__Original Research Article__

Minimum Transversal Eccentric Dominating Energy of Graphs

ABSTRACT

For a graph $G = (V, E)$, the minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G)$ is the sum of the eigenvalues obtained from the minimum transversal eccentric dominating $n \times n$ matrix $\mathbb{M}_{ted}(G) = (m_{ij})$. In this paper $\mathbb{E}_{ted}(G)$ of standard graphs are computed. Properties, upper and lower bounds for $\mathbb{E}_{ted}(G)$ are established.

Keywords: Transversal domination, eccentricity, transversal eccentric domination number, minimum transversal eccentric dominating set, eigenvalues, energy.

1. INTRODUCTION

In 1978 I. Gutman[1] introduced energy of a graph. Inspired by Gutman many authors have explored different types of energy in graph theory. M.R. Rajesh Kanna et al[2] found the minimum dominating energy of a graph. For a graph $G = (V, E)$, let $A = (a_{ij})$ be the minimum dominating matrix defined by

$$(a_{ij}) = \begin{cases} 1, if\ v_i v_j \in E, \\ 1, if\ i = j\ and\ v_i \in D, \\ 0, otherwise \end{cases}$$

and let $\lambda_1, \lambda_2, \lambda_3, \dots \lambda_n$ are the eigenvalues of A, then minimum dominating energy is $E_D(G) = \sum_{i=1}^{n} |\lambda_i|$. Eccentric domination was introduced by T.N. Janakiraman et al[3] in 2010. For a graph $G = (V, E)$, a set $S \subseteq V$ is said to be a dominating set, if every vertex in $V - S$ is adjacent to some vertex in S. The eccentricity $e(v)$ of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v): u \in V\}$. For a vertex v, each vertex at a distance $e(v)$ from v is an eccentric vertex. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G): d(u, v) = e(v)\}$. A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric vertex of v in D. An eccentric dominating set with minimum cardinality is called a minimum eccentric dominating set. The eccentric domination number $\gamma_{ed}(G)$ of a graph G equals the minimum cardinality of an eccentric dominating set. Nayaka S.R et al[4] introduced transversal domination in graphs. Riyaz Ur Rehman A and A Mohamed Ismayil [5] introduced transversal eccentric domination in graphs. Tejaskumar R, A Mohamed Ismayil and Ivan Gutman[8] introduced 'minimum eccentric dominating energy of graphs'. Inspired by Tejaskumar et al we introduce minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G)$ of graphs. In this paper we find $\mathbb{E}_{ted}(G)$ of standard graphs.

2. THE MINIMUM TRANSVERSAL ECCENTRIC DOMINATING ENERGY-$\mathbb{E}_{ted}(G)$

Definition 2.1: Let $G = (V, E)$ be a simple graph where $V(G) = \{v_1, v_2, \dots v_n\}$ where $n \in \mathbb{N}$ is the set of vertices and E is the set of edges. Let D be a minimum transversal eccentric dominating set of G then the minimum transversal eccentric dominating matrix of G is a $n \times n$ matrix defined by $\mathbb{M}_{ted}(G) = (m_{ij})$, where

$$(m_{ij}) = \begin{cases} 1, if\ v_i\ or\ v_j \cap D \neq \emptyset\ and\ v_i \in E(v_j)\ or\ v_j \in E(v_i), \\ 1, if\ i = j\ and\ v_i \in D, \\ 0, otherwise \end{cases}$$

Definition 2.2: The characteristic polynomial of $\mathbb{M}_{ted}(G)$ is denoted by $\mathcal{P}_n(G, \alpha) = \det(\mathbb{M}_{ted}(G) - \alpha I)$.

Definition 2.3: The minimum transversal eccentric dominating eigenvalues of G are the eigenvalues of minimum transversal eccentric dominating matrix $\mathbb{M}_{ted}(G)$. Since $\mathbb{M}_{ted}(G)$ is symmetric and real. We label the eigen values in non-increasing order $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$.

Definition 2.4: The minimum transversal eccentric dominating energy of G is defined by $\mathbb{E}_{ted}(G) = \sum_{i=1}^{n} |\alpha_i|$.

Remark 2.1: The trace of $\mathbb{M}_{ted}(G)$=Transversal eccentric domination number.

