TheSakilianradius-to-massratiopostulatehasbeenusedtodeterminethemassorradiusofaplanetorbitingthesun,aplanetarysatellite,andagivenstarintheMilkyWay.Sakhalinradius-to-mass ratio postulate applied to the calculation of the mass or the radius of a sun's planet, of a planetary satellite and of a given star belonging to the Milky Way

Comment [Bs1]: If the author accepts, it twill be better if he decreases the number of words.

Abstract: In this work, it is demonstrated that the ratio of the radius of a satellite to that of its center of rotation, is equal to the ratio of the mass of the satellite to that of its center of rotation raised to α power.This new radius-to-mass ratio relationship postulated,is to as thereferred as Sakholian radius-to-mass ratio (SRMR) law. For a given satellite,The SRMR-law indicates clearly that the sSolar system contains three categories of planets: terrestrial planets (Mercury, Venus, Earth,and Mars:$\alpha \approx$ 0.40), dwarfplanets (Ceres, Pluto, Haumea, Makemake, and Eris; α = 0.34), and giant planets (Jupiter, Saturn, Uranus, and Neptune :α = 0.33).Thevalue of αequal to 0.34 is a theoretical argument in favor of the status of dwarf planet attributed to Pluto since the very controversial Prague 2006 IAU vote.In addition,SRMR-law is applied in the calculations of the massand the density (volumic mass) of 64small regular planetary moons: 24 for Jupiter (α = 0.331), 12 for Saturn (α = 0.330),22 for Uranus (α = 0.334), and 6 for Neptune (α = 0.326).For all these 64 satellites,it is seen that $\alpha \approx$ 0.33. Excellentagreements are obtained with the literature masses of small regular satellites calculated assuming a constant density and using a given radius. Besides,it is demonstrated that the SRMR-lawcan be applied to the calculation of the mass or the radius of a given star belonging to the Milky Way.For particular cases of fourth stars,calculations of the β-parameter give β = 0.662for both Alpha Centauri B and Rigel andβ = 0.390 for both Alpha Centauri A and Capella A. These primary results indicate the possibility to use the SRMR-law to estimate the mass or the radius of a given star inofthe Milky Way, which containscontaining between 200 andto 400 billionbillions stars.For all the the bodies in the Solar System Solar system bodies (satellites and planets), the radius-to-mass ratio condition is 0.3<α< 0.4. Out of this range, the mass or the radius determined must be revised. Then,α may be very useful parameter for modeling the size (diameter or mass) of a given celestial satellite.

Keyword.radiusRadius-to-mass ratio relationship, SRMR-law, satellite, Solar System, terrestrial planets, dwarf planets, giant planets, planetary moons, star, Milky Way.

1. Introduction

There are currently six recognized types of solar system bodies. These classifications are asteroids, comets, planets, dwarf planets, planetary moons, and stars. Before the IAU General Assembly in Prague in 2006, the planets in the solar system were divided into three categories based on their size and composition. Mercury, Venus, Earth, Mars, and Pluto are classified as terrestrial planets, with Jupiter and Saturn being gas giants, and Uranus and Neptune being the planets. giant ice crystals. But according to the 2006 IAU resolution, the solar system is made up of only eight planets:

Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. Dwarf planets are also classified as an entirely new, unique class of celestial bodies [1-5].For the Solar system,six categories of astronomical bodies are currently accepted. These categories are star, planet, dwarf planet,planetary moon (or satellite), comet, and asteroid.Based on their size and composition,the planets of the Solar system were grouped into three categories before the Prague 2006 IAU General Assembly.These categories were terrestrial planets (Mercury, Venus, Earth, Mars and Pluto),gas giants (Jupiter and Saturn) and ice giants (Uranus and Neptune). But,the 2006 IAU Resolution officially stated that the Solar system consists only of eight planets which are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. In addition,dwarf planets were also defined asa new distinct class of celestial objects [1-5].

The IAU decided that "planets" and other celestial bodies in the solar system, with the exception of satellites, should be divided into three groups:

(1) Planets:

“Planet” is a celestial body that meets the following criteria:

(a) it is in orbit around the Sun;

(b) it has enough mass to overcome the rigid forces of the body and is in hydrostatic equilibrium (almost circular); and

(c) it has wiped out the area in and around its orbit.

(2) Dwarf planet:

A “dwarf planet” is an object that meets the following criteria:

it is in orbit around the sun, it has enough mass for its own gravity to overcome the rigid forces of the object and it has an almost circular hydrostatic equilibrium, it has not cleared the area around the orbit its and this is not a satellite. .

(3) Small Solar System objects is the global term for all other objects, except satellites, that orbit the Sun. On the other hand, all the planets in the solar system have satellites, except Mercury and Venus:

Earth has one moon, Mars has two, Jupiter has 95, Saturn has 83, Uranus has 27, Neptune has 14 and Pluto has 5. Mass of an object in the solar system, which is known to be a more difficult quantity to measure in astronomical contexts, is one of its important properties [6].

