Original Research Article

A MIXED STRATEGY TO COVER CONTINUOUS ROI

ABSTRACT. Coverage in wireless sensor networks (WSNs) is a well-studied problem. In most of the situation WSNs have two-dimensional bounded continuous domain. In practice, sensors usually dropped randomly from air on previously determined points (called, vertices) of the domain (called Region of Interest or simply ROI). But since the deployment is random in manner the sensors will not place on the target vertices in most of the times. Hence ROI will not cover by the deployed sensors. The question is, how we reduced the area which is not covered by the sensors? Usually, extra sensors are dropped on some randomly chosen vertices to minimize the uncovered area. In our previous work we developed an alternative strategy, we reduced the distance between two adjacent vertices and drop exactly one sensor on each vertex. The amount of reduction depends on the number of extra sensors used. In that paper we compare uncovered area for the two strategies (the old one and the alternative one), for two distributions (uniform and normal), and for several number of excess sensors. Simulation result shows that alternative strategy is better for lower variance of the randomness but old one is better for higher variance. In this paper, we partitioned the ROI in regular hexagons and develop a new strategy, which is mixing of the above two strategies, for deployment of extra sensors. We divide the total number of extra sensors in two parts. One part is used for reducing the distance between the two adjacent vertices and other parts is used for deploying two sensors on some randomly selected vertices. Simulation results suggest the proper balancing between these two parts with respect to the variance of two distributions (uniform and normal).

1. Introduction

WSNs usually contain a huge number of small sensors, with some wireless receiver and processing circuit. Usually, the sensors are small in size with low battery capacity, processing power and radio power. The sensors are eligible for measures direction, humidity, speed, distance, temperature and other physical quantities. The most important feature of a WSNs is, they can be dropped randomly in an inaccessible region [6]. They also give opportunities for the military and civilian applications; like military tactical surveillance, industrial automation, emergency health care, security of nation, etc. [14]. Sensors are now used in IoT based smart physiotherapy system also [13]. Abdallah et.al. [1] described deployment of sensors for wireless connected things in indoor.

The main aim of WSNs is monitoring their nearby region for object tracking and event detection. For this reason, coverage is important for any wireless sensor network. To fulfil the task, a WSNs should cover the ROI, without any sensing hole [5]. A sensor can detect an event inside a circular region (known as sensing disc) of a prefixed radius (known as sensing radius). A point in ROI will not be covered by a WSN if that point is outside the sensing radius of the sensors of the WSN. We want to place the sensors in such a manner that there will be no sensing hole. However, we cannot expect that the sensors will be placed in a pre-defined vertex, as sensors are usually randomly dropped from the air. Wrong placement may happen due to some operational factors. Sensors are deployed in a continuous bounded subset of $\mathbb{R}^2$. Our object is to find a strategy to deploy the sensors in the ROI, such that the total area of sensing hole is minimized.

Key words and phrases. Sensor coverage; Wireless sensor; Sensor networks; Random deployment.

There are two different ways for deployment of sensors: (1) placement of sensors in a deterministic way and (2) dropping sensors from the air on the target points or vertices. In the first type of placement, ROI may be covered totally by an enough number of sensors. On the other hand, many points of ROI will not be covered even if a large number of sensors are randomly deployed. Observed that when the sensors are placed in a deterministic way, in a continuous domain, coverage problem is a geometrical problem.

In case of deployment from air, in general, robots (actuators) are used to cover the ROI or to reduce the uncovered area. The above type of network is usually referred as wireless sensor and actuator network (WSAN). In this network, sensors are deterministically placed and relocated by actuators [10]. In some situation, there are movable sensors, and the sensors can put themselves without the help of actuators. But movement of sensors need a huge amount of battery or other energy source, so movement assisted sensor placement is preferred [4]. In either situation our aim is to calculate the amount of uncovered area. In this paper, we develop a new strategy to deploy the sensors which minimize the uncovered area of ROI.

There are several algorithms in literature to place sensors efficiently for covering a convex region in $\mathbb{R}^2$. If the ROI is a bounded convex subset, the problem of covering of ROI is known as *coverage problem* or *covering problem*. Many variations of coverage problem is found in [3]. A survey on the above topics can be found in [8]. Younis et al. [16] gave a survey on the models and strategies that affect the sensor deployment. Analysis of maximum and expected distance covered by the actuators to achieve the full coverage can be found in [7, 10]. In all the previous literature, the uncovered region is covered, either by dropping extra sensors or using one or more robots, or by activating a group of passive sensors. Nandi et al. develop an new algorithm for robot to minimize the uncovered region [10].

