Original Research Article

On the effect of a fixed cost on an Inventory model with Time –dependent parameters

Abstract

An economic order quantity model with a time-dependent demand, a fixed cost and a time-dependent holding cost is developed. It provides quantitative insight into a serious practical problem where costs are incurred even when an order is not placed. The effect of a fixed cost on the inventory model is examined. Previous models incorporating time-dependent demand rate assume that the holding cost is constant for the entire inventorycycle.The holding cost is considered as an increasing function of time spent in storage. Differential calculus is used for finding the optimal solution. A numerical example is used to validate the proposed model. Sensitivity analysis is carried out to analyze the effect of changes in the optimal solution with respect to changes in various parameters.

Keywords: Inventory model, Time-dependent demand, Time-dependent holding cost, Fixed cost, Optimization, fixed cost effect

1. Introduction

Customer patronage in any business is important but not without a proper inventory cost management. This is because inventory management is the soul of any business venture. More so, inventory control for physical goods is a common problem to all enterprise within any sector of an economy.The usefulness of an inventory model in managerial decision making requiressome of its usual parameters to be decision variables. In traditional inventory models, the demand rate and the holding cost are assumed to be a given constant. However, it is frequently observed in real life that the demand for a particular product as well as its holding cost can indeed be influenced by internal factors such as price, time, quantity and availability. A change in demand in response to inventory or marketing decisions is commonly referred to as demand elasticity.Most inventory models consider the holding cost with regards to its variable cost component assuming that the fixed cost exerts no influence. Due to ranging peculiarities of inventory management, it is observed that the holding cost is strongly influenced by its fixed cost component in Nigeria. This is particularly true in the storage of items where the price of the item changes every day and when delivery may take some time longer than expected.

Since Harris (1915) presented an economic order quantity (EOQ) model, many researchers have been made to adjust their assumptions to more realistic situations in inventory management. For instance,Rathod and Bhathawala (2013) investigated an inventory model with inventory-level dependent demand rate, a variable holding cost and shortages.They considered the holding cost as a decreasing step function of the quantity in storage and discovered that both the optimal quantity and the cycle time decreases when the holding cost increases.KariKari and Noutchie (2013) developed an inventory control system for determining optimal quantity, optimal total inventory cost and optimal cycle time under a retroactive holding cost.The retroactive cost model was used to model 20% concentrate poultry feed of a poultry feed company. They discovered that the optimal quantity and the cycle time experienced a reduction in value when the holding cost rises in value. The demand also increases resulting in the increase of the optimal quantity and decrease in the cycle time. Rathod and Bhathawala (2012) developed an inventory model with a linear inventory-level dependent demand rate and a variable holding cost.They considered two types of holding cost variation in terms of storage time: retroactive increase and stepwise incremental increase. They discovered that both the optimal quantity and the cycle time decreases when holding cost increases.Alfares (2007) developed an inventory model with inventory-level dependent demand rate and variable holding cost. He considered holding cost as a step function of storage time in two cases: retroactive holding cost increase and incremental holding cost increase.Ghasemi and Nadjafi (2013) developed inventory models with varying holding cost.They developed two models. The first model was optimized without shortages while the second model considered shortages.Khan, et al. (2020) studied inventory models for perishable items with advance payment, linearly time-dependent holding cost and demand dependent on advertisement and selling price. They developed two models. The first model does not consider shortages while partially backlogged shortages were considered for the second model. Adak and Mahapatra(2022) studied the effect of reliability on varying demand and holding cost on inventory system incorporating probabilistic deterioration. They discovered that the demand rate is related to cash in hand and the holding cost is dependent on the reliability of the item.Singh and Rani (2020) considered an inventory model for exponential time-sensitive demand and parabolic time linked holding cost under inflation and shortages with salvage value. They discovered that total cost is sensitive to deterioration rate and holding coefficient while less sensitive to salvage value.Macias-Lopez et al. (2021) developed an inventory model for perishable items with price, stock and time-dependent demand rate considering shelf-life and non-linear holding cost. They discovered that increasing the value of the shelf-life results in an increment in price, inventory circle time, quantity ordered and profits that are generated for all price demand functions.Mohan and Venkateswarlu (2013) proposed an inventory model with a variable holding cost and a salvage value.They discovered that the salvage value for deteriorated items is insignificant in the total optimal cost of the system. Yang (2014) studied an inventory model with both stock-dependent demand rate and stock-dependent holding cost rate.He formulated two models. The first model considered shortage cost while the second was optimized without shortages.Kumar and Tripathi (2010) developed an inventory model with a time-dependent demand and a quantity-dependent holding cost function in payments.They developed two models. The first model considered a non-linear quantity-dependent holding cost while the second model presented holding cost as a power function of time.