~~The IAU resolves that ‘planets’ and other bodies in the Solar system, exceptsatellites, be defined into three distinct categories:(1) *Planet*: a ‘planet’ is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces sothat it assumes a hydrostatic equilibrium (nearly round) shape, and (c) has cleared the neighborhood around its orbit,(2) *Dwarf planet*: a ‘dwarf planet’ is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape, (c) has not cleared the neighborhood around its orbit, and (d) is not a satellite,(3) *Small Solar system bodies*: all other objects except satellites orbiting theSun shall be referred to collectively as Small Solar System Bodies.On the other hand, except Mercury and Venus, the planets of the Solar system have satellites, one moon for Earth; two for Mars; 95 for Jupiter; 83 for Saturn; 27 for Uranus, 14 for Neptune, and 5 for dwarf planet Pluto. One of the important characteristics of a Solar system body is its mass which is known to be more awkward quantity to measure in an astronomical context [6].~~

Many studies have been conducted to determine the physical characteristics of objects in the solar system, including the physical characteristics of dwarf planets [7-10] and planetary moons such as Jupiter [11]. -12], Saturn [13-18], Uranus [19]. -22] and Neptune [23-26], among others. In general, for satellites, the unmeasured mass is calculated using the assumptions of constant spherical volume and density. In this essay, we intend to give the theoretical justification for treating Pluto as a dwarf planet since the controversial IAU vote in Prague in 2006 and propose a method Simple to calculate mass and radius of a planetary satellite. Determining the mass-radius relationship, or the relationship between the radius R of a main sequence star or exoplanet (extrasolar planet) and its mass, M, has been key. subject of many studies. [27-33] (to name a few). The widely used Equation of State (EOS), used in many studies, is used to determine the relationship between R and M.

~~Determination of physical characteristics of Solar system bodies were the subjects of many researches as well as for dwarf planets [7-10] than for planetary moons such as those of Jupiter [11, 12], Saturn [13-18], Uranus [19-22], and of Neptune [23-26], to name a few references.In general,satellites, the masses not measured are estimated by assuming a spherical volume and a constant density.In this paper, we aim to give a theoretical argument in favor of the status of dwarf planet attributed to Pluto since the very controversial Prague 2006 IAU vote and to present a simple way for the calculation of the mass and the radius of a planetary moon. In the past, many investigations [27-33](to name a few) have been devoted to the determination of mass-radius relation which is a relationship between the radius, *R*, of a main-sequence star or an exoplanet (extrasolar planet) and its mass, *M*. For many studies, the determination of the relationship between *R* and *M* is derived from the widely used equation of state (EOS).~~

The aim of this study was not to develop an association between R and M that could be assessed based on previous studies on the topic. The goal of the current work is quite original. The ratio between the mass of the orbiting body and the radius of the central body shall be determined. Sakholic's law of radius-to-mass ratio (SRMR), which describes the relationship between the radius-to-mass ratio of a circular object and a central object with greater mass, is considered to be the such a first relationship. Then there is previous work in this area for comparison purposes. In addition, we aim to establish radius-to-mass ratio conditions for every object in the solar system, including planets and satellites, as a standard by which to judge the accuracy of measurements or an estimate of the mass or radius of an object. The size (diameter or mass) of a particular astronomical satellite can be modeled very efficiently using this condition. The present study builds on our previous studies [34, 35] in the same field. This is how the paper is configured:

The theoretical part of the study is presented in section 2. The results obtained will be presented in section 3 along with their presentation. In part 4, we're done.

~~The present study is not focused on the establishment of a relation between *R* and *M*to be compared to previous works on the matter. The goal of the present work is absolutely novel. The relationship to establish is a radius ratio and mass ratio of an orbiting body to a central body. This relationship referred as Sakholian radius-to-mass ratio (SRMR) law is believed to be the first relation between radius ratio and mass ratio of an orbiting body to a central more massive body. Subsequently, there are any previous works in the matter for comparison. In addition, we aim to present for all the Solar system bodies (satellites and planets), a radius-to-mass ratio condition as a criteria for appreciating accuracy on the measurements of on the calculations of the mass or the radius of a given determined must be revised. This condition may be very useful for modeling the size (diameter or mass) of a given celestial satellite.The present research follows our previous works [34, 35] in the same area .The paper is organized as follows. Section 2 presents the theoretical part of the work. The results obtained are presented in section3 along with their discussion. We conclude in section 4.~~

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2. Theory

2.1. Hydrostatic equilibrium model

~~Hydrostatic equilibrium is originally the final state of an ideal liquid apart of extern forces. It results in a perfectly spherical shape. The shape can be altered by rotation, inhomogeneity and/or the gravitational attraction of other bodies[4].The equilibrium shape of a rotating and orbiting fluid body is generally that of a triaxialellipsoid. That behavior is often expected also for solid bodies as planets, asteroids and satellites, provided they have mass enough to overcome rigid body forces [10, 36, 37]. For most of the satellite considered in this work, the shape is not perfectly spherical. For example, Adrastea, satellite of Jupiter, has an irregular shape and measures 20×16×14 km across [19].~~
A perfect fluid, unaffected by external forces, is in its final state when hydrostatic equilibrium is reached. This creates a completely spherical shape. The rotation, inhomogeneity, and/or gravity of other bodies can all change shape [4]. The equilibrium shape of a rotating and orbiting liquid body is usually a three-axis ellipse. For solid bodies such as planets, asteroids, and satellites, as long as they have enough mass to resist the force of the solid, this behavior is usually predictable [10, 36, 37]. The shape is not quite spherical for most of the satellites accounted for in this study. For example, Jupiter Adrastea is irregular in shape and has a diameter of 20×16×14 km [19].