2. Motivations

Many research was done on the following problem: 'Whether the ROI is totally covered or not?'. Moreover, if the region is not fully covered by sensors, then there are methods to cover the region using robots, extra sensors, movable sensors etc. But there are few works on the following problems: (i) 'How the uncovered region changed with respect to the number of nodes?' and (ii) 'How the uncovered region depends on the strategy of random dropping of the nodes?'. Note that we cannot give guarantee on the full coverage of the region even if we use extra sensors unless we relocate of the nodes either by movable sensors or by actuator(s). In this paper, we consider that there is no mobile sensor or actuator. Hence our main target is to find a strategy of deployment of sensors to minimized the uncovered area.

Observed that it is sufficient to cover each point of the ROI by not more than one sensor. Hence if some portion of the region is covered by greater than or equal to two sensors then it is in some sense 'wastage'. But since the sensing area of a node is a disc, hence we cannot have zero wastage. So, our goal is to reduce the wastage portion of the region. One general idea is, deploy the nodes in some deterministic points of ROI, such that if they are really placed on those pre-fixed points, in that case, the wastage is minimum. But after deployment there will be uncovered area due to the random deployment of the nodes. So we require some extra sensors.

Now the problem can be stated as follows, how we drop the sensors such that the wastage is minimum, i.e., how we use the extra nodes to reduce the uncovered area. Nandi and Sarkar find a solution of the above problem in $\mathbb{R}^2$ [9] and Nandi find the solution of the above problem in $\mathbb{R}^3$ [11]. In both papers, they assume that a sensor can be dropped at an arbitrary point of ROI. They also assume, the distance between the point where the sensor placed and the target point where we want to place, is a random variable. Now when we drop extra sensors at some randomly chosen point then the proportion of the uncovered area will decrease. On the other hand, if we decrease the distance between the neighbouring target points, but keeping the sensing radius unchanged, and placed exactly one sensor at each point, then also the uncovered area will decrease. The idea is, they use the extra sensors in 2 different ways in 2 different strategies. Two basic differences between the 2 strategies are as follows:

- In the first strategy, (say, St.1), they target to deploy two sensors on some randomly chosen centers and one sensor on to the rest. In the second one, (say, St.2), they deploy exactly one node or sensor on the target points. The sensing radius and number of sensors are equal for both the strategies.
- Let in St.1, there are n hexagons and k extra sensors are used, that is, total $n+k$ sensors is used in St.1. If the length of the sides is a in St.1, then in St.2 the length of the side will be b such that $(n+k)b^2 = na^2$. Hence the total area target to cover is same in the both cases. The distance between two target vertices is less in St.2.

In the above two papers they consider ROI as convex bounded subsets of $\mathbb{R}^n$, for $n = 2, 3$. The distance between the target point for a sensor and the point where the sensor is placed after deployment, considered as a random variable (D_i) whose probability distribution is either normal or uniform. They simulate the proportion of the uncovered area for above two distributions and for both strategies.

In coverage problem, usually hexagonal or square partition of the region is used for 2 dimensions. It is known that partitioning the ROI into regular hexagons is better than the any other. But after random deployment of extra nodes, hexagonal partition may not be better than the strategy of partitioning the ROI into congruence squares. In those papers, they consider hexagonal partition and cube centered partition only for 2 and 3 dimensions respectively. They show that the first strategy is better for distributions whose have higher variance and the second strategy is better for distributions whose have smaller variance.

In this paper, we partitioned the ROI in regular hexagons and develop a new strategy, which is mixing of the above two strategies, for deployment of extra sensors. We divide the total number of extra sensors in two parts. One part is used for reducing the distance between the two adjacent vertices and other parts is used for deploying two sensors on some randomly selected vertices. Simulation results suggest the proper balancing between these two parts with respect to the variance of two distributions (uniform and normal).

