The Proposed Buys-Ballot Estimates for Multiplicative Model with the Error Terms

Abstract: This article presents the condition(s) under which the multiplicative model with error terms and variances best describes the pattern in an observed time series, while comparing it with those of the additive and mixed models. The method of estimation is based on the periodic, seasonal and overall averages and variances of time series data arranged in a Buys-Ballot table. The method assumes that (1) the underlying distribution of the variable, X_{ij}, $i = 1, 2, ..., m$, $j = 1, 2, ..., s$, under study is normal. (2) the trending curve is linear (3) the decomposition method is multiplicative. Result shows that, under the stated assumptions, the seasonal variances of the Buys-Ballot table, for multiplicative model, a function of column (j) through the seasonal component s_j^2 with error variance.
Keywords: Time Series Decomposition, Trend-Cycle Component, Multiplicative Model, Buys-Ballot Estimates, Appropriate Model.

1.0 Introduction

Description method involves examination of trend, seasonality, cycles, turning points, changes in level, trend and scale and so on that may influence the series. This is also very vital preliminary to modelling, when it has to be decided whether and how to seasonally adjust, to transform, and to deal with outliers and whether to fit a model. In the examination of trend, seasonality and cycles, a time series is often described as having trends, seasonal effects, cyclic pattern and irregular or random component. Theoretically, a time series contains four components, namely, the trend, the seasonal variation, the cyclical and irregular variation. The trend may be loosely defined as the long term change in the mean and refers to the general direction in which the graph of the series appeared to be going over a long interval of time. Trend shows the presence of factors that persist for a considerable duration. These factors include changes in population, fluctuation in price level, improvements in technology and several conditions that are peculiar to individual investments or establishment. Abrupt or sudden changes in trend may be caused by introduction of new element into or elimination of an old factors from forces affecting the series [1].

1.1 Buys-Ballot Procedure for Time Series Decomposition

According to [2] the successive periodic mean $\left(\bar{X}_i\right)$ provide a simple description of the underlying trend from the methods of monthly or quarterly means. It was shown that the estimates of the seasonal indices can be obtained from the column means $\bar{X}_{.j}$. Therefore, while the periodic means give estimate of the trend, the column means gives estimates of seasonal indices. It has been observed that a time series theoretically contains four components, However, if short period time are involved, the trend components is estimated into the cyclical and trend cycle component is obtained and denoted by m_t. Under these conditions, it can be stated that estimates of trend-cycle and seasonal components can be obtained from the row and column means, respectively, of the Buys-Ballot table. These estimates have been designated "Buys-Ballot" estimates in this study the details of the procedure for estimation of the trend-cycle component (m_t) are presented in section 2 for the multiplicative model.

The Buys - Ballot table helps in the assessment of the trend – cycle and seasonal indices of time series data. The row means ($\bar{X}_{i.}$) estimate trend, and the differences ($\bar{X}_{.j} - \bar{X}_{..}$) or the ratio $\left(\frac{\bar{X}_{.j}}{\bar{X}_{..}}\right)$ between the column means ($\bar{X}_{.j}$) and the overall mean ($\bar{X}_{..}$) estimate the seasonal indices. [2] proposed the use of the Buys - Ballot table for inspecting time series data for the presence of trend and seasonal effects. [3] provided a new estimation procedure based on row, column and overall averages of the Buys - Ballot table. This method, called Buys – Ballot estimation procedure uses the periodic mean ($\bar{X}_{i.}$, $i =$

$1,2,\ldots,m$) and the overall mean ($\bar{X}_{..}$) to estimate the trend component. Seasonal means ($\bar{X}_{.j}, j = 1,2,\ldots,s$) and the overall mean ($\bar{X}_{..}$) are used to estimate the seasonal indices.

The method of coefficient of variation of seasonal differences and quotient was proposed by Justo and Rivera [4]. The seasonal differences was calculated by taking the difference between a certain season of a period and the same season from the period before while the seasonal quotient was calculated as the quotient of a certain season of the period and the same season from the period before. Iwueze and Nwogu [5] demonstrated that when the trend cycle component is linear, the seasonal variances of the Buys-Ballot are constant for the additive model, but contain the seasonal indices for the multiplicative model. Therefore, choice between additive and multiplicative models reduces to test for constant variance can be used to identify the additive model. Therefore, they suggested that any test of constant variance can be used to identify the test that admits the additive model. This is an improvement over what is in existence. However, this approach can only identify the additive model when the column variance is constant, but does tell the analyst the alternative model when the variance is not constant.