Let us then consider in this work the bodies of the solar system (the sun, planets, and satellites) in hydrostatic equilibrium like fluid bodies. The shape of each of these bodies is then considered perfectly spherical. Let us also consider a rotating body of mass M_S and radius R_S orbiting a more massive body of mass M_C and radius R_C (fig.1).~~Let us then consider in this work,Solar system bodies (Sun,planets and satellites)in hydrostatic equilibrium like fluid bodies.The shape of each of these bodies is then considered as perfectly spherical.Let us besides consider a rotating body of mass M_S and of radius R_S orbiting a more massive body of mass M_C and of radius R_C (fig.1).~~

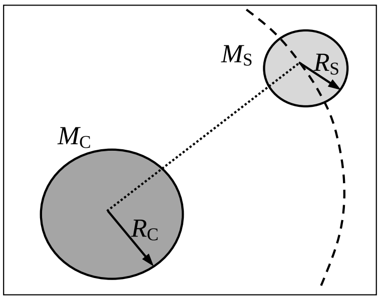

Fig.1.Satellite of mass M_S and of radius R_S orbiting a more massive body of mass M_C and of radius R_C. The two bodies are considered to be in hydrostatic equilibrium conferring them spherical shapes.

According to the Kepler's Third Law, the squares of the orbital periods T of the planets are directly proportional to the cubes of the semi-major axes r of their orbits. From this law, we deduce the mass M of a planet or star:star :

$$M = \frac{4\pi^2 r^3}{GT^2}. \quad (1)$$

Formula (1) permits calculating the mass, M, given the orbital period, T, and orbital radius, r, of an object that is moving along a circular orbit around But this formula cannot be used to calculate the mass of a planetary satellite. For some moons, the size (mass and diameter) is often measured. For example, Doppler data from the Galileo spacecraft's encounter with Amalthea, one of Jupiter's small inner moons, on November 5, 2002, yielded a mass of (2.08 ± 0.15) × 10^{18} kilograms. Images of Amalthea from two Voyager spacecraft in 1979 and Galileo imaging between November 1996 and June 1997 yield a volume of (2.43 ± 0.22) × 10^6 cubic kilometers [38]. But for most of the planetary moons, masses are generally calculated assuming a constant density and using a given radius. In addition, with an assumed constant density, the radius of a moon can be calculated using its measured mass. This is the particular case of Adrastea (the moon of Jupiter). Assuming that its mean density is like that of Amalthea, around 0.86 g/cm^3 [38, 39], its mass can be estimated at about 2 × 10^{15} kg by assuming a spherical volume with a diameter of 16.4 km [40].

Formula (1) permits to calculate the mass, M given the orbital period, T, and orbital radius, r, of an object that is moving along a circular orbit around it. But, this formula cannot be used to calculate the mass of a planetary satellite. For some moons, the size (mass and diameter) are often measured. For example Radio Doppler data from the Galileo spacecraft's encounter with Amalthea, one of Jupiter's small inner moons, on 5 November 2002 yield a mass of (2.08 ± 0.15) × 10^{18} kilograms. Images of Amalthea from two Voyager spacecraft in 1979 and Galileo imaging between November 1996 and June 1997 yield a volume of (2.43 ± 0.22) × 10^6 cubic kilometers [38]. But most of the planetary moons, masses are generally calculated assuming a constant density and using given radius. In addition, with an assumed constant density, the radius of a moon can be calculated using its measured mass. This is the particular case of Adrastea (moon of Jupiter). Assuming that its mean density is like that of Amalthea around 0.86 g/cm^3[38, 39] its mass can be estimated at about 2 × 10^{15} kg by assuming a spherical volume with diameter of 16.4 km [40].

Let us then move of establishing a new procedure for the calculation of the mass or the radius of a planetary satellite.

2.2. Radius-to-mass ratio relationship

As well known, the gravitational force is due to the interaction between masses.In the case of the present model describe in figure 1, these masses are concentered in a spherical shape characterized by its radius. Intuitively,they may be a relationship between the ratios M_S/R_SandM_C/R_C. As $R_S/R_C< 1$ and $M_S/M_C< 1$, we postulate that for a given system {satellite (R_S,M_S) – central body orbited (R_C,M_C)}, it is satisfied the relationship

$$\frac{R_S}{R_C} = \left(\frac{M_S}{M_C}\right)^{\alpha}. \quad (2)$$

Equation (2) corresponds to the Sakholian radius-to-mass ratio (SRMR) Postulate. From the SRMR law (2), we pull the α-parameter

$$\alpha = \frac{\ln\left(R_S / R_C\right)}{\ln\left(M_S / M_C\right)}. \quad (3)$$

If α is known,the SRMR postulate (2) permits to calculate the mass M_S of given satellite if its radius is known and vice versa. For a center of rotation (Sun of planet) in the Solar System,M_C and R_C are known. Fora planet of mass M_S and of radius R_S,M_C and R_C denote the mass and the radius of the Sun. For a planetary moon of mass M_S and of radius R_S,M_C and R_C are the mass and the radius of the central body (here planet) about which the satellite orbits.

