Model Development For Residual Chlorine

Abstract

The research is to develop a mathematical model for the decay of residual chlorine at the nodes and compare with results from Epanet 2.0. All the influencing parameters of initial chlorine $C_{cl(init(o))}$, bulk decay (K_b), wall decay ($\mathbf{K_W}$) and travel time (t), were well represented and the equation is developed applying the differential form of the first order chlorine decay model, describing reactions occurring in the bulk fluid and at the pipe wall, which was transformed in to chlorine concentration-based integral expression$C_{cl(t)}=C_{cl(o)}.e^{-kt}$, $C_{cl(t)}$is the free chlorine (HOCL) concentration (mg/l) at time (t), which is also called the residual chlorine. $C_{cl(o)}$ is the chlorine concentration at time zero, which is the initial chlorine concentration in the entire water distribution systems. K_W is the pipe wall decay coefficient of the particular water distribution pipe network directly affected by initial concentration. K is expressed in the study as the decay due to the quality of water, known as bulk decay coefficient (k_b), per unit hour (t=1) at any particular node per hour.he developed mathematical model was a reflection of the reality in the study area $\mathbf{R_{cl}(n)=C_{cl}}$ **(**init**).** $\mathbf{K_W.}$ $\mathbf{e^{-Kb}}$ **.**The results of the model at nodes 31, 12, 13, 14, 18, 10, 17, 15, 20 and 21 are 0.18 mg/l, 0.19 mg/l, 0.19 mg/l, 0.19 mg/l, 0.19 mg/l, 0.19 mg/l, 0.20 mg/l, 0.19 mg/l, 0.20 mg/l, 0.19mg/l, respectively.Epanet 2.0 results for the same nodes were determine to be 0.17 mg/l, 0.21 mg/l, 0.22 mg/l, 0.20 mg/l, 0.21 mg/l, 0.21 mg/l, 0.25 mg/l, 0.24 mg/l, 0.20 mg/l, 0.20 mg/l, 0.18 mg/l. The correlation between the developed mathematical model residual chlorine results and Epanet 2.0 results gives moderate correlation of 0.561, signifying 60% pearson correlation and 0.01, 2-tailed significance level at 99%.

Keywords: Mathematical model, Residual chlorine, Epanet 2.0, Bulk decay and Wall decay coefficients.

1.0 INTRODUCTION

Water quality model is a reliable tool only if it is able to predict the real system behavior (Yi Wu, 2006). In addition, water quality modeling within water distribution systems is not an easy task because a hydraulic model analysis has to be performed previously in order to provide the resulting flow distribution to the water quality module to transport the chlorine through the system. The predicted chlorine concentrations within a distribution system are governed by bulk and wall reaction parameters. Bulk decay coefficients for chlorine depend on the nature of the source water and the treatment it has received while wall decay coefficients depend on the pipe material and its condition. It is generally assumed that chlorine evolution in a water distribution system can be described by a first-order kinetic model. But the bulk decay parameter can also be non-first-order and some more reliable alternatives have been presented in some recent works (Boccelli *et al*., 2003). Temperature is one of the most important factors affecting chlorine decay rates in drinking water supply systems (Laura et, al, 2015).

The reduction of chlorine residual in water supply system take place in the bulk water as well as the walls of pipes and surfaces such as storage tanks present along the distribution system. The bulk decay is a volume-based reduction process whereas the wall decay is a surface area based process. In addition, the nature of reaction and the types of reactants involved in these two processes are different. Water quality models for the reduction of chlorine, therefore, require separate steps for determination of the wall and bulk decay coefficients. The bulk decay is determined by the laboratory batch test on sample of water taken from the water treatment system ready for disinfection. The wall decay rate is often determined by a calibration process as a difference between the chlorine consumption observed in the distribution system and the chlorine consumption due to bulk decay alone determined by laboratory tests. The wall decay

depends on pipe conditions including the materials from which pipes are made In general, laboratory and pilot tests alone cannot adequately represent the chlorine decay process in the actual distribution system. (Rossman, 2000)

