The Physical Application of Motion Using Single Step Block Method

Abstract

The manuscript proposed a physical application of single step block method using the interpolation and collocation procedure for the direct solution second order physical initial value problem. The properties of the new scheme which include error constant, order, zero-stability, consistency and convergent are established and satisfied. The new scheme was tested on some second order initial value problems and compared with the existing works in literature, and later the new scheme revealed its superiority by producing less error if compared. Therefore, the new scheme does not required much computation when compared with predictor corrector methods.

***Keywords*:** Physical application, Single Step, Power Series, Predictor Corrector, Mass Spring and Simple Harmonic Motion.

1 Introduction

The numerical application of general second order initial value problems given as

$$y''(t) = f(t, y, y') \tag{1.1}$$

With initial condition $y(t) = y_1,\ y'(t) = y_2$ is consider in this manuscript.

"The previous efforts have been made by eminent researchers to solve higher order initial value problems specifically, the second order ordinary differential equation. In exercise, this class of problem (1.1) is usually reduced to system of first order differential equation and numerical methods for first order ODEs then employ to solve them, these researchers" [1-3] showed that "reduction of higher order equations to its first order has a serious implication in the results; hence it is necessary to modify existing algorithms to handle directly this class of problem (1.1)". [4] demonstrate "a successful application of LMM methods to solve directly a general second order odes of the form (1.1)". The few researchers also contributed immensely to the development of block hybrid method for the direct solution of second order initial value problems, among others are [5-7] [2] just to mention a few. [5 and 6] employ a power series polynomial to developed double step hybrid linear multistep method for solving second order initial value problems (1.1). While [7] proposed Numerical Simulation of One Step Block Method for Treatment of Second Order Forced Motions in Mass-Spring Systems. They results is better when reduced to first order.

Block methods which are widely used by many researchers for solving (1.1) were first announced by Milne [8] and later by [9] mainly to provide starting values for predictor-corrector algorithms. Those methods produced better accuracy than the usual step by step methods. [10], on the other hand, extended Milne's idea to develop block methods for solving initial value problems (1.1). In order to obtain higher order methods and hence to increase the accuracy of the approximate solution, [11] proposed the direct simulation of higher order initial value problems on single step block method.

Different researchers such as [12-16] have applied hybrid methods to solve (1.1) but their solutions have lower order of accuracy.

The aim of this manuscript is to develop the physical application of motion using single step block method using the power series polynomial. While the objectives are

i. To developed the method using interpolation and collocation method
ii. To analyzed the basic properties of the method and
iii. To compare the method with the existing once in literature.

2 Methodology

The one step block hybrid method was developed using the power series polynomial as a basic function, for solving (1.1). Let the power series

$$y(t)=\sum_{j=0}^{u+v-1} a_j t^j \qquad (2.1)$$

Be the approximate solution of (1.1) where $t \in [0,1]$, u are number of interpolation and v are number of collocation.

Differentiating ((2.1) twice, yield

$$y''(t)=\sum_{j=0}^{u+v-1} j(j-1)a_j t^{j-2} \qquad (2.2)$$

Substituting (2.2) into (1.1) yield

$$\sum_{j=0}^{u+v-1} j(j-1)a_j\, t^{j-2} = f(t, y, y') \qquad (2.3)$$

Equation (2.1) is interpolated at $u = \frac{1}{8}, \frac{1}{4}$ while equation (2.3) at $v = 0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, 1$ lead to a system of equation reformed in matrix form as

$$T\,D = K \qquad (2.4)$$

where

$$T = \begin{pmatrix} 1 & \frac{1}{8} & \frac{1}{64} & \frac{1}{512} & \frac{1}{4096} & \frac{1}{32768} & \frac{1}{262144} & \frac{1}{2097152} \\ 1 & \frac{1}{4} & \frac{1}{16} & \frac{1}{64} & \frac{1}{256} & \frac{1}{1024} & \frac{1}{4096} & \frac{1}{16384} \\ 0 & 0 & \frac{2}{h^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{h^2} & \frac{3}{4h^2} & \frac{3}{16h^2} & \frac{5}{128h^2} & \frac{15}{2048h^2} & \frac{21}{16384h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{3}{2h^2} & \frac{3}{4h^2} & \frac{5}{16h^2} & \frac{15}{128h^2} & \frac{21}{512h^2} \\ 0 & 0 & \frac{2}{h^2} & \frac{9}{4h^2} & \frac{27}{16h^2} & \frac{135}{128h^2} & \frac{1215}{2048h^2} & \frac{5103}{16384h^2} \\ 0 & 0 & \frac{2}{h^2} & 3h^2 & 3h^2 & \frac{5}{2h^2} & \frac{15}{8h^2} & \frac{21}{16h^2} \\ 0 & 0 & \frac{2}{h^2} & 6h^2 & 12h^2 & 20h^2 & 30h^2 & 42h^2 \end{pmatrix}$$

$$D = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \end{bmatrix}^T$$

$$K = \begin{bmatrix} y_{n+\frac{1}{8}} & y_{n+\frac{1}{4}} & f_n & f_{n+\frac{1}{8}} & f_{n+\frac{1}{4}} & f_{n+\frac{3}{8}} & f_{n+\frac{1}{2}} & f_{n+1} \end{bmatrix}^T$$