As a postulate, the law (2) is true because of the consequences that one draws from it. That's what we'll check through the discussion section.

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3. Results and discussion

3.1. Categorization of the planets of the Solar system, radius-to-mass ratio condition

Due to its size, Pluto is not a giant planet. So, its classification must be between the categories of terrestrial planets and dwarf planets. During the Prague 2006 IAU General Assembly, the vote came after eight days of contentious (antagonistic) debate that involved four separate proposals at the group's meeting in Prague. Only 424 astronomers were allowed to vote out of some 10.000 professional astronomers around the globe [2]. The status of Pluto as a terrestrial or dwarf planet must be concluded via scientific argument and not by vote. Using (3), we can close the debate regarding this status. For this purpose, we classify Pluto into the category of dwarf planet, as stated during the Prague 2006 IAU meeting. Let us then calculate (3) for the bodies of the solar system (satellites, dwarf planets, and regular planets). The results obtained are quoted in Tables 1–3.

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Table 1: Values of the α-parameter for the terrestrial planets of the Solar System.

Planet	Radius R_p (km)	Mass M_p ($\times 10^{24}$ kg)	α
Mercury[a]	2 439.7	0.3301	0.3621 ≈ 0.40
Venus[a]	6 051.8	4.8675	0.3672 ≈ 0.40
Earth[a]	6 378.1	5.9724	0.3690 ≈ 0.40
Mars[a]	3 396.2	0.6417	0.3561 ≈ 0.40
Sun[b]	695 700	1 988 500	-

[a][41], [b] https://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html, 2022.

Table 2: Values of the α-parameter for the dwarf planets of the Solar System.

Dwarf planets	Radius R_p (km)	Mass M_p ($\times 10^{24}$ kg)	α
Pluton[a]	1 151.253	0.013 143	0.3400 ≈ 0.34
Ceres[b]	476.740	0.000 950	0.3395 ≈ 0.34
Haumea[c]	780	0.004 000	0.3393 ≈ 0.34
Eris[d]	1 163	0.016 608	0.3437 ≈ 0.34
Makemake[e]	715	0.003 106	0.3393 ≈ 0.34
Sun	695 700	1 988 500	-

[a]https://www.princeton.edu/~willman/planetary_systems/Sol/Pluto/.2023
[b]https://fr.wikipedia.org/wiki/(1)_C%C3%A9r%C3%A8s#cite_note-Pitjeva2005-4.
2023, [c] [8], [d] [9,42], [e][7,43]

Table 3: Values of the α-parameter for the giant planets of the Solar System.

Planet[a]	Radius R_p (km)	Mass M_p ($\times 10^{24}$ kg)	α
Jupiter	71 492	1 898.19	0.3270 ≈ 0.33
Saturn	60 268	568.34	0.2998 ≈ 0.30
Uranus	25 559	86.813	0.3291 ≈ 0.33
Neptune	24 764	102.413	0.3379 ≈ 0.34
Sun	695 700.000	1 988 500	-

[a][41]

As stated above. Pluto is not enough massive to be classified in the category of giant planets. It is either a dwarf planet either a terrestrial planet. The results listed in the last column of each Table show clearly that Pluto is indeed a dwarf planet for which $\alpha \approx$ 0.34 (Table 2) compared with the corresponding value at 0.40 for all the terrestrial planets (Table 1). For the giant planets, we find $\alpha \approx$ 0.33 for Jupiter and Uranus and $\alpha \approx$ 0.30 and $\alpha \approx$ 0.34 respectively for Saturn and Neptune respectively. These discrepancies are not due to character gas giants (Jupiter $\alpha \approx$ 0.33 and Saturn $\alpha \approx$ 0.30) or ice giants (Uranus $\alpha \approx$ 0.33 and Neptune $\alpha \approx$ 0.33). This indicates that, the accurate value of α for all the giant planets may be $\alpha \approx$ 0.33.An accurate value of Neptune may be equal to 0.334 ≈ 0.33 instead of 0.339. The anomalous value for Saturn may indicate that, the hydrostatic equilibrium model adopted is not appropriated for Saturn. Over all, we can state to the radius-to-mass ratio condition

$$0.3 < \alpha < 0.4 \quad (4)$$

Out of this range. the mass or the radius determined (measured of calculated) must be revised. The important result (4) is a tangible proof of the validity of postulate (2).

Let us given an importance consequence of condition (4) to confirm the validity of postulate (2).