3. Assumptions and Definitions

Now we state our problem formally as follows: For an arbitrary index set J, consider the set of unit balls $\{C_j \subseteq \mathbb{R}^2 : j \in J\}$, which cover a convex and bounded subset of $\mathbb{R}^2$. This set is considered to be ROI. Consider a collection of 2-dimensional random vectors $\{Y_j : j \in J\}$. Let D_j is the distance between Y_j and the center of C_j, for $j \in J$. Assume that D_js are i.i.d. with p.d.f. $f(\cdot)$. Now the question is 'what portion of ROI will be uncovered?' Here we consider two probability distributions for D_i, normal and uniform.

Assume the ROI is partitioned into a number of congruent regular hexagons of side length a. To cover each hexagon using exactly one sensor we must take $a \leq r$ (r be the sensing radius). If $r = a$ each hexagon will be covered by exactly one sensor when it is placed on the center of the hexagon. Assume that the sensors are too small and that can be think as a point.

In this paper we consider a new strategy for dropping or deployment of the extra sensors. We divide the total number of extra sensors in two parts. One part is used for reducing the distance between the two adjacent vertices and other parts is used for deploying two sensors on some randomly selected vertices. Let in there are n hexagons and l extra sensors are used, that is, total $n+l$ sensors is used. Let $k(< l)$ number of sensors is used for reducing the distance between two adjacent vertices and $l-k$ sensors are used for dropping two sensors on $l-k$ randomly chosen vertices. If the initial length of the sides of the hexagon (i.e., the distance between two adjacent vertices) is a, then the length of the side will be reduced to b where $(n+k)b^2 = na^2$. Hence the total area target to cover is same in the both cases. Next we define few important terms:

- *Node* is that point where a particular sensor is placed after the deployment. We use the word 'node' to mean that point where a typical sensor placed, as well as the respective sensor.
- *Vertex* is that point where a sensor is to target to place.
- $N(W)$ is the node which corresponds to a vertex W, that is, a sensor is placed on $N(V)$ but the target was to place at W.
- $V(M)$ is the respective vertex of a node M.
- *Sensing Disc* S_N of a node N is a closed disc of radius r with center N, which is covered by the sensor placed at that node.
- The radius of the disc, r is known as *sensing radius.* Sensing radius is assumed to be same for all discs. Throughout the paper, by the word 'disc' we consider closed discs only. In higher dimensions we call it as *sensing ball.*
- *Adjacent vertex* of a vertex is that vertex which is at the distance not more than twice of the sensing radius from the aforesaid vertex. Therefore, the sensing disc of a node has non empty intersection with the sensing disc of its adjacent nodes and empty intersection with the sensing disc of a node which is not an adjacent node.
- $\mathcal{W}$ is set of all vertices and Adj_W is set of all the adjacent vertices of vertex W. Similar definitions and notations apply for nodes also. The respective notations are $\mathcal{N}$ and Adj_N for $N \in \mathcal{N}$.
- The distance between two points A and B is denoted by $d(A, B)$.
- A point $A \in \mathbb{R}^n$ is said to be covered by a node N if $d(A, N) \leq r$ and the point A is said to be covered by a set of nodes $\mathcal{N}$ if A is covered by at least one node in $\mathcal{N}$. A point $A \in \mathbb{R}^n$ is said to be uncovered by a node N if it not covered by N and the point P is said to be uncovered by $\mathcal{N}$ if P is not covered by any nodes in $\mathcal{N}$.
- *Sensing hole* in ROI (respectively, $\mathbb{R}^n$) is a connected subset of ROI (respectively, $\mathbb{R}^n$) whose elements are uncovered by $\mathcal{N}$.
- *Adjacent sensing hole* of a node means the sensing hole whose boundary intersects with the boundary of the sensor disc of that node.
- ROI will be called covered by a set of nodes if every point of ROI is covered by at least one node.
- Area of a set S will be denoted as $\mathrm{A}(S)$.

Observe that if there is no randomness, that is, if a sensor is placed on exact point, then a vertex and its corresponding node is same, $V(N) = N$ and $N(V) = V$.