The unknown values of $a'_j s,\ j = 0(1)7$ are obtained by applying Gaussian elimination method and substituted into (2.1) to produce a continuous implicit hybrid one step method with its derivatives of the form:

$$y(t) = \sum_{j=\frac{1}{8},\frac{1}{4}} \alpha_j^{i}(t) y_{n+j} + \sum_{j=0}^{1} \beta_j(t) f_{n+j} + \sum_{j=\frac{1}{8},\frac{1}{4},\frac{3}{8},\frac{1}{2}} \beta_j(t) f_{n+j} \qquad (2.5)$$

Where the values of α_j and β_j are

$$
\left.\begin{aligned}
\alpha_{\frac{1}{8}} &= 3-20t+32t^{2}\\
\alpha_{\frac{1}{4}} &= -3+32t-64t^{2}\\
\alpha_{\frac{3}{8}} &= 1-12t-32t^{2}\\
\beta_{0} &= -\frac{7}{655360}+\frac{1787}{1720320}t-\frac{51529}{2580480}t^{2}+\frac{1}{6}t^{3}-\frac{53}{72}t^{4}+\frac{11}{6}t^{5}-\frac{23}{9}t^{6}+\frac{64}{35}t^{7}-\frac{32}{63}t^{8}\\
\beta_{\frac{1}{8}} &= -\frac{401}{430080}+\frac{3679}{225792}t-\frac{16027}{188160}t^{2}+\frac{32}{21}t^{4}-\frac{1856}{315}t^{5}+\frac{448}{45}t^{6}-\frac{17408}{2205}t^{7}+\frac{1024}{441}t^{8}\\
\beta_{\frac{1}{4}} &= -\frac{257}{245760}+\frac{7199}{645120}t-\frac{449}{35840}t^{2}-\frac{4}{3}t^{4}+\frac{328}{45}t^{5}-\frac{664}{45}t^{6}+\frac{4096}{315}t^{7}-\frac{256}{63}t^{8}\\
\beta_{\frac{3}{8}} &= +\frac{1}{20480}+\frac{3}{17920}t-\frac{757}{80640}t^{2}+\frac{32}{45}t^{4}-\frac{64}{15}t^{5}+\frac{448}{45}t^{6}-\frac{1024}{105}t^{7}+\frac{1024}{315}t^{8}\\
\beta_{\frac{1}{2}} &= -\frac{13}{983040}+\frac{1}{73728}t-\frac{893}{430080}t^{2}-\frac{1}{6}t^{4}+\frac{47}{45}t^{5}-\frac{118}{45}t^{6}+\frac{128}{45}t^{7}-\frac{64}{63}t^{8}\\
\beta_{1} &= +\frac{1}{13762560}+\frac{11}{36126720}t-\frac{89}{6021120}t^{2}+\frac{1}{840}t^{4}-\frac{1}{126}t^{5}+\frac{1}{45}t^{6}-\frac{64}{2205}t^{7}+\frac{32}{2205}t^{8}
\end{aligned}\right\}
$$

Evaluating (2.5) non interpolating points to obtain the continuous form as,

$$
\begin{pmatrix} y_{n} \\ y_{n+\frac{3}{8}} \\ y_{n+\frac{1}{2}} \\ y_{n+1} \end{pmatrix} = h^{2}
\begin{pmatrix}
2 & -1 & \frac{49}{40960} & \frac{181}{13440} & \frac{3}{5120} & \frac{1}{1920} & -\frac{3}{20480} & \frac{1}{860160} \\
-1 & 2 & -\frac{1}{15360} & \frac{1}{640} & \frac{97}{7680} & \frac{1}{640} & -\frac{1}{15360} & 0 \\
-2 & 3 & -\frac{19}{122880} & \frac{43}{13440} & \frac{407}{15360} & \frac{31}{1920} & \frac{73}{61440} & -\frac{1}{860160} \\
-6 & 7 & \frac{10567}{122880} & -\frac{6239}{13440} & \frac{17189}{15360} & -\frac{1987}{1920} & \frac{37531}{61440} & \frac{1945}{172032}
\end{pmatrix}
\begin{pmatrix} y_{n+\frac{1}{8}} \\ y_{n+\frac{1}{4}} \\ f_{n} \\ f_{n+\frac{1}{8}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{3}{8}} \\ f_{n+\frac{1}{2}} \\ f_{n+1} \end{pmatrix} \quad (2.6)
$$