For Amalthea the measured mass is M_S = (2.08 ± 0.15) × 10^{18} kg along with a measured volume V= (2.43 ± 0.22) × 10^6 cubic km[38].Comparison with available experimental and/or theoretical results can enlighten accuracy in the measurements for Amalthea. But without any comparison (andparticularly if no literature data are available), it is possible via the present work to enlighten accuracy in the measurements as far as Amalthea is concerned. If the α-parameter for Amalthea is out of the range (3), the measurements are note precise (mass and/or volume). To verify this assertion, let us deduce the radius of Amalthea from its volume since $V = (4/3)\pi R^3$. So R_S = 83.40km.knowing that Amalthea orbits Jupiter, we get from Table 3 $M_C = M_J$ = 1 898.19 × 10^{24} kg and $R_C = R_J$ = 71 492 km. Using (3), we find

$$\alpha = \frac{\ln(83{,}4/71492)}{\ln(2{,}08/1898190000)} = 0.3273.$$

The above result is well in the range (4). So the measured mass and volume of Amalthea can be considered as accurate. This is the main important consequence of condition (4) and subsequently the validity of postulate (2).

In addition, let us demonstrate that the SRMR ~~law~~Law (2) can lead to the mass-radius relationship between the radius, R, of a main-sequence star, and its mass, M.For this purpose, we consider the particular case of the very rough version for main-sequence stars relates the radius to an exponent of the mass (http://astro.vaporia.com/start/massradius.html) (in units of R_{sun} and M_{sun}):

$$R = M^{0.8}. \qquad (5.a)$$

Let us then rewrite postulate (2) in solar units (R_C = 1 and M_C = 1). Let us also put $R_S = R$ and $M_S = M$. We get from postulate (2)

$$R = M^{\alpha} \qquad (5.b)$$

Equation (5.b) is equal to equation (5.a) if α = 0.8. Although the similitude is excellent, it should be reminded that postulate (2) is defined for the radius ratio and mass ratio of an orbiting body to a central body and not for the mass-radius relationship of amain-sequence star. Finally, the SRMR postulate (2) can be applied to an exoplanet and its central star. Comparison of equations (5.a) and (5.b) indicates clearly that postulate (2) lead to the mass-radius relationship between the radius, R, of a main-sequence star. This is a striking proof of the validity of theSakholian radius-to-mass ratio postulate.

3.2. The α-parameter for various moons of Jupiter.Saturn. Uranus and Neptune

Let us calculate α-parameter for various moons of Jupiter, Saturn, Uranus and Neptune using (3). The results obtained are listed in Table 4.

Table 4: α-parameter for various moons of Jupiter, Saturn, Uranus and Neptune.

Satellite[a]	Radius R_S (km)	Mass M_S ($\times 10^{15}$ kg)	α
Deimos	6.200	1.4762	0.3170 ≈ 0.32
Phobos	11.200	10.659	0.3190 ≈ 0.32
Mars	3 396.2	641 700 000	
Satellite[b]	Radius R_S (km)	Mass M_S ($\times 10^{20}$ kg)	
Io	1 821.5	897.0	0.3685 ≈ 0.37
Europa	1 561.0	478.4	0.3612 ≈ 0.36
Ganymede	2 631.0	1 495	0.3495 ≈ 0.35
Callisto	2 410.5	1 076.4	0.3469 ≈ 0.35
Jupiter	71 492	18 981 900	
Satellite[c]	Radius R_S (km)	Mass M_S ($\times 10^{21}$ kg)	α
Miranda	236	0.0659	0.3325 ≈ 0.33
Ariel	579	1.353	0.3419 ≈ 0.34
Umbriel	586	1.172	0.3367 ≈ 0.34
Titania	790	3.527	0.3438 ≈ 0.34
Oberon	762	3.014	0.3420 ≈ 0.34
Uranus	25 559	86 813	
Satellite[d]	Radius R_S (km)	Mass M_S ($\times 10^{15}$ kg)	α
Mimas	198.2	37493	0.3458 ≈ 0.34
Enceladus	252.1	108022	0.3539 ≈ 0.35
Tethys	531.1	617449	0.3445 ≈ 0.34
Dione	561.4	1095452	0.3553 ≈ 0.35
Rhea	763.8	2306518	0.3519 ≈ 0.35
Lapetus	734.3	1805635	0.3482 ≈ 0.35
Phoebe	106.5	8292	0.3513 ≈ 0.35
Saturne	60 268	568 340 000 000	
Satellite[e]	Radius R_S (km)	Mass M_S ($\times 10^{16}$ kg)	α
Naiad	29	≈ 13	0.3295 ≈ 0.33
Thalassa	40	≈ 35	0.3297 ≈ 0.33
Despina	74	≈ 170	0.3245 ≈ 0.32
Galatea	79	≈ 280	0.3300 = 0.33
Larissa	96	≈ 380	0.3245 ≈ 0.32
Hippocamp	17.4	≈ 2.2	0.3555 ≈ 0.35
Proteus	208	≈ 3900	0.3233≈ 0.32
Triton	1353	2139000	0.3406 ≈ 0.34
Neptune	24 764	10 241 300 000	

[a]https://starwalk.space/en/news/mars-moons-phobos-deimos.2023
[b] [11], [c][13], [d][14,43].
[e]https:// sites.google.com/carnegiescience.edu/sheppard/moons/neptunemoons
The masses for Neptune moons are taken from[24].

For all the satellites considered, the data listed in table 3 indicate clearly that the radius-to-mass ratio condition (4)is rigorously satisfied. It should be underlined that, the value of α approximately equal to 0.34 for Ariel,Umbriel,Titania and Oberon doesn't means that these moons are in the category of dwarf planets as indicated in Table 2. These moons are orbiting the Uranus planet while the dwarf planets are orbiting the Sun.

