Modelling of Earthquake b- and a-values using Least Squares and Maximum Likelihood Estimate Methods in Different Tectonic Regions of the World

ABSTRACT

Aims: This study modelled a- and b-values of earthquakes employing the least squares regression and maximum likelihood estimate methods.

Methodology: Data used in the study were obtained from the International Seismological Centre (ISC), an earthquake catalogue of the United Kingdom. The time window was from 1st January 1988 to 31st December 2010 (30 years) with earthquake focal depth of 0-700km and magnitude Mb ≥1.3. Ten different locations were selected and a total of 149,965 events were used. The acquired data were processed and analysed using Microsoft Excel and the hypothesis was tested using independent t-test statistics with the aid of Statistical Software for Social Sciences (SPSS) version 23.0.

Results: The findings of the study revealed that the b- and a-values calculated using the least squares regression method were higher than the ones obtained using the maximum likelihood estimate method. Also, the hypothesis revealed that there is a significant difference between the use of the least squares regression method and the maximum likelihood estimate method in the determination of b- and a-value of earthquakes in a given region.

Conclusion: The maximum likelihood estimate gives a better estimate of b- and -a values than the least squares regression method.

Keywords: Modelling; a- and b-values; least squares regression; maximum likelihood; tectonic region

INTRODUCTION

One of the most common methods of dealing with problems in seismology that involves the statistical of earthquakes is the Gutenberg- Richter's relation. It was developed by Richter and Gutenberg in 1944 in California. Before now, A similar formula was developed in Japan in 1939 using the amplitude of earthquakes by Iida and Ishimoto. Later Utsu (1965) developed the maximum likelihood estimate method. The b- and a- values which appear in the two methods in seismology are referred to as the properties of the seismic medium and level of seismicity or productivity respectively.

[1[and [2] reported that the b- and a-values can be estimated either by linear least square regression or by maximum likelihood method. However some researchers believe that the maximum likelihood is the most robust and generally accepted method for estimating b-values ([1]; [3[).

The two constants b- and a-values are very important parameters in seismological studies, but the b-value is the most studied. Experimental rock effects have proven that the b-value is inversely proportional to the stress magnitude and that low b - value regions have excessive strain accumulations ([4]; [5[; [6[). Precise, there's a clear pattern of decreasing b -values before a rock rupture or an earthquake tremor.

Many seismologists have studied the physical significance of the b - -value extensively, and lots of case research has corroborated the phenomenon of decreased b - value frequently going on around and adjacent areas before earthquakes. For instance, [7] studied the adjustments within the b-values before an M s-6.0 earthquake in Changning, China, and observed that low b -value anomalies (≤0.85) had been present inside the epicentre area and adjoining regions before the Changning earthquake, with a lower inside the b -values close to the epicentre fewer months earlier than the earthquake. [7] examined the fluctuations within the b-value in and around the source vicinity of the magnitude of 6.9 and 6.8 earthquakes off the coast of Miyagi Prefecture, Japan. The authors determined that the earthquakes occurred close to an area with very small b - values, even after the earthquake. Also [9] examined the pre-seismic b-value variation of a 6.0 magnitude earthquake in Luxian, China, and observed that anomalous low b -value features came about in and across the source region.

[10] studied the characteristics of the pre-earthquake b - value anomaly in Jiuzhaigou, China, for a 7.0-magnitude earthquake and found that this area had extensively low b -value anomalies before the earthquake. Also [11] examined spatial and temporal b -value for large- and small-scale regions and found that b -values in the large-scale region ranged from 0.689 to 1.169, with a mean value of 0.928, while the b -values in the small-scale region ranged from 0.694 to 1.223, with a mean value of 0.925. The b -values in the study area were below the mean value before the medium and strong earthquake occurrence, and all associated with the unusual feature of a sudden drop-low peak rise.

Hypothesis

HO: There is no significant difference between the use of the least squares regression method and the maximum likelihood estimate method in the determination of b- and a-value of the earthquake in a given region.

HA: There is a significant difference between the use of the least squares regression method and the maximum likelihood estimate method in the determination of b- and a-value of the earthquake in a given region.

