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**Original Research Article**

**The Method of the Probability Analysis of Area  
with Dissolved gases in Power Transformer  
Insulating Oil**

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**ABSTRACT**

Transformer insulating oil plays an extremely important role in power transformers and is used for insulation, arc suppression, cooling and other purposes. However, its other focus is on dissolved gases analysis in the oil to monitor internal operating conditions, which is the responsibility of preventive maintenance. This paper uses the normal distribution theory and the American National Standard Specification (hereinafter referred to as ANSI/IEEE C57.104) to reorganize as a diagnostic tool. The range of each gas from three stages of the specification - normal, caution, and abnormal are incorporated to a stage of abnormal, which is divided into equal different values of 1000 as the maternal body of a qualitative normal distribution. Then the benchmark is been calculated from those parameters of a qualitative normal distribution. The data of detection has to pass through "Gas chromatography" to generate dissolved gases. At last, the benchmark value compares with dissolved gases to find the abnormal probability value as a diagnostic tool in maintenance evaluation of equipment. This method was developed using EXCEL application software, and repeatedly tested and verified with examples to confirm its feasibility and high accuracy. Therefore, this diagnostic method can replace the traditional multi-stage judgment and be represented by a single probability value to show. Based on technology sharing, the development process is specially written into a technical article as referenced by scholars and electrical maintenance personnel in the field of power engineering.

*Keywords: Insulating Oil, Dissolved gases Analysis, ANSI/IEEE C57.104, Normal Distribution.*

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## 22 1. INTRODUCTION

23 The internal insulating oil of transformer plays an  
24 important role in the operation of power transformers.  
25 It is related to the quality of power and the safety of  
26 power system supply. So that, we are very cautious  
27 in monitoring and control, especially internal testing.  
28 When it comes to internal testing, it falls on dissolved  
29 gases analysis from the insulating oil. The dissolved  
30 gases analysis (DGA) is a crucial method for  
31 identifying incipient faults in power transformers. It is  
32 been dissolved from the insulating oil of transformer  
33 which is running condition. When the data of  
34 detection of insulating oil must pass through "Gas  
35 chromatography" to obtain various dissolved gases  
36 of the flammability properties such as - hydrogen  
37 (H<sub>2</sub>), methane (CH<sub>4</sub>), ethane (C<sub>2</sub>H<sub>6</sub>), ethylene  
38 (C<sub>2</sub>H<sub>4</sub>), acetylene (C<sub>2</sub>H<sub>2</sub>) and carbon monoxide (CO)  
39 However, how to ~~interpret~~ ~~interpretation~~ — ~~those~~ the  
40 analysis number of ones in insulating oil are based  
41 on the ANSI/IEEE C57.104 [1,2]. When it comes to  
42 ANSI/IEEE C57.104 - the specification covers a wide  
43 range of areas: 1) The relationship between the  
44 insulating oil in the transformer; 2) Purpose and  
45 application; 3) Quality verification and limitations; 4)  
46 Interpretation and specifications; 5) Failure type  
47 definition and identification; 6) Case studies and  
48 explanation examples. Its purpose is to provide  
49 operators or maintenance personnel with  
50 maintenance information and introduce various  
51 diagnostic techniques, such as Key gases, Rogers  
52 ratio, Duval triangle method and other methods. This  
53 paper only proposes improvements from the fault  
54 definition and identification of the fifth paragraph. It  
55 uses all the characteristics of the "Normal  
56 Distribution" theory and applies the number covering  
57 each gas from normal to abnormal in the  
58 ANSI/IEEE C57.104 as the maternal body to make a  
59 qualitative normal distribution. The method is been  
60 used with the data of detection compared with as  
61 diagnosis, so that it is called the probabilistic method.  
62 In addition to this paragraph, there are also  
63 paragraphs such as literature review, research steps,  
64 program testing, case verification, discussion,  
65 conclusion and references.

