods of fractions as well as their application in practical problems, and put forward higher requirements for the application of computational algorithms. Therefore, Japanese textbooks use part-whole model in the significance model of fraction division, and design multiple methods for one model to help students understand the consistency of the significance of fraction division.

German curriculum standards focus on cultivating students' multidimensional understanding and application ability of fraction division, and the equal division model adopted by them emphasizes the concept of equal distribution and part-whole, which is applicable to many problems involving equal distribution. The inclusion model, on the other hand, emphasizes partial inclusion and unequal distribution, and is suitable for more complex problems, such as allocating different amounts of resources or calculating unequal ratios. Through the introduction of these two types of models, the new curriculum standards in Germany aim to enable students to cope with different types of fraction division problems in a more comprehensive way. This kind of relatively balanced method is helpful to satisfy the learning needs of different students, help students better understand mathematical concepts, solve practical problems, and promote the overall development of mathematics education.

Singapore's curriculum standards emphasize not only the development of multidimensional application skills, but also a deep understanding of fraction division. In the TM and MC versions of the textbook, the four models of equal division, inclusion, part-whole, product and factor are involved, so that students can deal with different types of fraction division problems more comprehensively, promote the development of mathematical thinking and problem solving ability, and cultivate their mathematical literacy and creative thinking.

Chinese curriculum standards emphasize the effectiveness and depth of mathematics education. The introduction of the equal division model and part-whole model provides students with an intuitive and effective way of learning, which helps students better grasp the knowledge and skills of fraction division, so as to help students deeply understand the concept, establish a solid mathematical foundation, cultivate mathematical thinking and the ability to deal with practical problems.

To sum up, curriculum standards are not only a guide for the preparation of textbooks, but also a basis for evaluation. As the main carrier of curriculum standards,textbook carries the core value of education. Through in-depth analysis of curriculum standards in various countries, we find that: in the selection of materials, the arrangement of each chapter and detail of the textbook is tightly tied to the requirements of the national curriculum standards, reflecting the training requirements of the core literacy of the country, which is conducive to the guidance of teachers and the exploration of students.

3.2 The revealing process of fraction division theory

See Table 3 for the detailed models of the fraction division calculation revealing process in each textbook.

Table 3 shows the model of the process revealed by fraction division

Nation	Textbook version	Using models
China	Peoples Education Press	The intuitive geometry
		Deductive reasoning supplemented by intuitive geometry
America	CM	Intuitive geometry Associated with Fraction Multiplication
	M5	The intuitive geometry
	HM	The intuitive geometry
Singapore	TM	The intuitive geometry
		Deductive Reasoning Based on Intuitive geometry
	MC	The intuitive geometry
		Deductive Reasoning Based on Intuitive geometry
		Deductive reasoning supplemented by intuitive geometry
Japan	Math 634	Deductive reasoning
		Deductive reasoning supplemented by intuitive geometry
		Deductive Reasoning Based on Intuitive geometry
	Math 631	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
	Math 638	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
	Math 636	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
	Math 633	Deductive reasoning
		Intuitive geometry Associated with Fraction Multiplication
		Deductive reasoning supplemented by intuitive geometry
Germany	WE	The intuitive geometry
	NA	The intuitive geometry
	MB	The intuitive geometry
South Korea	TC	The intuitive geometry

The models used in American textbooks are basically geometric and intuitive, with a small part associated with fraction multiplication and less deduction. Germany and South Korea are similar to the United States in that they basically focus on complete intuitive geometry, which is partly related to the emphasis on practical problem solving in the curriculum standards of these countries.

In Singapore, there are three types of models for explaining the computational process of

fractional division, and there are also different ways to explain the computational method of students' fractional division, which is relatively comprehensive and complete, echoing the Singapore curriculum standards that emphasizes students' multidimensional understanding of fraction division.

Fraction division in the teaching material of Peoples Education Press presents intuitive geometry, but the teaching material is more inclined to reveal the theory through deductive reasoning, and intuitive geometry is only an auxiliary means of deductive reasoning[2]. In the process of reasoning, two types of reasoning are used, intuitive geometry and deductive reasoning supplemented by intuitive geometry, and more attention is paid to the level of reasoning, which is closely related to the Chinese curriculum standard that emphasizes the effectiveness of mathematics education and the depth of mathematics learning[20].