Example 2.1:

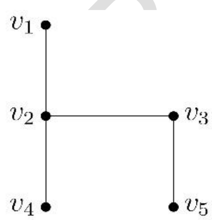


Fig 1. Fork graph

Table 1. The vertex with Eccentricity $e(v)$ and Eccentric vertex $E(v)$

Vertex	Eccentricity $e(v)$	Eccentric vertex $E(v)$
v_1	3	v_5
v_2	2	v_5
v_3	2	v_1, v_4
v_4	3	v_5
v_5	3	v_1, v_4

The minimum transversal eccentric dominating sets of fork graph are $D_1 = \{v_1, v_2, v_5\}$, $D_2 = \{v_1, v_4, v_5\}$, $D_3 = \{v_2, v_3, v_5\}$ and $D_4 = \{v_2, v_4, v_5\}$.

1. $D_1 = \{v_1, v_2, v_5\}$,

$$\mathbb{M}_{ted}(G) = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 5\alpha + 1$.

Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.705$, $\alpha_2 \approx 1.3835$, $\alpha_3 \approx 0.5102$, $\alpha_4 \approx -0.4599$, $\alpha_5 \approx -1.1388$.
Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.1974$.
2. $D_2 = \{v_1, v_4, v_5\}$,

$$\mathbb{M}_{ted}(G) = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 5\alpha + 1$.
Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.705$, $\alpha_2 \approx 1.3835$, $\alpha_3 \approx 0.5102$, $\alpha_4 \approx -0.4599$, $\alpha_5 \approx -1.1388$.
Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.1974$.

3. $D_3 = \{v_2, v_3, v_5\}$,

$$\mathbb{M}_{ted}(G) = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 6\alpha^2 + \alpha + 1$.
Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.5563$, $\alpha_2 \approx 1.5063$, $\alpha_3 \approx 0.5896$, $\alpha_4 \approx -0.3342$, $\alpha_5 \approx -1.138$.
Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.3044$.

4. $D_3 = \{v_2, v_4, v_5\}$,

$$\mathbb{M}_{ted}(G) = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 6\alpha^2 + \alpha + 1$.
Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.5563$, $\alpha_2 \approx 1.5063$, $\alpha_3 \approx 0.5896$, $\alpha_4 \approx -0.3342$, $\alpha_5 \approx -1.138$.
Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.3044$.

Remark 2.2: The minimum transversal eccentric dominating energy depends on the minimum transversal eccentric dominating set.

3. MINIMUM TRANSVERSAL ECCENTRIC DOMINATING ENERGY OF SOME STANDARD GRAPHS

In this section we find the $\mathbb{E}_{ted}(G)$ of complete graph, cocktail party and crown graph.

Theorem 3.1: For a complete graph K_n where $n > 2$ the minimum transversal eccentric dominating energy of a complete graph is $\mathbb{E}_{ted}(K_n) = n$.
Proof: Let K_n be a complete graph with the vertex set $V = \{v_1, v_2, \dots v_n\}$. The minimum transversal eccentric dominating set is $D = \{v_1\}$, then

$$\mathbb{M}_{ted}(K_n) = \begin{pmatrix} 1 & 1 & 1 & & 1 & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 0 & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & 0 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 & 1 \\ 1 & 1 & 1 & & 1 & 1 & 0 \end{pmatrix}_{n\times n}$$

Characteristic polynomial is $\mathcal{P}_n(K_n, \alpha) = \det(\mathbb{M}_{ted}(K_n) - \alpha I)$.

$$= \begin{vmatrix} 1-\psi & 1 & 1 & & 1 & 1 & 1 \\ 1 & -\psi & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & -\psi & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & -\psi & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & -\psi & 1 \\ 1 & 1 & 1 & & 1 & 1 & -\psi \end{vmatrix}$$

The characteristic equation is $\mathcal{P}_n(K_n, \alpha) = (-1)^n \alpha^n - (-1)^n n \alpha^{n-1}$.

The minimum transversal eccentric dominating eigenvalues are

$$\alpha = 0,$$
$$\alpha = n.$$

The minimum transversal eccentric dominating energy of the complete graph K_n is given by $\mathbb{E}_{ted}(K_n) = n$.

Theorem 3.2: For a cocktail party where $n \geq 4$ and crown graph G where $n \geq 6$ the minimum transversal eccentric dominating energy

$\mathbb{E}_{ted}(G) = 2 + \left|\frac{1+\sqrt{5}}{2}\right| \left[\left(\frac{n}{2}\right) - 1\right] + \left|\frac{1-\sqrt{5}}{2}\right| \left[\left(\frac{n}{2}\right) - 1\right]$.