METHODOLOGY

Source of data

The data employed in this study was acquired from an earthquake catalogue called International Seismological Centre (ISC) hosted by Piper Lane, Thatchman, Berkshire, United Kingdom on the website (http://www.isc.ac.uk/). The time window was from 1st January 1988 to 31st December 2010 (30 years) with earthquake focal depth of 0-700km and magnitude $M_b \geq 1.3$.

Study Areas

The study areas covered the Mediterranean, South Africa, Western Europe, Western Pacific, Southern Australia, Southern Pacific, West South America, West of North America, Artic and Japan. The coordinates of the respective regions are as shown in Table 1 and Fig.1

Table 1: Study areas

S/N	Location	Symbol	Min. Lat	Max. Lat	Min Long.	Max. Long
1	Mediterranean	L1	20^0N	47.5^0N	17.5^0W	45.5^0E
2	Southern Africa	L2	35^0S	5^0S	5^0E	55^0E
3	Western Europe	L3	55^0N	70^0N	0^0	49^0E
4	Western Pacific	L4	20^0S	5^0N	99^0E	150^0E
5	Southern Australia	L5	45^0S	20^0S	110^0E	160^0E
6	Southern Pacific	L6	45^0S	20^0S	160^0E	180^0E
7	West of South America	L7	45^0S	5^0N	90^0W	60^0W
8	West of North America	L8	20^0N	50^0N	140^0W	10^0W
9	Arctic	L9	50^0N	68^0N	160^0W	130^0W
10	Japan	L10	30^0N	55^0N	130^0E	170^0E

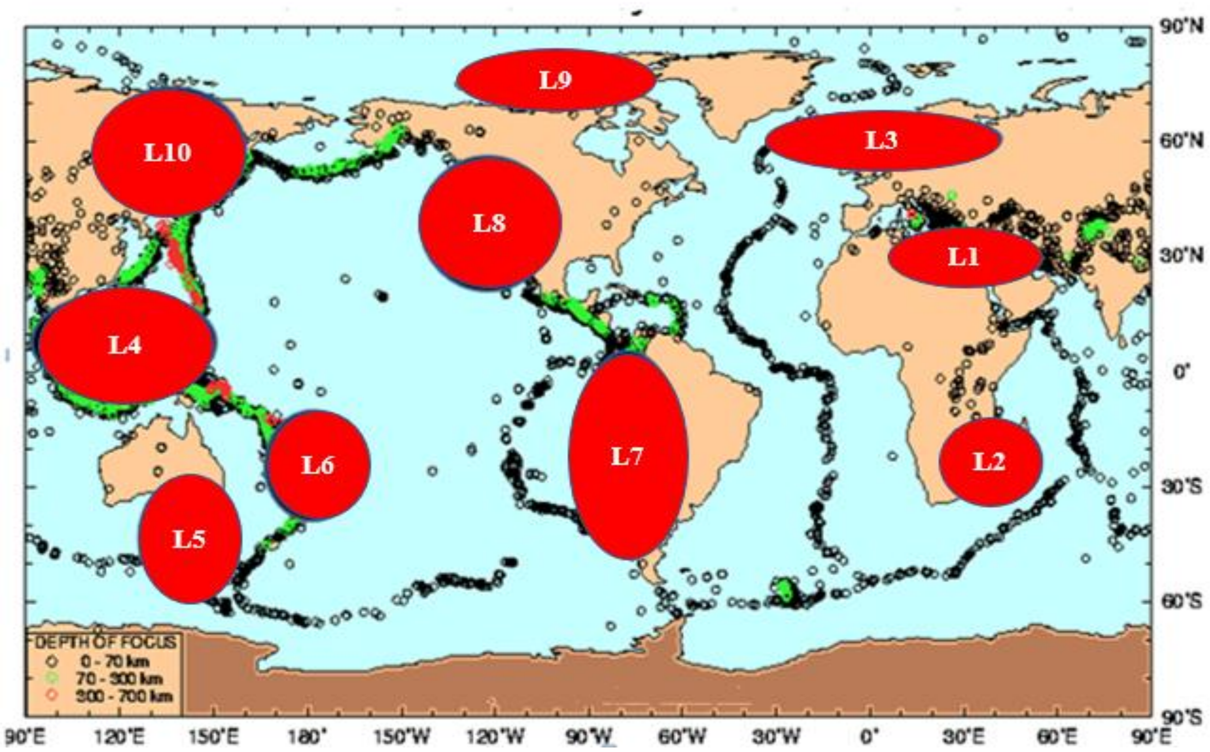

Fig.1. Map showing the study areas across the world modified from ([12]

