__Original Research Article__

Steady State Conditions in Tractable Markov Manpower Model for an Extended Manpower System

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ABSTRACT

In this work, a formulation of manpower structure in discrete-time homogeneous Markov model is done for a multilevel manpower system. The structure of the manpower system is first extended in a departmentalized framework and the features of the extended structure utilized to create a scenario of personnel membership in three classes: the active, non-active and external classes. This allows for the inclusion of different units of the system in the model. Specifically, a pool of members in absorbing states with respect to intra-class transitions is included, which forms a second channel of recruitment. The first channel of recruitment is from the external class and all recruits go to the active class. All states of the active class are non-absorbing and give rise to various intra-class and inter-class transitions. Different probability matrices that form the main components of the Markov manpower model are constructed from probabilities of these transitions. One-step steady state condition on the manpower structure is considered, based on the formulated model, and established to be invariant with respect to varying proportion of recruits into the system.

Keywords: *Statistical manpower planning, Markov model, control, steady state condition, transition probability*

1. Introduction

The application of statistical techniques in manpower planning has been a long standing practice(Ossai et al., 2023a). The dynamics of a manpower system, arising from the interplay of manpower flows such as promotion, transfer, recruitment and wastage, present the platform for statistical manpower planning (Bartholomew et al., 1991). This, depending on the goal, model type and structural framework of the manpower system, can be in form of description, prediction, modelling or control of the attendant issues of the system, (Bartholomew 1971; McClean et al. 1992). Three basic stochastic models that have been employed more in statistical manpower modelling are Markov, semi-Markov and renewal models (Bartholomew, 1982; Bartholomew et al., 1991; McClean et al., 2004; Vassiliou, 1981, 1998; Dimitriou et al., 2013; Udom & Ebedoro, 2021; Georgiou et al., 2021). The choice and treatment of any of these models are governed by a number of factors, which include: the nature of transitions and promotion flow – whether it is of 'push' or 'pull' type (Georgiou & Tsantas, 2002; Guerry & De Feyter, 2011), discrete or continuous time space (McClean, 1978; Davies, 1985; McClean et al., 1998), system homogeneity or non-homogeneity (Hougaard, 1984; Uche, 1990; Vassiliou, 1998; Ugwuowo & McClean, 2000), model tractability, descriptive ability and practicability (Barsnet & Ellison, 1998). Though Markov and semi-Markov models are push types, they have relative advantages over one another. Markovian manpower models are mathematically tractable, simple and yields more readily to approximations. These properties of tractability and simplicity are important characterizations of good manpower models. Semi-Markov models have good descriptive ability but they soon become complex and mathematically intractable (McClean, 1991). Again, renewal models, which are pull types, soon also become complex and intractable. One way out is a combination of these models to strike a balance between the true picture of manpower system dynamics and model tractability. To this end, the need for manpower models that combine the desirable qualities of Markov and semi-Markov models in Markov framework has been advocated (McClean, 1991). This idea is important for modeling manpower systems with obvious complex semi-Markov features (McClean, 1991; Ossai & Uche, 2009).

Another factor in mathematical manpower model development, which is important to the current work, is the structure and structural framework of a manpower system. In the early development of mathematical manpower models, a manpower system was considered as a simple collection of employees

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with common target, stocked in different grades or cadres with the opportunity of vertical transition (promotion or demotion) from grade to grade, attrition or wastage and accommodation of new members in form of recruitment. In order words, the system was viewed as one large system of individuals in different homogeneous grades (Bartholomew, 1971). This kind of consideration guided most of the earlier developments. Despite this, some earlier researchers also recognized that a manpower system might be really represented as a group of many departments or subsystems. Bartholomew (1971), presented the idea of a "system composed of several non-interacting subsystems", where transition is possible only within each subsystem but not between subsystems. Butlers (1971) called manpower subsystems departments, but also considered interactions between departments in form of internal movement (transfer) from department to department; however, only the hypothesis of Poisson transfer from department to department was discussed in this direction.

