Stochastic Analysis on the Value of Assets for Capital Investments

Abstract

The benefit of monetary assets cannot be over emphasizedbecause it stands as an engine room to every investment which accumulates wealthperiodically such as daily, weekly, monthly and yearly etc. In this study, a closed formsolution of Stochastic Differential Equation (SDE) was successfully exploited for the analysis of asset values and other stock market quantities. The solutions of stock variables were critically observed by simulations which describe the behavior of asset values with respect to their maturity periods. Finally, the skewness and kurtosis of the asset values were obtained to give ~~investors~~ proper directions to the investors in terms of decision making.

Key words: Asset value, Kurtosis, Skewness, Stochastic Analysis, Prices.

Introduction

An asset in the financial market is a financial asset and is one of the instruments that accumulates return rate to an investor or commercial traders. Therefore, assessment of a financial asset involves the ~~is a~~ procedure of defining the market value of the asset price~~s~~ on its ~~an~~ investment. Asset price determination has turned out to be so influential for impelling economic variations, distributing economic resources to subdivisions with time and controlling the assets of the whole system that produces different returns. Return on investment for capital market is an estimate which correctly examines the profit of an investment for effective running and good management of the business.

On the contrary, while considering this kind of exercises~~problems~~, analytical method which offers exact solution for suitable mathematical expectation is ~~therefore~~ required to enhance efficiency in terms of decision making for future developmental plans. Problems which are related to asset price and its rate of return, appropriate formulation and accurate analytical solutions are necessary requirements for the analysis of return rates in time varying investment; however, following the basic feature of the problem under-study, the analytical solution is exploited.

Though, stock market prices have been considered in different ways and results obtained in divers ways by scholars. For instance,Adeosun,M.E, S.O and Ugbebor, O.O (2015)[1] studied the stochastic analysis of stock market price model, and considered the stochastic analysis of the behavior of stock prices using a proposed log-normal distribution model. Their results showed that the proposed model is efficient for the production of stock prices.Osu and Amadi(2022)[2] studied the stochastic analysis of stock market expected returns for investors. In their results the variances of four different stocks indicated that stock1 is the best among the stocks of different companies, which is consistent with the work of Ofomata,A.I.O; Inyama, S.C., Umana, R.A. and Omame,A.(2017)[3]. Also, Amadi, I.U; Igbudu, R. and Azor, P.A.(2022)[4] investigated a stochastic analysis of stock market expected returns and growth-rates .

In trying to study stochastic model Davis,I., Amadi, I.U. and Ndu, R.I. (2019)[5] considered the stability analysis of stochastic model for stock market prices and did analysis of the unstable nature of stock market forces applying a new differential equation model that can impact the expected returns of investors in stock exchange market with a stochastic volatility in the equation. ~~While~~ Farnoosh, R, Razazadeh, H; Sobhani, A and Behboud, M. (2015)[6] suggested in their study, of some analytical solutions of stochastic differential equations with respect to Martingale processes and discovered that the solutions of some SDEs are related to other stochastic equations with diffusion part. The second technique is to change SDE to ODE that are tried to omit diffusion part of stochastic equation by using Martingale processes.

On the other hand, Bayram, M; partal, T. and Buyukoz, G.S.(2018)[7] looked at the numerical techniques of solving stochastic differential equations like the Euler-Maruyama and Milstein methods based on the truncated Ito-Taylor expansion by solving a non-linear stochastic differential equation and approximated numerical solution using Monte Carlo simulation for each scheme. Their results showed that if the discretization value N is increasing, the Euler-Maruyama and Milstein techniques were closed to exact solution. LI, Q. and GAN, S. (2012)[8] worked on the stability of both analytical and numerical solutions for non-linear stochastic delay differential equations with jumps and they observed that the compensated stochastic methods inherit stability property of the correct solution. Amadi, I.U. and Anthony, C. (2022)[9] studied the solution of differential equations and stochastic differential equations of time varying investment returns and obtained precise conditions governing asset price returns rate via multiplicative and multiplicative inverse trend series. The proposed model showed an efficient and reliable multiplicative inverse trend series than the multiplicative trend in both deterministic and stochastic system. Osu, B .O.(2010)[10] studied stochastic model of the fluctuations of stock market price and obtained precise conditions for determining the equilibrium price. The model constrains the drift parameters of price process in a manner that is adequately characterized by the volatility. WU, Q.(2016)[11] examined the stability behaviours of stochastic differential equations(SDEs) driven by time changed Brownian motions. The study also found the ~~And a~~ connection between the stability of the solution to the time -changed stochastic differential equations and their corresponding non-time-changed stochastic differential equations were shown using the duality theorem. Adewole, Y.O.(2016)[12] studied stochastic methods in practical delineation of financial models and suggested the Euler-Maruyama method as the stochastic differential equation expressions as potentially useful for delineation of asset stock price and volatility.