3.3. Calculation of the mass of various moons of Jupiter. Saturn. Uranus and Neptune

Using (3), we calculate the mass of various moons of Jupiter, Saturn, Uranus and Neptune. For each planet,an average value of $\alpha$is used and calculated for some satellites. We obtain then $\alpha \approx 0.3310$ for the moons of Jupiter,$\alpha \approx 0.3342$ for the moons of Uranus,$\alpha \approx 0.330$ for the moons of Saturn and $\alpha \approx 0.326$ for the moons of Neptune. The masses of the moons are calculated using the following formulas

- For the moons of Jupiter

$$M_S \approx M_J \left(\frac{R_S}{R_J}\right)^{\frac{1}{\alpha}} = 0.0004101184\,70063 \times R_S^{\frac{1}{0.3310}} \times 10^{16}\ \text{kg}. \qquad (6)$$

- For the moons of Uranus

$$M_S \approx M_U \left(\frac{R_S}{R_U}\right)^{\frac{1}{\alpha}} = 0.0005626559\,02780 \times R_S^{\frac{1}{0.3342}} \times 10^{16}\ \text{kg}. \qquad (7)$$

- For the moons of Saturn

$$M_S \approx M_{Sat} \left(\frac{R_S}{R_{Sat}}\right)^{\frac{1}{\alpha}} = 0.0018599307\,28562 \times R_S^{\frac{1}{0.330}} \times 10^{15}\ \text{kg}. \qquad (8)$$

- For the moons of Neptune

$$M_S \approx M_N \left(\frac{R_S}{R_N}\right)^{\frac{1}{\alpha}} = 0.0003407039\,65463 \times R_S^{\frac{1}{0.3260}} \times 10^{16}\ \text{kg}. \qquad (9)$$

The results obtained using the above equations are presented in Tables 5-8.

Table 5: Mass, radius and density of some moons of Jupiter.

Satellite	Radius R_S (km)[a]	Mass M_S ($\times 10^{16}$ kg)[b]	Mass M_S ($\times 10^{16}$ kg)[c]	density ρ (g/cm^3)[c]
Metis	21.5	≈ 3.6	4.35	1.03
Adrastea	8.2	≈ 0.2	0.24	1.04
Amalthea	83.5	208	262.18	1.08
Thebe	49.3	≈ 43	54.02	1.08
Themisto	4.5	≈ 0.07	0.038	1.00
Leda	10.75	≈ 0.52	0.53	1.02

Ersa	1.5	≈0.0014	0.0013	0.92
Pandia	1.5	≈0.0014	0.00014	0.10
Lysithea	21.1	≈ 3.6	4.11	1.04
Elara	39.95	≈ 27	28.27	1.06
Dia	2	≈ 0.0034	0.0033	0.98
Carpo	1.5	≈0.0014	0.0014	0.99
Valetudo	0.5	≈ 0.000052	0.000050	0.95
Euporie	1.0	≈ 0.00042	0.00041	0.98
Mneme	1.0	≈ 0.00042	0.00041	0.98
Euanthe	1.5	≈0.0014	0.0014	0.99
Praxidike	3.5	≈ 0.018	0.0180	1.00
Ananke	14.55	≈ 1.3	1.34	1.04
Iocaste	2.5	≈ 0.0065	0.0065	0.99
Came	23.35	≈ 5.3	5.58	1.05
Kalike	3.45	≈ 0.017	0.0173	1.01
Pasiphae	28.9	≈ 10	10.63	1.05
Sinope	17.5	≈ 2.2	2.33	1.04
Callirrhoe	4.8	≈ 0.046	0.047	1.01
Jupiter: M_J = 189 819 000 000 $\cdot 10^{20}$ kg			R_J = 71 492 km	

[a] [40]
[a.b] https://en.wikipedia.org/wiki/Moons_of_Jupiter.2023
[c] present calculations

Note: the only satellite with measured mass is Amalthea. The masses of the inner satellites are estimated by assuming a density similar to Amalthea's (0.86 g/cm^3)[38], while the rest of the irregular satellites are estimated by assuming a spherical volume and a density of 1 g/cm^3(https://en.wikipedia.org/wiki/Moons_of_Jupiter.2023).

Table 6: Mass, radius and density of some moons of Uranus.