Differentiating (2.5) once, yields

$$
y'(g)=a'_{\frac{1}{8}}(g)y_{n+\frac{1}{8}}+\alpha'_{\frac{1}{4}}(g)y_{n+\frac{1}{4}}+h^{2}\left[\sum_{j=0}^{1}\beta'_{j}(g)f_{n+j}+\beta'_{v_i}(g)f_{n+v_i}\right], v_i=0,\frac{1}{8},\frac{1}{4},\frac{3}{8},\frac{1}{2} \quad (2.7)
$$

On evaluating (2.7) at $0,\frac{1}{8},\frac{1}{4},\ \frac{3}{8},\frac{1}{2}$ and1, so that the following discrete schemes are obtained as

$$\begin{pmatrix} hy'_n \\ hy'_{n+\frac{1}{8}} \\ hy'_{n+\frac{1}{4}} \\ hy'_{n+\frac{3}{8}} \\ hy'_{n+\frac{1}{2}} \\ hy'_{n+1} \end{pmatrix} = h^2 \begin{pmatrix} -8 & 8 & -\frac{26191}{645120} & -\frac{11177}{70560} & \frac{2131}{80640} & -\frac{197}{10080} & \frac{1501}{322560} & \frac{149}{4515840} \\ -8 & 8 & \frac{61}{40320} & -\frac{2791}{70560} & -\frac{251}{8064} & \frac{83}{10080} & -\frac{127}{80640} & \frac{1}{112896} \\ -8 & 8 & -\frac{767}{645120} & \frac{251}{14112} & \frac{4259}{80640} & -\frac{17}{2016} & \frac{493}{322560} & -\frac{37}{4515840} \\ -8 & 8 & \frac{1}{8064} & \frac{541}{70560} & \frac{5129}{40320} & \frac{559}{10080} & -\frac{239}{80640} & \frac{1}{112896} \\ -8 & 8 & -\frac{1103}{645120} & \frac{1367}{70560} & \frac{7507}{80640} & \frac{319}{2016} & \frac{2809}{64512} & -\frac{149}{4515840} \\ -8 & 8 & \frac{62141}{129024} & -\frac{183209}{70560} & \frac{460883}{80640} & -\frac{57541}{10080} & \frac{904669}{322560} & \frac{104623}{903168} \end{pmatrix} \begin{pmatrix} y_{n+\frac{1}{8}} \\ y_{n+\frac{1}{4}} \\ f_n \\ f_{n+\frac{1}{8}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{3}{8}} \\ f_{n+\frac{1}{2}} \\ f_{n+1} \end{pmatrix} \quad (2.7)$$

Equation (2.5) and (2.7) are simultaneously combined, to obtain the new schemes as

$$\begin{pmatrix} y_{n+\frac{1}{8}} \\ y_{n+\frac{1}{4}} \\ y_{n+\frac{3}{8}} \\ y_{n+\frac{1}{2}} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{8} & \frac{20017}{5160960} & \frac{715}{112896} & -\frac{2509}{645120} & \frac{31}{16128} & -\frac{1123}{2580480} & \frac{107}{36126720} \\ 1 & \frac{1}{4} & \frac{361}{40320} & \frac{461}{17640} & -\frac{29}{4032} & \frac{11}{2520} & -\frac{41}{40320} & \frac{1}{141120} \\ 1 & \frac{3}{8} & \frac{8007}{572440} & \frac{2979}{62720} & \frac{153}{71680} & \frac{15}{1792} & -\frac{477}{286720} & \frac{9}{802816} \\ 1 & \frac{1}{2} & \frac{191}{10080} & \frac{152}{2205} & \frac{4}{315} & \frac{8}{315} & \frac{1}{1008} & \frac{1}{70560} \\ 1 & 1 & \frac{79}{630} & -\frac{704}{2205} & \frac{344}{315} & -\frac{64}{63} & \frac{191}{315} & \frac{5}{441} \end{pmatrix} \begin{pmatrix} y_n \\ hy'_n \\ h^2 f_n \\ h^2 f_{n+\frac{1}{8}} \\ h^2 f_{n+\frac{1}{4}} \\ h^2 f_{n+\frac{3}{8}} \\ h^2 f_{n+\frac{1}{2}} \\ h^2 f_{n+1} \end{pmatrix}$$