Theoretical background

(i) Least squares regression (LSR) method

Least squares regression method was developed by Gutenberg and Richter. It expresses the relationship between number of earthquakes and magnitude in the form:

$$\text{Log } N = a - b\ (M_0;\ M \geq M_0) \qquad 1$$

Where N (M) is the cumulative number of earthquakes with magnitudes ($M \geq 0$), M_0 is the minimum magnitude above which all earthquake within a particular region are recorded, a and b are constants. a = the level of seismicity or productivity and b = the tectonic parameter that describes the properties of the seismic medium.

(ii) Maximum likelihood estimate method

Eqn(1) indicates that the magnitude is distributed exponentially as given by[13]..

$$P(m) = \beta e^{-\beta(M - Mc)}\ ;\ M \geq M_c \qquad 2$$

Where $\beta = bLn(10)$, p(m) is the probability density function of M

Maximum likelihood can be developed according to [14] as:

(i) $$L(M) = \prod_{I=1}^{n} \beta e^{-\beta(M_I - M_0)} \qquad 3$$

$$L(M) = \beta^n e^{-\beta \sum_{I=1}^{n} (M_I - M_0)} \qquad 4$$

(ii) The natural logarithm of likelihood function is given as:

$$LnL(M) = n\lambda v\beta - \beta \sum_{i=1}^{n} (M_I - M_0) \qquad 5$$

(III) Define the quantity β which optimized the logarithm likelihood function by taking the derivative and equating to zero.

$$\frac{\partial LnL(M)}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^{n} \left(M_I - M_0\right) = 0 \qquad 6$$

$$\beta = \frac{1}{\bar{M} - M_0} \qquad 7$$

Putting $\beta = bLn(10)$ in eqn (7) yields

$$\overset{*}{b} = \frac{1}{\bar{M} - M_0} \qquad 8$$

The parameter β also known as b-value and the modified b-value is given by:

$$\overset{*}{b} = \frac{Loge}{\bar{M} - (M_0 - \frac{\Delta M}{2})} \qquad 9$$

From eqn (9), in practice the magnitude is rounded into ΔM normally set $\Delta M = 0.1$

$$b^{(u)} = \frac{Log_e}{\left[\left(\bar{M}\right) - \left(M_C - \frac{\Delta M}{2}\right)\right]} = \frac{0.4343}{\left[\bar{M} - \left(M_C - 0.05\right)\right]} \qquad 10$$

Where $\bar{M}$ is the average magnitude for a particular region, M_C is the magnitude of completeness

The error or uncertainty in b is given by:

$$\sigma_b{}^{(u)} = \frac{b^{(u)}}{\sqrt{N}} \qquad 11$$

The a-value is given by [15]) as:

$$a = Log\left(\frac{N}{T}\right) + b.M_C \qquad 12$$

Where T = time interval employed in the study.

The error or uncertainty is given by:

$$\sigma_a = \left[(M_C \sigma_b)^2 + (b\Delta M)^2\right]^{\frac{1}{2}} \qquad 13$$

Where ΔM = 0.1

σ_b =error in the b-value

Results and discussion

Table 2 Summary of **b-** and **a-** values of earthquakes by Least squares method and Maximum likelihood method in the study area