In standard settings a closed form analytical solution of variable coefficient problem of SDE is not a difficult one. The problem of it is when the analysis is being sought. So the capability to properly comprehend the analytical solution to suit the explanations of capital investments is not an easy one to comprehend as to apply satisfactorily. Hence, the aim of this paper is adopting a closed form analytical solution of stochastic differential equationandassessing empirically on the value of assets for capitalinvestments in order to comple~~i~~ment previous efforts; therefore this paper extends the work of Okpoye, O.T., Amadi, I.U. and Azor, P.A(2023)[13] as this will increase the scope of this dynamic area of financial mathematics.

This paper is prearranged as follows: Section 2.1 presents the formulation of the problem, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2.1 . Formulation of the Problem

We consider linear stochastic differential equation with the dynamics of stock variables which is said to have a complete probability space $(\Omega, \mathrm{F}, \wp)$ such that a finite time investment horizon $T > 0$ since any investment will span some period being longer than the point of time of investment. Such dynamics are ~~is~~ governed by the following for a non-dividend paying stock as:

$$dS(t) = \mu S(t)dt + \sigma S(t)dz(t) \ , \quad (1)$$

Where S denotes the asset value, μ is the stock rate of return (drift) which is also known as the average rate of the growth of asset price and σ denotes the volatility otherwise called standard deviation of the returns. The $dz(t)$ is a Brownian motion or Wiener process which is defined on probability space $(\Omega, \mathrm{F}, \wp)$, [16]. However, stock price follows the Ito's process and the drift rate is stated as follows:

$$\mu = \left(\frac{\partial f}{\partial S_t} a_t + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S_T^2} b_t^2 \right), \quad (2)$$

$$\sigma^2 = \frac{\partial^2 f}{\partial S_t^2} b_t^2 \quad (3)$$

Ito's process is a stochastic process $\{X_t, t \geq 0\}$ known as Ito's process which follows:

$$X_t = X_0 + \int_0^t (\mathrm{t}, \varpi) d\tau + \int_0^t b(t, \varpi) dz_t \ . \quad (4)$$

Where $a(t, \varpi)$ and $b(t, \varpi)$ are adapted random function, [15].

(Ito's lemma). Let $f(S, t)$ be a twice continuous differential function on $[0, \infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 \ ,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higer order terms } (h.\mathrm{o}, \mathrm{t}) \ ,$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t} (a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (a_t dt + bdz(t))^2 \quad (5)$$

$$= \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2 dt, \quad (6)$$

$$= \left(\frac{\partial F}{\partial S_t}a_t + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2\right)dt + \frac{\partial F}{\partial S_t}b_t dz(t) \quad (7)$$

More so, given the variable $S(\mathrm{t})$ denotes stock price, then following GBM implies (1) and hence, the function $F(S,t)$,Ito's lemma gives:

$$dF = \left(\mu S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma S\frac{\partial F}{\partial S}dz(t) \quad (8)$$

Todeterminea stock price S which follows a random process as

$$dS(t) = \mu S(t)dt + \sigma S(t)dz(t)$$

Let $F(S,t) = \ln S$ the partial derivatives are:

$$\frac{\partial F}{\partial S} = \frac{1}{S}, \frac{\partial^2 F}{\partial S^2} = -\frac{1}{S^2} \ and \frac{\partial F}{\partial t} = 0.$$

Putting the above values into (8) yields the following

$$d(\ln S) = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dz(t),$$

Integrating both sides and talking upper and lower bounds as 0 to t gives

$$\int_0^t d(\ln S) = \int_0^t\left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma\int_0^t dz(t),$$

$$\ln S(t) - l\,\mathrm{n}\,S_0 = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\left(dz(\mathrm{t}) - dz(0)\right),$$

$$S(t) = S_0\exp\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma dz(t). \quad (9)$$

where dz is a standard Brownian Motion.The expected value of the solution gives:

$$
\left.\begin{aligned}
E\left(S(t)\right) &= E\left(S(0)e^{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma dZ(t)+\sigma\,\mathrm{dZ(t)}} \right) = S(0)e^{\left(\mu-\frac{1}{2}\sigma^2\right)tE\left(e\sigma dZ(t)\right)} \\
&= S(0)e^{\left(\mu-\frac{1}{2}\sigma^2\right)te^{\frac{1}{2}\sigma^2 t}} \quad = S(0)e^{\mu t}
\end{aligned}\right\} \qquad (10)
$$

The variance of the solution is given as:

$$
\left.\begin{aligned}
&Var\left(S(t)\right) = Var\left(S(0)e^{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma dZ(t)} \right) = S^2(0)e^{2\left(\mu-\frac{1}{2}\sigma^2\right)tVar\left(e^{\sigma dZ(t)}dZ(t)\right.} \\
&= S^2(0)e^{2\left(\mu-\frac{1}{2}\sigma^2\right)t[E((e^{\sigma dZ(t)}dZ(t)} \quad -\left(E\left(e^{\sigma dZ(t)-Z(t_0)} - \frac{1}{2}\sigma^2(t-t_0)/\mathrm{f}_{to} \right)\right) \\
&S^2{}_0 e^{-\sigma^2(t-t_0)}\left[E\left(e^{2\sigma Z(t-t_{to})/S(t_0)=S_0}\right)-E^2\left(e^{\sigma dZ(t)-Z(t_{t-to})}/S(t)=S_0\right)\right] = S_0^2\left[e^{\sigma 2(t-t_0)}-1\right]
\end{aligned}\right\}. \qquad (11)
$$