Thus, the problem of determining the optimal inventory policy of an inventory model with a time-dependent demand rate, a fixed cost and a time-dependent holding cost is addressed in this paper.Further, we investigate the impact of a fixed cost on the EOQ model.The principal contribution of this paper is addressing the limiting assumptions discussed above apart from the model.The layout of this paper is organised as follows.In Section 2, we give relevant notations and assumptions for the proposed model. In section 3, we present a mathematical formulation of the model. Numerical example is given in section 4.The effect of fixed cost on the EOQ model was evaluated in section 5 followed bysensitivity analysis in section 6. Finally, we summarize our findings in section 7 and provide some suggestions for future research.

2. Notations and Assumptions

The following are notations applied in the development of the model:

λ_o – Constant annual demand rate

I(t) – Inventory on-hand at time, t

K – Ordering cost per order

T – Cycle time

Q – Ordering quantity

h – Holding cost of the item

β – Demand parameter indicating elasticity, $0 < \beta < 1$

h(t) – Time-dependent holding cost of the item at time, t

R(t) – Time-varying demand

C_o – fixed cost of keeping the item

t – Reorder time

In addition, the following assumptions are made in developing our mathematical model:

i. The demand rate R(t) for the product is a decreasing function of time

R(t) = $\lambda_o t^{-\beta}$, $\lambda_o > 0$, $0 < \beta < 1$, $0 \leq t \leq T$

ii. A single item is considered

iii. The replenishment lead time is zero and planning horizon is infinite

iv. shortages are not allowed

v. The holding cost is time-dependent, h(t) = ht + $C_o T$

vi. Fixed cost incurred during cycle time, $C_o T$

3. Model formulation

In this section, an Economic Order Quantity model is developed under a condition where the holding cost is time-dependent but with a fixed cost. The holding cost depends on the length of the storage used. The total inventory cost per unit time is expressed as:

$$\text{TIC} = \frac{K}{T} + \frac{1}{T}\int_0^T h(t).I(t)dt \quad (1)$$

$$\text{Given that } h(t) = ht + C_o T \quad (2)$$

$$I(t) = \frac{\lambda_o}{(1-\beta)}\left(T^{1-\beta} - t^{1-\beta}\right) \quad (3)$$

$$T = \left\{\frac{(1-\beta)Q}{\lambda_o}\right\}^{\frac{1}{(1-\beta)}} \quad (4)$$

Then, the total inventory cost (TIC) becomes:

$$TIC = \frac{K}{T} + \frac{1}{T}\int_o^T \left(ht + C_o T\right) I(t)\,dt \quad (5)$$

$$= \frac{K}{T} + \frac{I}{T}\int_o^T (ht)\, I(t)\, dt + \frac{1}{T}\int_o^T \left(C_o T\right) I(t)\, dt \quad (6)$$

Substituting for I(t) from (3) into (6), we have

$$= \frac{K}{T} + \frac{1}{T}\int_o^T (ht)\left(\frac{\lambda_o}{1-\beta}\left[T^{1-\beta} - t^{1-\beta}\right]\right) dt + \int_o^T C_o\left(\frac{\lambda_o}{1-\beta}\left[T^{1-\beta} - t^{1-\beta}\right]\right) dt \quad (7)$$

$$= \frac{K}{T} + \frac{1}{T}\left(\frac{\lambda_o h}{1-\beta}\right)\int_o^T \left(T^{1-\beta} t - t^{2-\beta}\right) dt + \left(\frac{C_o \lambda_o}{1-\beta}\right)\int_o^T \left(T^{1-\beta} - t^{1-\beta}\right) dt \quad (8)$$