Proof: Let G be a graph with the vertex set $V = \{v_1, v_2, \dots v_n\}$. The minimum transversal eccentric dominating set is $D = \left\{v_1, v_2, \dots, v_{\frac{n}{2}}\right\}$, $|D| = n/2$ then

$$\mathbb{M}_{ted}(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & & 0 & 1 & 0 & 0 \\ & \vdots & & & \ddots & & \vdots & & \\ 0 & 0 & 1 & 0 & & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & & 0 & 0 & 0 & 0 \end{pmatrix}_{n\times n}$$

Characteristic polynomial is $\mathcal{P}_n(G, \alpha) = \det(\mathbb{M}_{ted}(G) - \alpha I)$.

$$= \begin{vmatrix} 1-\alpha & 0 & 0 & 0 & & 0 & 0 & 1 & 0 \\ 0 & 1-\alpha & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1-\alpha & 0 & & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\alpha & & 0 & 1 & 0 & 0 \\ & \vdots & & & \ddots & & \vdots & & \\ 0 & 0 & 1 & 0 & & 1-\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & -\alpha & 0 & 0 \\ 1 & 0 & 0 & 0 & & 0 & 0 & -\alpha & 0 \\ 0 & 1 & 0 & 0 & & 0 & 0 & 0 & -\alpha \end{vmatrix}$$

The characteristic equation is $\mathcal{P}_n(G, \alpha) = \alpha(\alpha - 2)(\alpha^2 - \alpha - 1)^{\frac{n-2}{n}}$.

The minimum transversal eccentric dominating eigenvalues are

$$\alpha = 0,$$
$$\alpha = 2,$$

$$\alpha = \frac{1+\sqrt{5}}{2} \left[\left(\frac{n}{2}-1\right) \, times\right],$$
$$\alpha = \frac{1-\sqrt{5}}{2} \left[\left(\frac{n}{2}-1\right) \, times\right],$$

The minimum transversal eccentric dominating energy of the cocktail party and crown graph G is given by

$\mathbb{E}_{ted}(G) = 0 + 2 + \frac{1+\sqrt{5}}{2}\left[\left(\frac{n}{2}\right) - 1\right] + \frac{1-\sqrt{5}}{2}\left[\left(\frac{n}{2}\right) - 1\right].$

$\mathbb{E}_{ted}(G) = 2 + \frac{1+\sqrt{5}}{2}\left[\left(\frac{n}{2}\right) - 1\right] + \frac{1-\sqrt{5}}{2}\left[\left(\frac{n}{2}\right) - 1\right].$

4. Properties of Minimum Transversal Eccentric Dominating Eigenvalues

In this section we discuss the properties of eigenvalues of $\mathbb{M}_{ted}(G)$ for complete, crown and cocktail party graphs.

Theorem 4.1: Let D be a minimum transversal eccentric dominating set and $\alpha_1, \alpha_2, \dots \alpha_n$ are the eigenvalues of minimum transversal eccentric dominating matrix $\mathbb{M}_{ted}(G)$ then

1. For any graph G, $\sum_{i=1}^{n} \alpha_i = |D|$,
2. For a complete graph K_n where $n > 2$, $\sum_{i=1}^{n} \alpha_i^2 = |D| + (n)(n-1)$,
3. For a crown and cocktail party graph G then $\sum_{i=1}^{n} \alpha_i^2 = |D| + n$.

Proof:

1. We know that the sum of eigenvalues of $\mathbb{M}_{ted}(G)$ is the trace of $\mathbb{M}_{ted}(G)$.

$$\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} m_{ii} = |D|.$$

2. Similarly, for a complete graph K_n sum of square of eigen values of $\mathbb{M}_{ted}(G)$ is trace of $[\mathbb{M}_{ted}(G)]^2$

$$\sum_{i=1}^{n} \alpha_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij} m_{ij}$$
$$\sum_{i=1}^{n} \alpha_i^2 = \sum_{i=1}^{n} (m_{ii})^2 + \sum_{i\neq j} m_{ij} m_{ij}$$
$$\sum_{i=1}^{n} \alpha_i^2 = \sum_{i=1}^{n} (m_{ii})^2 + 2\sum_{i<j} \left(m_{ij}\right)^2$$
$$\sum_{i=1}^{n} \alpha_i^2 = |D| + (n)(n-1)$$

Since for a complete graph K_n, $2\sum_{i<j}\left(m_{ij}\right)^2 = (n)(n-1)$.