Satellite	Radius R_S (km)	Mass M_S ($\times 10^{16}$ kg)[a]	Mass M_S ($\times 10^{16}$ kg)[c]	density ρ (g/cm^3)[c]
Cordelia	20.3	≈ 4.4	4.59	1.31
Ophelia	21.5	≈ 5.3	5.46	1.31
Bianca	25.5	≈ 9.2	9.10	1.31
Cressida	40	≈ 34	34.99	1.31
Desdemona	32	≈ 18	17.95	1.31
Juliet	47	≈ 56	56.69	1.30
Portia	67.5	≈ 170	167.46	1.30
Rosalind	36	≈ 25	25.53	1.31
Cupid	9	≈ 0.38	0.40	1.31
Belinda	45	≈ 49	49.77	1.30
Perdita	15	≈ 1.8	1.86	1.32
Puck	81	≈ 290	288.97	1.30
Mab	12.5	≈ 1.0	1.08	1.32
Francisco	11	≈ 0.72	0.73	1.31
Caliban	36	≈ 25[b]	25.53	1.31

Stephano	16	≈ 2.2	2.25	1.31
Trinculo	9	≈ 0.39	0.40	1.31
Sycorax	75	≈ 230[b]	229.53	1.30
Margaret	10	≈ 0.54	0.55	1.31
Prospero	25	≈ 8.5	8.57	1.31
Setebos	24	≈ 7.5	7.59	1.31
Ferdinand	10	≈ 0.54	0.55	1.31
Uranus: $M_U = 86813 \cdot 10^{21}$ kg		$R_U = 25\,559$ km		

[a]*https://en.wikipedia.org/wiki/Moons_of_Uranus.2023.*Masses of all moons were calculated assuming a density of 1.3 g/cm^3 and using given radii.
[b]https://web.archive.org/web/20100105183741/http://nssdc.gsfc.nasa.gov/planetary/factsheet/uraniansatfact.html.2007
Radii are taken from https://nssdc.gsfc.nasa.gov/planetary/factsheet/uraniansatfact.html

Table 7: Mass, radius and density of some moons of Saturn.

Satellite	Radius* (km)[a]	Mass ($\times 10^{15}$ kg)[a]	Mass ($\times 10^{15}$ kg)[b]	Density ρ (g/cm^3)[b]
Pan	14.1	5	5.65	0.48
Atlas	15.1	6.6	6.95	0.48
Prometheus	43.1	159.5	166.90	0.50
Pandora	40.7	137.1	140.30	0.50
Epimetheus	58.1	526.6	412.56	0.50
Janus	89.5	1897.5	1527.96	0.51
Methone	1.45	≈ 0.0063	0.0057	0.45
Aegaeon	0.33	≈ 0.000073	0.000065	0.43
Pallene	2.2	≈ 0.023	0.020	0.45
Telesto	12.4	≈ 4.0	3.83	0.48
Calypso	10.7	≈ 2.5	2.45	0.48
Polydeuces	1.3	≈ 0.0038	0.0041	0.45
Saturn : R_S = 60 268 km ; M_S = 568 340 000 000.10^{15} kg				

[a]https://en.wikipedia.org/wiki/Moons_of_Saturn.2023
Masses of small regular satellites were calculated assuming a density of 0.5 g/cm^3.
[a][43,44], [b]Present calculations.

Table 8: Mass, radius and density of some moons of Neptune.

Satellite	Radius (km)[a]	Mass ($\times 10^{16}$ kg)[b]	Mass ($\times 10^{16}$ kg)[c]	density ρ (g/cm^3)[c]
Nereid	172	≈ 2400	2453.73	1.15
Nereid	185	≈ 2400	3068.26	1.16
Halimede	≈ 30.5	≈ 12	12.17	1.02
Sao	≈ 20	≈ 3.4	3.34	1.00
Laomedeia	≈ 20	≈ 3.4	3.34	1.00
Psamathe	≈ 19	≈ 2.9	2.85	0.99
Neso	≈ 30	≈ 11	11.57	1.02

Neptune : R_N = 24 764 km M_N = 10 241 300 000 $\times 10^{16}$ kg

[a] https://sites.google.com/carnegiescience.edu/sheppard/moons/neptunemoonsb
[b] https://en.wikipedia.org/wiki/Moons_of_Neptune#cite_note-Karkoschka2003-22**.2023**
[c] Present calculations
The masses of all irregular moons of Neptune were calculated assuming a density of 1 g/cm^3.
https://en.wikipedia.org/wiki/Moons_of_Neptune.2023

The results quoted in Tables 5-8 indicate good agreement between the present calculates masses and the literature data.It should be underlined that, for Nereid, the diameter d = (357± 13) km gives two values, lower radius(357- 13)/2 = 172 kmand upper radius(357+ 13)/2 = 185 km, as shown in Table 7.The present mass at 2453.73 $\times 10^{16}$ kgis seen to agree best with the mass estimated with the lower radius at 172 km.

Besides,the masses of the irregular satellites of Jupiter are estimated assuming a density of 1.0 g/cm^3 (Table 5). The masses of all moons of Uranuswere calculated assuming a density of 1.3 g/cm^3(Table 6) and that of the small regular satellites of Saturn were calculated assuming a density of 0.5 g/cm^3(Table 7).Finally, the masses of all irregular moons of Neptune were calculated assuming a density of 1 g/cm^3(Table 8). Comparison indicate very good agreement between the present calculated density and what is assumed for all the moons considered in Table 5-8.These agreements are compatible with the spherical shape adopted for the satellites studies in this work.

3.4. Radius-to-mass ratio relationship for Alpha Centauri B, Rigel,Alpha Centauri A and Capella A stars

For the bodies of the Solar system, the above study has indicated that for all the satellites (planets or planetary moons),the radius-to-mass ratio condition$0.3<\alpha< 0.4$ is thoroughly satisfied. In this section, we aim to establish a similar condition in the case of theMilky Way possesses between 200 to 400 billion stars.