We shall now define the most important term called 'wastage'. Let S be any bounded set in $\mathbb{R}^2$, which is covered by a finite set of sensors or nodes $\mathcal{N}$. The wastage in S for $\mathcal{N}$ is define as follows

$$W_{\mathcal{N}}(S) = \frac{\sum_{M \in \mathcal{N}} \mathrm{A}(S \cap S_M) - \mathrm{A}(S)}{\sum_{M \in \mathcal{N}} \mathrm{A}(S \cap S_M)}.$$

If $\mathcal{N}$ is a set so that $|S_{N_1} \cap S_{N_2} \cap S_{N_3}| \leq 1$ for distinct $N_1, N_2, N_3 \in \mathcal{N}$, then

$$W_{\mathcal{N}}(S) = \frac{\sum_{N_1 \neq N_2 \in \mathcal{N}} \mathrm{A}(S \cap S_{N_1} \cap S_{N_2})}{\sum_{M \in \mathcal{N}} \mathrm{A}(S \cap S_M)}.$$

Intuitively, the denominator of the above expression is the total area, which is common with S, and the numerator is the wastage in area. Here 'wastage' represent the ratio of wastage area and the total area.

Let $\mathcal{N}$ be the set of sensors or nodes which cover whole $\mathbb{R}^2$ and $\mathcal{N} \cap S$ is a finite set for all bounded set S. We define wastage in $\mathbb{R}^2$ for $\mathcal{N}$ as follows

$$W_{\mathcal{N}}(\mathbb{R}^2) = \lim_{y \to \infty} W_{\mathcal{N} \cap B_y}(B_y),$$

where B_y be the ball in $\mathbb{R}^2$ of radius y and centered at origin.

Intuitively, wastage in $\mathbb{R}^2$ is the proportion of wastage volume in $\mathbb{R}^2$. Note that we can take an increasing sequence of sets whose limit (union) is $\mathbb{R}^2$ other than B_x. As for example, partitioned $\mathbb{R}^2$ into hexagons and then take an increasing sequence of union of finitely many such hexagons with the property that limit (union) of this sequence is $\mathbb{R}^2$. In that case we can define wastage similarly. It can be proved that these two definitions are equivalent.

4. Simulation Result

In this section, we describe the simulation procedure and the data we get from these simulations.

Observed that the radius of the sensing disc (r) has no role in simulation study. We consider, in our simulation, 10000 nodes with $r = 1$. Partition ROI as regular hexagon and try to deploy a sensor at the center of each regular hexagon. Clearly the total area is $10000 \times \frac{3\sqrt{3}}{2}$ unit. Also two adjacent vertices has distance $\sqrt{3}$ unit. 100 sensors are placed in each row. We assume that the distance D_i between the target vertex and respective node are i.i.d. either uniform or normal. The uniform distribution whose p.d.f. is $f(x) = \frac{2x}{t^2} I_{(0,t)}$ denoted by $U(t)$ and $N(0, t^2)$ be the normal distribution with expectation 0 and standard deviation t.

Let $(p_1 + p_2)\%$ extra sensors have been used, hence the total number of sensors is $10000 \times \left(1 + \frac{p_1+p_2}{100}\right)$ where $(p_1 + p_2) \in [0, 100]$. The new strategy is as follows:

Partition ROI in to $10000\left(1 + \frac{p_1}{100}\right)$ regular hexagon of side $\sqrt{\frac{100}{100+p_1}}$. Consider $10000\left(1 + \frac{p_1}{100}\right)$ centers of these hexagons as vertices and deploy one sensor exactly for every vertices. Observe that the area of the whole region is $10000\left(1 + \frac{p_1}{100}\right) \times \frac{3\sqrt{3}}{2}\left(\sqrt{\frac{100}{100+p_1}}\right)^2 = 10000 \times \frac{3\sqrt{3}}{2}$.

Next pick $100p_2$ vertices uniformly and randomly from the $10000\left(1 + \frac{p_1}{100}\right)$ and deploy two sensors on to each of the selected vertices and deploy one sensor for other $(10000+100p_1-100p_2)$ vertices. We simulate the ratio of uncovered area of ROI with different values of p_1, p_2 and t for uniform and normal distribution, where p_1, p_2, t are fraction between 0 and 1. We repeat the simulation for a fixed set of three parameters p_1, p_2 and t for 10000 times and take the average of the ratios. Then we find the average of ratios for different sets of three parameters p_1, p_2, t and for uniform or normal distribution.

We have the following tables 1 to 4 showing the proportion of uncovered area of ROI (written in the body) with different values of p_1, p_2 for $U(0.5), U(1.0), N(0, 0.1)$ and $N(0, 0.25)$ respectively. Here first row represent the value of p_1 and first column represent the value of p_2.