In the current paper, the manpower system is considered as a conglomerate of several sub units called departments, which are homogeneous with respect to personnel state configuration. This leads to more inter state and inter department transitions. To ensure tractability, as stated above, the more tractable Markov chain is employed to build a homogeneous Markov manpower model which incorporates the various state transitions arising from the different structural extensions. One of such structural extensions is the non-active or limbo class (Ossai et al. 2023b), which holds members who face wastage, but are still useful, in a non-active and limbo position. According to Guerry (2011), wastage analysis is very essential in manpower planning because wastage governs, to a large extent, the need for promotion and recruitment. According to Ossai et al. (2023b), the inclusion of the limbo class is a form of wastage control. It also ensures two recruitment channels, one from the non-active class and the other from the external recruitment pool or environment. The steady state or maintainability condition (Ossai et al., 2023b) on the manpower structure shall then be considered along these two channels of recruitment into the active class.

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2. Model Specification

Consider a u-graded, $(k+1)$-departmentalized manpower system having manpower stock vector for department $i, i = 1, \dots, k+1$, at time t, given by $n^i(t) = [n_1^i(t), \dots, n_u^i(t)] \in \mathbb{R}^u$, where $n_g^i(t), g = 1, \dots, u$, represents the number of members of department i in grade g at time t. The vector containing the manpower overall stock in each department and each grade at t is represented by $n(t) = [n^1(t), \dots, n^{k+1}(t)] \in \mathbb{R}^{u,k+1}$. The vector $n(t)$ is a classified stock vector in that it can be expressed as $n(t) = [n_1(t), n_2(t)]$, where all members in $n_1(t) = [n^1(t), \dots, n^k(t)]$ are in active class or service in the system and all members in $n_2(t) = n^{k+1}(t)$ are in non-active class or limbo class (Ossai et al., 2023b). The non-active position of members in $n_2(t)$ can be characterized in different ways. In the current paper, we assume that:

(i) members in $n_2(t)$ come from $n_1(t)$ only

(ii) with respect to grade-to-grade transitions, all states in the non-active class are absorbing states

Membership in the system is such that each personnel must be in only one of the states d_{is}, i = 1, …, k+1; s = 1, …, u, at a time. Being a member of d_{is} means being in department i with grade s. Those who end their membership in these active and non-active states enter the wastage state, which is external to the system, denoted by $d_{k+2,s}$. There are manpower transitions from state to state within and across the departments with the following assumptions and probabilities.

i. Transition from state to state within each active department (intra-active department transition). The intra-active department transition probability from grade s to grade r within department i *is* given by: $p_{ii(sr)} = \text{P(transition from state } d_{is} \text{ to } d_{ir})$, i = 1, …, k; s,r = 1, …, u. It is assumed that there is no transition within the non-active class. Thus, $p_{k+1,k+1(sr)} = 0$, for all $r \neq s$.

ii. Transition from one department i to another department j with or without change of grade (inter active department transition). The inter active department transition probability from grade s in department i to grade r in department j *is* given by: $p_{ij(sr)} = \text{P(transition from state } d_{is} \text{ to } d_{jr})$, $i = 1, \dots, k+$

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1; $s, r = 1, \ldots, u; i \neq j$. It is assumed that there is no grade change for all transitions to and from the non-active class. Thus, $p_{i,k+1(sr)} = 0$ and $p_{k+1,i(sr)} = 0$ for all i and all $r \neq s$.

iii. Wastage transition from grade s in department i to the external environment, with probability $p_{i,k+2(ss)} = \mathrm{P}(\text{transition from state } d_{is} \text{ to } d_{k+2,s})$, $i = 1, \ldots, k+1$; $s = 1, \ldots, u$, with the assumption that $p_{i,k+2(sr)} = 0$, for all i and $r \neq s$; so, $p_{k+2,k+2(ss)} = 1$, for all s.

With the above definitions and assumptions and the process $\{X_t, t \geq 0\}$ representing the membership of the state of the system, the various transition probabilities, homogeneous in time, can generally be represented by $\mathrm{p}_{\mathrm{ij}(\mathrm{sr})}$, where:

$$p_{ij\,(sr)} = \mathrm{P}(X_{t+1} = d_{jr} \,|X_t = d_{is}),$$

$$i, j = 1, \ldots, k+2;\; s, r = 1, \ldots, u$$

$j = i$ for intra-active department transition

$i \neq j$ for inter department transition

$j = k+2$ and $r = s$ for transition (wastage) to external environment.