2.0 Methodology

The estimation procedure for multiplicative model with the error terms in this study are done using Buys-Ballot method often referred to in the literature. This method adopted in this study assumed that the series are arranged in a Buys-Ballot table with m rows and s columns. For details of this method see Wei [6], Nwogu *et.al* [7], Dozie and Ihekuna [8], Dozie *et.al* [9], Dozie and Nwanya [10], Dozie [11], Dozie and Ijeomah [12], Dozie and Ibebuogu (13), Dozie and Uwaezuoke [14], Dozie and Ihekuna [15] Dozie and Ibebuogu [16] Akpanta and Iwueze [17], Iwueze and Nwogu [5]

2.1 Estimation Procedure of Means for Multiplicative Model with the Error Terms

This method is developed for short term of period in which the trend and cyclical component are jointly combined with the consideration of error term. For multiplicative model with the error terms, we obtain

$$X_{(i-1)s+j} = M_{(i-1)s+j} \times S_{(i-1)s+j} \times e_{(i-1)s+j} \qquad (1)$$

Using Buys-ballot table with m-rows and s- columns;

$$t = (i-1)s + j, \quad i = 1,2,...,m, \quad j = 1,2,...,s$$

For convenience, let

$$X_{ij} = X_{(i-1)s+j}, \; M_{ij} = M_{(i-1)s+j}, \; e_{ij} = e_{(i-1)s+j}$$

$$M_{ij} = a + b[(i-1)s + j] \text{ and } X_{ij} = M_{ij}.S_j.e_{ij}$$

$$= \{ \, a + b[(i-1)s + j] \, \} S_j \times e_{ij} \qquad (2)$$

In deriving an expression for row average with error term, we make use of the assumption

$\sum_{j=1}^{s} S_{t+j} = s$, Now, the ith row average is given as

$$\bar{X}_{i.} = \sum_{j=1}^{s} \left[\, \{ \, a + b[(i-1)s + j] \, \} S_j \times e_{ij} \, \right]$$

$$= s\left[a - b\left(s - \frac{1}{s}\sum_{j=1}^{s} jS_j \right) + (bs)i \right]$$

$$= \left[a - bs + \frac{b}{s}\sum_{j=1}^{s} jS_j + bsi \right] \bar{e}_{i.} \qquad (3)$$

Next, we obtain an expression for the column mean with error term. With

$\sum_{j=1}^{s} S_{t+j} = mS_j$, the jth column mean becomes

$$\bar{X}_{.j} = \frac{1}{m}\sum_{i=1}^{m} M_{ij} \times S_j \times \bar{e}_{.j} \qquad (4)$$

$$= \sum_{i=1}^{m} \{a + b[(i-1)s + j]\} S_j \times e_{ij}$$

$$= m\left[a + b\left(\frac{n-s}{2}\right) + \frac{b}{s}\sum_{j=1}^{s} jS_j \right]$$

$$= \left[a\bar{e}_j + \frac{bs}{m}\sum_{i=1}^{m} ie_{ij} - bs\bar{e}_{.j} + bj\bar{e}_{.j} \right] S_j \qquad (5)$$

Furthermore, the grand mean is obtained to be

$$\bar{X}_{..} = \frac{1}{m}\sum_{i=1}^{m} \bar{X}_{i.} \qquad (6)$$

$$= \sum_{i=1}^{m} \left\{ s\left[a - b\left(s - \frac{1}{s}\sum_{j=1}^{s} jS_j \right) + (bs)i \right] \right\}$$

$$= n\left[a + \frac{b}{2}(n-s) + \frac{b}{s}\sum_{j=1}^{s} jS_j \right]$$

Thus, the grand mean is

$$= a + \frac{b}{2}(n-s) + \frac{b}{2}\sum_{j=1}^{s} jS_j \qquad (7)$$

The estimates of row, column and overall averages of the Buys-Ballot table for multiplicative model with the error terms are given in Table 1