## 66 2. LITERATURE REVIEW

67 Before discussing the analysis of various gases in oil,  
68 we need to introduce the " Gas  
69 chromatography." This instrument can be traced back  
70 to the work of Russian scientist Mikhail  
71 Semenovitch Tswett in 1903 and German graduate  
72 student Fritz Prior invented gas-solid layer analysis  
73 in 1947. In 1950, gas-liquid layer analysis was  
74 invented. Until Archer John Porter Martin laid the  
75 foundation for the development of gas  
76 chromatography and won the Nobel Prize for his  
77 contribution because the development of liquid-liquid  
78 chromatography in 1941 and paper-published in  
79 1944. Then named after "Gas chromatography", it is  
80 a chromatography technology that separates and  
81 analyzes flammable organic chemical mixtures that  
82 are not easily decomposed. It must be separated by  
83 an inert gas (such as helium) or a less reactive gas  
84 (such as nitrogen), with a thin layer of liquid or  
85 polymer attached to the surface of an inert solid  
86 carrier. Those different gas substances are eluted at  
87 different times and temperatures. The "Normal  
88 Distribution" was used by the great mathematician  
89 Gauss in 1794 to describe the small error in repeated  
90 measurements of the same variable. However, the  
91 curve of the error resembles a bell and is called the  
92 bell curve. It was not until 1872 that the American  
93 logician Peirce discovered that it is a continuous  
94 probability distribution. The curve drawn by its  
95 density function is also called the normal curve and  
96 has been verified. Its function is shown in Formula 1  
97 as follows.

$$f(x) = (1/\sigma\sqrt{2\pi})e^{-(x-\mu)^2/2\sigma^2} \quad (1)$$

98 Then, those related articles were selected from  
99 numerous literatures, and each characteristic is  
100 summarized as follows: [3] Apply a lot of historic  
101 number to construct a new qualitative normal  
102 distribution and then convert it into a probability value  
103 as a tool of judgement according to its characteristics  
104 to judge whether the opening and closing stroke time  
105 of the power circuit breaker is normal or not. To  
106 utilize the relationship between the mean, variation,  
107 standard deviation and its height in the generalized  
108

109 normal distribution parameters, and propose  
 110 evaluation and real number sets for analysis and  
 111 explanation. [4] The new deviation from normal  
 112 distribution parameters is not mainly three but one  
 113 that can be used in life and industry. Increase the  
 114 flexibility of its use by adding random variables. As  
 115 for [5 - 12], those literatures describe the normal  
 116 distribution, compilation instructions, bell curve  
 117 drawing and other related rules. Based on the above  
 118 reference number and by standardizing the gas  
 119 ranges of ANSI/IEEE C57.104, a novel detection  
 120 method - " A New Method for Identification within  
 121 Dissolved gases - Probability Method " was  
 122 developed.

### 123 3. RESEARCHMETHODS

124 Taking "Normal Distribution Theory" and "ANSI/IEEE  
 125 C57.104" as the main axis, taking the number range  
 126 of all stages as the maternal body of normal  
 127 distribution to construct a qualitative normal  
 128 distribution. Those parameters are calculated by  
 129 EXCEL application software, and then draw the bell  
 130 shaped curve area according to the characteristics of  
 131 normal distribution, and converting it into a  
 132 probability value in percentages as a diagnostic  
 133 benchmark to compare with the detection data. After  
 134 the above evolution and attributes are described:

#### 135 3.1 NORMAL DISTRIBUTION

136 The normal distribution is used to describe the small  
 137 error in repeated measurements of the same variable.  
 138 However, its shape curve of errors value to make a  
 139 bell-shaped, so that is called a "bell curve." It is a  
 140 continuous probability distribution and the density  
 141 function has been verified and it is been drawn bell  
 142 curve and also called the normal curve. The curve  
 143 area is calculated based on the maternal body of  
 144 a qualitative normal distribution from those  
 145 parameters - mean, variation, standard deviation,  
 146 and is then converted into standard scores and  
 147 probability values through the EXCEL application  
 148 software, as shown in Fig.1. As for, how to convert  
 149 and calculate from those formulas each other, such  
 150 as formulas 2, 3, 4, 5, 6 and the symbols are

explained as follows:

$$X \sim N(\mu, \sigma^2) \quad (2)$$

$$\mu = \sum_{i=1}^n (x_i) / n \quad (3)$$

$$\sigma^2 = \sum_{i=1}^n (x_i - \mu)^2 / n \quad (4)$$

$$\sigma = \sqrt{\sum_{i=1}^n (x_i - \mu)^2 / n} \quad (5)$$

$$Z = (X - \mu) / \sigma \quad (6)$$

where  $\sigma \neq 0$ .