Integrating (9) with respect to t, we have:

$$= \frac{K}{T} + \frac{1}{T}\left(\left(\frac{h\lambda_o}{1-\beta}\right)\left[T^{1-\beta}\frac{t^2}{2} - \frac{t^{3-\beta}}{3-\beta}\right]_o^T\right) + \frac{C_o \lambda_o}{(1-\beta)}\left[T^{1-\beta} t - \frac{t^{2-\beta}}{2-\beta}\right]_o^T \quad (10)$$

Evaluating (10), we have the total inventory cost (TIC) as:

$$TIC = \frac{K}{T} + \frac{h\lambda_o T^{2-\beta}}{2(3-\beta)} + \frac{C_o \lambda_o T^{2-\beta}}{(2-\beta)} \quad (11)$$

Putting the value of T from (4) into equation (11), we have that:

$$TIC = KQ^{-\frac{1}{1-\beta}}\left(\frac{\lambda_o}{1-\beta}\right)^{\frac{1}{1-\beta}} + \frac{h\lambda_o}{2(3-\beta)}Q^{\frac{2-\beta}{1-\beta}}\left(\frac{1-\beta}{\lambda_o}\right)^{\frac{2-\beta}{1-\beta}} + \frac{C_o\lambda_o}{(2-\beta)}\left(\frac{1-\beta}{\lambda_o}\right)^{\frac{(2-\beta)}{(1-\beta)}}Q^{\frac{2-\beta}{1-\beta}} \quad (12)$$

We differentiate (12) with respect to Q and equating to zero to get the following:

$$\frac{d(TIC)}{dQ} = -\frac{K}{1-\beta}Q^{-\frac{2-\beta}{1-\beta}}\left(\frac{\lambda_o}{1-\beta}\right)^{\frac{1}{1-\beta}} + \frac{h\lambda_o(2-\beta)}{2(3-\beta)(1-\beta)}Q^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{\lambda_o}\right)^{\frac{2-\beta}{1-\beta}} + \frac{C_o\lambda_o}{(1-\beta)}\left(\frac{1-\beta}{\lambda_o}\right)^{\frac{2-\beta}{1-\beta}}Q^{\frac{1}{1-\beta}} \quad (13)$$

We simplify (13) to have:

$$\frac{-K}{1-\beta}Q^{-\frac{(2-\beta)}{(1-\beta)}}\left(\frac{\lambda_o}{1-\beta}\right)^{\frac{1}{1-\beta}} + Q^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{\lambda_o}\right)^{\frac{2-\beta}{1-\beta}}\left[\frac{h\lambda_o(2-\beta)}{2(3-\beta)(1-\beta)} + \frac{C_o\lambda_o}{(1-\beta)}\right] = 0 \quad (14)$$

$$\frac{-K}{1-\beta}Q^{-\frac{(2-\beta)}{(1-\beta)}} + Q^{\frac{1}{1-\beta}}\left[\frac{h(2-\beta)}{2(3-\beta)} + C_o\right]\left(\frac{\lambda_o}{1-\beta}\right)\left(\frac{1-\beta}{\lambda_o}\right)^{\frac{3-\beta}{1-\beta}} = 0 \quad (15)$$

$$Q^{-\frac{(3-\beta)}{(1-\beta)}} = \left(\frac{1-\beta}{K}\right)\left[\frac{h(2-\beta)}{2(3-\beta)} + C_o\right]\left(\frac{1-\beta}{\lambda_o}\right)^{\frac{2}{(1-\beta)}} \quad (16)$$

