

# **Examining the Efficacy of Break for Time Series Components (BFTSC) and Group for Time Series Components (GFTSC) with Volatile Simulated and Empirical Data**

## ***Abstract:***

*The main reason for this study is to know the performance of BFTSC (break for time series components) and GFTSC (group for time series components) in identification of time series components using volatile simulated and empirical data. BFTSC was created to capture the trend, seasonal, cyclical and irregular components and presented them in a time series plot. While GFTSC was designed to capture all the four time series components together with the equations that produces each components of time series. BFAST (break for additive, seasonal and trend) only identifies trend and seasonal components while considering all other left over components as random, identification of trend and seasonal components alone is not enough to have a clear image of all the time series components in a time series data. Performance through evaluation using low and high volatile simulated and empirical data was conducted to evaluate the performance of both techniques. For yearly sample size of 8, 16 and 24 years were for small medium and large sample size. For the monthly data, 48, 96 and 144 months were used as small, medium and large sample size. Each of the sample size was replicated 100 times each. Finally, GFTSC and BFTSC performance was very good for large sample size with linear trend for both monthly and yearly data (approximately 100%). While the performance drops with highly volatile data such as trend with curve trend line (such as quadratic and cubic). These findings indicate that BFTSC and GFTSC can provide a better alternative to manual technique and BFAST for data associated with linear trend, hence BFTSC and GFTSC are recommended for public.*

**Keywords:** Automation, Break for Time Series Components, Trend, Seasonal, Cyclical, Irregular.

## **Introduction**

The purpose of this study is to evaluate the performance of BFTSC and GFTSC in identification of time series components. The technique BFAST was created to identify trend and seasonal time series components only while BFTSC was created for identification of the four time series components (such as trend, seasonal, cyclical and irregular time series components) and GFTSC was created to identify the four time series components together with the equations that produces each time series components (Ajare & Suzilah, 2019). Both GFTSC and BFTSC automated time series components identification were created from BFAST (Ajare & Suzilah, 2019). BFTSC and GFTSC are both improved BFAST. BFAST is a technique used for identification of trend and seasonal components only, this was first suggested by Verbesselt et al. (2010) and was utilized by Jong, Verbesselt, Schaepman and Bruin (2012). recommended an approach of basic swing identification to spot time series component. The technique BFAST was for recognizing breaking points with the help of seasonal and trend decomposition using loess (STL), it facilitates the detection of trend change in a given information. The elementary aims of the BFAST technique is the splitting of time series into seasonal, trend and also remnants element by the approach for breaks detecting software in R studio core 2012 (Cunha, 2013).

Banas and Kozuch (2019) recommended the use of manual time series decomposition for identification of time series components in complex data such as variety of timber price and supply data. Multiplicative model was utilized as the product of four components while additive model is the addition of the four components (such as trend, seasonal, cyclical and irregular). The components of the time series were determined by means of the Census X11 technique but the cyclical component was detached from the trend by utilizing the Hodrick–Prescott filter (Banas & Kozuch, 2019). This is further described in the literature review.

## **Literature Review**

Idrees, Alam and Agarwal (2019) described univariate time series components identification as a very essential components for projection of data. Importantly, owing to its wide applications/uses in various practical domains. Stock market forecasting involves uncovering the market trends with respect to time, being highly sensitive and volatile to quick changes, this suggested the use of ARIMA approach to be good enough for handling such data for prediction. The main future study of stock-trend prediction is to develop new automated innovative model that can help to forecast the future stocks profits.

Sen (2019) utilized partial trend identification by change-point successive average methodology (SAM). The very important enquiry here is to how to identify the time series components, using trends of different durations and slopes (through automation). SAM was recommended, firstly, for a set of trends visual inspection and then their quantitative duration. The most important and vital advantage of SAM is that preliminary assumptions is not necessary and

trends identifications are not straightforward (Sen, 2019). The limitation of SAM is that it is time consuming.

Xu, Yu, Peng, Zhao, Cheng, Liu and Gong (2020) examined the annual 30-m land use/land cover maps of China for 1980 until 2015. The length of each sample size was recommended to be within 5 to 15 years for breaks to be detected by the different break test approaches. Annual land use land cover (LULC) change information at medium sample size resolution is essential subjects for object model identification. Annual LULC observations is not always available at continental national scale due to insufficient remote sensing information coverage and lack of computational capabilities. The reliable classification of land change dynamics for China can be advance and studied using more years (16 years and above) for scientific research and to support land management for policy-makers.

Verbesselt, Hyndman, Zeileis and Culvenor (2010) used BFAST to examine the vegetation change. This was done using different terrestrial cover types and seasonal amplitude to determine the signal-to-noise ratio. BFAST is very capable of identifying change independently by employing the full time series tools. The technique is globally applicable within the setting of thresholds to detect change within a time series. Long term vegetation changes can be detected (e.g. grassland, woodlands and deciduous forests) having a seasonal frequencies higher than the noise level. The technique can be applied to any time series data and not only NDVI data.

Verbesselt, Hyndman, Zeileis and Culvenor (2010) recommended that BFAST is capable of detecting and grouping spatial, temporal vegetation changes. BFAST steps are iterated until the number and position of the breakpoints are detected.

Sang, Wang and Liu (2014) investigated the performances of two techniques of time series components identification. Mann–Kendall (MK) technique and the primary model decomposition (EMD) technique for time series components identification in hydrology time series. Identification of time series components was a vital problems in hydrology time series study, but it was also very challenging task due to the various performances of diverse techniques. Examination of both synthetic and observed time series components identification techniques indicated a better than EMD when compared with the MK technique.

The outcomes confirmed that pre-whitening can't improve time series component identification with the use of MK technique, but produced non accurate results occasionally. If the trend time series component is analyzed with small magnitude of other time series components, it can't be correctly detected by the MK technique, this is due to the fact that can submerged too severely by other time series components but trend component would be accurately identified. When analyzed series has short length, its trend cannot be accurately identified by the MK technique (Sang, Wang & Liu, 2014). Comparatively, the procedure can be adjusted to determine nonlinear trend time series component by considering statistical significance.

Bonakdari, Moeeni, Ebtehaj, Zeynoddin, Mahoammadian and Gharabaghi (2019) investigated a new insights into soil temperature, time series modeling in linear and nonlinear. The spectral analysis technique is utilized, time series data from the time domain is transmitted to the new linear data and presented in the methodology. A methodology was offered based on stochastics time series method addressing the problem of Daily Soil Temperature (DST). Based on the outcomes of comparison of the three methods which was applied to various time series data, it reveals that spectral analysis system combined with that of stochastic outperformed the seasonal standardization technique.

Ambrosino, Thinová, Briestenský and Sabbarese (2019) examined the anomalies in identification of time series components in earth's rotation rate. The chosen hybrid techniques are Empirical Model Decomposition (EMD), Support Vector Regression (SVR), Singular Spectrum Analysis (SSA) and Forecasting Method (FM). Mutually hybrid techniques combined to form the 1<sup>st</sup> part of decomposition and 2<sup>nd</sup> part of s modeling. The hybrid procedure are produced by the combination of diverse procedures. The chosen earthquakes period occurrence are estimated to show a direct reliability of hybrid technique. The EMD + SVR techniques has been proven to be the best for non linear time series data.

Parmezan, Souza and Batistaa (2019) examined the performance of statistics and machine learning models for time series modeling, identifying the best conditions for the use of each model. The preferred model, for a specific phenomenon is the most important time series modeling. Modeling are similar to other data mining tasks, uses empirical evidence to select the most suitable model for a current problem since no modeling technique can be considered as the best. Only few technical research publications rigorously focus on the benefits and limitations of the most common algorithms for univariate [time series](#) processing. However, there are limited performance record of these models when applied to complex and highly nonlinear data. The outcomes indicated that SARIMA is one of the best statistical technique to outperform other techniques in terms of time series trend component identification. Though, without a statistical difference machine learning procedures like artificial neural network (ANN), support vector machine (SVM) and Kth neural network (**K**NN) precision comes with the application of larger number of parameters. The findings reveals that they helps in providing a clearer sight into time series selection of model , parameterized setting, testing, evaluation and experimental system.

Awty, Bunting, Hardy and Bell (2019) investigate the performance of four time series components identification techniques using simulated data which comes from univariate time series. Established on the study done by Verbesselt et al. (2010) the commonly simulated data ranges from 5 years until 24 years. 10 years' time series data that contained a 16 day time-based resolution gives roughly 23 observations in a year and per curve is cantered on the center of the year. The noise component was included indiscriminately to produce more detailed realistic time series.