For i or $j = k+1$ or $k+2, r = s$, otherwise $p_{ij\,(sr)} = 0$.

Consider the observed flow of manpower between states of the system from time t to $t+1$. Let $n_{ij\,(sr)}(t)$ be such flow from state d_{is} to d_{jr}, so that $n_{ij}(t) = [n_{ij\,(s1)}(t), \ldots, n_{ij\,(su)}(t)]$ is the observed flow vector for flows from grade s in department i to all other grades in department j. For all $j = 1, \ldots, k+2$, $n_{i(s)}(t) = [n_{i1}(t), \ldots, n_{ik}(t), n_{ik+1}(t), n_{i,k+2}(t)]$ is the block vector of $n_{ij}(t)$'s for all observed flows emanating from d_{is}; where $n_{i,k+1}(t) = n_{i,k+1(ss)}(t)$ and $n_{i,k+2}(t) = n_{i,k+2(ss)}(t)$ by the assumptions above. Hence, $n_s^i(t)$, the observed stock in d_{is} at t, *is* given by

$$n_s^i(t) = \sum_{r=1}^{u} n_{ii\,(sr)}(t) + \sum_{\substack{j=1 \\ j \neq i}}^{k+2} \sum_{r=1}^{u} n_{ij\,(sr)}(t)$$

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$$= \sum_{j=1}^{k+2} \sum_{r=1}^{u} n_{ij\,(sr)}(t) \qquad (2.1)$$

The transition probabilities, represented in general by $p_{ij\,(sr)}$, of the Markov chain can be estimated by the well known maximum likelihood (ML) method. Given observed flow or stock data over times, $t = 1, \ldots, T$, the ML estimator of $p_{ij\,(sr)}$ is given by

$$\hat{p}_{ij\,(sr)} = \frac{\sum_{t=1}^{T} n_{ij\,(sr)}(t)}{\sum_{t=1}^{T} n_s^i(t)} \qquad (2.2)$$

For the manpower system of interest, let matrix $\boldsymbol{P}$be definedas

$$\boldsymbol{P} = \left(\begin{array}{c|c} \boldsymbol{A}_{uk\times uk} & \boldsymbol{B}_{uk\times u} \\ \hline \boldsymbol{C}_{u\times uk} & \boldsymbol{I}_{u\times u} \end{array} \right)$$

Where $\boldsymbol{B}$ is a zero matrix, $\boldsymbol{C} = \boldsymbol{B}'$ and $\boldsymbol{I}$ is an identity matrix. Then,

$$\boldsymbol{A} = \begin{array}{cc} & \begin{array}{ccccccc} & d_1 & \cdots & & & d_k & \\ g_1 & \cdots & g_u & \cdots & g_1 & \cdots & g_u \end{array} \\ \begin{array}{cc} & g_1 \\ d_1 & \vdots \\ & g_u \\ \vdots & \vdots \\ & g_1 \\ d_k & \vdots \\ & g_u \end{array} & \left(\begin{array}{ccccccc} p_{11\,(11)} & \cdots & p_{11\,(1u)} & \cdots & p_{1,k\,(11)} & \cdots & p_{1,k\,(1u)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ p_{11\,(u1)} & \cdots & p_{11\,(uu)} & \cdots & p_{1,k\,(u1)} & \cdots & p_{1,k\,(uu)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ p_{k,1\,(11)} & \cdots & p_{k,1\,(1u)} & \cdots & p_{k,k\,(11)} & \cdots & p_{k,k\,(1u)} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ p_{k,1\,(u1)} & \cdots & p_{k,1\,(uu)} & \cdots & p_{k,k\,(u1)} & \cdots & p_{k,k\,(uu)} \end{array} \right) \end{array}$$