$$\begin{pmatrix} y'_{n+\frac{1}{8}} \\ y'_{n+\frac{1}{4}} \\ y'_{n+\frac{3}{8}} \\ y'_{n+\frac{1}{2}} \\ y'_{n+1} \end{pmatrix} = y'_n + \begin{pmatrix} \frac{3881}{92160} & \frac{599}{5040} & -\frac{221}{3840} & \frac{1}{36} & -\frac{287}{46080} & \frac{3}{71680} \\ \frac{227}{5760} & \frac{37}{210} & \frac{19}{720} & \frac{1}{90} & -\frac{1}{320} & \frac{1}{40320} \\ \frac{417}{10240} & \frac{93}{560} & \frac{129}{1280} & \frac{3}{40} & -\frac{39}{5120} & \frac{3}{71680} \\ \frac{7}{180} & \frac{8}{45} & \frac{1}{15} & \frac{8}{45} & \frac{7}{180} & 0 \\ \frac{47}{90} & -\frac{256}{105} & \frac{256}{45} & -\frac{256}{45} & \frac{14}{5} & \frac{73}{630} \end{pmatrix} \begin{pmatrix} y_n \\ hy'_n \\ h^2 f_n \\ h^2 f_{n+\frac{1}{8}} \\ h^2 f_{n+\frac{1}{4}} \\ h^2 f_{n+\frac{3}{8}} \\ h^2 f_{n+\frac{1}{2}} \\ h^2 f_{n+1} \end{pmatrix} \quad (2.8)$$

3 Analysis of the Block Method

In this section, we the analysis of the block method, which includes the order, error constant, consistency, zero stability, convergence and region of absolute stability of the method.

3.1 Order and error constant

Let the linear operator defined on the method be $\ell[y(t);h]$, where,

$$\Delta\{y(t):h\} = A^{(0)}Y_m^{(i)} - \sum_{i=0}^{k} \frac{jh^{(i)}}{i!} y_n^{(i)} - h^{(3-1)}\left[d_i f(y_n) + b_i F(Y_m)\right], \quad (3.1)$$

Expanding Y_m *and* $F(Y_m)$ in Taylor series and comparing the coefficients of h according to [2, 4] to gives

$$\Delta\{y(t):h\} = C_0 y(t) + C_1 y'(t) + \cdots + C_p h^p y^p(t) + C_{p+1} h^{p+1} y^{p+1}(t) + C_{p+2} h^{p+2} y^{p+2}(t) + \cdots \qquad (3.2)$$

Definition 3.1: The linear operator L and the associate block method are said to be of order p if $C_0 = C_1 = \cdots = C_p = C_{p+1} = 0, \quad C_{p+2} \neq 0.$ C_{p+2} is called the error constant and implies that the truncation error is given by $t_{n+k} = C_{p+2} h^{p+2} y^{p+3}(x) + 0h^{p+3}$

$$L\{y(t):h\} = C_0 y(t) + C_1 y'(t) + \cdots + C_p h^p y^p(t) + C_{p+1} h^{p+1} y^{p+1}(t) + C_{p+2} h^{p+2} y^{p+2}(t) + \cdots$$

$$\begin{bmatrix}
\sum_{j=0}^{\infty} \frac{\left(\frac{1}{8}\right)^j}{j!} - y_n - \frac{1}{8} h y'_n - \frac{20017}{5160960} h y''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[\frac{715}{112896}\left(\frac{1}{8}\right) + \frac{461}{17640}\left(\frac{1}{4}\right) + \frac{2979}{62720}\left(\frac{3}{8}\right) + \frac{152}{2205}\left(\frac{1}{2}\right) - \frac{704}{2205}(1) \right] \\
\sum_{j=0}^{\infty} \frac{\left(\frac{1}{4}\right)^j}{j!} - y_n - \frac{1}{4} h y'_n - \frac{361}{40320} h y''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[-\frac{2509}{645120}\left(\frac{1}{8}\right) - \frac{29}{4032}\left(\frac{1}{4}\right) + \frac{153}{71680}\left(\frac{3}{8}\right) + \frac{4}{315}\left(\frac{1}{2}\right) + \frac{344}{315}(1) \right] \\
\sum_{j=0}^{\infty} \frac{\left(\frac{3}{8}\right)^j}{j!} - y_n - \frac{3}{8} h y'_n - \frac{8007}{573440} h y''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[\frac{31}{16128}\left(\frac{1}{8}\right) + \frac{11}{2520}\left(\frac{1}{4}\right) + \frac{15}{1792}\left(\frac{3}{8}\right) + \frac{8}{315}\left(\frac{1}{2}\right) - \frac{64}{63}(1) \right] \\
\sum_{j=0}^{\infty} \frac{\left(\frac{1}{2}\right)^j}{j!} - y_n - \frac{1}{2} h y'_n - \frac{191}{10080} h y''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[-\frac{1123}{2580480}\left(\frac{1}{8}\right) - \frac{41}{40320}\left(\frac{1}{4}\right) - \frac{477}{286720}\left(\frac{3}{8}\right) - \frac{1}{1008}\left(\frac{1}{2}\right) + \frac{191}{315}(1) \right] \\
\sum_{j=0}^{\infty} \frac{(1)^j}{j!} - y_n - h y'_n - \frac{79}{630} h y''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[\frac{107}{36126720}\left(\frac{1}{8}\right) + \frac{1}{14120}\left(\frac{1}{4}\right) + \frac{9}{802816}\left(\frac{3}{8}\right) + \frac{1}{70560}\left(\frac{1}{2}\right) + \frac{5}{441}(1) \right]
\end{bmatrix} \qquad (3.3)$$