3. Similarly, for a crown and cocktail party graph G sum of square of eigenvalues of $\mathbb{M}_{ted}(G)$ is trace of $[\mathbb{M}_{ted}(G)]^2$

$$\sum_{i=1}^{n} \alpha_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij} m_{ij}$$
$$\sum_{i=1}^{n} \alpha_i^2 = \sum_{i=1}^{n} (m_{ii})^2 + \sum_{i\neq j} m_{ij} m_{ij}$$
$$\sum_{i=1}^{n} \alpha_i^2 = \sum_{i=1}^{n} (m_{ii})^2 + 2\sum_{i<j} \left(m_{ij}\right)^2$$
$$\sum_{i=1}^{n} \alpha_i^2 = |D| + n$$

Since for a crown and cocktail party graph G, $2\sum_{i<j}\left(m_{ij}\right)^2 = n$.

5. Bounds for Minimum Transversal Eccentric Dominating Energy

Similarly to McClelland's[6] bounds for energy of a graph, bounds for $\mathbb{E}_{ted}(G)$ are given in this section.

Theorem 5.1: For a complete graph K_n where $n > 1$, if D be the minimum transversal eccentric dominating set and $W = |\det \mathbb{M}_{ted}(G)|$ then
$\sqrt{|D| + n(n-1) + n(n-1)W^{2/n}} \leq \mathbb{E}_{ted}(K_n) \leq \sqrt{n(n(n-1) + |D|)}$.
Proof: By Cauchy schwarz inequality $(\sum_{i=1}^{n} g_i h_i)^2 \leq (\sum_{i=1}^{n} g_i^2)(\sum_{i=1}^{n} h_i^2)$. If $g_i = 1$ and $h_i = \alpha_i$ then

$$\left(\sum_{i=1}^{n} |\alpha_i|\right)^2 \leq \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} \alpha_i^2\right)$$
$$(\mathbb{E}_{ted}(G))^2 \leq n(|D| + n(n-1))$$
$$\Rightarrow \mathbb{E}_{ted}(G) \leq \sqrt{n(|D| + n(n-1))}$$

Since the arithmetic mean is not smaller than geometric mean we have

$$\frac{1}{n(n-1)}\sum_{i\neq j} |\alpha_i||\alpha_j| \geq \left[\prod_{i\neq j} |\alpha_i||\alpha_j|\right]^{\frac{1}{n(n-1)}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j} |\alpha_i||\alpha_j| = \left[\prod_{i=1}^{n} |\alpha_i|^{2(n-1)}\right]^{\frac{1}{n(n-1)}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j} |\alpha_i||\alpha_j| = \left[\prod_{i=1}^{n} |\alpha_i|\right]^{\frac{2}{n}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j} |\alpha_i||\alpha_j| = \left[\prod_{i=1}^{n} \alpha_i\right]^{\frac{2}{n}}$$
$$\frac{1}{n(n-1)}\sum_{i\neq j} |\alpha_i||\alpha_j| = |\det \mathbb{M}_{ted}(G)|^{\frac{2}{n}} = W^{\frac{2}{n}}$$
$$\sum_{i\neq j} |\alpha_i||\alpha_j| \geq n(n-1)W^{\frac{2}{n}}$$

Now consider

$$(\mathbb{E}_{ted}(G))^2 = \left(\sum_{i=1}^{n} |\alpha_i|\right)^2$$
$$(\mathbb{E}_{ted}(G))^2 = \left(\sum_{i=1}^{n} |\alpha_i|\right)^2 + \sum_{i\neq j} |\alpha_i||\alpha_j|$$
$$(\mathbb{E}_{ted}(G))^2 = (|D| + n(n-1)) + n(n-1)W^{\frac{2}{n}}$$
$$\mathbb{E}_{ted}(G) \geq \sqrt{(|D| + n(n-1)) + n(n-1)W^{\frac{2}{n}}}$$

Theorem 5.2: For a crown graph where $n \geq 6$ and cocktail party graph G where $n \geq 4$, be the minimum transversal eccentric dominating set and $W = |\det \mathbb{M}_{ted}(G)|$ then
$\sqrt{|D| + n + n(n-1)W^{\frac{2}{n}}} \leq \mathbb{E}_{ted}(G) \leq \sqrt{n(n + |D|)}$.
Proof: The proof follows on the similar lines of theorem-5.1.