Table 1: Estimates of Means for Multiplicative Model with the Error Terms

Measures	Multiplicative Model $(M_t = a + bt)$
$\bar{X}_{i.}$	$\left[a - bs + \frac{b}{s}\sum_{j=1}^{s} jS_j + bsi \right] * \bar{e}_{i.}$
$\bar{X}_{.j}$	$\left[a\bar{e}_{.j} + \frac{bs}{m}\sum_{i=1}^{m} ie_{ij} - bs\bar{e}_{,j} + bj\bar{e}_{.j} \right] * S_j$

$\overline{X}_{..}$	$a + b\left(\frac{n-s}{2}\right) + bC_1$

Source: Nwogu *et al* (2019) and Dozie, *et al*, (2020)

$$C_1 = \frac{1}{s}\sum_{j=1}^{s} jS_j$$

2.1 Estimation Procedure of Variances for Multiplicative Model with the Error Variances

From equations (1) and (2), we derive the expressions of row, column and overall variances

for multiplicative model with the error variances.

$$\hat{\sigma}_{i.}^2 = \frac{1}{s-1}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{i.}\right)^2 \quad (8)$$

$$(s-1)\hat{\sigma}_{i.}^2 = \sum_{j=1}^{s}\left(M_{ij} \times S_j \times e_{ij} - \frac{1}{s}\sum_{j=1}^{s} M_{ij} \times S_j \times \bar{e}_{i.}\right)^2$$

$$M_{ij}S_j = a + b[(i-1)s + j]S_j + bjS_j$$

$$\frac{1}{s}\sum_{j=1}^{s} M_{ij}S_j = [a + bs(i-1)s] + \frac{b}{s}\sum_{j=1}^{s} jS_j$$

$$(s-1)\hat{\sigma}_{i.}^2 = \sum_{j=1}^{s}\left(M_{ij}S_j e_{ij} - \frac{1}{s}\sum_{j=1}^{s} M_{ij}S_j\,\bar{e}_{i.}\right)^2$$

$$= \sum_{j=1}^{s}\left\{ [a + bs(i-1)]S_j e_{ij} + bjS_j e_{ij} - \left([a + bs(i-1)s] + \frac{b}{s}\sum_{j=1}^{s} jS_j\,\bar{e}_{i.} \right) \right\}^2$$

$$= -b\left(\frac{s-1}{2}\right) + b_j + S_j + \left(e_{ij} - \bar{e}_{i.}\right)$$

$$= b^2 s\left(\frac{s+1}{2}\right) + \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \sigma_1^2 \quad (9)$$

Therefore, the row mean is

$$= \left[\; [a + bs(i-1) + bc_1]^2 + \text{var}[[a + bs(i-1)]s_j + bjS_j] \right] \; \sigma_2^2 \qquad (10)$$

For the seasonal variance, we have

$$\hat{\sigma}_{.j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij} - \bar{X}_{.j} \right)^2 \qquad (11)$$

$$= \frac{1}{m-1}\sum_{i=1}^{m}\left\{ \; [\; a + bs(i-1) + bj \;\;]S_j \times .e_{ij} - \left(a + b\left(\frac{n-s}{2}\right) + bj + S_j + \bar{e}_{.j} \right) \right\}^2$$

$$= \frac{1}{m-1}\sum_{i=1}^{m}\left\{ \; \left(a + bs(i-1) - a - b\left(\frac{n-s}{2}\right) \; \right)S_j + bjS_j + \left(e_{ij} - \bar{e}_{.j} \right) \right\}^2$$

Hence, the seasonal mean is

$$= \left[\frac{b^2\left(n^2 - s^2\right)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + b_j \right]^2 \right] S_j^2 \; \sigma_2^2 \qquad (12)$$

Finally, the grand variance is obtained as

$$\hat{\sigma}_x^2 = \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{..} \right)^2$$

$$= \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left\{ \; [\; a + bs(i-1) + b_j \;\;] + S_j e_{ij} - a - \frac{bs(m-1)}{2} - \frac{b}{s}\sum_{j=1}^{s} jS_j \, \bar{e}_{..} \;\; \right\}^2$$

$$(n-1) \;\; \hat{\sigma}_x^2 = \sum_{i=1}^{m}\sum_{j=1}^{s}\left\{ \; [\; a + bs(i-1) \;\;]S_j e_{ij} + bjS_j e_{ij} - a - \frac{bs(m-1)}{2} - \frac{b}{s}\sum_{j=1}^{s} jS_j \, \bar{e}_{..} \;\; \right\}^2$$