Chlorine substance is universally believed to be relatively cheap and readily available chemical that, when dissolved in clarified water in sufficient quantities destroys most disease causing organisms without being detrimental to people. Chlorine however, is consumed as organisms are destroyed. If sufficient chlorine is added, there will be some residual in water, after all the organisms have been destroyed, this is referred to as free chlorine. Free chlorine usually remain in the water until, it is either lost as exfiltration or used up destroying new contamination. Therefore, remanence of free chlorine in water when test water, proves that most dangerous organisms in water have been removed and it is portable to drink and it is called chlorine residual measuring. (Hallam, 2000)

2.0 MATERIAL AND METHODS

2.1 Residual Chlorine Model Development

The decay order in the research work is determine to be first order with a total of 86 out of the 120 samples found to be straight lines (1^{st} order) which constitute 71.7% and applying the differential form of the first order chlorine decay model, this includes expressions to describe reactions occurring in the bulk fluid and at the pipe wall.

$$\frac{dC}{dt} = -k.C \quad (2.0)$$

Where C is the chlorine concentration at time (t) per (day) and

K is the decay rate constant per (day^{-1}).

Equation 1.0 above is solved as chlorine concentration-based expression:

$$C_{cl(t)} = C_{cl(o)}\, e^{-kt} \quad (2.1)$$

Where, $C_{cl(t)}$ is the free chlorine (HOCL) concentration (mg/l) at time (t), which is also called the residual chlorine or chlorine residual.

Time (t) is expressed in unit hour i.e. per hour (t =1), establishing the basic fact that Epanet 2.0 time interval is expressed in per hour and also the decay rate coefficient is considered in per hourly bases within the system.

$C_{cl(o)}$ is the chlorine concentration at time zero, which is the initial chlorine concentration in the entire water distribution systems or the chlorine concentration at age zero within the network.$k_b(1)$ is the first order rate coefficient per day at unit hour.

K(t) is expressed in the study as the decay due to the quality of water, known as bulk decay coefficient (k_b) and t is 1 as elucidated by Clark et al, (1994), Powell et al (2000) and Saidan et al (2017).

Among all the parameters at the nodes and links of any distribution network, initial chlorine $\mathbf{C_{cl(o)}}$, bulk chlorine decay coefficient (k_b), wall chlorine decay coefficient (k_w) and travel time (t) are the most important influencers for the determination of Residual Chlorine at the network junctions.

Therefore, the research expresses equation 2.1 as thus;

$$\mathbf{R_{cl}(n) = C_{cl}\,(init).\, K_W.e^{-Kb}} \quad (2.2)$$

Where $\mathbf{R_{cl}(n)}$ is the residual chlorine at time (t) per unit hour

$C_{cl(init(o))}$ is the initial chlorine concentration at age zero in the entire water distribution systems.

k_w is the pipe wall decay coefficient of the particular water distribution pipe network.

k_b is the bulk decay due to the quality of the water.

Both k_w and k_b are dependents on the water temperature as explain by Hua et al, 1999 and Clark et al, 1994.

3.0 RESULTS AND DISCUSSIONS

3.1 Residual Chlorine Mathematical Model Computation and Results

Sample points 1, 2, 3, 4, 5, or sample nodes 1, 2, 3, 4, 5, 6, 7, 32 and 33 are source and reservoir nodes with initial chlorine values in the distribution system. The computation of the network residual chlorine does not include the source nodes and the reservoir nodes. As can be seen in eqn. 3.6, developed residual chlorine model for Sokoto water distribution networks;

$$\mathbf{R_{cl}(n)=C_{cl}\ (}\text{init}\mathbf{).\ K_W.e^{-Kb}} \qquad (3.0)$$

$R.C_{cl(n)}$= ?