3. 1 Results and Discussion

This Section presents the solutions from (9)-(11), Hence the following Tables values were generated:

Table 1:Asset values with various volatility in respect to their trading days through the solution below: $S(t) = S_0 \exp\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma dz(t).\ S_0 = 20.23,\ \ dz = 1$

drift (μ)	Volatility (σ)	Weekly trading (2)	Monthly trading (3)	Yearly trading (7)
0.5	0.1 0.2 0.3 0.4	60.17 64.53 67.84 69.91	98.71 104.29 106.93 106.40	714.92 711.35 659.95 570.87
0.75	0.1 0.2 0.3 0.4	99.20 106.40 111.85 115.26	208.96 220.78 226.37 225.24	4114.06 4093.55 3797.76 3285.15
Skewness **Kurtosis**		**0.0664** **1.1909**	**0.0154** **1.0314**	**NaN** **NaN**

Table 2: Expected asset values with their ~~in~~ respective trading days through the solution below:

$$E\left(S(t)\right) = S_0 e^{\mu t}$$

drift (μ)	Initial stock prices (S_0)	Weekly trading (2)	Monthly trading (3)	Yearly trading (7)
0.5	2.0000 4.0000 6.0000 8.0000	6.60 13.19 19.79 26.38	9.89 19.79 29.68 39.57	23.08 46.16 69.25 92.33
0.75	2.0000 4.0000 6.0000 8.0000	8.47 16.94 25.40 33.87	12.70 25.40 38.11 50.81	23.08 46.16 69.25 118.55
Skewness **Kurtosis**		**1.3430** **3.3659**	**0.1815** **1.9006**	**0.4452** **2.1576**

Table 3: Variance of asset values with their ~~in~~ respective trading days through the solution below:

$$Var\left(S(t)\right) = S_0^2\left[e^{\sigma 2(t-t_0)} - 1\right]$$

Volatility (σ)	Initial stock prices (S_0)	Weekly trading (2)	Monthly trading (3)	Yearly trading (7)
0.1	2.0000 4.0000 6.0000 8.0000	0.081 0.323 0.727 1.293	0.12 0.49 1.09 1.95	0.29 1.16 2.61 4.64
0.4	2.0000 4.0000 6.0000 8.0000	1.51 6.03 13.58 24.14	2.46 9.86 22.18 39.43	8.26 33.04 74.34 132.15
Skewness **Kurtosis**		**0.0597** **1.0764**	**1.3505** **3.3870**	**1.3430** **1.3659**

Clearly in Table 1, the time to maturity of underlying assets significantly increases the value of asset. This is realistic because as time continues to grow valuable assets appreciate in value.

Which is also due to the accretion of the returns and the value to the asset. This is to say that asset value is a function of time. However,stock volatility measures the degree of changes of asset over time, which otherwise is a stochastic formation on the price history of financial market, see column 2 of Table 1.Generally when assets are traded yearly it accumulates higher increase:The same is observed at 714.92 with a lower volatility of 10 percent throughout investment durations; see column 5.

Table 2 describes an anticipated average value of an investment at some point in the future. Therefore investors or traders use expected value to quantify the worth of investments thereby predicting the future and also as a guild for decision making. The durations of asset values as can be seen in the above; informs an investor on when to invest more on time varying investments.

Table 3 shows that increase in initial stock prices also increases the level of variance on asset value with their respective time to maturity periods. This implies that, time significantly influences asset valuations. This is discernable since as the time elongates the valuations of the assets change, both with respect to the returns and with respect to the variance measured in forms such as standard deviation.

More so, the skewness and kurtosis seen in Tables 1,2 and 3 were found to be 0.066,0.015: 1.191,1.0314: 1.34 ,0.18,0.445,3.37,1.90,2.16:0.060,1.35,1.34,1.076,3.39 and 1.37 . These indicate the distributions were skewed to the right. This remark has a financial implication which implies that the investments are profit maximizing. While the kurtosis of the asset values was observed to be describing that the distribution is more heavily-tailed in comparison to the normal distribution. This description informs an investor more reliable and effective ways of decision making.

4.1 Conclusion

In this study, a closed analytical solution of Stochastic Differential Equation (SDE) was successfully exploited for the analysis of asset values and other stock quantities. The solutions of stock variables were critically observed by simulations which describe the behavior of asset values with respect to their maturity periods in the following ways:time to maturity of underlying assets significantly increases the value of assets, volatility measures the changes in respect to asset values, yearly trading outperformed more increase than weekly and monthly respectively, the skewness and kurtosis of asset values shows the right direction of profit maximization and expected asset values were predic~~a~~ted in time varying investments. However, we shall be looking at incorporating control terms to the SDE model in the next study. This study helps the investors in understanding the decision making related to financial investments with reference to time horizon of their respective investments of each of their financial assets.

To this end, the skewness and kurtosis of the asset values were obtained to give investors proper directions in terms of decision making.

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