Thus, the grand variance is

$$= \left[\begin{array}{l} \frac{b^2\left(n^2 - s^2\right)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + c_1 \right]^2 + \left[a^2 + 2ab\left(\frac{n-s}{2}\right) + \frac{b^2(n-s)(2n-s)}{6} \right] \text{var}\left(S_j\right) \\ + b^2 \, \text{var}\left(jS_j\right) + 2b\left[a + b\left(\frac{n-s}{2}\right) \right] \text{cov}\left(S_j, jS_j\right) \end{array} \right] \sigma_2^2$$

The estimates of row, column and overall averages and variances of the Buys-Ballot table for multiplicative model with the error variances are given in Table 2

Table 2: Estimates of Variances for Multiplicative Model with the Error Variances

Measures	Multiplicative Model $\left(M_t = a + bt\right)$
$\hat{\sigma}^2_{i.}$	$\left\{\left[(a + bs(i-1)) + bC_1\right]^2 + \text{var}\begin{bmatrix}[a + bs(i-1)]S_j \\ + bjS_j\end{bmatrix}\right\} \sigma^2_2$
$\hat{\sigma}^2_{.j.}$	$\left\{\frac{b^2(n^2 - s^2)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + bj\right]^2\right\} S^2_j \ \sigma^2_2$
$\hat{\sigma}^2_{x}$	$\left\{\begin{array}{l}\frac{b^2(n^2 - s^2)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + C_1\right]^2 \\ + \left[a^2 + 2ab\left(\frac{n-s}{2}\right) + \frac{b^2(n-s)(2n-s)}{6}\right] \text{Var}(S_j) \\ + b^2 \text{Var}(jS_j) + 2b\left[a + b\left(\frac{n-s}{2}\right)\right] \text{Cov}(S_j, jS_j)\end{array}\right\} \sigma^2_2$

Source: Nwogu *et al* (2019) and Dozie, *et al*, (2020)

$$C_1 = \frac{1}{s}\sum_{j=1}^{s} jS_j$$

2.2 Seasonal Variances with Decomposition Models

Iwueze and Nwogu [5], Nwogu, *et al* [7] and Dozie, *et al* [9] proposed the estimation procedure for the seasonal variances with decomposition models of the Buys-Ballot table with the error variances for linear trending curve are given in equations (13), (14) and (15) respectively.

Additive Model $$\hat{\sigma}^2_{.j.} = \frac{b^2 n(n+s)}{12} + \sigma^2_1 \qquad (13)$$

Multiplicative Model $$\hat{\sigma}^2_{.j.} = \left\{\frac{b^2(n^2 - s^2)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + bj\right]^2\right\} S^2_j \ \sigma^2_2 \qquad (14)$$

Mixed Model $$\hat{\sigma}^2_j = b^2 \frac{n(n+s)}{12} S^2_j + \sigma^2_1 \qquad (15)$$

The column variance is, for additive model and equation (13), a product of trending series with the error variance. For the multiplicative and equation (14), a quadratic function of the season j and square of the seasonal effect $S^2_{j,}$ and for the mixed model and equation (15), a constant multiple of square of the seasonal effect only.

2.3 Cochran's Test for Constant Variance

To test the null hypothesis that the variances are equal, that is

$H_0 = \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

Against the alternative

$H_1 \neq \sigma_1^2 \neq \sigma_2^2 \neq \ldots \neq \sigma_k^2$ $(Atleast\ one\ \text{var}iance\ is\ different\ \ from\ others)$

Cochran has shown that the statistic

$$C = \frac{\max\left(S_j^2\right)}{\sum_{j=1}^{k} S_j^2} \tag{16}$$

Where, $\max\left(S_j^2\right)$ is the maximum variance among all column variances

$\sum_{j=1}^{k} S_j^2$ is the sum of the variances

S_j has the range j = 1, 2, …, s, which are the variances of the j[th] sub-group.

Using the parameters of the Buys-Ballot table: $S_j^2 = \hat{\sigma}_j^2$, the statistic in (16) is then given as;

$$C = \frac{\max\left(\hat{\sigma}_j^2\right)}{\sum_{j=1}^{k} \hat{\sigma}_j^2} \tag{17}$$

2.4 Chi-Square Test

The seasonal variance of the Buys-Ballot table for the mixed model with error variance

$\sigma_{zj}^2 = \dfrac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$ is reduces to that of test null hypothesis.