Table 1. Simulation result of proportion of the coverage area for $U(0.5)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.97090	0.97103	0.97371	0.97811	0.98251	0.98502	0.98527	0.98642	0.98831	0.98890	0.99150
0.05	0.96901	0.97392	0.97734	0.98012	0.98521	0.98585	0.98641	0.98841	0.98907	0.99155	0.99159
0.10	0.97590	0.97712	0.97998	0.98531	0.98618	0.98651	0.98854	0.98920	0.99156	0.99161	0.99173
0.15	0.97752	0.97976	0.98582	0.98681	0.98690	0.98860	0.98952	0.99159	0.99165	0.99182	0.99202
0.20	0.98112	0.98601	0.98729	0.98695	0.98862	0.98982	0.99161	0.99168	0.99190	0.99211	0.99359
0.25	0.98527	0.98829	0.98892	0.98893	0.98990	0.99160	0.99169	0.99194	0.99243	0.99361	0.99447
0.30	0.98789	0.98875	0.98897	0.98991	0.99161	0.99172	0.99197	0.99256	0.99364	0.99453	0.99529
0.35	0.98865	0.98998	0.99021	0.99168	0.99178	0.99202	0.99298	0.99370	0.99458	0.99531	0.99572
0.40	0.98992	0.99096	0.99171	0.99180	0.99200	0.99301	0.99378	0.99462	0.99538	0.99587	0.99651
0.45	0.99098	0.99180	0.99185	0.99200	0.99300	0.99389	0.99461	0.99540	0.99597	0.99686	0.99701
0.50	0.99179	0.99189	0.99201	0.99312	0.99397	0.99477	0.99541	0.99601	0.99699	0.99741	0.99800

TABLE 2. Simulation result of proportion of the coverage area for $U(1.0)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.92341	0.92918	0.93350	0.93680	0.94500	0.94876	0.95321	0.95676	0.95753	0.95932	0.96247
0.05	0.92904	0.93412	0.93762	0.94519	0.94901	0.95410	0.95782	0.95799	0.96071	0.96350	0.96765
0.10	0.93380	0.93942	0.94597	0.94998	0.95512	0.95802	0.95843	0.96226	0.96454	0.96854	0.97139
0.15	0.94231	0.94625	0.95016	0.95601	0.95897	0.95957	0.96352	0.96589	0.96932	0.97199	0.97731
0.20	0.94601	0.95321	0.95754	0.95964	0.96187	0.96432	0.96672	0.97031	0.97267	0.97719	0.98001
0.25	0.95676	0.95807	0.96002	0.96352	0.96517	0.96749	0.97110	0.97318	0.97801	0.98062	0.98236
0.30	0.95997	0.96187	0.96529	0.96602	0.96831	0.97192	0.97400	0.97893	0.98135	0.98311	0.98471
0.35	0.96302	0.96754	0.96725	0.96932	0.97275	0.97532	0.97971	0.98211	0.98389	0.98542	0.98666
0.40	0.96896	0.96843	0.97076	0.97327	0.97687	0.98056	0.98301	0.98467	0.98608	0.98732	0.98831
0.45	0.97003	0.97197	0.97401	0.97802	0.98148	0.98399	0.98542	0.98692	0.98809	0.98902	0.99001
0.50	0.97300	0.97521	0.97901	0.98231	0.98489	0.98611	0.98755	0.98897	0.98999	0.99053	0.99102

TABLE 3. Simulation result of proportion of the coverage area for $N(0, 0.1)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.96569	0.96881	0.97079	0.97591	0.97649	0.97870	0.97976	0.98231	0.98421	0.98520	0.98694
0.05	0.96849	0.97021	0.97542	0.97772	0.97983	0.98001	0.98341	0.98534	0.98597	0.98705	0.98721
0.10	0.96951	0.97387	0.97875	0.98099	0.98081	0.98403	0.98607	0.98653	0.98749	0.98897	0.98911
0.15	0.97230	0.97997	0.98189	0.98205	0.98571	0.98701	0.98712	0.98764	0.98812	0.99001	0.99119
0.20	0.97968	0.98264	0.98310	0.98610	0.98826	0.99839	0.99841	0.98901	0.99100	0.99197	0.99208
0.25	0.98230	0.98421	0.98735	0.98921	0.98953	0.98960	0.98998	0.99241	0.99243	0.99241	0.99294
0.30	0.98543	0.98812	0.99033	0.99034	0.99035	0.99058	0.99098	0.99241	0.99278	0.99307	0.99452
0.35	0.98901	0.99021	0.99035	0.99036	0.99081	0.99149	0.99258	0.99301	0.99378	0.99498	0.99537
0.40	0.98997	0.99025	0.99041	0.99173	0.99261	0.99309	0.99372	0.99450	0.99501	0.99568	0.99619
0.45	0.99054	0.99060	0.99281	0.99324	0.99380	0.99421	0.99471	0.99513	0.99581	0.99674	0.99765
0.50	0.99132	0.99219	0.99310	0.99388	0.99412	0.99490	0.99541	0.99601	0.99721	0.99804	0.99900