			Number of earthquakes	Least squares regression method (LSR)				Maximum likelihood estimate method(MLE)			
S/N	Location	Label	Number of earthquakes	b	σ_b	a	σ_a	b	σ_b	a	σ_a
1	Mediterranean	L1	16613	1.00	0.05	7.60	0.23	0.41	0.03	4.22	0.22
2	South Africa	L2	1230	0.79	0.06	5.85	0.27	0.40	0.05	3.09	0.25
3	Western Europe	L3	191	0.77	0.03	4.64	0.12	0.49	0.02	2.37	0.11
4	Western Pacific	L4	53907	0.92	0.06	7.98	0.28	0.39	0.04	4.70	0.22
5	Southern Australia	L5	257	0.72	0.05	5.03	0.21	0.40	0.04	2.33	0.20
6	Southern Pacific	L6	10676	0.81	0.05	6.62	0.24	0.38	0.04	3.84	0.22
7	West of South America	L7	11096	0.82	0.05	6.67	0.24	0.35	0.03	3.72	0.19
8	West of North	L8	8297	0.76	0.05	6.34	0.23	0.43	0.04	3.90	0.22

	America										
9	Arctic	L9	4613	0.86	0.03	6.63	0.15	0.34	0.02	3.41	0.14
10	Japan	L10	43085	0.84	0.05	7.32	0.25	0.31	0.03	4.18	0.17
	Total		**149965**								

Table 2 indicates that the **b-** and **a-**values evaluated using the least squares regression method were higher than the ones obtained using the maximum likelihood estimate method. The distribution of the results is shown in Fig. 2 and Fig. 3. Both the least squares regression method and maximum likelihood estimate method are based on the fitting process. The least squares regression method was used to evaluate the b-and a-value by carrying out the linear regression of log N against M_b. The maximum likelihood estimate method gives a less biased and less unlikely estimate than the weighted least squares regression method [15]. Also, the assumption that every data point has the same weight and residuals are Gaussian-distributed makes the least square method biased [16].

It was also observed that the standard error σ_b obtained for b-and standard error σ_a obtained for a-values using the maximum likelihood estimate method was smaller than the ones obtained with the least squares regression method. This implies that the smaller the standard error the better the accuracy. The findings in this study are in line with [3] and [17] who concluded after their comparative analysis of b- and a- values that the maximum likelihood method is more robust as compared to the least squares regression method in the estimation of b-and –a values in a given region.