$H_0:\ \sigma_j^2 = \sigma_{zj}^2$

and the appropriate model is mixed

$H_1:\ \sigma_j^2 \neq \sigma_{zj}^2$

and the appropriate model is not mixed

$\sigma_j^2 = (j=1,\ 2,...s)$ *is the true variance of the jth season.*

$$\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (18)$$

and σ_1^2 *is the error variance assumed to be equal to* 1

Therefore, the statistic is $\chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{zj}^2}$ (19)

follows the chi-square distribution with $m-1$ degree of freedom, m is the number of observations in each column and s is the seasonal lag.

The interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ *contains the statistic* (19) with $100(1-\alpha)\%$ degree of confidence.

2.5 Choice of Appropriate Transformation

For time data arranged in Buys-Ballot table Akpanta and Iwueze [17] provided the slope of the regression equation of log of group standard deviation on log of group mean as given in equation (20) is what is needed for choice of appropriate transformation. Some of the values of slope β and their implied transformation are stated in Table 3

$$\log_e \hat{\sigma}_i = a + \beta \log_e \hat{X}_{i.} \qquad (20)$$

Table 3: Bartlett's Transformation for Some Values of β

S/No	**1**	**2**	**3**	**4**	**5**	**6**	**7**
β	**0**	$\frac{1}{2}$	**1**	$\frac{3}{2}$	**2**	**3**	**-1**
Transformation	**No transformation**	$\sqrt{X_t}$	$\log_e X_i$	$\frac{1}{\sqrt{X_t}}$	$\frac{1}{X_t}$	$\frac{1}{X_t^2}$	X_t^2

The method of Akpanta and Iwueze [17] is used in choosing the appropriate transformation, the natural logarithm of standard deviation will be used to regress against the natural logarithm of periodic means and the result of the β - value will determine the type of transformation.

3.0 Empirical Example

The time series data presented in the summary table (Table 4) is analysed using the Buys-Ballot method. The series is used to determine the appropriate model for decomposition of the study series.

Table 4: Buys-Ballot Estimates for Means and Standard Deviations

Year	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec	$\bar{X}_{i.}$	$\sigma_{i.}$
2012	5	7	8	8	11	8	12	9	10	9	16	28	10.92	6.05
2013	2	5	8	9	9	10	5	12	8	12	11	32	10.25	7.48
2014	4	5	3	9	8	9	6	7	8	8	16	22	8.75	5.31
2015	6	2	8	10	9	6	9	9	11	12	12	33	10.58	7.61
2016	2	7	12	13	20	8	12	5	13	13	16	26	12.25	6.52
2017	3	8	15	12	13	8	10	4	17	10	11	19	10.83	4.80
2018	2	6	17	22	16	11	10	9	16	13	12	25	13.25	6.45
2019	5	8	13	12	22	17	16	12	13	19	18	22	14.75	5.24
2020	3	8	28	14	22	15	9	12	18	13	21	28	15.92	7.77
2021	3	13	19	12	21	16	11	14	13	15	21	22	15.00	5.36
2022	20	17	8	10	12	8	13	9	10	15	32	52	17.17	12.89
$\bar{X}_{.j}$	5.00	7.82	12.64	11.91	14.82	10.55	10.27	9.27	12.45	12.64	16.91	28.09		
$\sigma_{.j}$	5.16	4.07	6.93	3.83	5.56	3.75	3.10	3.10	3.45	3.08	6.19	9.05		

3.1 Choice of Model

The test statistic given (17) is used to determine if the data is additive model. The null hypothesis that the data is additive model is rejected, if C is greater than the tabulated value C_{tab} {k, V, 1 – α}. level of significance, or do not reject null hypothesis otherwise

From Appendix A and Table 5

$$m=12,\ \max \hat{\sigma}_j^2 = 81.890,\ \ \sum_{j=1}^{k} \hat{\sigma}_j^2 = 311.671$$

$$C = \frac{81.890}{311.671} = 0.2601$$

Reject H_0 if $C > C_{tab}$

{11,12:0.05)