Evaluating the correctness and weakness of these techniques can be tedious because validated data are not readily available and commonly depends on human clarification. Data generated through time series simulation offer an unbiased technique for comparison amid change recognition algorithms. In total, 151,300 monthly generated/simulations data to represent a range of rapid, regular, and seasonal changes. Exponentially Weight-Moving Average Change Detection (EWMACD) and Break for Additive, Seasonal and Trend (BFAST) performed very well. EWMACD correctly identified the accurate date of change in 78.9% of cases. Continuous Change Detection and Classification (CCDC) and Continuous Change Detection and Classification plus Cross Validation (CCDC+CV) performed worst. BFAST Monitor performed better when some data were removed or data reduced. Though BFAST could only correctly identify less than 10% of seasonal changes but 100% of linear trend. All in all the techniques showed some reduction in performance with augmented noise (with highly volatile data) and missing data. The following recommendations are made from the limitations of each techniques as a preliminary point for future studies. EWMACD ought to be utilized for detection of lesser magnitude trend changes and changes in seasonality. CCDC ought to be utilized for robust detection of comprehensive land cover class changes. EWMACD and BFAST are appropriate for noisy datasets and they are both recommended as the best time series components identification technique. BFAST can be extended to detect cyclical and irregular mechanisms in addition to linear trend and seasonal components. CCDC can be used where there are high numbers of missing data. The replicated datasets have been made freely obtainable online as a underpinning for future work (Ajare & Suzilah, 2019).

Verbesselt, Hyndman, Zeileis and Culvenor (2010) recommended BFAST technique for public use. BFAST has primarily been utilized to monitoring disturbance but can also be used for more robust data, and not general land cover scenarios. First, an Ordinary-Least-Squares-Moving-Sum (OLS-MOSUM) experiment is conducted to determine breakpoints. If the OLS-MOSUM assessment indicates substantial ( $p < 0.05$ ) change, then the number and location of breakpoints is assessed distinctly for the seasonal- trend components using OLS fitting. The BFAST bundle robotically fits a 3<sup>rd</sup>-order harmonic model. The outcome was a set of piecewise season and trendy models which reduce inaccuracy across the entire time series. The smallest detachment between disruptions was set to 3 to 6 years (48 observations), which streamline with the procedures given by Verbesselt et al. (2010) and ties the both-training period that was utilized for the other techniques.

BFAST Monitor was established as a near-real period substitute to BFAST (Verbesselt, Zeileis and Herold (2012). BFAST Monitor is analogous to BFAST, it was mostly been functional to forest monitoring. It is constructed on the statistic that modification can be detected by looking for aberration of new clarifications from an established data past period. BFAST Monitor is constructed for separating seasonal-trend components. The season-trend model is form fitted to the stable past period using OLS. Whenever new information are obtainable, the residual

numbers are estimated using the fit model and Moving-Sums (MOSUMs) of the residuals are used to appear for variability which would designate structural change. BFAST Monitor was run using the R package the same manner as BFAST is being run. Given that all replications were calculated with a break after five to six-years of stability, a stable history period of two years (46 observations) was used. BFAST Monitor can also be useful on datasets with omitted values which provide it better improvement over BFAST. BFAST Monitor practises the use of difference in medians concerning the history historical data and observing period to estimate break points. The basic limitation of BFAST monitor is the R implementation which doesn't permitted the continuous monitoring and could not also identify cyclical and irregular time series components (Verbesselt, Zeileis and Herold, 2012).

Continuous Change Detection and Classification (CCDC) was built to primarily focus on univariate time series components identification in land cover time series (Verbesselt, Zeileis and Herold, 2012). Classification time series component wasn't employed because the replicated data were not structured to relate directly to some specific land time series. CCDC is analogous to BFAST Monitor, CCDC focus on sensing breaks in close-real period. The model utilized by CCDC is very close to the season-trendy- model used by BFAST Monitor, except that CCDC practises an adaptive progression to reduce model overfitting while also vigorously identifying the seasonal periods. Lasso Regression Model (LRM) instead of OLS to elude overfitting of higher-order models to season and trend to the historical period (Verbesselt, Zeileis & Herold, 2012). LRM lessens over-fitting by restrictive total absolute value coefficients. As a effect, some coefficients was forced to come as zero and will have no effect on the model. There wasn't easily accessible application of CCDC fit for use with generated data as suitable application was coded in the Python programming language. Most of the time series components identification technique depends on some levels parameter regulation to realise the best results. The major limitation of CCDC is due to Lasso fitting levels, CCDC is somehow dependent on the expert to select the numbers of harmonics power of the historical period and it is not consistent due to this facts that it is dependent on users.

CCDC through Cross validation can be applied to find the best value for suitable fit numerous models with diverse values and relating them. These two tactics are denoted to as CCDC and CCDC through Cross Validation (CV). For the immovable style, a value 0.01 was selected and founded on lesser scale trying. Some revisions have described values of 20. The time series replicas was tailored to surface reflectance rather than NDVI. CCDC with CV gave the third best performance among BFAST, BFAST monitor, CCDC, CCDC+CV and EWMACD (Deng & Zhu, 2020). According to Deng and Zhu (2020), the performance of CCDC, CCDC with CV, and EWMACD were very similar at guesstimating trend break detection. This technique of break estimation is very robust to omitted data but fewer operative with enlarged noise. The influence of uproar is always affecting the performance of this techniques, this techniques depends on remaining values, the further noisy the observations, the lower to reflect the accurate break size. One of

the limitations of CCDC with CV is that it required much longer time to run than the other three techniques. Final findings reveals that using CV build from CCDC can serve more detect true breaks, but also has the probability of detecting at minimum one, false break in a time series data.

The technique BFAST had a reduced RMSE and that makes it a little robust against uproar noise, Hence BFAST is recommended as one of the best trend break detection. One of the limitation of CCDC with CV is that its algorithm was made complicated, unlike CCDC, CCDC and CV did not really streamlined with RMSE amount of changes and noise. CCDC and CV was less accurate compared to BFAST (Zhu, Zhang, Yang, Aljaddani Cohen, Qiu & Zhou, 2020). Another limitation is also in terms of noise, with improved noise, the method was lower to detect accurate effects and the likelihood of identifying at smallest non true change remained continuous. However, the breaks in RMSE reveals to us that the real number of non-true breaks were noticed to be greater at extremes of uproar. At great intensities of noise, models are most likely to be subjective by complex points and may be fit too much noise. This is possibly the reasons for some techniques to less in terms of RMSE quantity of changes at high noise levels. The unique pattern shown by CCDC with CV suggests that it must also detect more breaks if there is very little noise (Zhu, Zhang, Yang, Aljaddani Cohen, Qiu & Zhou, 2020).

EWMACD was built to focus on indirect breaks, such as regulated breaks within pixels (Brooks, Wynne, Thomas, Blinn, Coulston, 2019). Just like CCDC and BFAST Monitor, EWMACD also identifies disorder (increasing/declining trend) variations because it only fits a season trend without its term. EWMACD is embedded with some specific statistics- control-chart, the EWMA chart, to rapidly help in identification of time series component.

Zhu, Zhang, Yang, Aljaddani, Cohen, Qiu and Zhou (2020) developed a new univariate time series components identification method known as Continuous Monitor of Land Disturb (COLD) expending [Landsat](#) time series data. COLD identifies some time series constituent such as trend and seasonal. COLD can also detect land disturbance uninterruptedly as new pattern is composed and likewise provide historic land disorder history. Evaluation of the trend detection ability and land disturbance, different kinds of data are utilized. The COLD procedure was established and standardized centred on all the trainings learned. The correctness valuation displays that COLD outcomes were very precise for identifying trend and seasonal as land disturbance with an omission error of 27% and a commission error of 28%. The limitation of COLD was inability to detect time series components accurately with large noise.

Statistics Control Chart (SCC) was established as a method of univariate time series components identification method and used as mechanism limits to regulates the time series data whenever it deviates from a controlled state. The Moving Sum (MOSUM) and Cumulative Sum (CUSUM) graphs used by BFAST and BFAST Monitor are additional illustrations of statistics control chart. EWMACD computes the left over components for a given experimental period grounded on a season-model fit with OLS (Brooks, Wynne, Thomas, Blinn and Coulson, 2013).

To match BFAST Monitor, a 2<sup>nd</sup>-order season-model and a 2yrs histories of the same set of data were used. This yields a set of habitually distributed, autonomous observations appropriate for use with a EWMA chart. EWMACD has a free available version in goggle scholar. The limitation was that the version doesn't permit for uninterrupted monitoring and therefore we also described from a later application of EWMACD called vigorous EWMACD. As of January 2015, BFAST was still one the most widely utilized time series components identification technique and is freely accessible online: <http://cran.r-project.org/web/packages/bfast/bfast.pdf>. The identification of time series components are summarised starting from 1960 to date. The strengths and weakness of each period and BFTSC and GFTSC was created (Ajare E.O. & Suzilah, 2019).