$C_{cl(init(o))}$ = 2.13mg/l (summation of initial chlorines at the sources and reservoirs)

K_W= 0.078 m/h(Pipe wall decay coefficient for steel pipes in the distribution system)

.K_b= Bulk decay coefficient m/h for each node within a particular WDN

So, applying the values for all the remaining 25 sampling points; Table 1 summarizes the results

Table 1: Mathematical Model Residual Chlorine Results

S/N	Sample Nodes	Computations	Output Results
1	Sample point 6	$R_{cl_{31}} = 2.13 * 0.078 * e^{-(-0.06)} = 0.1764$	**0.18mg/l**

2	Sample point 7	$R_{cl_{12}} = 2.13 * 0.078 * e^{-(-0.12)} = 0.1873$	**0.19mg/l**
3	Sample point 8	$R_{cl_{14}} = 2.13 * 0.078 * e^{-(-0.144)} == 0.1873$	**0.19mg/l**
4	Sample point 9	$R_{cl_{13}} = 2.13 * 0.078 * e^{-(-0.168)} = 0.1918$	**0.19mg/l**
5	Sample point 10	$R_{cl_{18}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.1873$	**0.19mg/l**
6	Sample point 11	$R_{cl_{10}} = 2.13 * 0.078 * e^{-(-0.096)} = 0.1918$	**0.19mg/l**
7	Sample point 12	$R_{cl_{17}} = 2.13 * 0.078 * e^{-(-0.216)} = 0.1951$	**0.20mg/l**
8	Sample point 13	$R_{cl_{15}} = 2.13 * 0.078 * e^{-(-0.168)} = 0.1918$	**0.19mg/l**
9	Sample point 14	$R_{cl_{20}} = 2.13 * 0.078 * e^{-(-0.12)} = 0.1965$	**0.20mg/l**
10	Sample point 15	$R_{cl_{21}} = 2.13 * 0.078 * e^{-(-0.12)} = 0.1914$	**0.19mg/l**
11	Sample point 16	$R_{cl_{30}} = 2.13 * 0.078 * e^{-(-0.192)} = 0.1829$	**0.18mg/l**
12	Sample point 17	$R_{cl_{22}} = 2.13 * 0.078 * e^{-(-0.168)} = 0.2062$	**0.21mg/l**
13	Sample point 18	$R_{cl_{25}} = 2.13 * 0.078 * e^{-(-0.096)} == 0.1965$	**0.20mg/l**
14	Sample point 19	$R_{cl_{16}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.1873$	**0.19mg/l**
15	Sample point 20	$R_{cl_{11}} = 2.13 * 0.078 * e^{-(-0.12)} = 0.1873$	**0.19mg/l**
16	Sample point 21	$R_{cl_{24}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.2013$	**0.20mg/l**
17	Sample point 22	$R_{cl_{27}} = 2.13 * 0.078 * e^{-(-0.192)} = 0.1965$	**0.20mg/l**
18	Sample point 23	$R_{cl_{24}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.1829$	**0.18mg/l**

19	Sample point 24	$R_{cl_{19}} = 2.13 * 0.078 * e^{-(-0.168)} = 0.1914$	**0.19mg/l**
20	Sample point 25	$R_{cl_{24}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.1873$	**0.19mg/l**
21	Sample point 26	$R_{cl_{24}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.1914$	**0.19mg/l**
22	Sample point 27	$R_{cl_{27}} = 2.13 * 0.078 * e^{-(-0.192)} = 0.2013$	**0.20mg/l**
23	Sample point 28	$R_{cl_{24}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.1914$	**0.19mg/l**
24	Sample point 29	$R_{cl_{19}} = 2.13 * 0.078 * e^{-(-0.168)} = 0.1965$	**0.20mg/l**
25	Sample point 30	$R_{cl_{24}} = 2.13 * 0.078 * e^{-(-0.1416)} = 0.1914$	**0.19mg/l**

3.2: Residual Chlorine Mathematical Model Comparison and Discussions

The mathematical model developed was the reflection of the reality in the study area, with the model adequately representing all the influencing parameters of initial chlorine, bulk decay, wall decay and time travel. The results obtained from the developed mathematical model were compared with field measured and Epanet 2.0 results as in Table 2

Table 2: Residual Chlorine Results Comparison; Field, Epanet 2.0 and the Mathematical Model