Comparing the coefficient of h, according to [2] the order p of the new scheme and the error constant are given respectively by $p = [5 \quad 5 \quad 5 \quad 5 \quad 5]^T$ and $C_{p+2} = \left[-1.1274\times10^{-8} \quad -7.3165\times10^{-9} \quad -1.0644\times10^{-8} \quad -4.0367\times10^{-9} \quad -3.2294\times10^{-6}\right]$

3.2 Consistency of the Method

A numerical method is said to be consistent if the following conditions are satisfied.

i. The order of the method must be greater than or equal to zero to one i.e. $p \geq 1$.

ii. $\sum_{j=0}^{k} \alpha_j = 0$

iii. $\rho(r) = \rho'(r) = 0$

iv. $\rho'''(r) = 3!\sigma(r)$

Where $\rho(r)$ *and* $\sigma(r)$ are first and second characteristics polynomials of our method. According to [4], the first condition is a sufficient condition for the associated block method to be consistent. Hence the scheme is consistent.

3.3 Zero Stability of the Method

Definition 3.2: the numerical method is said to be zero-stable, if the roots $z_s,\ s = 1, 2, \cdots, k$ of the first characteristics polynomial $\rho(z)$ defined by $\rho(z) = \det\left(zA^{(0)} - E\right)$ satisfies $|z_s| \le 1$ and every root satisfies $|z_s| = 1$ have multiplicity not exceeding the order of the differential equation, [2]. The first characteristic polynomial is given by,

$$\rho(z) = \left| z\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right| = \left\| \begin{bmatrix} z & 0 & 0 & 0 & -1 \\ 0 & z & 0 & 0 & -1 \\ 0 & 0 & z & 0 & -1 \\ 0 & 0 & 0 & z & -1 \\ 0 & 0 & 0 & 0 & z-1 \end{bmatrix} \right\| = z^4(z-1)$$

Thus, solving for z in

$$z^4(z-1) \tag{3.4}$$

gives $z = 0, 0, 0, 0, 1$. Hence the scheme is said to be zero stable.

3.4 Convergence of the Block Method

Theorem 3.1: the necessary and sufficient conditions for linear multistep method to be convergent are that it must be consistent and zero-stable. Hence the scheme is consistent [4].

3.5 Region of Absolute Stability of our Method

Definition 3.3: the region of absolute stability is the region of the complex z plane, where $z = \lambda h$ for which the method is absolute stable. To determine the region of absolute stability of the block method, the methods that compare neither the computation of roots of a polynomial nor solving of simultaneous inequalities was adopted. Thus, the method according to [1] is called the boundary locus method. Applying this method we obtain the stability polynomial as

$$\bar{h}(w) = h^{10}\left(-\frac{53}{9909043200}w^4 - \frac{1}{52848230400}w^5\right) + h^8\left(-\frac{348679}{79272345600}w^4 - \frac{23}{11324620800}w^5\right)$$
$$+ h^6\left(-\frac{686209}{990904320}w^4 - \frac{41}{141557760}w^5\right) + h^4\left(-\frac{11}{114688}w^4 - \frac{11297}{344064}w^5\right) + h^4\left(-\frac{1}{56}w^4 - \frac{27}{56}w^5\right)$$
$$- w^4 + w^5$$

(3.5)

Applying the stability polynomial, we obtain the region of absolute stability in figure below.