X is a random variable of normal distribution.

$\mu$  is the mean value of the sample group.

$\sigma^2$  is the variation (variance) of the sample group.

$\sigma$  is the standard deviation of sample group.

Z is the standard score.

P is the probability of the area of the bell shape.

If  $\sigma=1$ , the Z value is approximately from -3 to +3.

The area of the bell shape is 1 of the total value of the probability, as shown in the Fig. 1 below.

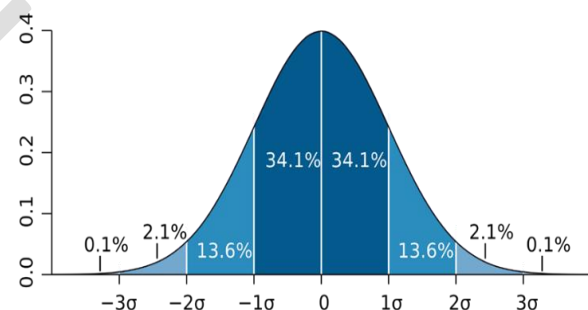


Fig 1. Bell-shaped curve [12]

Because the probability value can be converted to the standard score or the curve area any, using Formula 6 and Formula 7, the probability conversion value can be calculated because,  $Z \sim N(0,1)$ .

$$P(a \leq X \leq b) = P((a - \mu) / \sigma \leq (X - \mu) / \sigma \leq (b - \mu) / \sigma) \\ P((a - \mu) / \sigma \leq Z \leq (b - \mu) / \sigma) \quad (7)$$

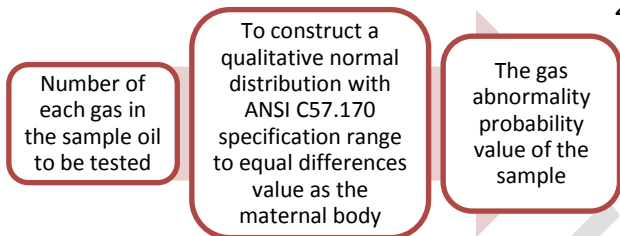
#### 3.2 ANSI/IEEE C57.104 STANDARD

The American National Standards Institute (ANSI) describes detailed procedures for interpreting the

178 results of dissolved gases analysis of insulating oils  
 179 This paragraph only discusses the definition of fault  
 180 stages and the content of each gas (ppm) in the  
 181 identification, divided into four stages - normal,  
 182 caution, abnormal and dangerous, the number  
 183 ranges of each stage according to the specification  
 184 to provide a basis for judgment on equipment  
 185 maintenance, and it is an important element that is  
 186 been discussed in this paper.

### 187 3.3 BLOCK FLOW CHART

188 There are three blocks of schematic chart to show  
 189 what it is been developed, procedure in this section,  
 190 as shown in Fig.2. the details is as described in behind.



191  
192 **Fig 2. Block flow chart**

### 193 3.4A QUALITATIVE NORMAL DISTRIBUTION

194 This section includes: 1) Each gas range and column  
 195 location are been set, 2) Each gas related parameter  
 196 is been calculated by formula  
 197 and described the result value in, 3) Bell  
 198 curve area is been drawn with command description,  
 199 4) How to convert between standard score and  
 200 probability through formula command, and result  
 201 value explain, 5) the result value of the detection data  
 202 is explained.

203 3.4.1 The amount is sum from the range of the  
 204 normal, caution and abnormal, taking it from the  
 205 range of each gas in the ANSI/IEEE C57.104. Then  
 206 divided it into 1000 equal differences value, as the  
 207 maternal body of a qualitative normal distribution. As  
 208 for formulas of the column location explain, such as  
 209 H<sub>2</sub> (from normal stage 0 to abnormal stage maximum  
 210 value 1800) and the command of EXCEL application  
 211 software is shown in Table 1, such as H<sub>2</sub> (A5:A1005)