The optimal quantity becomes

$$Q^* = \left(\frac{K}{1-\beta}\right)^{\frac{(1-\beta)}{(3-\beta)}}\left[\frac{h(2-\beta)}{2(3-\beta)} + C_o\right]^{-\frac{(1-\beta)}{(3-\beta)}}\left(\frac{\lambda_o}{1-\beta}\right)^{\frac{2}{(3-\beta)}} \quad (17)$$

From equation (4),

$$T^* = \left\{\frac{(1-\beta)}{\lambda_o}Q^*\right\}^{\frac{1}{1-\beta}} \quad (18)$$

From equation (11)

$$TIC^* = \frac{K}{T^*} + \frac{h\lambda_0 T^{*(2-\beta)}}{2(3-\beta)} + \frac{C_0\lambda_0 T^{*(2-\beta)}}{(2-\beta)}$$

The total inventory cost without a fixed cost developed by Tripathi (2013) is defined as:

$$TIC^* = \frac{K}{T^*} + \frac{h\lambda_0 T^{*(2-\beta)}}{2(3-\beta)} \qquad (19)$$

4. Numerical example

To illustrate the proposed model and the results derived, data set was obtained from a cement wholesaler ECHIDIEGWU Ltd (pseudonym) in Benin City, Nigeria.

C_0 = 228000 naira / year, λ_0= 43200 units / year, K = 1600000 naira per order,

β = 0.2, Q = 900 units / order, Staffing cost = 900 000 naira /year,

Utilities = 1 040 000 naira / year

Table 1: Values of optimal T = T*, Q = Q* and TIC = TIC* for different values of 'h' (**model with fixed cost)**

h	Q = Q*	T = T*	TIC = TIC*
10	4464.11	0.0443279	5.61473
12	4464.11	0.0443278	5.61473
14	4464.11	0.0443278	5.61474
16	4464.10	0.0443277	5.61474
17	4464.10	0.0443277	5.61475
18	4464.10	0.0443277	5.61475
19	4464.10	0.0443277	5.61475
20	4464.10	0.0443276	5.61476
23	4464.09	0.0443276	5.61476
26	4464.09	0.0443275	5.61478
30	4464.08	0.0443274	5.61479
34	4464.07	0.0443273	5.61480
35	4464.07	0.0443273	5.61480
36	4464.07	0.0443273	5.61480
37	4464.07	0.0443273	5.61481
38	4464.06	0.0443272	5.61481
39	4464.06	0.0443272	5.61481

40	4464.06	0.0443272	5.61483
45	4464.05	0.0443271	5.61483
46	4464.05	0.0443271	5.61483
48	4464.05	0.0443270	5.61484
50	4464.04	0.0443270	5.61485
55	4464.03	0.0443269	5.61486
57	4464.03	0.0443268	5.61487
60	4464.02	0.0443268	5.61488

TIC values are all in standard form ($\times 10^7$). This Table represents optimal values for **model**

Table 1 gives the optimal solutions for selected values of h ranging from 10 to 60. It is clear from table 1 that the optimal total inventory cost TIC* is directly associated with the holding cost. More so, the optimal order quantity Q* and the optimal cycle time T* are inversely related to holding cost. This implies that T* and Q* decrease steadily while TIC* increases steadily as the holding cost increases.

Table 2: Values of optimal $T = T_t^*$, $Q = Q_t^*$ and $TIC = TIC_t^*$ for different values of 'h' (**model without fixed cost**)

h	$Q = Q_t^*$	$T = T_t^*$	$TIC = TIC_t^*$
10	101860	2.21062	1.04546
12	96690.2	2.07127	1.1158
14	92524.1	1.96032	1.17895
16	89060.6	1.86902	1.23653
17	87531.3	1.82899	1.2636
18	86113.4	1.79203	1.28966
19	83559.7	1.72585	1.33911
20	82403	1.69604	1.36265
23	79318.4	1.61706	1.42921
26	77525	1.57148	1.47065
30	74419.3	1.49319	1.54777
34	72420.1	1.44322	1.60136
35	71212.7	1.4132	1.6324
36	70641.9	1.39906	1.65191
37	70091	1.38543	1.66815
38	69559	1.3723	1.68411
39	69044.7	1.35963	1.69981
40	68547	1.34739	1.71525
45	66278.6	1.29189	1.78894
46	65863.7	1.28179	1.80304
48	65067.7	1.26245	1.83065
50	64313.2	1.24418	1.85754
55	62585.5	1.20254	1.92186
57	61950	1.1873	1.94653