## Material and Methods

BFAST is the technique used in identifying the time-series variations by separating the trend and seasonal sections during time series disintegration. Given the conventional time series additive model as:

$$= \quad (2.1)$$

Where - observed, - trend, while .seasonal, cyclical and - irregular component all with time period p (Maggi, 2018 ; Zhao, Li, Mu, Wen, Rayburg, & Tian, 2015).

From (2.1) BFAST identifies only trend - seasonal component and the rest is known as random ()

$$= \quad (2.2)$$

(Zdravevski, Lameski, Mingov, Kulakov & Gjorgjevikj, 2015).

To produce the trendy components by means of BFAST, we requires a piecewise linear model approach.

$$= + P$$

(See *Ajare E.O. & Suzilah, 2019*)

To produce seasonal components with BFAST, we need a simple harmonic model.

$$\text{Sin} ( + ) \quad (2.3)$$

Where  $k = 1 \dots q$ ,  $p$  and also , (Zeileis, Kleiber, Krämer & Hornik, 2003).

To produce random components, any leftover data sonal is classified random.

$$= + \quad (2.4)$$

=

According to Ajare and Suzilah (2019) the new technique called BFTSC and GFTSC considered splitting the random into cyclical components and irregular components which is an extension of BFAST. Cyclical components can be calculated through the regression cyclical movement. The regression function at the breakpoint maybe discontinuous but the model can be written in such a way that the function continues at all point including breakpoints. To calculate cyclical components, center moving average is involved (Bornhorst, Dobrescu, Fedelino, Gottschalk & Nakata, 2011).

The new equation becomes

$$= ++ \} + \{ \} \quad (2.5)$$

=

(Ajare and Suzilah, 2019)

For identification of , , , and (See equation 2.1)

BFTSC and GFTSC technique considers every vital component of time series (Ajare & Suzilah, 2019). BFAST is known to be weak in identifying and breaking random components, also very weak in applicability to other types of empirical data (Flicek & Birney 2009). The delinquent of time series mechanisms detection is a problem that need to be addressed immediately earliest stage of time series forecasting (Flicek & Birney 2009). BFTSC followed similar derivative steps like BFAST but deviated in the addition of cyclical and irregular components. BFTSC is the technique used in investigating the simplification of time series data by mining out the trend, seasonal, cyclical and irregular components during time series decomposition. GFTSC followed similar derivative steps like BFTSC but in addition to identification of trend, seasonal, cyclical and irregular components GFTSC also have the capability of generating the equations that produces each component. The residual component in BFAST now converted to contained cyclical and irregular component in GFTSC and BFTSC. Both BFTSC and GFTSC are automated time series components identification. In BFAST only random component can be observed but in BFTSC & GFTSC, the cyclical and irregular components were included (Ajare & Suzilah, 2019).

## Data Simulation

This study compare the performance of BFTSC and GFTSC using both simulated and empirical data. In the simulation study, monthly and yearly data were replicated 100 times based on 3 sample sizes (small, medium and large) and by embedding the four time series components as the simulation conditions. Percentages were calculated in identifying the correct time series components that existed in

the simulation. Simulation of 8, 16 and 24 years were used as year simulated sample sizes. Simulation of 48, 96 and 144 months were used for monthly sample sizes. Three types of trends were used: linear, quadratic and cubic for both yearly and monthly simulations. The data were replicated 100 times for each condition percentages of identifying correct time series components were computed to evaluate BFTSC and GFTSC respectively. Each of the simulated set of data contains component combinations of different form; where monthly data has Trend (T), Trend and Seasonal (TS), Trend and Irregular (TI), Trend and Cyclical (TC), Trend, Seasonal and Irregular (TSI), Trend, Seasonal and Cyclical (TSC), Trend, Seasonal, Irregular and Cyclical (TSIC). As for yearly data has Trend (T), Trend and Irregular (TI), Trend and Cyclical (TC), Trend, Irregular and Cyclical (TIC). Evaluation of BFTSC and GFTSC is based on the ability to identify the correct time series components in 100 replications. Though the issue of how large is large and maximum data accepted by BFAST is yet to be addressed (Van Leeuwen, Huete and Laing, 1999).

Table 1 lists the equations and conditions for monthly and yearly data simulation. First, three different types of trend (linear, quadratic and cubic) were generated randomly based on different values of the coefficient a, b, c, d by using the time variable (t) to replicate 100 set of data for trend component condition. Next, the trend equations were adjusted accordingly by adding other components conditions which were seasonal, cyclical and irregular for monthly data; and cyclical and irregular for yearly data. The seasonal, cyclical and irregular values were obtained from the average, maximum and double maximum values respectively, from the trend randomly generated data. Then, the data was simulated by incorporating 12 sets of seasonal, 3 sets of cyclical, 2 sets of irregular and embedded in 3 sample sizes of small, medium and large of monthly (48, 96 and 144 months respectively) and yearly data (8, 16 and 24 years respectively). The seasonal adjustment was implemented by deducting average value to the trend values at 12 places. As for cyclical, maximum value was added to the trend values at 3 places and for irregular, double maximum value was added at 2 places. Table 1 displays the strategy use in generating the trend (with linear, quadratic and cubic) for yearly and monthly time series data.

*Table 1*

*Monthly and Yearly Data Simulation (Equations and Conditions)*

<b>Time Series Components</b>	<b>Types of Trend</b>	<b>Equations</b>
<b>Trend (T)</b>	Linear	100 200, 201 300

	Quadratic	100 200, 201 300, 301 400
	Cubic	100 200, 201 300, 301 400. 401 500
<b>Seasonal ()</b>	Adjusted at 12 places using average value of the trend generated data	
<b>Cyclical ()</b>	Adjusted at 3 places using maximum value of the trend generated data	
<b>Irregular ()</b>	Adjusted at 2 places using double maximum value of the trend generated data	
<b>Overall Equation</b>	(monthly) (yearly)	

The main model used in generating subsequent trend data, the data was adjusted to add other components appropriately. The first trend data is in one hundred replicates involving only trend component. The second set of data, 12 seasonal components were added to the trend to form trend and seasonal. The third set of data, 2 irregular components was added to the trend to form trend and irregular. The fourth set of data, 3 cyclical components was added to the trend to form trend and cyclical and the last set of data is the combination of the four time series components. The same approach above will be use in generating other subsequent monthly trend data for 96 months and 144 months, such that they all contain only trend and are replicated in 100 places.

### **Evaluation using simulated data**

Evaluation of both techniques is the process of examining the efficiency of BFTSC and GFTSC using data generated from the equations in Table 1 through the simulation study. Table 2 and 3 display the simulation results based on linear trend as the basis for time series components combinations using both monthly and yearly data respectively. Both BFTSC and GFTSC performed very well in the large sample size (144 months) together with all different conditions of time series components. Both techniques successfully identified 100% of the correct

components (Table 2). However, the small and medium sample sizes have percentages of 81% and 90%, respectively, when the combinations became more complex (Linear Trend, Seasonal and Cyclical (); and Linear Trend, Seasonal, Irregular and Cyclical ()). As for the yearly simulated data, BFTSC managed to obtain 100% correct identification for only two conditions which were Linear Trend (), and Linear Trend and Irregular () for all sample sizes (small (8 years), medium (16 years) and large (24 years)) as shows in Table 3. However, when the conditions were Linear Trend and Cyclical (), and Linear Trend, Irregular and Cyclical () with small (8 years) and medium (16 years) sample sizes; the performance of BFTSC deteriorated 2% and 1% respectively. This is due to the small and medium sample sizes of 8 and 16 data points in capturing the complexity of the time series components conditions where involving not just linear trend but 3 sets of cyclical and 2 sets of irregular.