Theorem 5.3: If $\alpha_1(G)$ is the largest minimum transversal eccentric dominating eigenvalue of $\mathbb{M}_{ted}(G)$ then

1. For a complete graph K_n, $\alpha_1(G) \geq \frac{|D|+n(n-1)}{n}$,

2. For a crown and cocktail party graph G, $\alpha_1(G) \geq \frac{|D|+n}{n}$.

Proof:

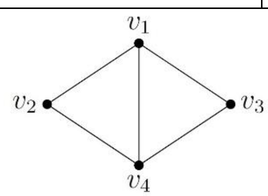
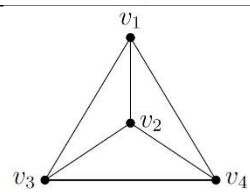
1. Let Y be a non-zero vector, then by ref.[7], we have $\alpha_1\left(\mathbb{M}_{ted}(G)\right) = \max_{Y \neq 0} \frac{Y^T \mathbb{M}_{ted}(G) Y}{Y^T Y}$.

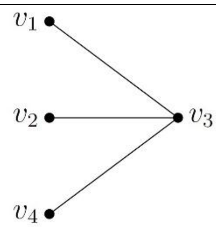
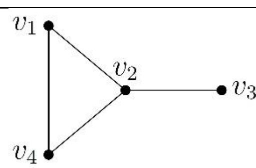
$\alpha_1\left(\mathbb{M}_{ted}(G)\right) \geq \frac{U^T \mathbb{M}_{ted}(G) U}{U^T U} = \frac{|D|+n(n-1)}{n}$ where U is the unit matrix.

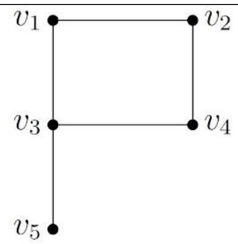
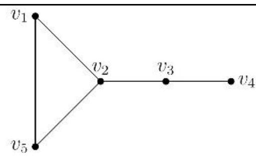
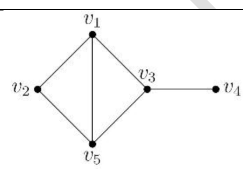
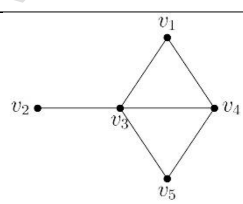
2. Let Y be a non-zero vector, then by ref.[7], we have $\mu_1\left(\mathbb{M}_{ted}(G)\right) = \max_{Y \neq 0} \frac{Y^T \mathbb{M}_{ted}(G) Y}{Y^T Y}$.

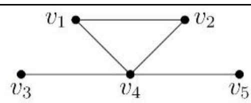
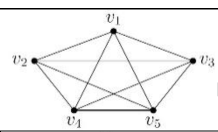
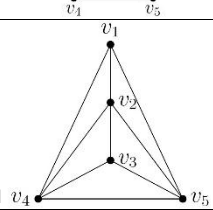
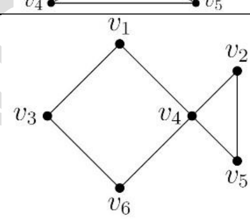
$\mu_1\left(\mathbb{M}_{ted}(G)\right) \geq \frac{U^T \mathbb{M}_{ted}(G) U}{U^T U} = \frac{|D|+n}{n}$ where U is the unit matrix.

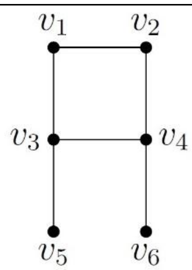
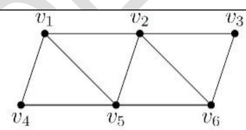
Table 2. Characteristic equation $\mathcal{P}_n(G,\alpha)$, Roots $\alpha(G)$ and Energy $\mathbb{E}_{ted}(G)$ of Minimum TED sets of various standard graphs are tabulated.