From Table 6 and Appendix B

$$m=12,\ \max \hat{\sigma}_j^2 = 0.4520,\ \ \sum_{j=1}^{k} \hat{\sigma}_j^2 = 2.0442$$

$$C = \frac{0.4520}{2.0442} = 0.2211$$

Reject H_0 if $C > C_{tab}$

{11,12 : 0.05}

Table 5: Seasonal means ($\overline{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$)

j	$\overline{X}_j$	$\hat{\sigma}_j^2$
1	5.00	26.60
2	7.82	16.56
3	12.64	48.05
4	11.91	14.69
5	14.82	30.96
6	10.55	14.07

7	10.27	9.62
8	9.27	9.62
9	12.45	11.87
10	12.64	9.46
11	16.91	38.29
12	28.09	81.89

The test statistic (0.2601) is greater than, when compared with the tabulated value (0.2353), suggesting that the data does not accept additive time series model.

Having stated that the data is not additive model, we have to choose between multiplicative and mixed models. The null hypothesis that the data admits the mixed model is rejected, if the statistic defined in equation (19) lies outside the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ which for $\alpha = 0.05$ level of significance and $m-1=10$ degrees of freedom, equals (3.8, 21.9) or do not reject H_0 otherwise, and from equation (19) the calculated values, χ^2_{cal} given in Table 7 are obtained. With the critical values (3.8 and 21.9), all the calculated values are outside the range, showing that the model structure is not mixed.

However, there is indication the choice of appropriate model may be affected by violation of the underlying assumptions, therefore, it is required to evaluate data for transformation to meet the constant variance and normality assumptions in the distribution. When the seasonal variances of the transformed data given in Table 7 are tested for constant variance, the computed test statistic from equation (17) is 0.2211 and that of the critical value is 0.2353 at $\alpha = 0.05$ level of significance and $m-1=10$ degrees of freedom. This shows that the variance is constant and the transformed data is additive model.

Table 6: Seasonal means ($\bar{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$)

j	$\bar{X}_j$	$\hat{\sigma}_j^2$
1	1.34	0.45
2	1.93	0.30
3	2.39	0.35
4	2.44	0.08
5	2.63	0.15
6	2.30	0.12
7	2.28	0.11
8	2.17	0.15
9	2.49	0.08
10	2.51	0.06
11	2.78	0.11
12	3.30	0.08

Table 7: Seasonal effects (S_j), estimate of the column variance ($\hat{\sigma}_j^2$) and Calculated Chi-square $\left(\chi_{cal}^2\right)$

j	1	2	3	4	5	6	7	8	9	10	11	12
S_j	1.69	0.77	0.79	1.16	2.16	0.54	0.32	0.68	0.68	0.97	0.90	1.28
$\hat{\sigma}_j^2$	0.45	0.30	0.35	0.08	0.15	0.12	0.11	0.15	0.08	0.06	0.11	0.08
χ_{cal}^2	1.06	1.08	0.07	1.09	1.08	0.20	1.17	1.18	1.55	1.33	0.39	1.16

From appendix B and Table 7, $\sigma_1^2 = 1,\ b = 0.1143,\ n = 144,\ m = 12$

Therefore, from (7), $\sigma_{zj}^2 = (0.1143)^2 \times 144\left(\frac{144+12}{12}\right)S_j^2 + 1$

4.0 Summary, Conclusions and Recommendations

This article has presented estimates of multiplicative model with the error terms and variances, while comparing it with those of the additive and mixed models. The method of estimation is based on the periodic, seasonal and overall averages and variances of time series data arranged in a Buys-Ballot table. The method assumes that (1) the underlying distribution of the variable, X_{ij}, $i = 1, 2, ..., m$, $j = 1, 2, ..., s$, under study is normal. (2) the trending curve is linear (3) the decomposition method is multiplicative. Results show that, under the stated assumptions, the seasonal variances of the Buys-Ballot table, for multiplicative model, a function of column (j) through the seasonal component s_j^2 with error variance. For additive model, a product of trending series with the error variance and for the mixed model, a constant multiple of square of the seasonal effect with error variance

A real life data is applied to determine the appropriate model for decomposition of the study data given in table 4. Result from table 7 indicates that the variance is constant and the transformed data is additive model. This further confirms that the appropriate model of the original data is multiplicative. There is indication that choosing the appropriate model may be affected by violation of underlying assumptions, hence, it is recommended that a study series should be evaluated for normality assumptions in the distribution, before employing test for choice of appropriate model.