60	61048.7	1.16595	1.98252

TIC values are all in standard form ($\times 10^6$). This table represents optimal values for **model 2.**

In **Table 2**, the optimal solutions for selected values of h ranging from 10 to 60 are examined. The result shows that the optimal order quantity Q_t* and the optimal cycle time T_t* are inversely associated with holding cost whereas the optimal total inventory cost TIC_t* is directly related to the holding cost. It implies that as holding cost increases steadily the TIC_t* will be increasing while Q_t* and T_t* will be decreasing.

5. The effect of fixed cost on the EOQ model

In the above discussions, we implicitly assume that the parameters of the problem are known with certainty. However, in real world, these parameters are usually estimated. Hence, it is important to investigate what range of values for these parameters is worthwhile.

Table 3: Summary of the optimal solutions for Model 1 and Model 2

Model type		Model 1		Model 2		%
β	h	Q*	TIC*	Q_t*	TIC_t*	loss
0.2	10	4464.11	5.61473	101860	1.04546	81.38005
0.2	12	4464.11	5.61473	96690.2	1.1158	80.12727
0.2	14	4464.11	5.61474	92524.1	1.17895	79.00259
0.2	16	4464.10	5.61474	89060.6	1.23653	77.97707
0.2	17	4464.10	5.61475	87531.3	1.2636	77.49499
0.2	18	4464.10	5.61475	86113.4	1.28966	77.03086
0.2	19	4464.10	5.61475	83559.7	1.33911	76.15014
0.2	20	4464.10	5.61476	82403	1.36265	75.73093
0.2	23	4464.09	5.61476	79318.4	1.42921	74.54548
0.2	26	4464.09	5.61478	77525	1.47065	73.80752
0.2	30	4464.08	5.61479	74419.3	1.54777	72.43405
0.2	34	4464.07	5.61480	72420.1	1.60136	71.47966
0.2	35	4464.07	5.61480	71212.7	1.3537	75.8905

0.2	36	4464.07	5.61480	70641.9	1.65191	70.57936
0.2	37	4464.07	5.61481	70091	1.66815	70.29018
0.2	38	4464.06	5.61481	69559	1.68411	70.00593
0.2	39	4464.06	5.61481	69044.7	1.69981	69.72631
0.2	40	4464.06	5.61483	68547	1.71525	69.45143
0.2	45	4464.05	5.61483	66278.6	1.78894	68.13902
0.2	46	4464.05	5.61483	65863.7	1.80304	67.8879
0.2	48	4464.05	5.61484	65067.7	1.83065	67.39622
0.2	50	4464.04	5.61485	64313.2	1.85754	66.91737
0.2	55	4464.03	5.61486	62585.5	1.92186	65.7719
0.2	57	4464.03	5.61487	61950	1.94653	65.33259
0.2	60	4464.02	5.61488	61048.7	1.98252	64.69168

Note:

- '% loss' denotes the % loss of total inventory cost by comparing the values of the total inventory cost of model 2 with model 1.

From **table 3**, we see that in both models, for an inelastic demand (fixed value of β) as the holding cost increases both Q* and Q_t* decreases steadily while TIC* and TIC_t* increases.

High values of TIC* shows the effect of the fixed cost on the model developed when compared to TIC_t* for the existing model. In **model 1**, for fixed value of β as the holding cost increases; the percentage loss of the total inventory cost begins to decrease steadily. The maximum percentage decrease (64.69) of the total inventory cost occurs at the holding cost value, h = 60. This shows that model 1 is an improvement of model 2 and performs better.