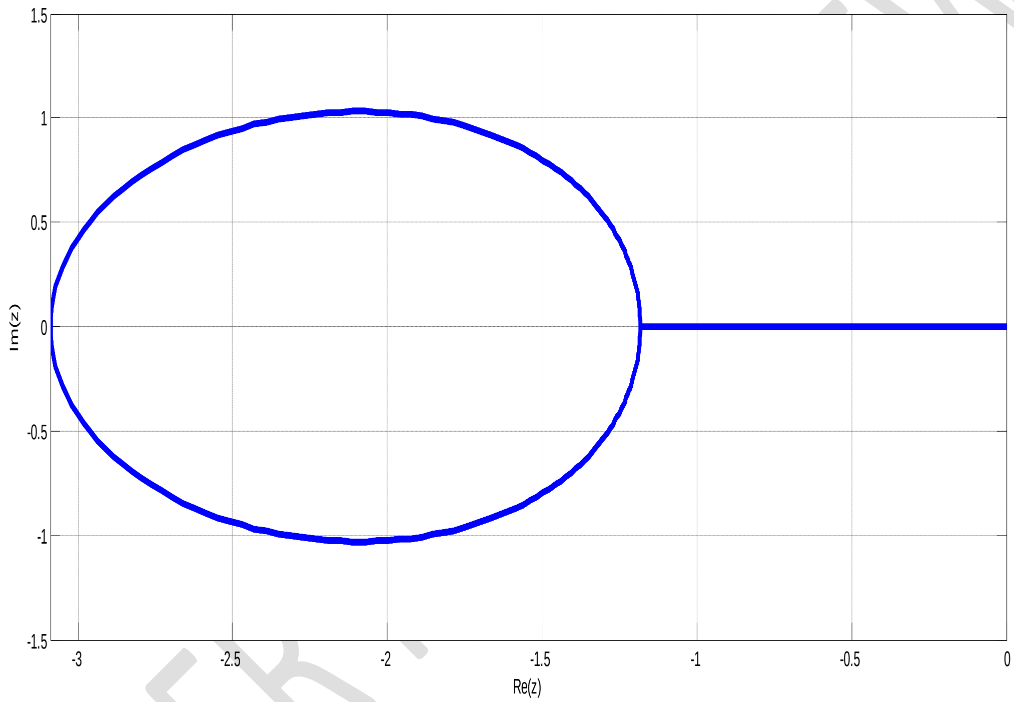

Fig 1. Region of absolute stability

4 Numerical Application and Results

4.1 Numerical Application of the Method

The Single step block method of order five was tested on some systems of second order initial value problems. Therefore, the conditions of solving (1.1) using the first order initial value problem has some computational burden which affect the accuracy of the scheme in terms of error and times constraints. To show performance of the scheme, this research has proposed the direct method.

Problem 1: The system of second order Mass Spring Motion real-life initial value problem defined below is considered.

A 128*lb* weight is attached to a spring having a spring constant of $64lb/ft$. The weight is started in motion with no initial velocity by displacing it $6inches$ above the equilibrium position and by simultaneously applying to the weight an external force $F_4(t) = 8\sin 4t$. Assuming no air resistance, compute the subsequent motion of the weight at $t : 0.01 \le t \le 0.10$.

Now, we model this problem into a mathematical model and then apply our method to compute the motion on the weight attached to the spring. Here,

$m = 4, k = 64, b = 0, \text{ and } F_4(t) = 8\sin 4t$

Thus, problem 1 boils down to

$$\frac{d^2y}{dt^2} + 16y = 2\sin 4t,\ y(0) = -\frac{1}{2},\ y'(0) = 0 \tag{4.1}$$

with the exact solution of (4.1) is given by,

$$y(t) = -\frac{1}{2}\cos 4t + \frac{1}{16}\sin 4t - \frac{1}{4}t\cos 4t \tag{4.2}$$

Source: [17].

Problem 2: The second order Simple Harmonic Motion is a linear real-life initial value problem defined as

An object stretches a spring 6 inches in equilibrium.

i. Set up the equation of motion and find its general solution.
ii. Find the displacement of the object for $t > 0$, if it's initially displaced 18 inches above equilibrium and given a downward velocity of $3\frac{ft}{s}$.

From Newton's second law of motion, we have

$$my'' + cy' + ky = F \tag{4.3}$$

By setting $c = 0$ and $F = 0$, we get

$$my'' + ky = 0 \Rightarrow y'' + \frac{k}{m}y = 0 \tag{4.3}$$

The equation of the weight of the object is given as follow:

$$mg = k\Delta l \Rightarrow \frac{k}{m} = \frac{g}{\Delta l} \tag{4.4}$$

Substituting $g = 32\frac{ft}{s^2}, \Delta l = \frac{6}{12} ft$ into (4.4) we obtain

$$\frac{k}{m} = \frac{32}{\frac{6}{12}} = 64 \tag{4.5}$$

Substituting equation (4.5) into the equation (4.3) we get

$$y'' + 64y = 0 \tag{4.6}$$

The initial upward displacement of 18 inches is positive and must be expressed in feet. The initial downward velocity is negative; thus, $y(0) = \frac{3}{2}, \ y'(0) = -3$ and $h = 0.1$. We make use of (4.6) as

$$dsolver\left(\left\{y''(t) + 64y(t) = 0, \ y(0) = \frac{3}{2}, \ y'(0) = -3\right\}\right) \tag{4.7}$$

We obtain the exact solution (4.7) as

$$y(t) = -\frac{3}{8}\sin(8t) + \frac{3}{2}\cos(8t) \tag{4.8}$$

Source [18].