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Chart 1 Original number of campari drink at Vandoz Enterprise, Owerri, Imo State, Nigeria

Year	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec	$\bar{X}_{i.}$	$\sigma_{i.}$
2012	5	7	8	8	11	8	12	9	10	9	16	28	10.92	6.05
2013	2	5	8	9	9	10	5	12	8	12	11	32	10.25	7.48
2014	4	5	3	9	8	9	6	7	8	8	16	22	8.75	5.31
2015	6	2	8	10	9	6	9	9	11	12	12	33	10.58	7.61

2016	2	7	12	13	20	8	12	5	13	13	16	26	12.25	6.52
2017	3	8	15	12	13	8	10	4	17	10	11	19	10.83	4.80
2018	2	6	17	22	16	11	10	9	16	13	12	25	13.25	6.45
2019	5	8	13	12	22	17	16	12	13	19	18	22	14.75	5.24
2020	3	8	28	14	22	15	9	12	18	13	21	28	15.92	7.77
2021	3	13	19	12	21	16	11	14	13	15	21	22	15.00	5.36
2022	20	17	8	10	12	8	13	9	10	15	32	52	17.17	12.89
$\overline{X}_{.j}$	5.00	7.82	12.64	11.91	14.82	10.55	10.27	9.27	12.45	12.64	16.91	28.09		
$\sigma_{.j}$	5.16	4.07	6.93	3.83	5.56	3.75	3.10	3.10	3.45	3.08	6.19	9.05		

Source: Vandoz Enterprise, Owerri, Imo State, Nigeria

Chart 2 Transformed series of number of campari drink at Vandoz Enterprise, Owerri, Imo State, Nigeria

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{y}_{i.}$	$\sigma_{i.}$
2012	1.6094	1.9459	2.0794	2.0794	2.3979	2.0794	2.4849	2.1972	2.3026	2.1972	2.7726	3.3322	2.290	0.191
2013	0.6932	1.6094	2.0794	2.1972	2.1972	2.3025	1.6094	2.4849	2.0790	2.4849	2.3979	3.4657	2.133	0.433
2014	1.3863	1.6094	1.0986	2.1972	2.0794	2.1972	1.7918	1.9459	2.0794	2.0794	2.7726	3.0910	2.027	0.297
2015	1.7917	0.6932	2.0794	2.3026	2.1972	1.7918	2.1972	2.1972	2.3970	2.4849	2.4819	3.4965	2.176	0.410
2016	0.6932	1.9459	2.4848	2.5650	2.9953	2.0794	2.4849	1.6094	2.5649	2.5650	2.7726	3.2581	2.335	0.466
2017	1.0986	2.0794	2.7080	2.7726	2.5650	2.0794	2.3026	1.3863	2.8332	2.3929	2.3979	2.9444	2.289	0.322
2018	0.6932	1.7918	2.8332	3.0910	2.7726	2.3979	2.3026	2.1972	2.7726	2.5650	2.4849	3.2189	2.427	0.452
2019	1.6094	2.0794	2.5650	2.4849	3.0910	2.8332	2.7726	2.4849	2.5650	2.9444	2.8904	3.0910	2.618	0.186
2020	1.0986	2.0794	3.3322	2.6391	3.0910	2.7081	2.1972	2.4849	2.8904	2.5650	3.0445	3.3322	2.622	0.391
2021	1.0986	2.5650	2.9444	2.4849	3.0445	2.7726	2.3979	2.6391	2.5650	2.7081	3.0445	3.0910	2.613	0.282
2022	2.9957	2.8332	2.0794	2.3026	2.4849	2.0794	2.5650	2.1972	2.3026	2.7081	3.4657	3.9512	2.664	0.331
$\bar{y}_{.j}$	1.3430	1.9300	2.3890	2.4651	2.6290	2.3020	2.2820	2.1660	2.4866	2.5095	2.7753	3.2975		
$\sigma_{.j}$	0.4520	0.3040	0.3590	0.0872	0.1510	0.1150	0.1120	0.1500	0.0790	0.0804	0.1088	0.0758		

Source: Vandoz Enterprise, Owerri, Imo State, Nigeria