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The matrix $\boldsymbol{A}$contains all probabilities of transition within the active class across all the uk grades of the k departments. Considering the transitions within and across the departments and from the departments to the outside environment, the following equation holds

$$\boldsymbol{P}\mathbf{1}'_{1\times u(k+1)} + [\boldsymbol{P}_1|\boldsymbol{0}_{1\times u}]' = \mathbf{1}'_{1\times u(k+1)} \tag{2.3}$$

In (2.3), P_1 is a vector of wastage probabilities, $p_{i,k+2(ss)}$, for all the uk states of the active departments, $\mathbf{1}$ is a vector of one's and $\boldsymbol{0}$ is a vector of zero's. Equation (2.3) represents the stochastic condition of the transition matrices of the system. Let also P_2 be the vector of transition probabilities $p_{i,k+1(ss)}$ and $\boldsymbol{P}_3$ be the vector of wastage probabilities $p_{k+1,k+2(ss)}$. So, P_2 contains all the probabilities of transiting from states of the active class to the non-active class and P_3 contains all the probabilities of going from the non-active class to the external environment. New members enter the active states from the non-active states and/or the external environment to fill vacancies, arising from wastage or system expansion, in form of recruitment inputs. The proportion of inputs from both sources and the choice of which source is considered first differ according to system needs or management decision. For more on the vacancy analysis, proportion and priority of input see Ossai et al. (2023b), Georgiou & Tsantas (2002) and Dimitriou & Tsantas (2009). For the current paper, M is the number of available vacancy in the active class, ψ is the proportion of intake from the external environment, R_1 and R_2 are the probability vectors for input from the external environment and non-active class respectively. With the above definitions, assumptions and functional relations, and basic homogeneous Markov manpower model formulation (see, for instance, Bartholomew, 1982; Bartholomew et al., 1991), the current manpower structure in homogeneousMarkov model is given as in equation (2.4) following.

$$n(t) = \begin{cases} n(t-1)P + \ M[\psi R_1|-P_3^*] \ + M(1-\psi R_1\mathbf{1}')[R_2|-q], \ \text{if } \ M > 0 \\ n(t-1)P - M[-P_2|P_2^* - P_3^*], \ \text{if } M \le 0 \end{cases}$$

(2.4)

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Equation (2.4) expresses the current structure, $n(t)$, in terms of the immediate past structure, $n(t-1)$, in two forms: when there is need for intake of new members into the active class, $M > 0$, and when there is no need for intake of new members, $M \leq 0$. The $u \times u$vector P_2^* distributes all members of the active class coming into the non-active class through P_2 into the u states. Likewise, the $u \times u$vector q ensures removal of members of the non-active class from the u states who are then distributed through R_2 in the active class. Also, P_3^* is a function of P_3 and M given by

$$P_3^* = n^{k+1}(t-1)diag(P_3)|M|^{-1}, M \neq 0 \tag{2.5}$$

Where,

$$diag(P_3) = \begin{pmatrix} p_{k+1,k+2(11)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p_{k+1,k+2(uu)} \end{pmatrix}$$

The following two proposition adapted from Ossai et al. (2023b) apply to the Markov manpower model of the type in equation (2.4) for the steady state condition of the manpower structure $n(t)$.

Proposition2.1

Let $M > 0$ and $\boldsymbol{I}$a $uk \times uk$ identity matrix. The manpower structure n^m attains a steady state in one step through control by R_1 under priority recruitment from the external environment if the number of recruits from the external environment,$M^m\psi$, is equal to $[n^m(I-P)]\mathbf{1}' + M^m P_3^{*m}\mathbf{1}'$.

The superscript 'm' appended to the already defined symbols indicates they are evaluated using the maintained manpower structure, n^m. The second such proposition is as follows:

Proposition 2.2

Let $M > 0$ and $\boldsymbol{I}$a $uk \times uk$ identity matrix. The manpower structure n^m attains a steady state in one step through control by R_2 under priority recruitment from the external environment if the number of recruits from the external environment,$M^m(1-\psi)$, is equal to $[n^m(I-P)]\mathbf{1}' + M^m P_3^{*m}\mathbf{1}'$.

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Corollary 2.1

Varying the proportion of intake,ψ, from the external environment or the non-active class does not affect the steady state (maintainability) condition through R_1 and R_2.