**Table 1. Each gas range and column location**

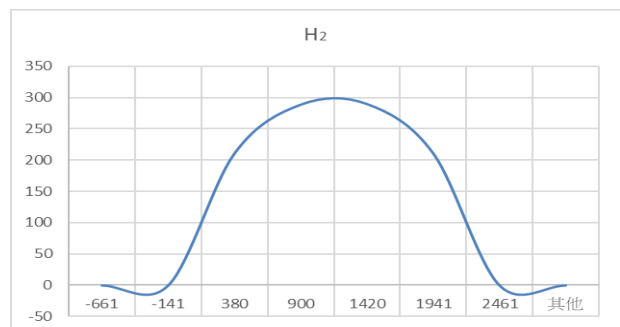
Gas	Range(ppm)	Column location
H <sub>2</sub>	0 ~ 1800	A5:A1005
CH <sub>4</sub>	0 ~ 1000	B5:B1005
C <sub>2</sub> H <sub>2</sub>	0 ~ 35	C5:C1005
C <sub>2</sub> H <sub>4</sub>	0 ~ 200	D5:D1005
C <sub>2</sub> H <sub>6</sub>	0 ~ 150	E5:E1005
CO	0 ~ 1400	F5:F1005

213 3.4.2 Taking H<sub>2</sub> gas column location for an example,  
 214 calculate the qualitative normal distribution of each  
 215 gas parameter - mean, variation, standard deviation  
 216 and other calculation formula symbol description  
 217 column location from A5 to A1005, those formulas  
 218 been shown =AVERAGE (A5:A1005) is 900,  
 219 =VARPA (A5:A1005) is 270540, and =STDEV  
 220 (A5:A1005) is 520, as shown in Table 2 below.

221 **Table 2. Results of various parameters of H<sub>2</sub> gas**

Parameters	Formula (EXCEL)	Result
Mean	=Avera(A5:A1005)	900
Variation	=Varpa(A5:A1005)	270540
Standard deviation	=Stdev(A5:A1005)	520

222 3.4.3 To draw a "bell curve area" after constructing  
 223 the qualitative normal distribution, taking H<sub>2</sub> gas for  
 224 example. First, using relevant parameters to calculate  
 225 the results of group boundaries and frequency  
 226 according to those parameters formulas in Table 3,  
 227 that is, use the mean (\$I\$5) and standard deviation  
 228 (\$I\$9) to plot a symmetrical bell shape, as shown in  
 229 Fig.3.



**Fig 3. Bell curve area of H<sub>2</sub> gas a qualitative normal distribution**

3.4.4 The bell curve area, the standard score and the probability value of the relationship between is

235 obtained from the calculation formula in Table 4.  
 236 The standard score is calculated from column  
 237 location (A5) subtracting the mean of the gas (\$I\$5)  
 238 and then dividing the standard deviation (\$I\$9) of  
 239 the gas. The formula is  $= (A5 - \$I\$5) / (\$I\$9)$ . Taking H2  
 240 for an example, column location (A5) is 0, the  
 241 calculation result is -1.729 (Z), as for the formula  
 242 been shown  $= \text{Norm.Dist}(A5, 900, 520, \text{true})$  and the  
 243 probability value is 0.04 (P). The rest can be  
 244 deduced by analogy. If the column location (A1005)  
 245 of the H2 is 1800, its value is 1.729 (Z), and the  
 246 probability value becomes 0.96 (P). Another  
 247 description of the formula symbols: \$I\$5 is the  
 248 rows of I column, which is 900, which is the mean of  
 249 H2 gas; \$I\$9 is the 9 rows of I column, which is 520  
 250 which is the standard deviation of the H2 gas.

600	Z	$= \text{Norm.Dist}(600, 900, 520, \text{true})$	-0.57
	P	$= (600 - 900) / 520$	0.28

The above description uses EXCEL application software to divide the range values of each gas in the normal, caution and abnormal stages of the ANSI/IEEE C57.104 into 1000 parts in an arithmetic series, as the maternal body of the qualitative normal distribution, and taking the range of H2 gas stage as an example - normal (0 to 100), attention (101 to 700), abnormal (701 to 1800) stages to sum one stage abnormal (0 to 1800), then find those parameters of H2 gas - mean, variation, standard deviation, and then convert it into standard scores and probability values serve as diagnostic benchmarks.