Problem 3: The highly stiff second order linear initial value problem

$$y'' - y' = 0, \quad y(0) = 0, \ y'(0) = -1, \ h = 0.1 \tag{4.9}$$

is consider, with analytic solution is given by

$$y(t) = 1 - \exp(t) \tag{4.10}$$

Source [19, 20].

4.2 Results and Discussion

Table 1: Showing the results for second order Mass Spring Motion (4.1).

t	**Exact Solution**	**Computed Solution**	**EOM**	**E[17]**
0.1	- 0.49959872021047678004	-0.49959872021047678187	1.8300e-18	1.6621e-09
0.2	- 0.49839019330974949646	-0.49839019330974951095	1.4490e-17	1.1586e-08
0.3	- 0.49636836974027966301	-0.49636836974027970138	3.8370e-17	2.9743e-08
0.4	- 0.49352852660817937130	-0.49352852660817944509	7.3790e-17	5.6076e-08
0.5	- 0.48986728796894500998	-0.48986728796894513108	1.2110e-16	9.0504e-08
0.6	- 0.48538264289709933476	-0.48538264289709951530	1.8054e-16	1.3291e-07
0.7	- 0.48007396129056685722	-0.48007396129056710958	2.5236e-16	1.8317e-07
0.8	- 0.47394200736436189072	-0.47394200736436222748	3.3676e-16	2.4110e-07
0.9	- 0.46698895079202783994	-0.46698895079202827380	4.3386e-16	3.0653e-07
1.0	- 0.45921837545722401274	-0.45921837545722455653	5.4379e-16	3.7922e-07

See [17].

Table 2: Showing the results for second order Simple Harmonic Motion (4.7).

t	**Exact Solution**	**Computed Solution**	**EOM**	**E[18]**
0.1	0.77605152993342709579	0.77605164672277503203	1.1679e-07	3.3496e-07
0.2	- 0.41863938459249752594	- 0.41863866774780550794	7.1685e-07	1.6371e-06
0.3	- 1.3593892660185498469	- 1.35938835559453437100	9.1042e-07	3.2716e-06
0.4	- 1.4755518599067871611	- 1.47555182275661788490	3.7150e-08	3.5979e-06
0.5	- 0.69666449555494477770	- 0.69666609961950345728	1.6041e-06	1.3589e-06
0.6	0.50481020347261010590	0.50480740714035305148	2.7963e-06	2.9143-06
0.7	1.4000738069674951883	1.40007152976286630760	2.2772e-06	6.7226e-06
0.8	1.4460714263183540043	1.44607159476884231050	1.6845e-07	7.0589e-06
0.9	0.61490152285494961183	0.61490477930018535131	3.2565e-06	2.6543e-06
1.0	- 0.58925939319668845548	- 0.58925453185468652875	4.8613e-06	4.6056e-06

See [18].

Table 3: Showing the results for highly stiff second order initial value problem (4.9)

t	**Exact Solution**	**Computed Solution**	**EOM**	**E[19]**	**E[20]**
0.1	-0.1051709180756476248	-0.1051709180756476248	5.7670e-15	3.2482e-12	7.5650e-11
0.2	-0.2214027581601698339	-0.22140275816021784595	4.8012e-14	8.5643e-11	1.6017e-10
0.3	-0.3498588075760031040	-0.34985880757601930108	1.6197e-14	3.4401e-10	1.7600e-10
0.4	-0.4918246976412703178	-0.49182469764154388788	2.7357e-13	7.4251e-10	6.0784e-10
0.5	-0.6487212707001281468	-0.64872127070060400673	4.7586e-13	1.3785e-09	1.4729e-09
0.6	-0.8221188003905089749	-0.82211880039126191378	7.5294e-13	2.2193e-09	2.5336e-09
0.7	-1.0137527074704765216	-1.01375270747159482250	1.1183e-12	3.3875e-09	4.7876e-09
0.8	-1.2255409284924676046	-1.22554092849405505530	1.5875e-12	4.8470e-09	7.2770e-09
0.9	-1.4596031111569496638	-1.45960311115912784190	2.1782e-12	6.7518e-09	1.0170e-08
1.0	-1.7182818284590452354	-1.71828182846195610190	2.9109e-12	9.0628e-09	1.4827e-08

See [19, 20].

The new scheme (one-step) was confirmed on three highly stiff initial value problem, viz. Mass Spring Motion, Simple Harmonic Motion and highly stiff second order initial value problem. The new scheme is was applied on problem 1, (i.e. second order Mass Spring Motion) the result on table 1 is obviously shown the better convergence of our method than [17]. The application of second order Simple Harmonic Motion on problem 2 was confirmed on the new method and minimized the error than [18] as seen on table 2. And problem 3 is a highly stiff second order initial value problem and the result are evidently shown on table 3 with that if [19, 20].