Proof

We show that the necessary and sufficient conditions, given in Ossai et. al (2023b), for equation type of (2.4) is unchanged with varying values of ψ. When the proportion ψ of intake is first made from external environment the necessary and sufficient condition for any structure n to be R_1 and R_2 maintainable is that

$$M^m \psi = [n^m (I - P)]\mathbf{1}' + M^m P_3^{*m} \mathbf{1}' \quad (2.6)$$

For the case of control by R_1, the condition in equation (2.6) arose from the following two necessary conditions:

i) $M^m \psi = [n^m (I - P)]\mathbf{1}' + M^m P_3^{*m} \mathbf{1}'$
ii) $0 \leq [n^m (I - P)]\mathbf{1}' + M^m P_3^{*m} \mathbf{1}'$

Similarly, for the case of control by R_2, the condition in equation (2.6) arose from the following two necessary conditions:

iii) $M^m \psi = [n^m (I - P)]\mathbf{1}' + M^m P_3^{*m} \mathbf{1}'$
iv) $M^m \psi - M^m (1 - \psi) \leq [n^m (I - P)]\mathbf{1}' + M^m P_3^{*m} \mathbf{1}'$

It can then be verified that for conditions i, ii, iii and iv, since $0 \leq \psi \leq 1$, no matter the value of ψ, having $M^m \psi = [n^m (I - P)]\mathbf{1}' + M^m P_3^{*m} \mathbf{1}'$ ensures that all the other two necessary conditions are satisfied.

Similarly, when the proportion ψ of intake is first made from the non-active class the necessary and sufficient condition for any structure n to be R_1 and R_2 maintainable is that

$$M^m (1 - \psi) = [n^m (I - P)]\mathbf{1}' + M^m P_3^{*m} \mathbf{1}' \quad (2.7)$$

Again, for the case of control by R_1, the condition in equation (2.7) arose from the following two necessary conditions:

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v) $M^m(1-\psi) = [n^m(I-P)]\mathbf{1}' + M^m P_3^{*m}\mathbf{1}'$
vi) $0 \le [n^m(I-P)]\mathbf{1}' + M^m P_3^{*m}\mathbf{1}'$

Similarly, for the case of control by R_2, the condition in equation (2.7) arose from the following two necessary conditions:

vii) $M^m(1-\psi) = [n^m(I-P)]\mathbf{1}' + M^m P_3^{*m}\mathbf{1}'$
viii) $M^m(1-\psi) - M^m\psi \le [n^m(I-P)]\mathbf{1}' + M^m P_3^{*m}\mathbf{1}'$

It can then be verified that for conditions v, vi, vii and viii, no matter the value of ψ having $M^m(1-\psi) = [n^m(I-P)]\mathbf{1}' + M^m P_3^{*m}\mathbf{1}'$ ensures that all the other two necessary conditions are satisfied. □

The result in Corollary 2.1 can be viewed as the invariant property of the maintainability condition of the manpower structure with respect to ψ.

Corollary 2.2

If equal proportion of intake,ψ, is made from the external environment and the non-active class then the maintainability conditions using $\boldsymbol{R}_1$and$\boldsymbol{R}_2$ are the same in all situations irrespective of which channel of recruitment is taken first.

Proof

The proof follows readily by putting $\psi = \frac{1}{2}$ in all the necessary and sufficient conditions for maintainability of any given manpower structure. □

3. Conclusion

The manpower system in this work has been presented in a sub-unit framework which allowed the extension to an annexed unit for special classes of members of the system. The extension creates the potential for considering different system scenarios depending on any imposed assumptions or prevailing working conditions. In other words, apart from the scenario of members of the annexed unit being in non-active or absorbing states, as assumed in the current work, other forms of consideration are possible and can likewise be incorporated in the model. This is intended for further research.

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The Markov chain approach utilized in the model building in the current work accommodated all the manpower interplay and transitions and gave rise to a tractable Markov manpower model, even with the seemingly complex system. The tractability enabled further consideration of the model under manpower control. Specifically, the steady state or maintainability condition on the system structure was established for the manpower model, and can be useful for planning and control purposes.

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