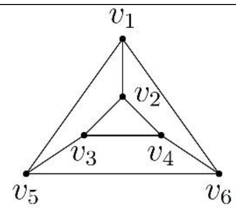
Graph	Figure	Minimum TED set	Characteristic equation $\mathcal{P}_n(G,\alpha)$	Roots $\alpha(G)$	Energy $\mathbb{E}_{ted}(G)$
Diamond graph		$\{v_2, v_3\}$,	$\alpha^4 - 2\alpha^3 - 4\alpha^2$.	$\alpha_1 = 3.2361$, $\alpha_2 = 0$, $\alpha_3 = -1.2361$.	4
Tetrahedral graph		$\{v_1, v_2, v_3, v_4\}$,	$\alpha^4 - 2\alpha^3$.	$\alpha_1 = 3.2361$, $\alpha_2 = 0$.	4

Claw graph		$\{v_1, v_3\}$,	$\alpha^4 - 2\alpha^3 - 4\alpha^2$,	$\alpha_1 = 3.2361$, $\alpha_2 = 0$, $\alpha_3 = -1.2361$.	4.4722
		$\{v_2, v_3\}$,	$\alpha^4 - 2\alpha^3 - 4\alpha^2 + \alpha + 1$,	$\alpha_1 = 3.1401$, $\alpha_2 = 0.5712$, $\alpha_3 = -0.4378$, $\alpha_4 = -1.2735$.	5.3866
		$\{v_3, v_4\}$.	$\alpha^4 - 2\alpha^3 - 4\alpha^2 + \alpha + 1$.	$\alpha_1 = 3.1401$, $\alpha_2 = 0.5712$, $\alpha_3 = -0.4378$, $\alpha_4 = -1.2735$.	5.3866
Paw graph		$\{v_1, v_3\}$,	$\alpha^4 - 2\alpha^3 - 3\alpha^2 + \alpha + 1$,	$\alpha_1 = 2.8794$, $\alpha_2 = 0.6527$, $\alpha_3 = -0.5321$, $\alpha_4 = -1$.	5.0642
		$\{v_2, v_3\}$,	$\alpha^4 - 2\alpha^3 - 3\alpha^2 + \alpha + 1$,	$\alpha_1 = 2.8794$, $\alpha_2 = 0.6527$, $\alpha_3 = -0.5321$, $\alpha_4 = -1$.	5.0642
		$\{v_3, v_4\}$.	$\alpha^4 - 2\alpha^3 - 3\alpha^2 + 2\alpha + 2$.	$\alpha_1 = 2.7321$, $\alpha_2 = 1$, $\alpha_3 = -0.7321$, $\alpha_4 = -1$.	5.4642

Banner graph		$\{v_2, v_5\}$.	$-\alpha^5 + 2\alpha^4 + 3\alpha^3 - 3\alpha^2 - 2\alpha$.	$\alpha_1 = 2.5962$, $\alpha_2 = 1.1826$, $\alpha_3 = 0$, $\alpha_4 = -0.5157$, $\alpha_5 = -1.2631$.	5.5576
(3,2)Tadpole graph		$\{v_1, v_4\}$, $\{v_4, v_5\}$.	$-\alpha^5 + 2\alpha^4 + 3\alpha^3 - 3\alpha^2 - 2\alpha$, $-\alpha^5 + 2\alpha^4 + 3\alpha^3 - 4\alpha^2 - \alpha + 1$.	$\alpha_1 = 2.5962$, $\alpha_2 = 1.1826$, $\alpha_3 = 0$, $\alpha_4 = -0.5157$, $\alpha_5 = -1.2631$. $\alpha_1 = 2.5231$, $\alpha_2 = 1$, $\alpha_3 = 0.4851$, $\alpha_4 = -0.5669$, $\alpha_5 = -1.4413$.	5.5576 6.0164
Kite graph		$\{v_2, v_4\}$.	$-\alpha^5 + 2\alpha^4 + 3\alpha^3 - 3\alpha^2 - 2\alpha$.	$\alpha_1 = 2.5962$, $\alpha_2 = 1.1826$, $\alpha_3 = 0$, $\alpha_4 = -0.5157$, $\alpha_5 = -1.2631$.	5.5576
Dart graph		$\{v_2, v_3\}$, $\{v_2, v_4\}$.	$-\alpha^5 + 2\alpha^4 + 6\alpha^3$, $-\alpha^5 + 2\alpha^4 + 6\alpha^3 - 2\alpha^2 - 2\alpha$.	$\alpha_1 = 3.6458$, $\alpha_2 = 0$, $\alpha_3 = -1.6458$.	5.2916 6.41