**Table 2. Evaluation of BFTSC and GFTSC with linear trend using small, medium and large sample size (months)**

*Percentages of correct identification (%)*

<i>Time Series Components</i>	<i>Small (48 months)</i>	<i>Medium (96 months)</i>	<i>Large (144 months)</i>
<i>Linear Trend ()</i>	<i>100 %</i>	<i>100 %</i>	<i>100 %</i>
<i>Linear Trend and Seasonal ()</i>	<i>100 %</i>	<i>100 %</i>	<i>100 %</i>
<i>Linear Trend and Irregular ()</i>	<i>100 %</i>	<i>100 %</i>	<i>100 %</i>
<i>Linear Trend and Cyclical ()</i>	<i>100 %</i>	<i>100 %</i>	<i>100 %</i>
<i>Linear Trend, Seasonal and Irregular ()</i>	<i>100 %</i>	<i>100 %</i>	<i>100 %</i>
<i>Linear Trend, Seasonal and Cyclical ()</i>	<i>81 %</i>	<i>90 %</i>	<i>100 %</i>
<i>Linear Trend, Seasonal, Irregular and Cyclical ()</i>	<i>81 %</i>	<i>90 %</i>	<i>100 %</i>
<i>Averages</i>	<i>95 %</i>	<i>97 %</i>	<i>100 %</i>

**Table 3. Evaluation of BFTSC and GFTSC with linear trend using small, medium and large sample size (years)**

*Percentages of correct identification (%)*

<i>Time Series Components</i>	<i>Small (48 months)</i>	<i>Medium (96 months)</i>	<i>Large (144 months)</i>
<i>Trend ()</i>	<i>0 %</i>	<i>0 %</i>	<i>0 %</i>
<i>Trend and Seasonal ()</i>	<i>50 %</i>	<i>55 %</i>	<i>58 %</i>
<i>Trend and Irregular ()</i>	<i>0 %</i>	<i>0 %</i>	<i>0 %</i>
<i>Trend and Cyclical ()</i>	<i>0 %</i>	<i>0 %</i>	<i>0 %</i>
<i>Trend, Seasonal and Irregular ()</i>	<i>33 %</i>	<i>36 %</i>	<i>38 %</i>

<i>Trend, Seasonal and Cyclical ()</i>	33 %	36%	38 %
<i>Trend, Seasonal, Irregular and Cyclical ()</i>	25 %	26%	28 %
<i>Averages</i>	20 %	22 %	23 %

**Table 4 Evaluation of BFTSC and GFTSC with Quadratic and Cubic trend using small, medium and large sample size (months)**

**Percentages of correct identification (%)**

<b>Time Series Components</b>	<b>Small (48 months)</b>	<b>Medium (96 months)</b>	<b>Large (144 months)</b>
<i>Trend ()</i>	0 %	0 %	0 %
<i>Trend and Seasonal ()</i>	50 %	55 %	58 %
<i>Trend and Irregular ()</i>	0 %	0 %	0 %
<i>Trend and Cyclical ()</i>	0 %	0 %	0 %
<i>Trend, Seasonal and Irregular ()</i>	33 %	36 %	38 %
<i>Trend, Seasonal and Cyclical ()</i>	33 %	36%	38 %
<i>Trend, Seasonal, Irregular and Cyclical ()</i>	25 %	26%	28 %
<i>Averages</i>	20 %	22 %	23 %

**Table 5. Evaluation of BFTSC and GFTSC with Quadratic and Cubic trend using small, medium and large sample size (years)**

**Percentages of correct identification (%)**

<b>Time Series Components</b>	<b>Small (8 years)</b>	<b>Medium (16 years)</b>	<b>Large (24 years)</b>
<i>Trend ()</i>	0 %	0 %	0 %
<i>Trend and Irregular ()</i>	0 %	0 %	0 %
<i>Trend and Cyclical ()</i>	0 %	0 %	0 %
<i>Trend, Irregular and Cyclical ()</i>	0 %	0 %	0 %

<i>Averages</i>	<i>0 %</i>	<i>0 %</i>	<i>0 %</i>
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Table 4 and 5 show the simulation results of both techniques based on monthly and yearly data generated from quadratic and cubic trends, respectively. The results were the same for both trends (quadratic and cubic); thus, they were combined. BFTSC and GFTSC performed poorly in quadratic and cubic trends for monthly data. They were only able to identify the time series components on average of 20% to 23% for all sample sizes (small (48 months), medium (96 months), and large (144 months)) as indicated in Table 4. Both techniques (BFTSC and GFTSC) also failed to identify any time series components in yearly data (0%) for all sample sizes, either small (8 years), medium (16 years), or large (24 years), as revealed in Table 5. These poor performances were due to the derivation of BFTSC and GFTSC was based on BFAST linear trend only and not other types of trends.

Both BFTSC and GFTSC performed very well (100%) in the simulation study of linear trend with different combination of time series components (seasonal, irregular and cyclical), and large sample size (144 months and 24 years) for monthly and yearly data. However, it was noticeable that the performances were affected by small and medium sample sizes with complex combination of time series components of linear trend for monthly and yearly data. The correct identification percentages decreased by 19% and 2% for small sample size and 10% and 1% for medium sample size for monthly and yearly data respectively. This is due to the limited number of data points (i.e., monthly: 48 and 96; yearly: 8 and 16 only) used in identifying the complex combination of linear trend, 12 sets of seasonal, 3 sets of cyclical and 2 sets of irregular. Nevertheless, BFTSC and GFTSC performed poorly when quadratic and cubic trend were used. This is due to the derivation of BFTSC and GFTSC was based on linear trend as in BFAST. Thus, to use BFTSC and GFTSC requires large sample size to identify the existence of linear trend and other time series components (seasonal, irregular, and cyclical) in a data set.

## **Evaluation using Empirical data**

Figure 1 displays the time series plot of Ibadan monthly rainfall from January 2007 until December 2018 using automated BFTSC. A total of 144 months, it was noticeable they were regular repeated patterns across the months and years. The high amount of rainfall was between May until August and low amount of rainfall between November until March due to wet and dry season respectively. This is because of the geographical location of Ibadan near to the lagoon. The plot (Figure 1) also shows slight increment in the regular fluctuation starting from 2011 onwards. These regular patterns closely related to seasonal component.

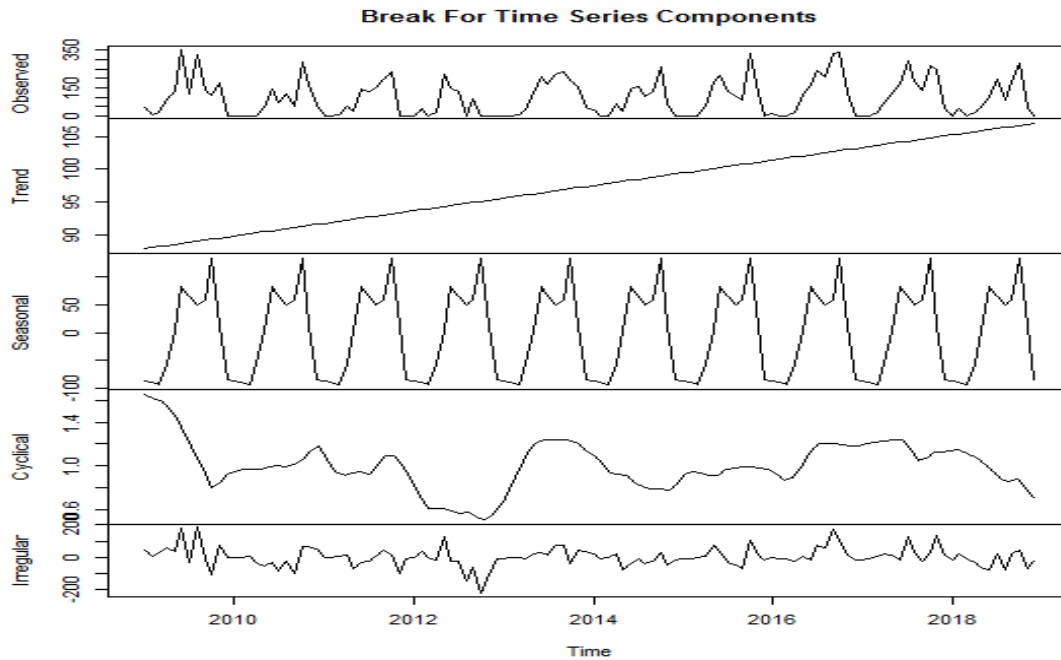
Figure 1 shows the plots produced by automated BFTSC when identifying the time series components in Ibadan rainfall data. BFTSC combined the 4 plots simultaneously for easy, straight forward and fast identification process. Figure 2 is the second plot which was the

automated GFTSC plot of actual monthly rainfall data; this give similar representative of the Ibadan monthly rainfall as BFTSC in figure 2. Figure 3 is the manual time plot of Ibadan monthly rainfall. These findings were the same as manual identification approach in plot 3. GFTSC separated the components and produced equation or time series component values on top of each plot.

The identification results obtained by GFTSC were the same as BFTSC because of using the same theoretical derivation as detailed earlier . Thus, GFTSC findings were also the same as the manual identification approach. The difference between BFTSC and GFTSC was in producing the plots where GFTSC provides more details by including the equation and time series component values which can be used for deeper understanding of the data and also for forecasting. Both BFTSC and GFTSC are more convenient to use by end users because of easy, straight forward and fast identification process.