6. Sensitivity analysis

In this section, we study the effect of the changes in the value of the parameters of the developed model such as the holding cost (h), the ordering cost (k) and the elasticity coefficient (β) on the optimal solution. The set of values of 'h', 'k' and 'β' are assumed to be h = 60, 55, 50, 45, 40, 35, 30, 25; k =1620, 1640, 1660, 1680, 1700, 1720, 1740, 1760, 1780, 1800 (each × 1000) and β = 0.1, 0.3, 0.5, 0.7, 0.9. Meanwhile the other parameter values follow those data mentioned in the numerical example. The results of the sensitivity analysis are given in **Tables 4** and **5**.

Table 4: Variation of the optimal solutions of Q = Q*, T = T* and TIC = TIC* with the variation of '**h**' and '**k**' keeping all other parameters constant.

h ↓	k→	1620	1640	1660	1680	1700	1720	1740	1760	1780	1800
	Q	4479.90	4495.63	4511.23	4526.69	4542.02	4557.22	4572.3	4587.26	4602.09	4616.8
60	T	0.0445239	0.0447194	0.0449134	0.0451059	0.045297	0.0454866	0.0456748	0.0458616	0.046047	0.0462312
	TIC	5.65989	5.70471	5.74934	5.79377	5.83802	5.88208	5.92596	5.96965	6.01318	6.05652
	Q	4479.91	4495.64	4511.23	4526.70	4542.03	4557.23	4572.31	4587.27	4602.1	4616.81
55	T	0.0445240	0.0447195	0.0449135	0.0451060	0.0452971	0.0454867	0.0456749	0.0458617	0.0460472	0.0462313
	TIC	5.65987	5.70469	5.74932	5.79376	5.838	5.88206	5.92594	5.96964	6.01316	6.05651
	Q	4479.91	4495.65	4511.24	4526.71	4542.04	4557.24	4572.32	4587.27	4602.11	4616.82
50	T	0.0445241	0.0447196	0.0449136	0.0451061	0.0452972	0.0454868	0.045675	0.0458618	0.0460473	0.0462314
	TIC	5.65986	5.70468	5.74931	5.79374	5.838	5.88205	5.92593	5.96962	6.01314	6.05649
	Q	4479.92	4495.66	4511.25	4526.72	4542.05	4557.25	4572.33	4587.28	4602.12	4616.83
45	T	0.0445242	0.0447197	0.0449137	0.0451063	0.0452973	0.0454869	0.0456751	0.0458619	0.0460474	0.0462315
	TIC	5.65984	5.70467	5.74929	5.79373	5.83797	5.88203	5.92591	5.96961	6.01313	6.05648
	Q	4479.93	4495.67	4511.26	4526.72	4542.06	4557.26	4572.34	4587.29	4602.13	4616.84
40	T	0.0445243	0.0447198	0.0449139	0.0451064	0.0452974	0.0454870	0.0456752	0.045862	0.0460475	0.0462316
	TIC	5.65983	5.70465	5.74928	5.79371	5.83796	5.88202	5.9259	5.96959	6.01311	6.05646
	Q	4479.94	4495.67	4511.27	4526.73	4542.07	4557.27	4572.35	4587.3	4602.14	4616.85
35	T	0.0445244	0.0447200	0.0449140	0.0451065	0.0452975	0.0454871	0.0456753	0.0458622	0.0460476	0.0462317
	TIC	5.65982	5.70464	5.74926	5.79370	5.83794	5.88200	5.92588	5.96958	6.0131	6.05645
	Q	4479.95	4495.68	4511.28	4526.74	4542.08	4557.28	4572.36	4587.31	4602.15	4616.86
30	T	0.0445245	0.0447201	0.0449141	0.0451066	0.0452977	0.0454873	0.0456755	0.0458623	0.0460477	0.0462318
	TIC	5.65980	5.70462	5.74925	5.79368	5.83793	5.88199	5.92587	5.96956	6.01308	6.05643
	Q	4479.96	4495.69	4511.29	4526.75	4542.08	4557.29	4572.37	4587.32	4602.15	4616.87
25	T	0.0445246	0.0447202	0.0449142	0.0451067	0.045298	0.0454874	0.0456756	0.0458624	0.0460478	0.046232
	TIC	5.65979	5.70461	5.74923	5.79367	5.83791	5.88197	5.92585	5.96955	6.01307	6.05642