S/N	Field Measured Residual Chlorine (mg/l)	Epanet 2.0 Residual Chlorine (mg/l)	Mathematical Model R. Chlorine (mg/l)
1	0.26	0.26	0.26
2	0.25	0.25	0.25
3	0.24	0.24	0.24
4	0.23	0.23	0.23
5	0.23	0.23	0.23
6	0.18	0.17	0.18
7	0.22	0.21	0.19

8	0.19	0.23	0.19
9	0.21	0.20	0.19
10	0.19	0.21	0.19
11	0.21	0.21	0.19
12	0.17	0.21	0.20
13	0.11	0.25	0.19
14	0.16	0.24	0.20
15	0.16	0.20	0.19
16	0.21	0.20	0.18
17	0.22	0.18	0.21
18	0.21	0.20	0.20
19	0.12	0.13	0.19
20	0.13	0.19	0.19
21	0.18	0.19	0.20
22	0.22	0.22	0.20
23	0.21	0.22	0.18
24	0.22	0.20	0.19
25	0.20	0.20	0.19
26	0.22	0.22	0.19
27	0.20	0.19	0.20
28	0.11	0.20	0.19
29	0.16	0.18	0.20
30	0.19	0.23	0.19

3.3: Residual Chlorine Model Statistical Analysis

The statistical analysis of the relationship using SPSS is hereby presented, Table 3; which shows the descriptive statistics of the three results showing their mean, standard deviations, standard

error, 95% confidence level of upper and lower bounds, minimum and maximum concentration of each set of the results. Table 3a; which shows analysis of variance (ANOVA) between and within the groups with F value of 2.158 and significant value of 0.122. Table 3b; uses Tukey HSD mean for groups in homogeneous subsets of 0.05 and it has significant value of 0.103. Table 3c; shows significance level of dependent results and independent results with field / mathematical model results, Epanet 2.0 / field results and mathematical model / Epanet 2.0 to have significance level of 0.686, 0.103 and 0.436 respectively.

Table 3d; shows 2-tailed Pearson correlation between the results; mathematical model cum Epanet 2.0 having 0.561, mathematical model cum field measured having 0.567 and Epanet 2.0 cum field measured having 0.423, and correlation is significant both at 0.01 and 0.05 levels (2-tailed). Table 3e; depicts the level of correlation between the developed mathematical model residual chlorine results and the simulated Epanet 2.0 in relation to the field results which gives moderate correlation of 0.561, signifying 60% pearson correlation and 0.01, 2-tailed significance level at 99%.

Table 3: Descriptive Relationship of the Residual Chlorine Results

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Field Results	30	0.1937	0.03943	0.00720	0.1789	0.2084	0.11	0.26
Epanet Results	30	0.2097	0.02697	0.00492	0.1996	0.2197	0.13	0.26
M Model Results	30	0.2001	0.02053	0.00375	0.1924	0.2078	0.18	0.26
Total	90	0.2011	0.03041	0.00321	0.1948	0.2075	0.11	0.26

Table 3 a: Analysis of Variance (ANOVA)for the Residual Chlorine Results

	Sum of Squares	Df	Mean Square	F	Significance
Between Groups	0.004	2	0.002	2.158	**0.122**
Within Groups	0.078	87	0.001		
Total	0.082	89			

Table 3 b: Tukey HSD Residual Chlorine Results

Division	N	Subset for alpha = 0.05
		1
Field Results	30	0.1937
M Model Results	30	0.2001
Epanet Results	30	0.2097
Significance		**0.103**

Means for groups in homogeneous subsets are displayed.
a. Uses Harmonic Mean Sample Size = 30.000.