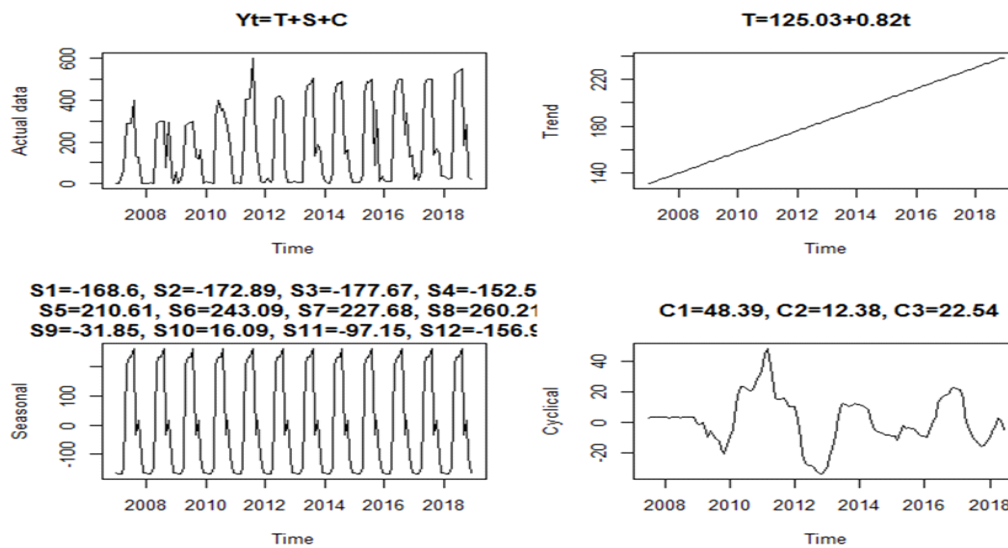
Figure 3 presents the manual time series plot of Ibadan yearly total rainfall from 2007 until 2018. This was a confirmation of BFTSC and GFTSC efficiency in identification of time series components. Now, it was much clearer, there was trend and also showing a linear trend which was very difficult to detect in Figure 3 above due to seasonal fluctuation. The amount of rainfall 2007 until 2009 was lower because of La Nina effect and higher in 2010 to 2011 due to El Nino. Then the rainfall decreased in 2012 was also due to La Nina. However the increment of rainfall in 2013 was due to global warming (Nucitteli, 2014). As for 2016, the high amount of rainfall was due to El Nino (Null, 2021).

***FIGURE 1. BFTSC OF 10 YEARS MONTHLY RAINFALL IN IBADAN***

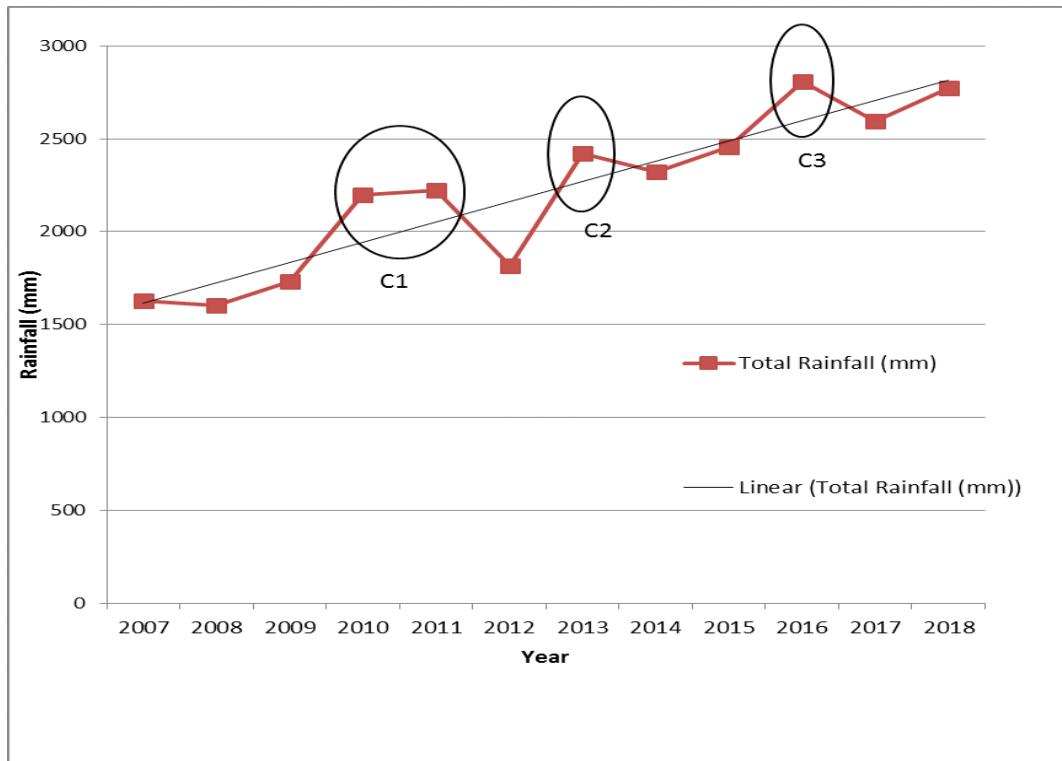


**FIGURE 2. GFTSC OF 10 YEARS MONTHLY RAINFALL IN IBADAN**

**Group for Time Series Components**



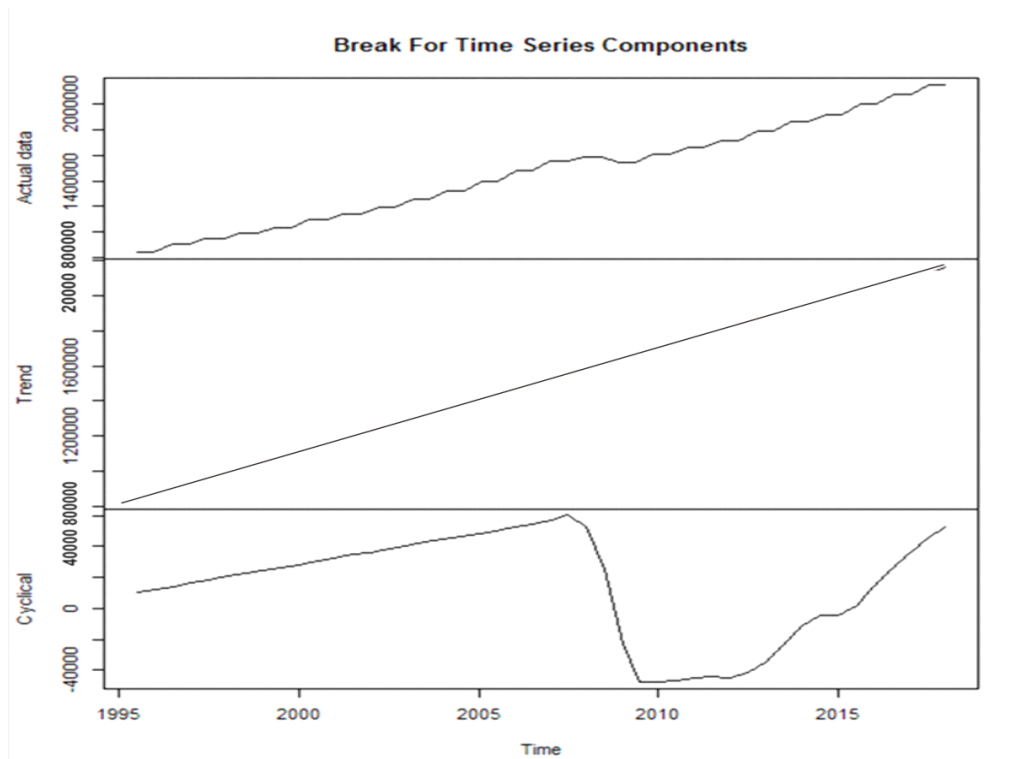
**Figure 3. TIME SERIES PLOT OF IBADAN YEARLY TOTAL RAINFALL**