251 **Table 3. Bell curve area drawing and formula**  
 252 **instructions**

Formulas	Result	boundaries	frequency
$= \$I\$5 - 3 * \$I\$9$	-661	-661	0
$= G17 + \$I\$9$	-141	-141	0
$= G18 + \$I\$9$	380	380	211
$= G19 + \$I\$9$	900	900	289
$= G20 + \$I\$9$	1420	1420	290
$= G21 + \$I\$9$	1941	1941	211
$= G22 + \$I\$9$			0

253 **Table 4. Standard score and probability value**  
 254 **conversion of H2**

Name	Formula	Result
Z	$= (A5 - \$I\$5) / \$I\$9$	-1.729
	$= (A1005 - \$I\$5) / \$I\$9$	1.729
P	$= \text{Norm.Dist}(A5, 900, 520, \text{true})$	0.04
	$= \text{Norm.Dist}(A1005, 900, 520, \text{true})$	0.96

255 3.4.5 When the diagnostic comparison is between  
 256 the detected data and the benchmark of a qualitative  
 257 normal distribution. Taking H2 for an example, if the  
 258 gas number is 600ppm, the result of the related  
 259 parameters is as shown in Table 5. Its standard  
 260 score is -0.57 and the probability value is 0.28, that is  
 261 the abnormality degree of H2 gas is 0.28 in the  
 262 detection equipment.

263 **Table 5. Detection number of H2& diagnosis**  
 264 **results**

Number	Name	Formula	Result

#### 4. VERIFICATION

This paragraph respectively describes – First, how to take those gases from the device. Secondly, how to construct a qualitative distribution to compare with the gases of the detection, and the diagnostic result is verified from this method of a qualitative normal distribution benchmark and the repair status of record from maintenance department. Finally, the pros and cons are to compare between the ANSI/IEEE C57.104 and this method.

4.1 The oil quality detected of #4 autotransformer for Taiwan Electric Power Company's Nanke Extra High Voltage Substation was regularly tested on October 9, 2012, and the data of detection was taken and passed through the "Gas chromatography," so that can obtain dissolved gases. those steps are must with the operating standard procedures. The number of detection gases are as follows: H2 (194), CH4 (602), C2H2 (1.8), C2H4 (579), C2H6 (204) and CO (75) unit (ppm), the results of this method to calculate with those parameters of a qualitative normal distribution are shown in Table 6. Green fonts are detected data, Red fonts are the probability values of each gas, and Black fonts are the relevant parameter values of each gas in the qualitative normal distribution. Such as the abnormal probability of CH4 gas has reached 0.64, and the probabilities of

305 C<sub>2</sub>H<sub>4</sub> and C<sub>2</sub>H<sub>6</sub> have reached 1 (in dangerous stage)  
 306 The verification results of this case are consistent  
 307 with the repair status of the maintenance department  
 308 at that time (A screw inside the transformer is  
 309 severely corroded).

310 4.2 To compare ~~with~~ between this probability method  
 311 and the ANSIC/IEEE C57.104, as shown in Table 7,  
 312 the later presents the abnormality degree of each  
 313 gas in percentage, while the former indicates which  
 314 range in, such as normal, attention or abnormal or  
 315 even dangerous stage appears. This probability  
 316 method is distinguished by the percentage value  
 317 within a single abnormal stage. Taking H<sub>2</sub> gas  
 318 number as an example, the probability value 0.04  
 319 corresponding to gas number 0 ppm, while the  
 320 probability value 0.96 corresponding to gas number  
 321 1800 ppm, that is the range of the probability from  
 322 0.04 to 0.96. If the probability value is greater than  
 323 0.96, it means the gas number is in a dangerous  
 324 stage.