				$\alpha_1 = 3.6458$, $\alpha_2 = 0.7018$, $\alpha_3 = 0$, $\alpha_4 = -0.4685$, $\alpha_5 = -1.7365$.	
Cricket graph		$\{v_3, v_4\}$,	$-\alpha^5 + 2\alpha^4 + 6\alpha^3$,	$\alpha_1 = 3.6458$, $\alpha_2 = 0$, $\alpha_3 = -1.6458$.	5.2916
		$\{v_4, v_5\}$.	$-\alpha^5 + 2\alpha^4 + 6\alpha^3 - 2\alpha^2 - 2\alpha$.	$\alpha_1 = 3.5032$, $\alpha_2 = 0.7018$, $\alpha_3 = 0$, $\alpha_4 = -0.4685$, $\alpha_5 = -1.7365$.	6.41
Pentatope graph		$\{v_1, v_2, v_3, v_4, v_5\}$.	$-\alpha^5 + 5\alpha^4$.	$\alpha_1 = 5$, $\alpha_2 = 0$.	5
Johnson solid skeleton 12 graph		$\{v_1, v_3\}$.	$-\alpha^5 + 2\alpha^4 + 6\alpha^3$.	$\alpha_1 = 3.6458$, $\alpha_2 = 0$, $\alpha_3 = -1.6458$.	5.2916
Fish graph		$\{v_2, v_3\}$,	$\alpha^6 - 2\alpha^5 - 4\alpha^4 + 4\alpha^3 + 4\alpha^2$,	$\alpha_1 = 2.7321$, $\alpha_2 = 1.4142$, $\alpha_3 = 0$, $\alpha_4 = -0.7321$, $\alpha_5 = -1.4142$.	6.2926
		$\{v_3, v_5\}$.	$\alpha^6 - 2\alpha^5 - 4\alpha^4 + 6\alpha^3 + 2\alpha^2 - 2\alpha$.		6.7824

				$\alpha_1 = 2.5772$, $\alpha_2 = 1.292$, $\alpha_3 = 0.522$, $\alpha_4 = 0$, $\alpha_5 = -0.6677$, $\alpha_6 = -1.7235$.	
A graph		$\{v_1, v_5, v_6\}$,	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 9\alpha^3 + \alpha^2 - 4\alpha - 1$,	$\alpha_1 = 3.0233$, $\alpha_2 = 1.3883$, $\alpha_3 = 1$, $\alpha_4 = -0.2853$, $\alpha_5 = -0.5198$, $\alpha_6 = -1.6065$.	6.823
		$\{v_2, v_5, v_6\}$.	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 10\alpha^3 - \alpha^2 - 3\alpha$.	$\alpha_1 = 2.9446$, $\alpha_2 = 1.541$, $\alpha_3 = 0.769$, $\alpha_4 = 0$, $\alpha_5 = -0.4862$, $\alpha_6 = -1.769$.	7.5092
4-polynomial graph		$\{v_3, v_4\}$.	$\alpha^6 - 2\alpha^5 - 4\alpha^4 + 4\alpha^3 + 4\alpha^2$.	$\alpha_1 = 2.7321$, $\alpha_2 = 1.4142$, $\alpha_3 = 0$, $\alpha_4 = -0.7321$, $\alpha_5 = -1.4142$.	6.2926

3-prism graph		$\{v_1, v_5, v_6\}$,	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 11\alpha^3 + 3\alpha^2 - 9\alpha - 4$,	$\alpha_1 = 2.5616$, $\alpha_2 = 1.618$, $\alpha_3 = 1.618$, $\alpha_4 = -0.618$, $\alpha_5 = -0.618$, $\alpha_6 = -1.5616$.	8.5952
		$\{v_2, v_3, v_4\}$.	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 11\alpha^3 + 3\alpha^2 - 9\alpha - 4$.	$\alpha_1 = 2.5616$, $\alpha_2 = 1.618$, $\alpha_3 = 1.618$, $\alpha_4 = -0.618$, $\alpha_5 = -0.618$, $\alpha_6 = -1.5616$.	8.5952

4. CONCLUSION

In this paper minimum transversal eccentric dominating energy of graph is introduced. The transversal eccentric dominating energy of some standard graphs are calculated. Results related to the upper and lower bound of the energy of standard graphs is stated and proved.

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