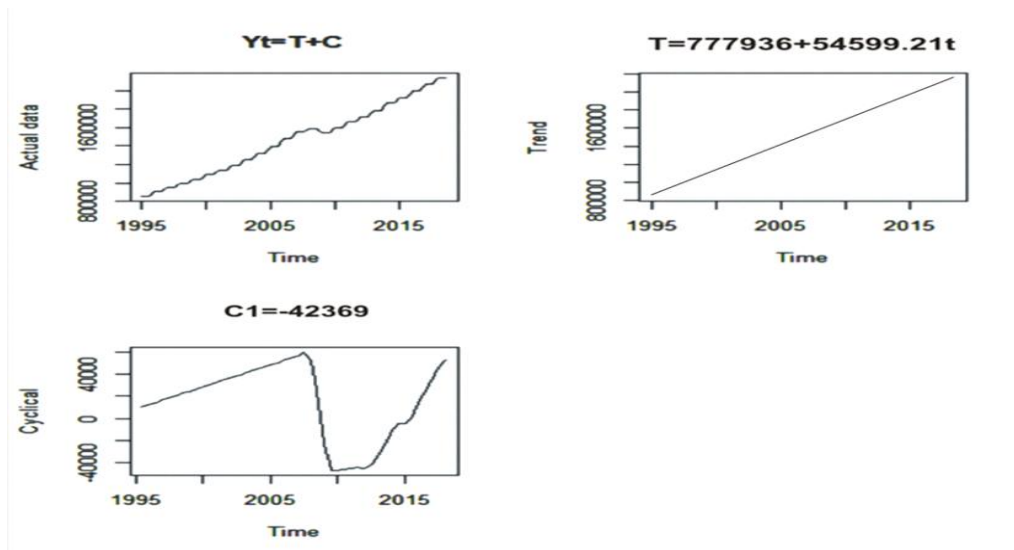
### BFTSC and GFTSC using UK GDP

Figure 4, 5 and 6 show the plots produced by BFTSC, GFTSC and time series plot for UK GDP yearly data respectively. BFTSC display simultaneously the plots of actual UK GDP, trend and cyclical as in Figure 4 which was easy, straight forward and fast in identifying the existence of time series components in the data. GFTSC also display the 3 plots but together with the trend equation and cyclical value (C1). This can enhanced further understanding regarding the UK GDP data and also can be used in forecasting. Both BFTSC and GFTSC successfully extract trend and cyclical components as identified by manual approach as in previous section

**Figure 4. BFTSC plots of Yearly UK GDP**



**Figure 5. GFTSC plots of Yearly UK GDP**



**Figure 6. UK GDP Yearly TIME SERIES PLOT**

Figure 6 displayed the United Kingdom Gross Domestic Product (UK GDP) from 1995 until 2018 which comprises of 24 years. The GDP value decreased from 1.59 million in 2008 to 1.55 million in 2009 with a sudden dropped of 2.7%. This was due to global economic crisis but has affected UK GDP severely and considered as the great recession because it was a long term effect and took 5 years to recover (Velma & James, 2018). This drop in 2009 has shifted the trend line about 6% lower and the GDP only bounced back in 2014; but still growing slowly onwards to 2018 (Cribb & Johnson, 2018). Based on the UK GDP time series plot (Figure 6) and also the reasons behind those values, we can confirmed that the UK GDP data has trend and cyclical components. The trend was linear and the cyclical (C1) was due to the great recession (Figure 6).

Two sets of linear trend data have been used earlier (Ibadan Monthly rainfall and UK GDP) where BFTSC and GFTSC performed well in both data sets. Next, BFTSC and GFTSC were evaluated using another two data sets having quadratic curve trend (the London Stock Exchange (LSE) and the United States (US) Stock Market).

### **London Stock Exchange (LSE) Data**

Figure 7, 8 and 9 shows the plots produced by BFTSC, GFTSC and time series plot for LSE monthly data respectively. Figure 7 is the time series plot for LSE monthly data using BFTSC. Figure 8 is the time series plot for LSE monthly data using GFTSC. Both BFTSC and GFTSC failed to identify curve trend and display linear trend instead. They also identify only one cyclical and no irregular which contradict with manual approach identification in figure 9 as in previously. These indicated the limitation of BFTSC and GFTSC when the trend deviated from linear which reflected similar findings as in the simulation study.

Figure 9 displays the manual time series plot of monthly LSE from January 2001 until December 2020. There was steady increment from 2001 until 2007 but dropped in 2008 due to economic crisis and slowly increased from 2009 to 2017. However, dropped once again in 2018 due to economic crisis. Then, it started to increase regularly up to 2019 but drastically dropped in 2020 due to COVID-19 pandemic (Moradi, Jabbari. & Rounaghi, 2021; Baker, Bloom, Davis, Kost, Sammon & Viratyosin, 2020). The first two dropped (in 2008 and 2018) related to economic crisis was considered as cyclical component (C1 and C2) and the third dropped (in 2020) related to COVID-19 pandemic was irregular component (I1). Figure 9 also shows a curve trend.

### **Figure 7. BFTSC plots of Monthly LSE**

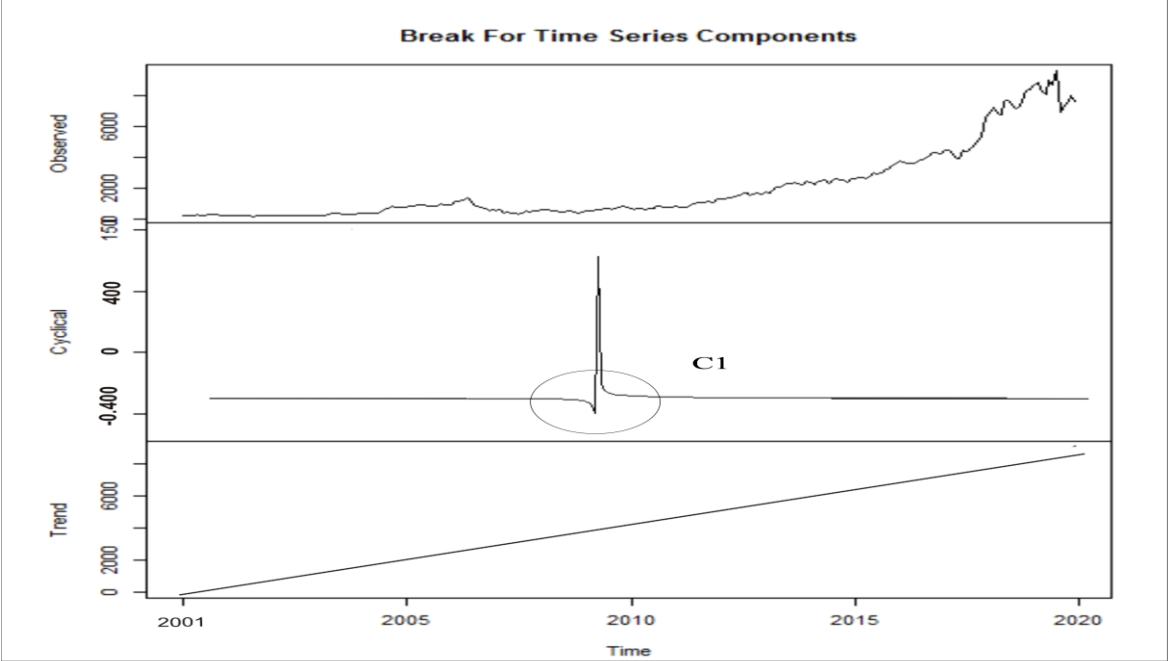
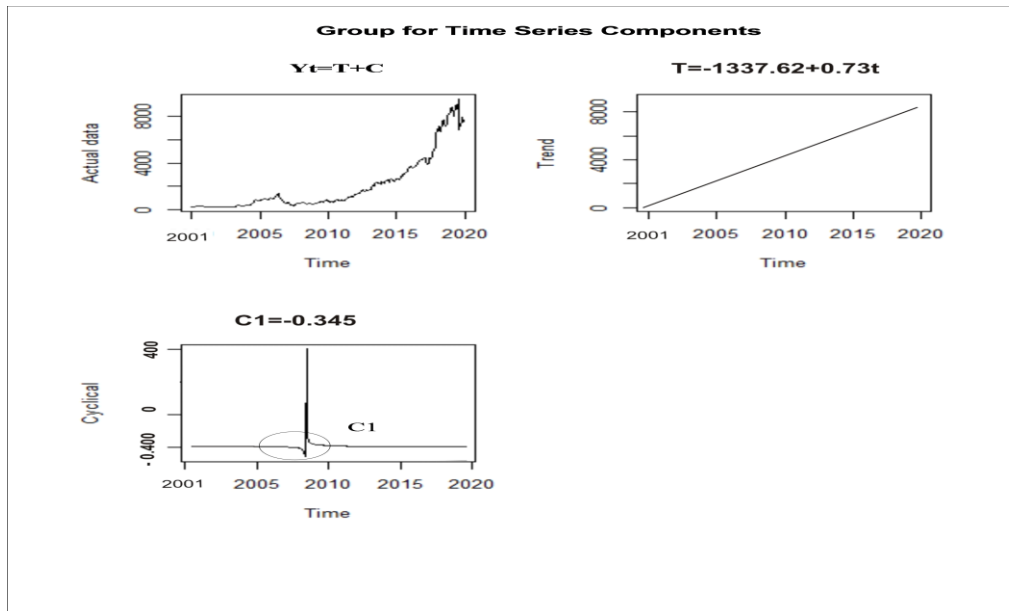


Figure 8. GFTSC plots of Monthly LSE



**Figure 9. TIME SERIES PLOT OF MONTHLY (LSE)**

### **United States (US) Stock Market Data**

Figure 10, 11 and 12 show the plots produced by BFTSC, GFTSC and time series plot for US stock market monthly data respectively. Figure 10 is the time series plot for US stock market monthly data using BFTSC. Figure 11 is the time series plot for US stock market monthly data using GFTSC. BFTSC and GFTSC has not performed well for US Stock Market monthly data respectively. Both BFTSC and GFTSC managed to identify one cyclical but failed to identify curve trend and display linear trend instead, which contradict with manual approach identification as in previously. These indicated the limitation of BFTSC and GFTSC when the trend deviated from linear which reflected similar findings as in LSE data and the simulation study

Figure 12 is the time series plot of monthly US Stock Market. In this study, the US Stock Market was a monthly data from January 2001 until December 2018 and a total of 18 years. The data was obtained from the Yahoo Finance using Nasdaq adjusted close data which was the closing price after adjustment for all applicable splits and dividend distribution (Tang, Xiao, Wahab & Ma, 2021). The measurement of US Stock Market is in United States Dollars (\$). There was steady increment from 2001 until 2007 but dropped in 2008 due to economic crisis and slowly increased from 2009 to 2018 (Shirvani, 2020). The dropped in 2008 that related to economic crisis was considered as cyclical component (C1). Figure 12 also shows a curve trend.