Note: TIC values in **Table 4** are all in standard form ($\times 10^7$)

Table 5: Variation of optimal solution of Q*, T* and TIC* with the variation of ‘h’ and ‘β’ keeping all other parameters constant

h ↓	β →	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**
	Q	3200.67	6426.37	15088.50	46142.60	285128
60	T	0.0493551	0.0394951	0.0304977	0.0225146	0.0156876
	TIC	4.94803	6.43416	8.74383	1.2573*	1.9471*
	Q	3200.67	6426.38	15088.60	46142.60	285128
55	T	0.0493553	0.0394952	0.0304978	0.0225147	0.0156877
	TIC	4.94801	6.43415	8.74381	1.2573*	1.9471*
	Q	3200.67	6426.39	15088.60	46142.70	285128
50	T	0.0493554	0.0394953	0.0304979	0.0225148	0.0156877
	TIC	4.94800	6.43413	8.74379	1.2573*	1.94709*
	Q	3200.69	6426.40	15088.60	46142.70	285128
45	T	0.0493555	0.0394954	0.0304979	0.0225148	0.0156878
	TIC	4.94799	6.43411	8.74376	1.25729*	1.94709*
	Q	3200.69	6426.41	15088.60	46142.70	285128
40	T	0.0493556	0.0394955	0.0304980	0.0225149	0.0156878
	TIC	4.94798	6.43410	8.74374	1.25729*	1.94708*
	Q	3200.70	6426.43	15088.60	46142.80	285128
35	T	0.0493558	0.0394956	0.0304981	0.0225149	0.0156879
	TIC	4.94796	6.43408	8.74372	1.25729*	1.94708*
	Q	3200.71	6426.44	15088.70	46142.80	285128
30	T	0.0493559	0.0394957	0.0304982	0.0225150	0.0156879
	TIC	4.94795	6.43406	8.74370	1.25728*	1.94707*
	Q	3200.72	6426.45	15088.70	46142.80	285128
25	T	0.0493560	0.0394958	0.0304982	0.0225151	0.0156879
	TIC	4.94794	6.43405	8.74367	1.25728*	1.94706*

Note: TIC values in **Table 3** are all in standard form ($\times 10^7$) but * is ($\times 10^8$)

Tables 2 and **3** show the result of the sensitivity analysis. On the basis of the results, the following observations were made:

From **Table 3**,

(i) Any increase in β results to an increase in the optimal order quantity Q* as well as TIC* whereas any decrease in the optimal cycle time T* does not change the holding cost.

(ii) Any increase in holding cost 'h' decreases the optimal order quantity Q* and the optimal cycle time, T* while the optimal total inventory cost TIC* increases.

In **Table 2**,

(i) An increase in the ordering cost 'k' results to an increase in optimal quantity Q*, optimal cycle time T* and optimal total inventory cost TIC*, keeping holding cost 'h' constant.

(ii) An increase in holding cost 'h' results to a decrease in the optimal order quantity Q* and the optimal cycle time T* while the optimal total inventory cost TIC* increases.

7. Conclusion and Future Research

In this paper, a model describing an inventory system with a time-dependent demand, a fixed cost and a time-dependent holding cost has been presented. The holding cost is considered as an increasing function of time spent in storage. A single item was considered. The newly developed model was examined using a numerical example. The preliminary result from the numerical example showed that the total inventory cost for the model increases while the cycle time and the order quantity decrease when the holding cost is increased. It was also observed from the sensitivity analysis on the developed model that the total inventory cost increases with an increase in the ordering cost 'k', the elasticity 'β' and the holding cost 'h'.

The model presented in this study provides a basis for several possible extensions. For future research, this model can be extended to include a variable ordering cost and a non-instantaneous receipt of orders. The case of the increasing holding cost considered in this paper applies to rented storage facilities.

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