TABLE 4. Simulation result of proportion of the coverage area for $N(0, 0.25)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.93848	0.94550	0.94750	0.95048	0.95502	0.95770	0.96321	0.96432	0.96764	0.97001	0.97052
0.05	0.94401	0.94732	0.95164	0.95709	0.95887	0.96398	0.96444	0.96814	0.97136	0.97156	0.97290
0.10	0.94700	0.95287	0.95851	0.95998	0.96405	0.96453	0.96887	0.97202	0.97245	0.97378	0.97463
0.15	0.95402	0.96021	0.96169	0.96483	0.96488	0.96853	0.97303	0.97343	0.97456	0.97524	0.97595
0.20	0.96184	0.96301	0.96557	0.96500	0.96961	0.97387	0.97456	0.97532	0.97609	0.9673	0.97747
0.25	0.96452	0.96638	0.96574	0.96050	0.97467	0.97572	0.97614	0.97692	0.97737	0.97831	0.97931
0.30	0.96712	0.96621	0.97153	0.97542	0.97654	0.97699	0.97769	0.97829	0.97907	0.97989	0.98042
0.35	0.96678	0.97128	0.97621	0.97731	0.97786	0.97852	0.97901	0.97982	0.98075	0.98134	0.98196
0.40	0.97055	0.97532	0.97802	0.97864	0.97932	0.97980	0.98051	0.98163	0.98222	0.98273	0.98328
0.45	0.97425	0.97898	0.97976	0.98021	0.98074	0.98163	0.98257	0.98301	0.98356	0.98414	0.98532
0.50	0.97873	0.97965	0.98045	0.98134	0.98245	0.98321	0.98389	0.98434	0.98509	0.98600	0.98623

From the table one can find the optimal choice of the p_1 and p_2 for a fixed value of $p = p_1 + p_2$ for 4 different distributions. As for example, consider the distribution $U(0.5)$.

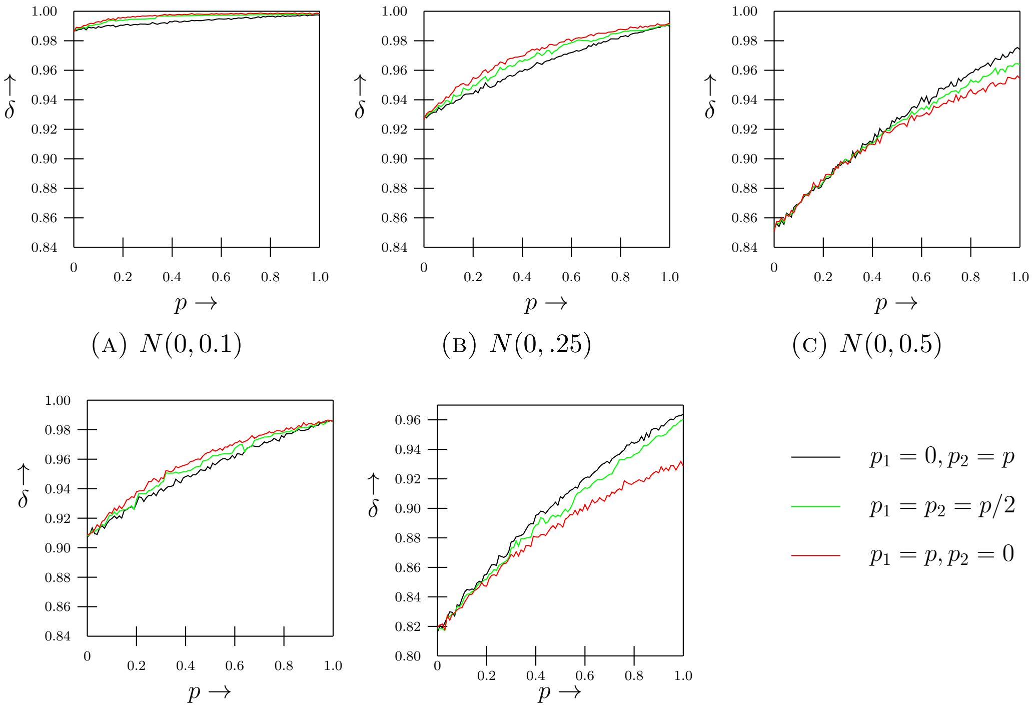
For $p = 0.05$ when $p_1 = 0$ and $p_2 = 0.05$ the proportion of covered area is 0.96901, whereas when $p_1 = 0.05$ and $p_2 = 0$ the proportion of covered area is 0.97103. Hence the second choice is better than the first one.