*Figure 10. BFTSC plots of Monthly US Stock Market*

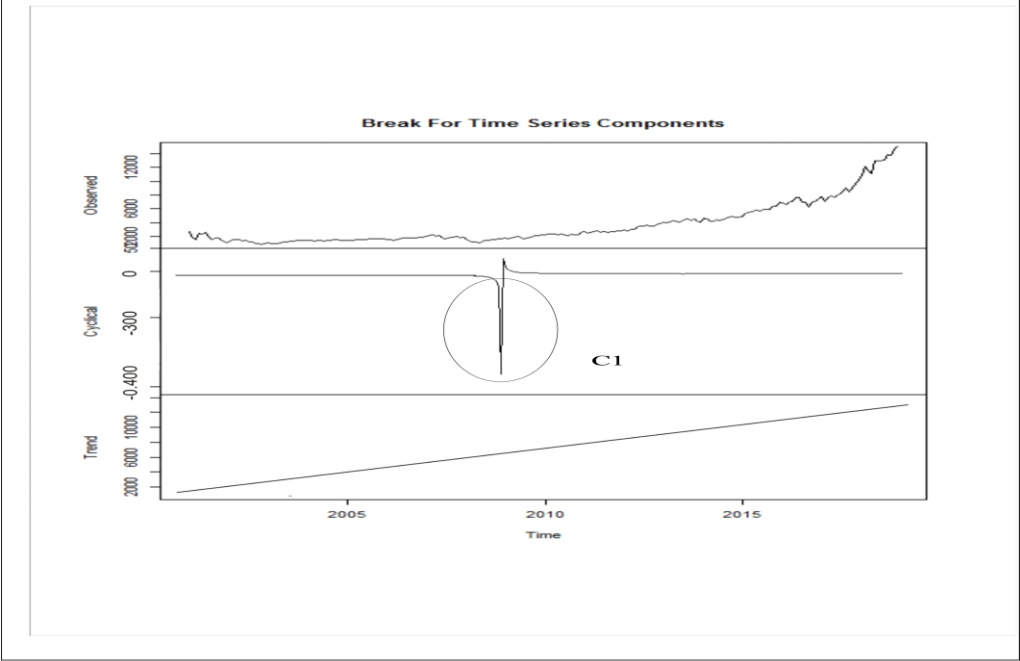


Figure 11. GFTSC plots of Monthly US Stock Market

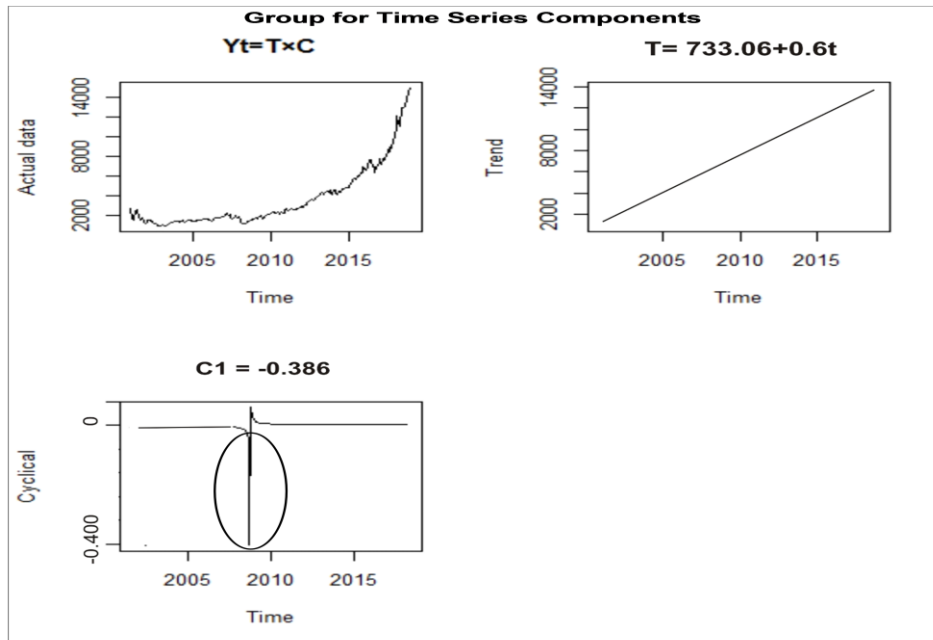


FIGURE 12 . Time Series plots of monthly US Stock Market

Overall BFTSC and GFTSC performed very well in identifying the time series components that embedded in the first two empirical data of Ibadan rainfall and UK GDP that showing linear trend. Thus, the displaying of the plots automatically and simultaneously makes time series identification process easy, straight forward and fast for end users. However, BFTSC and GFTSC performed poorly when using LSE and US Stock Market data that exhibiting curve trend. These indicated the limitation of BFTSC and GFTSC when the trend deviated from linear which reflected similar findings as in the simulation study. This is due to the derivation of BFTSC and GFTSC was based on linear trend as in BFAST. Thus, to use BFTSC and GFTSC requires large sample size to identify the existence of linear trend and other time series components (seasonal, irregular, and cyclical) in a data set.

## Discussion / Conclusion

Based on the every result in the simulated and the empirical analysis, BFTSC and GFTSC is the most appropriate for time series components identification, for this reason BFTSC and GFTSC is recommended as a good alternative to BFAST. This is because BFTSC and GFTSC identifies the four components of time series statistics which is one of the basic limitations of BFAST. GFTSC also outperform BFTSC with 0.2%. Based on the forecast value for a years ahead, it reveal no evidence of water disaster in the period 2019 in Ilorin Kwara state.

BFTSC and GFTSC produced the related plots automatically and showing them simultaneously. There was a slight difference between BFTSC and GFTSC when displaying the plots where BFTSC combined the plots while GFTSC separated them and included equation and time series components values on top of the respective plots. Hence, provides better understanding regarding the time series components and can be used in forecasting. Overall, based on the simulation and empirical findings, we can conclude that both BFTSC and GFTSC are performing very well in large data set that displaying linear trend which can bridge the gap between expert and end users in identifying the time series components because they are easy, straight forward and fast. These findings indicated that BFTSC and GFTSC automatic identification techniques are suitable for data with linear trend and require future extensions for other trends.

Therefore, from these discussion, BFAST was extended to a technique that can identify the four time series components. BFTSC is recommended for efficient time series components identification for an improved forecasting.

### **Limitation, implication and further research**

BFTSC and GFTSC were extended from BFAST which focus on linear trend only. Thus, BFTSC and GFTSC performed well when the data has linear trend and not other types of trend, which can be further expanded in the future.

Although BFTSC and GFTSC is as good as manual approach in identifying the times series components embedded in data that exhibited a linear trend, but the manual approach has more ability to provide reasons (or explanations) behind the time series components values. Thus, users of BFTSC and GFTSC are advised to find these reasons when referring to the BFTSC and GFTSC automatic plots. This is to obtain deeper meaning of the time series component values related to real world application (e.g., global warming or economic crisis etc.) which are very useful for the next several forecasting stages such as in selecting the appropriate forecasting techniques and also in justification of the forecast values accuracy if modification is needed due to cyclical reason if it is expected to happen in the near future.

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### **Authors Contributions**

Ajare Emmanuel Oloruntoba: Analyzing, producing the results and writing the paper.  
Suzilah bt Ismail: Supervising the contents and flow of the paper.

## Ethics

This is the original manuscript; there will be no expectation of any ethical problems after the publication. The two authors have read and approved the manuscript.