Table 3 c: Tukey HSD Dependent Variable Multiple Comparisons Results

(I) Division	(J) Division	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Field Result	Epanet Result	-.01600	0.00775	.103	-.0345	0.0025
	M Model Result	-.00643	0.00775	.686	-.0249	0.0120
Epanet Result	Field Result	.01600	0.00775	.103	-.0025	0.0345
	M Model Result	.00957	0.00775	.436	-.0089	0.0280
M Model Result	Field Result	.00643	0.00775	.686	-.0120	0.0249
	Epanet Result	-.00957	0.00775	.436	-.0280	0.0089

Table 3 d: Correlation Significance of Residual Chlorine Results

		Measured field Results	Epanet 2.0 Results	Mathematical Model Results
Measured field Results	Pearson Correlation	1	0.423*	0.567**
	Sig. (2-tailed)		0.020	0.001
	N	30	30	30
Epanet 2.0 Results	Pearson Correlation	0.423*	1	0.561**
	Sig. (2-tailed)	0.020		0.001
	N	30	30	30
Mathematical Model Results	Pearson Correlation	0.567**	0.561**	1
	Sig. (2-tailed)	0.001	0.001	
	N	30	30	30

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

Table 3 e: Correlation between Epanet 2.0 and Mathematical Model R. Chlorine Results

		Mathematical Model R	Epanet 2.0 R
Mathematical Model R	Pearson Correlation	1	.561**
	Significance (2-tailed)		.001
	N	30	30

**. Correlation is significant at the 0.01 level (2-tailed).

3.4 Developed Mathematical Model Program for Residual Chlorine

The developed mathematical model for the decay of residual chlorine in Sokoto water distribution network nodes gives 99% correlation significance at 0.01 levels and 60% moderate Pearson correlation in comparison with Epanet 2.0 results. A computer program was worked out using the developed mathematical model to compute the residual chlorine at each node of Sokoto water distribution pipe network. Table 3 f; shows the residual chlorine output run values from the developed program.

Table 3 f: Program Residual Chlorine Output Values

S/N	Sample point	Bulk decay coefficient	Residual Chlorine run values	
1	SP1/SN1	-	-	
2	SP2/SN2	-	-	
3	SP3/SN3	-	-	
4	SP4=SN4, SN5,SN32	-	-	
5	SP5=SN6, SN7 & SN33	-	-	
6	SP6/SN31	0.06	0.176414	
7	SP7/SN12	0.12	0.187322	
8	SP8/SN28	0.12	0.187322	
9	SP9/SN9	0.144	0.191872	
10	SP10/SN8	0.12	0.187322	
11	SP11/SN34	0.144	0.191872	
12	SP12/SN26	0.1608	0.195123	

13	SP13/SN14	0.144	0.191872	
14	SP14/SN13	0.168	0.196533	
15	SP15/SN18	0.146	0.192257	
16	SP16/SN10	0.096	0.18288	
17	SP17/SN17	0.216	0.206197	
18	SP18/SN15	0.168	0.196533	
19	SP19/SN20	0.12	0.187322	
20	SP20/SN21	0.12	0.187322	
21	SP21/SN30	0.192	0.201307	
22	SP22/SN22	0.168	0.196533	
23	SP23/SN25	0.096	0.18288	
24	SP24/SN16	0.146	0.192257	
25	SP25/SN11	0.12	0.187322	
26	SP26/SN24	0.146	0.192257	
27	SP27/SN27	0.192	0.201307	
28	SP28/SN29	0.146	0.192257	
29	SP29/SN19	0.168	0.196533	
30	SP30/SN23	0.146	0.192257	

4.0 CONCLUSIONS AND RECOMMENDATIONS:

4.1 Conclusions

1. A mathematical model for the decay of residual chlorine in Sokoto water distribution network nodes was develop with 99% level of significance at 0.01 and 60% correlation in comparison with Epanet 2.0 results.

10. A model program was also worked out using the developed mathematical model to compute the residual chlorine at each node of the network area.

4.1 Conclusions

4.2 Recommendations

- That, the developed mathematical model and computer program for the decay of residual chlorine in Sokoto water distribution network nodes be adopted in validating Epanet 2.0 results in line with the measured results.

4.3 Contribution To Knowledge

- A mathematical model and computer program for the decay of residual chlorine in Sokoto water distribution network nodes were developed which represent the actual situation in the study area.

5.0 References

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