Let us note by M_{GN} the mass of the galactic nucleus of the Galaxy (Milky Way). A star at the distance d of the center of the Milky Way orbits the galactic center with speed V given by the well-known relationship

$$V = \sqrt{\frac{GM_{GN}}{d}} \ . \qquad (10)$$

In equation (10),G = 6.67 × 10^{-11} m³/kg · s² is the universal gravitational constant.

Let us estimate M_{GN} in the case of the Sun orbiting the center of the galactic nucleusat the speed V = 250 km/s and the distance d = 28 000 ly. Using (10), we obtain

$$M_{GN} = 8.3 \times 10^{32} \text{ kg} = 414 \ M_\oplus . (11)$$

Let us underlined that.M_{GN} is not the mass of the supermassive black hole of the the Milky Way. In fact, the supermassive black hole (SMBH) at our galaxy's core. Sagittarius A* is sitting at the bottom of the central gravitational potential of the Milky Way with a mass about $4 \times 10^6 M_\oplus$[45].

For a given star of radius R_S and of mass M_S orbiting the galactic nucleus, we define the SRMR law as follows

$$\frac{R_S}{R_{GN}} = \left(\frac{M_S}{M_{GN}}\right)^\beta . (12)$$

As R_{GN} is not known. the β-parameter can be evaluated considering the Sun ($R_\oplus$.$M_\oplus$) and an other star (R_{OS},M_{OS}). We obtain from (12)

$$\begin{cases} M_\oplus = M_{NG}\left(\frac{R_\oplus}{R_{NG}}\right)^\beta \\ M_{OS} = M_{NG}\left(\frac{R_{OS}}{R_{NG}}\right)^\beta \end{cases} \Rightarrow \frac{M_{OS}}{M_\oplus} = \left(\frac{R_{OS}}{R_\oplus}\right)^\beta . \qquad (13)$$

Then

$$\beta = \frac{\ln(M_{OS} / M_\oplus)}{\ln(R_{OS} / R_\oplus)} . \qquad (14)$$

Let us then consider particular cases of fourth stars such as Alpha Centauri B,Rigel,Alpha Centauri Aand Capella A.Using (14), we obtain

- for Alpha Centauri B : R_{ACB} = 0.863$R_\oplus$,M_{ACB} = 0.907 $M_\oplus$
(Toliman (Alpha Centauri B) - Star Facts. https://www.star-facts.com › *toliman*.2020)

$$\beta = \frac{\ln(0.907/1)}{\ln(0.863/1)} = 0.6625 \approx 0.662 \ . \qquad (15)$$

- for Rigel: R_R = 78.9$R_\oplus$,M_R = 18 $M_\oplus$
(https://fr.wikipedia.org/wiki/Rigel. 2023)

$$\beta = \frac{\ln(18/1)}{\ln(78.9/1)} = 0.661688 \approx 0.662\,. \qquad (16)$$

- for Alpha Centauri A :R_{ACA} = 1.217$R_{\oplus}$,M_{ACA} = 1.079 $M_{\oplus}$[46]

$$\beta = \frac{\ln(1.079/1)}{\ln(1.217/1)} = 0.0.38716 \approx 0.390\,. \qquad (17)$$

- for Capella A (Aa): R_{CA} = 12.2$R_{\oplus}$,M_{CA} = 2.69 $M_{\oplus}$ (https://theplanets.org/stars/capella-star/).

$$\beta = \frac{\ln(2.69/1)}{\ln(12.2/1)} = 0.39558 \approx 0.390\,. \qquad (18)$$

Let us underline that. for Capella B (Ab), we find from (https://theplanets.org/stars/capella-star/) the data R_{CB} = 9.2 $R_{\oplus}$,M_{CB} = 2.56 $M_{\oplus}$. Equation (14) gives β = 0.42357 in contrast with the assumed correct value 0.662 for both Capella B (15) and Rigel (16). In addition, from the same reference, we get for Capella H the data R_{CH} = 0.54 $R_{\oplus}$,M_{CH} = 0.53 $M_{\oplus}$. Equation (14) gives then β = 1.030. These results indicate that the data for Capella B and H quoted in https://theplanets.org/stars/capella-star/ are probably inaccurate.However, this conclusion is too earlier. Systematic calculations of the β-parameter for the 100 stars closest to Earth in the Milky Way may be performed before drawing any conclusion.

4. Conclusion

We have presented in this paper a radius-to-mass ratio relationship applied to the calculations of various satellites belonging to the Solar system. Important results are obtained. Specially, it has been shown that. Pluto is indeed a dwarf planet. In addition,the radius-to-mass ratio condition 0.3<α< 0.4 is seen to be a very good criteria for appreciating accuracy of measured or calculated radius or mass of a given satellite of the Solar system.Besides, it is demonstrated that, the radius-to-mass ratio relationship can be applied to the calculation of the mass or the radius of a given star belonging to the Milky Way. Systematic calculations of the β-parameter for the 100 stars closest to Earth in the Milky Way is very challenging for the establishment of the radius-to-mass ratio condition for stars. Studies are in such direction.

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