For $p = 0.25$ when $p_1 = 0.05$ and $p_2 = 0.20$ the proportion of covered area is 0.98601, which is the optimal choice of p_1 and p_2. Note that if we refine the intervals of p_1 and p_2 we shall get more precise optimal choice of p_1 and p_2.

Next, we draw the 'proportion of covered area (δ)' vs. 'p' graphs for five distributions and three different values of p_1 and p_2 (see Figure 1a to 1e). The red, blue and black curves show the values of proportion of covered area for different values of p and for $(p_1, p_2) = (p, 0)$, $(p_1, p_2) = (p/2, p/2)$ and $(p_1, p_2) = (0, p)$ respectively. From the curves we can find the optimal choice of p_1 and p_2. Note that one can draw more curves for different choices of p_1 and p_2 to get more precise optimal choice of p_1 and p_2.

TABLE 5. Simulation result of proportion of the coverage area for $N(0, 0.5)$

$p_1 \rightarrow$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.00	0.92843	0.93503	0.93502	0.93948	0.94021	0.94377	0.95132	0.95343	0.95676	0.95900	0.96105
0.05	0.93344	0.93473	0.94051	0.94579	0.95588	0.95639	0.95644	0.95814	0.96071	0.96176	0.96372
0.10	0.93647	0.94152	0.94758	0.94859	0.95364	0.95464	0.95688	0.96072	0.96172	0.96378	0.96574
0.15	0.94454	0.94660	0.94761	0.95264	0.95464	0.95868	0.96473	0.96734	0.96874	0.96875	0.96975
0.20	0.95061	0.95363	0.95655	0.95665	0.96096	0.96473	0.96574	0.96753	0.96876	0.96973	0.97074
0.25	0.95645	0.95866	0.95965	0.96050	0.96774	0.96875	0.96914	0.97069	0.97373	0.97783	0.97951
0.30	0.96071	0.96162	0.96715	0.96875	0.96954	0.97169	0.97376	0.97282	0.97590	0.97698	0.98002
0.35	0.96067	0.96712	0.96762	0.96877	0.96877	0.96978	0.97090	0.97298	0.97580	0.97814	0.98019
0.40	0.96705	0.96753	0.968782	0.96864	0.97093	0.97398	0.97580	0.97816	0.97822	0.97982	0.98032
0.45	0.97025	0.97289	0.97197	0.97802	0.97980	0.98016	0.98025	0.98130	0.98235	0.98341	0.98452
0.50	0.97083	0.97296	0.97384	0.97813	0.97825	0.98032	0.98138	0.98434	0.98309	0.98460	0.98563



(A) $N(0, 0.1)$ (B) $N(0, .25)$ (C) $N(0, 0.5)$

(D) $U(0.5)$ (E) $U(1.0)$

FIGURE 1. Graph (from simulation data) of proportion of coverage area in $\mathbb{R}^2$

5. Conclusion

In the current paper, we try to solve the coverage or covering problem in $\mathbb{R}^2$. We consider that sensors or nodes may not be properly placed at the required target point but may be placed at any point in the plane. We assume that the distance between these two points follows i.i.d. We consider the distribution either uniform or normal. For these two distributions we have done computer simulations. To reduce the uncovered area, we have introduced a new strategies using extra sensors and have compared this strategy with the other strategies. We divide the total number of extra sensors in two parts. One part is used for reducing the distance between the two adjacent vertices and other parts is used for deploying two sensors on some randomly selected vertices. Simulation results suggest the proper balancing between these two parts with respect to the variance of two distributions (uniform and normal).