The new scheme displayed its superiority by producing less error if compared to the existing work of [17-20] as shown in tables 1 to 3.

Summary and Conclusion

We have developed a single step block method using the power series polynomial for the direct solution second order initial value problem. The properties of the new scheme which include error constant, order, zero-stability, consistency and convergent are established and satisfied.

The new scheme was tested on some second order initial value problems and compared with the existing method of [17-20], displayed its superiority by producing less error if compared to the work as shown in tables 1 to 3.

Conclusively, the new scheme does not required much computation when compared with predictor corrector methods.

Therefore, further research can consider the application of power series method on k-step block method for the direct solution of higher order initial value problems.

References

1. Fatunla, S.O. (1988). Numerical Methods for Initial Values Problems in Ordinary Differential Equations. New York: Academic Press
2. Lambert, J.D, (1973). Computational methods for ordinary differential equations. John Wiley,New York.
3. Awoyemi, D.O. (1999). A Class of Continuous Methods for General Second Order Initial Value Problems in Ordinary Differential Equations. International Journal of Computer Mathematics.72:29-37.
4. Skwame, Y., Sabo, J. & Mathew, M. (2019). The treatment of second order ordinary differential equations using equidistant one-step block hybrid. *Asian Journal of Probability and Statistics*. 5(3): 1-9.
5. Skwame, Y., Donald, J. Z., Kyagya, T. Y. & Sabo, J. (2019). The double step hybrid linear multistep method for solving second order initial value problems. *Asian Research Journal of Mathematics.* 15(2): 1-11.
6. Kyagya, *T. Y.*, Raymond, *D.* & Sabo, J. (2021). Numerical application of ordinary differential equations using power series for solving higher order initial value problems. *FUW Trends in Science and Technology. 6(3): 868-876.*
7. Sabo, J., Kyagya, T. Y. & Vashawa, W. J. (2021). Numerical Simulation of One Step Block Method for Treatment of Second Order Forced Motions in Mass-Spring Systems. *Asian Journal of Research and Reviews in Physics*. 5(2): 1-11.
8. Milne, W.E. (1953). Numerical Solution of Differential Equations, John Wiley and Sons.
9. Jator SN. (2007).A sixth order linear multistep method for the direct solution of $y'' = f(x, y, y')$. International Journal of Pure and Applied Mathematics. 40(4): 457-472
10. Henrici, P. (1962). Discrete variable methods for ODE's.John Wiley, New York.
11. Sabo, J., Skwame, Y., Kyagya, T. Y. & Kwanamu, J. A. (2021). The direct simulation of third order linear problems on single step block method. *Asian Journal of Research in Computer Science.* 12(2): 1-12.

12. Adeyeye, O. and Omar, Z. (2017). 4-step 5 point hybrid block method for the direct solution of second order initial value problems. *Journal of Mathematics and Computer science*. 17, 527-534.

13. Skwame, Y., Donald, J. Z., Kyagya, T. Y. and Sabo, J. (2019). The double step hybrid linear multistep method for solving second order initial value problems. *Asian Research Journal of Mathematics.* 15(2): 1-11.

14. Omar, Z. and Adeyeye, O. (2016). Solving two-second order boundary value problems using two-step block method with starting and non-starting values. *International Journal of Applied Engineering Research.* 11(4), 2407-2410.

15. Ma X, Xie D, Yao L, Xie S. (2022). Extensions of single-step method for equations of motion from multi body dynamics. Mechanism and Machine Theory. 177(11), 105034.

16. Kayode, S. J. and Adegboro, J. O. (2018). Predictor-Corrector linear multistep method for direct solution of initial value problems of second order ordinary differential equations. *Asian Journal of Physical and Chemical Sciences.* 6(1), 1-9.

17. Skwame, Y., Bakari, A. I. and Sunday, J. (2017). Computational method for the determination of forced motions in mass-spring systems. *Asian Research Journal of Mathematics*. 3(1), 1-2.

18. Omole, E. A. and Ogunware, B. G. (2018). 3-point single hybrid block method (3PSHBM) for direct solution of general second order initial value problem of ordinary differential equations. *Journal of Scientific Research & Reports.* 20(3), 1-11.

19. Adeyefa, E.O., Adeniyi, R.B., Udoye, A.M. and Odafi, N.O. (2018). Orthogonal based zero-table numerical integrator for second order IVPs in ODEs. *International Journal of Pure and Applied Mathematics.* 3, 329-337.

20. Areo, E. A. and Rufai, M. A. (2016). An efficient one-eight step hybrid block method for solving second order initial value problems of ODEs. *International Journal of Differential Equation and Application.* 15(2), 117-139.