325 **Table 6. Test results report**

H <sub>2</sub> u	CH <sub>4</sub> u	C <sub>2</sub> H <sub>2</sub> u	C <sub>2</sub> H <sub>4</sub> u	C <sub>2</sub> H <sub>6</sub> u	CO u
900	500	17.50	100	75	700
H <sub>2</sub> σ <sup>2</sup>	CH <sub>4</sub> σ <sup>2</sup>	C <sub>2</sub> H <sub>2</sub> σ <sup>2</sup>	C <sub>2</sub> H <sub>4</sub> σ <sup>2</sup>	C <sub>2</sub> H <sub>6</sub> σ <sup>2</sup>	CO σ <sup>2</sup>
270540	83500	102.288	3340	1879	163660
H <sub>2</sub> σ	CH <sub>4</sub> σ	C <sub>2</sub> H <sub>2</sub> σ	C <sub>2</sub> H <sub>4</sub> σ	C <sub>2</sub> H <sub>6</sub> σ	CO σ
520	289	10.12	58	43	405
H <sub>2</sub>	CH <sub>4</sub>	C <sub>2</sub> H <sub>2</sub>	C <sub>2</sub> H <sub>4</sub>	C <sub>2</sub> H <sub>6</sub>	CO
0.09	0.64	0.06	1.00	1.00	0.06
Gases data of the detection (ppm)					
H <sub>2</sub>	CH <sub>4</sub>	C <sub>2</sub> H <sub>2</sub>	C <sub>2</sub> H <sub>4</sub>	C <sub>2</sub> H <sub>6</sub>	CO
194	602	1.8	579	204	75

327 **Table 7. Comparison between probabilistic**  
 328 **method and ANSIC/IEEE C57.104**

Gas name	H <sub>2</sub>	CH <sub>4</sub>	C <sub>2</sub> H <sub>2</sub>	C <sub>2</sub> H <sub>4</sub>	C <sub>2</sub> H <sub>6</sub>	CO
ANSIC57.104	A	AB	N	D	D	N
P(%)	9	64	6	100	100	6

A : attention \ AB: abnormal \ D: danger \ N: normal .

329 4.3 The single abnormal probability method adapts  
 330 percentage compare with the stage range of the

ANSI/IEEE C57.104. The former can clearly  
 distinguish the severity of the abnormality based on  
 the percentage, while the latter only knows which  
 stage area it falls in, which is vaguer. So, the former  
 is better than the latter.

## 5. DISCUSSION

During the development and testing period, those  
 results were discovered what the probability method  
 value of a qualitative normal distribution resembled  
 the result of arithmetic method, it is very similar by  
 being compare the results. Taking the actual gases  
 data as an example, the number of each gas is  
 respectively CH<sub>4</sub> 602 ppm, C<sub>2</sub>H<sub>4</sub> 579 ppm, and  
 C<sub>2</sub>H<sub>6</sub> 204 ppm, the result of the arithmetic method is  
 0.60, 2.89 and 1.36, and the probability method is  
 0.64, 1 and 1. This is sufficient reason to prove that it  
 is ~~feasible~~ **feasibility** and ~~accurate~~ **accuracy** in this  
 probability method. In addition, the range of the  
 maternal body of each gas is not ~~be~~ the same range  
 (for an example, each range of gas takes from  
 ANSI/IEEE C57.104, H<sub>2</sub> is from 0 to 1800, CH<sub>4</sub> is  
 from 0 to 1000), but the standard score and  
 probability value are consistent according to rule of  
 the area of the bell curve to convert. ~~and~~ the number  
 of the group of maternal body (n) must ~~be~~ 1000 with  
 equal difference value and not equal value of ~~any~~  
 random.

## 6. CONCLUSION

Based on the above theory, use EXCEL application  
 software to compile available detection tools. First,  
 taking those number of all from the three stages of  
 normal, caution and abnormal in the  
 ANSIC/IEEE C57.104 to convert a stage of attention  
 which is been divided into 1000 by equal difference  
 level as the maternal body of a qualitative normal  
 distribution. Those related parameters of a  
 qualitative normal distribution converted into  
 standard scores and probability values as the  
 diagnostic benchmark, and then combined with case  
 gases to verify its feasibility and accuracy. The  
 comparison results prove that this method is superior  
 to the traditional judgment method. This method is  
 carefully developed under tried and tested a novel

374 probability value diagnosis method. It has been  
 375 verified and is worthy of being introduced to  
 376 colleagues in the field of power engineering as a  
 377 reference, also hopes that advanced professionals  
 378 and scholars will not hesitate to criticize and give  
 379 advice.

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