Here are some possible future works:

- In this paper we consider two dimensions. In future one may consider deployment of sensors for higher dimensional coverage problem.

- In future one may think ROI as a square grid structure and the distributions like two dimensional exponential distribution for deployment.
- In this paper we consider a new strategy. There are many others, which may be better, for other distributions. In future one can classify the distributions and strategies with respect to different types of partitions and distributions.
- In this paper we did not use the actuators. In future one may try to solve coverage problem using extra sensors and actuators.

References

[1] W. Abdallah, S. Mnasri and Thierry Val, Distributed Approach for the Indoor Deployment of Wireless Connected Objects by the Hybridization of the Voronoi Diagram and the Genetic Algorithm. J. Engg. Res. and Sci. 1(2), 10-23 (2022); 2022.

[2] D.S. Deif and Y.Gadallah, Classification of wireless sensor networks deployment techniques. IEEE Commun. Surv. Tutor. 16:834–855. doi: 10.1109/SURV.2013.091213.00018, 2013.

[3] R. Elhabyan, W. Shi, M. St-Hilaire, Coverage protocols for wireless sensor networks: Review and future directions. J. Commun. Netw. 2019;21:45–60. doi: 10.1109/JCN.2019.000005.

[4] R. Falcon, X. Li and A. Nayak, Carrier-Based Focused Coverage Formation in Wireless Sensor and Robot Networks. IEEE Trans. on Automatic Control. 56(10):2406–2417, 2011.

[5] A. Filippou, D.A. Karras and R.C. Papademetriou, Coverage Problem for Sensor Networks: An Overview of Solution Strategies. Proc. 17th Telecommunication Forum (TELFOR), 2009.

[6] J. Guo, H. Jafarkhani, Movement-Efficient Sensor Deployment in Wireless Sensor Networks With Limited Communication Range. IEEE Trans. Wirel. Commun. 18:3469–3484. doi: 10.1109/TWC.2019.2914199, 2019.

[7] E. Kranakis, D. Krizanc, O. M. Ponce, L. Narayanan, J. Opatrny and S. Shende, Expected Sum and Maximum of Displacement of Random Sensors for Coverage of a Domain. SPAA. 73-82, 2013.

[8] X. Li, A. Nayak, D. Simplot-Ryl and I. Stojmenovic, Sensor Placement in Sensor and Actuator Networks. Wireless Sensor and Actuator Networks: Algorithms and Protocols for Scalable Coordination and Data Communication, John Wiley and Sons. 2010.

[9] M. Nandi and S. Sarkar, Coverage Problem of sensor network in Continuous Region. IEEE Xplore. 13(1-2): 105-120, 2011.

[10] M. Nandi and X. Li, Coverage problem of sensor networks. Ad Hoc and Sensor Wireless Networks. 978-1-7281-1349-4-010, 2015.

[11] M. Nandi, Packing Problem and Random Coverage in Continuous Domain Asian Journal of Research in Computer Science. Volume 16, Issue 3, Page 76-86, 2023.

[12] M. Nandi, S. Parui and A. Adhikary. The domination numbers of cylindrical grid graphs. Applied Mathematics and Computation. 217: 4879–4889, 2011.

[13] A. A. Saleem, K. Zafar, M. A. Raza, Z. Kareem, Mui-zzud-din, H. Ur R. Siddiqui and S. Dudley. IoT Based Smart Physiotherapy System: A Review. J. Engg. Res. and Sci. 1(10), 45-55 (2022); 20022.

[14] R. Shakila, B. Paramasivan, Performance Analysis of Submarine Detection in Underwater Wireless Sensor Networks for Naval Application. Microprocess. Microsyst. doi: 10.1016/j.micpro.2020.103293. 2020.

[15] M. Srbinovska, C. Gavrovski, V. Dimcev, A. Krkoleva and V. Borozan, Environmental parameters monitoring in precision agriculture using wireless sensor networks. J. Clean. Prod. 88:297–307. doi: 10.1016/j.jclepro.2014.04.036. 2015.

[16] M. Younis and K. Akkaya, Strategies and techniques for node placement in wireless sensor placement in wireless sensor networks: A survey. Ad Hoc Networks. 6(4): 621-655, 2008.