Comparing the curriculum standards of China and South Korea,it is found that both countries pay attention to calculation. China focuses on intuitive geometryso as to help students understand

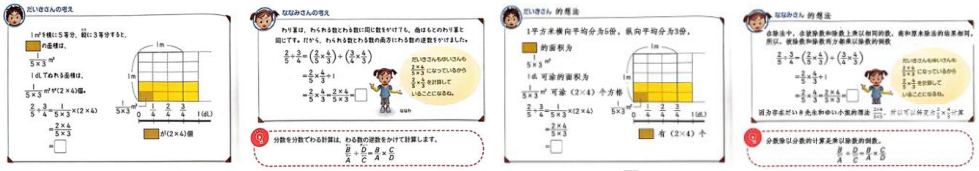

Figure 10The revealing process of fraction division theory of Japanese textbooks

fraction division theory, which is more abstract and general; South Korea puts more emphasis on the diversity of arithmetic and the use of "learning materials" in textbooks.

The revealing process of fraction division theory of Japanese textbooks is more special, and the author will take Japanese arithmetic 634 textbook [10] as an example to illustrate it.

The arrangement of Japanese textbooks has reached a high degree of consistency, there are more than three ways to explain the same topic, and the exploration of a topic is relatively profound. In the presentation of the algorithm, deductive reasoning supplemented by intuitive geometry and the reasoning associated with intuitive geometry and fraction multiplication are mainly used. Through role play to show different reasoning methods, from intuitive to abstract, progressive presentation, it is more conducive to students' deep understanding of fractional division theory, novel design.

Based on the curriculum standards, the research on the revealing ways of calculation in textbooks of various countries is conducive to the understanding and implementation of the meaning of fraction division, and is conducive to teachers' effective teaching according to students' cognitive level. The grade who learn fraction division is senior elementary school students, and the core quality of elementary school includes reasoning consciousness, while junior middle school students include reasoning ability. The development of mathematical ability of senior primary school students requires that reasoning consciousness be connected with junior middle school reasoning ability, but senior primary school students still stay in the concrete operation stage, and need geometric and intuitive assistance in the process of revealing and reasonably explaining calculation theory. However, the overall adoption of intuitive geometry will cause students to be afraid of difficulties when they encounter problems requiring high logical thinking such as reasoning proof, which hinders the development of students' deductive ability. Therefore, in the

mathematics textbooks of senior primary schools, based on the connection between psychological development stage and elementary school, how to deal with the proportion of intuitive geometry and deductive reasoning needs to be explored.

4 Suggestions

In the field of elementary mathematics education, fraction division is a crucial topic. By comparing the meaning of fraction division and arithmetic in textbooks of different countries, we provide the following beneficial suggestions for elementary school mathematics teachers in China to improve the teaching method of fraction division, so as to better satisfy the learning needs of students.

Firstly, when teaching "fraction division," it is recommended that elementary school mathematics teachers deepen their understanding of what fraction division is. Understanding fraction division is an abstract concept and therefore requires concrete example models and visual teaching to help students understand it. Teachers can draw on practical situations created when introduced in other countries to help students better understand the meaning of grades.

Secondly, in the application of teaching methods of fraction division, due to the great differences among different students, students have different degrees of acceptance of various models of "the significance of fraction division" and " Revelation Methods of Arithmetic Reasoning in Fraction Division ". In teaching, teachers should, according to the needs and understanding level of students, closely combine with the educational reality and national conditions in different regions of China, deeply study and compare the teaching materials of different countries, selectively absorb the teaching methods and concepts of different countries, clarify the characteristics and similarities and differences of each country, so as to choose the most suitable teaching methods for China's national conditions and regional characteristics, and ensure that they can deeply understand the concept of fractional division. For example, when designing teaching tasks, vivid examples and situations in foreign textbooks can be introduced to integrate some real life problems, so that students can learn fraction division under familiar background and master the calculation and algorithm of fraction division in the process of solving problems.

Finally, teachers should constantly reflect on and adjust their teaching methods, ensure that the teaching content is consistent with the curriculum standards and textbook requirements, and provide students with high-quality teaching experience.

It is hoped that the above suggestions can better provide theoretical support for the implementation of elementary school mathematics curriculum standards and textbooks, and contribute some useful thinking to the development of elementary school mathematics education in China.

5Conclusion

Through the comparative study of 15 elementary school mathematics textbooks "fraction division" from "China, Japan, South Korea, Singapore, America and Germany", this study analyzes that there are great differences in the representation Models of Fraction Division Meaning and the revealing process of fraction division theory.

Based on the curriculum standards of different countries, the author explains the reasons for the

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differences, and expounds the meaning model of fraction division in different countries and the relationship between the proportion of intuitive geometry and deductive reasoning according to the connection between the psychological development of students in the upper grades of elementary school , clarify the context so as to find a suitable learning path for students and a teaching method in line with the national conditions. At the same time, the author will select practical schools to carry out action research, and put the above theoretical results into practice, so as to effectively implement the new curriculum standards and teaching material requirements, and truly achieve the purpose of "light burden and high quality".

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