## References

- Abbes, A. B., & Farah, I. R. (2017). Prediction Changes for Nonstationary Multi-Temporal Satellite Images using HMM. In *Handbook of Research on Geographic Information Systems Applications and Advancements* (pp. 387-406). IGI Global. DOI: 10.4018/978-1-5225-0937-0.ch015
- Ajare, Emmanuel Oloruntoba , Suzilah bt. Ismail, (2019). Simulation of Data to Contain the Four Time Series Components in Univariate Forecasting. *Journal of Advanced Research in Dynamical and Control Systems*, 11(5), 1005 - 1010.
- Ajare, Emmanuel Oloruntoba , Suzilah bt. Ismail, (2019). Break for Time Series Components (BFTSC) and Group for Time Series Components (GFTSC) in Identification of Time Series Components in Univariate Forecasting. *Journal of Advanced Research in Dynamical & Control Systems*, 11(5), 995 - 1004.
- Box, G. E., Jenkins, G. M., Reinsel, G. C., & Ljung, G. M. (2015). *Time series analysis: forecasting and control*. John Wiley & Sons. Retrieve from [www.scirp.org/](http://www.scirp.org/)
- Box, G. E., & Jenkins, G. M. (1976). *Time series analysis: forecasting and control, revised ed.* Holden-Day. Oakland, California. Retrieve from [garfield.library.upenn.edu/classics](http://garfield.library.upenn.edu/classics)
- Bai, J., & Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of applied econometrics*, 18(1), 1-22. <https://doi.org/10.1002/jae.659>
- Bornhorst, F., Dobrescu, G., Fedelino, A., Gottschalk, J., & Nakata, T. (2011). When and how to adjust beyond the business cycle? A guide to structural fiscal balances. *IMF Technical Notes and Manuals*, 11(02). Retrieve from <https://www.imf.org/external/>
- Bohn, H. (1995). The sustainability of budget deficits in a stochastic economy. *Journal of Money, Credit and Banking*, 27(1), 257-271. DOI: 10.2307/2077862 <https://www.jstor.org/stable/2077862>

Buhalau, T. (2016). Detecting clear-cut deforestation using Landsat data: A time series analysis of remote sensing data in Covasna County, Romania between 2005 and 2015. *Student thesis series INES*. Retrieve from [https://lup.lub.lu.se/student\\_papers/search/publication/](https://lup.lub.lu.se/student_papers/search/publication/)

Cesta, A., Cortellessa, G., Pecora, F., & Rasconi, R. (2005, May). Monitoring Domestic Activities with Scheduling Techniques. In *Proceedings of the 2nd*. Retrieve from <https://www.researchgate.net/publication/238572582>

Cleveland, W. P., & Tiao, G. C. (1976). Decomposition of seasonal time series: A model for the Census X-11 program. *Journal of the American statistical Association*, 71(355), 581-587. Retrieve from

<https://amstat.tandfonline.com/doi/abs/>

Chen, C. (2006). CiteSpace II: Detecting and visualizing emerging trends and transient patterns in scientific literature. *Journal of the American Society for information Science and Technology*, 57(3), 359-377. <https://doi.org/10.1002/asi.20317>

Caiado, J. (2009). Performance of combined double seasonal univariate time series models for forecasting water demand. *Journal of Hydrologic Engineering*, 15(3), 215-222.

Cipra, T., & Romera, R. (1997). Kalman filter with outliers and missing observations. *Test*, 6(2), 379-395. Retrieve from

<https://link.springer.com/article/10.1007/BF02564705>

DeVries, B., Pratihast, A. K., Verbesselt, J., Kooistra, L., de Bruin, S., & Herold, M. (2013, June). Near real-time tropical forest disturbance monitoring using Landsat time series and local expert monitoring data. In *Analysis of Multi-temporal Remote Sensing Images, MultiTemp 2013: 7th International Workshop on the* (pp. 1-4). IEEE

**DOI:** [10.1109/Multi-Temp.2013.6866022](https://doi.org/10.1109/Multi-Temp.2013.6866022)

Ewing, B. T., & Malik, F. (2013). Volatility transmission between gold and oil futures under structural breaks. *International Review of Economics & Finance*, 25, 113-121. journal homepage: Retrieve from [www.elsevier.com/locate/iref](http://www.elsevier.com/locate/iref)

Flicek, P., & Birney, E. (2009). Sense from sequence reads: methods for alignment and assembly. *Nature methods*, 6(11s), S6. Retrieve from

<https://www.ncbi.nlm.nih.gov/pubmed/19844229>

Gorelick, N., Hancher, M., Dixon, M., Ilyushchenko, S., Thau, D., & Moore, R. (2017). Google Earth Engine: Planetary-scale geospatial analysis for everyone. *Remote Sensing of Environment*, 202, 18-27. <https://doi.org/10.1016/j.rse.2017.06.031>

Jong, R., Verbesselt, J., Schaepman, M. E., & Bruin, S. (2012). Trend changes in global greening and browning: contribution of short-term trends to longer-term change. *Global Change Biology*, 18(2), 642-655. DOI:10.1111/j.1365-2486.2011.02578.x

- Jain, A. K., Duin, R. P. W., & Mao, J. (2000). Statistical pattern recognition: A review. *IEEE Transactions on pattern analysis and machine intelligence*, 22(1), 4-37.  
DOI: [10.1109/34.824819](https://doi.org/10.1109/34.824819)
- Jamali, S., Jönsson, P., Eklundh, L., Ardö, J., & Seaquist, J. (2015). Detecting changes in vegetation trends using time series segmentation. *Remote Sensing of Environment*, 156, 182-195. doi.org/10.1016/j.rse.2014.09.010
- Porter, J., & Zhang, L. (2018). BisPin and BFAST-Gap: Mapping bisulfite-treated reads. *bioRxiv*, 284596. doi: <https://doi.org/10.1101/28459>
- Tolsheden, J. (2018). Detecting and testing for Seasonal breaks in Quarterly National accounts: Based on X-12-ARIMA and BFAST Methods. Retrieve from [diva-portal.org](http://diva-portal.org)
- Maggi, L. M. B. (2018). Times Series Analysis. In *Multiscale Forecasting Models* (pp. 1-29). Springer, Retrieve from <https://www.springer.com/us/book/9783319949918>
- Mok, T. S., Wu, Y. L., Ahn, M. J., Garassino, M. C., Kim, H. R., Ramalingam, S. S., ... & Lee, C. K. (2017). Osimertinib or platinum-pemetrexed in EGFR T790M-positive lung cancer. *New England Journal of Medicine*, 376(7), 629-640. DOI: 10.1056/NEJMoa1612674
- Maus, V., Câmara, G., Appel, M., & Pebesma, E. (2017). dtwSat: Time-Weighted Dynamic Time Warping for Satellite Image Time Series Analysis in R. *Journal of Statistical Software*. Retrieve from <https://cran.r-project.org/web/packages/dtwSat/ind...>
- Verbesselt, J., Hyndman, R., Newnham, G., & Culvenor, D. (2010). Detecting trend and seasonal changes in satellite image time series. *Remote sensing of Environment*, 114(1), 106-115. doi.org/10.1016/j.rse.2009.08.014
- Rikus, L. (2018). A simple climatology of westerly jet streams in global reanalysis datasets part 1: mid-latitude upper tropospheric jets. *Climate Dynamics*, 50(7-8), 2285-2310.
- Zhu, J. Y., Park, T., Isola, P., & Efros, A. A. (2017). Unpaired image-to-image translation using cycle-consistent adversarial networks. *arXiv preprint*. Retrieve from <https://arxiv.org/pdf/1703.10593>
- Zewdie, W., Csaplovics, E., & Inostroza, L. (2017). Monitoring ecosystem dynamics in northwestern Ethiopia using NDVI and climate variables to assess long term trends in dry land vegetation variability. *Applied Geography*, 79, 167-178. <https://doi.org/10.1016/j.apgeog.2016.12.019>
- Zdravevski, E., Lameski, P., Mingov, R., Kulakov, A., & Gjorgjevikj, D. (2015, September). Robust histogram-based feature engineering of time series data. In *Computer Science and Information*

*Systems (FedCSIS), 2015 Federated Conference on* (pp. 381-388). IEEE.

**DOI:** [10.15439/2015F420](https://doi.org/10.15439/2015F420)

Zhao, G., Li, E., Mu, X., Wen, Z., Rayburg, S., & Tian, P. (2015). Changing trends and regime shift of streamflow in the Yellow River basin. *Stochastic environmental research and risk assessment*, 29(5), 1331-1343. Retrieve from

<https://www.hindawi.com/journals/amete/2016/6130179/ref/>

Zeileis, A., Kleiber, C., Krämer, W., & Hornik, K. (2003). Testing and dating of structural changes in practice. *Computational Statistics & Data Analysis*, 44(1-2), 109-123.

[https://doi.org/10.1016/S0167-9473\(03\)00030-6](https://doi.org/10.1016/S0167-9473(03)00030-6)