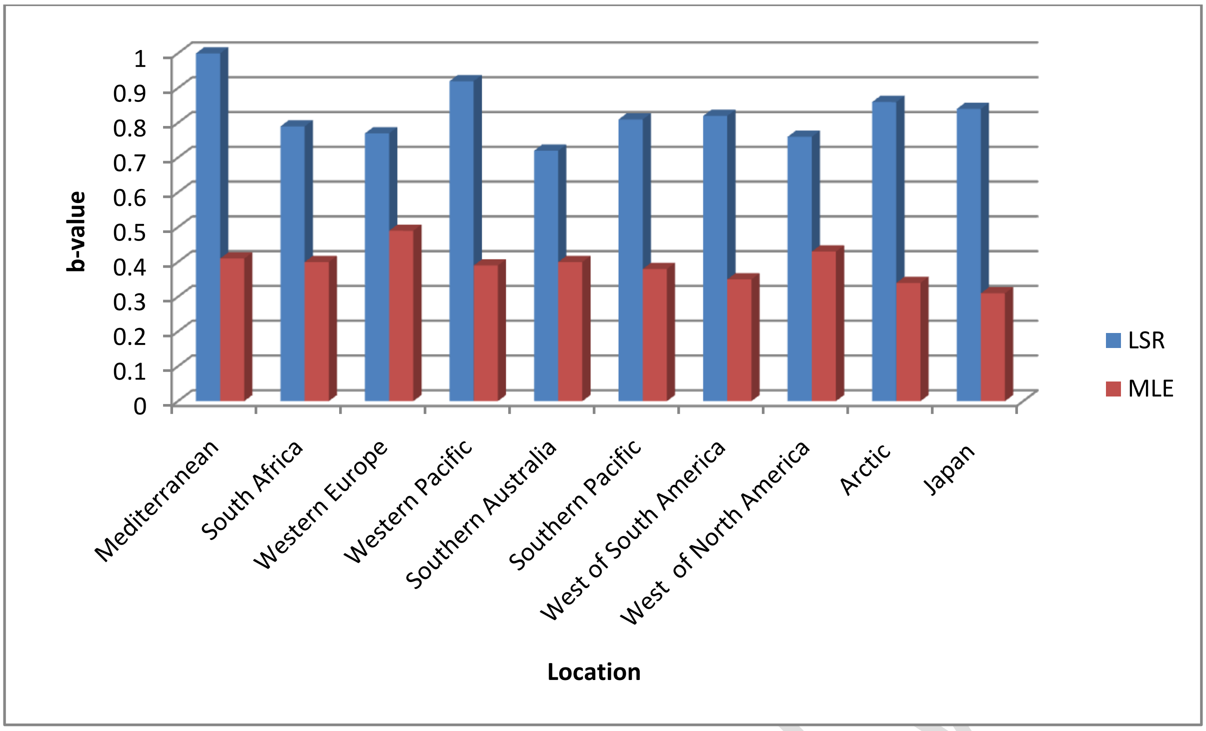

Fig. 2 Bar chart showing the b-values of the least squares regression method and maximum likelihood estimate methods

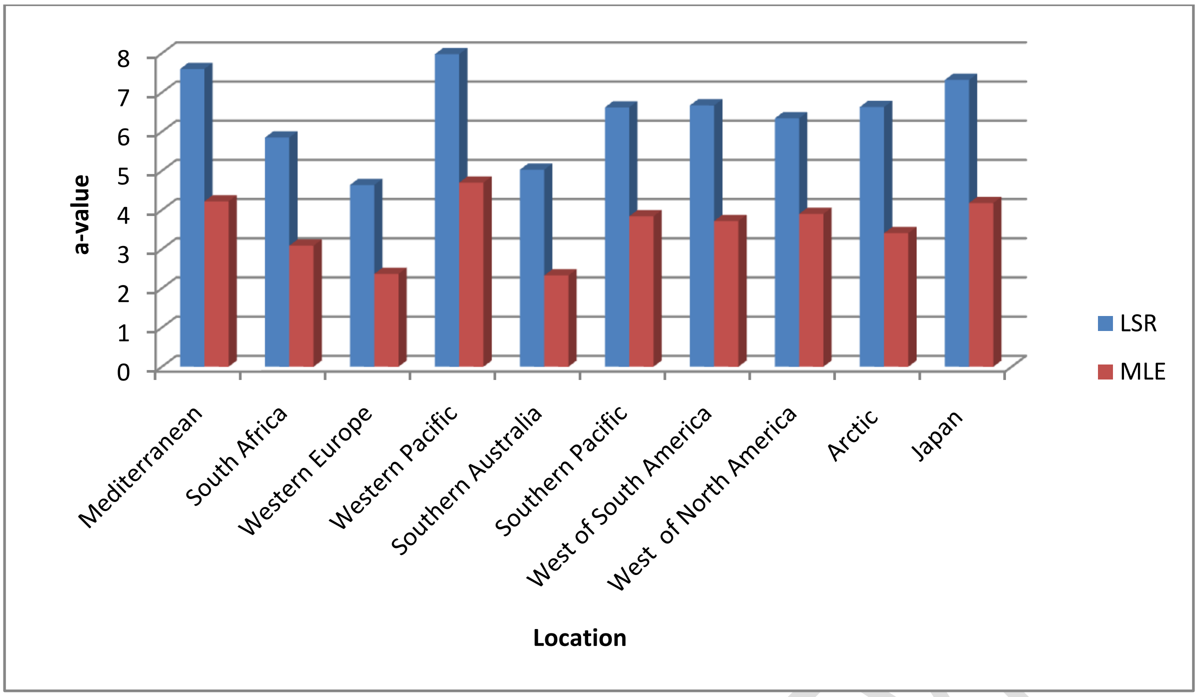

Fig.3 Bar chart showing the **a**-values of the least squares regression method and maximum likelihood estimate methods

Test of hypothesis

There is no significant difference between the use of the least squares regression method and the maximum likelihood estimate method in the determination of b- and a-value of the earthquake in a given region.

This hypothesis was tested using an independent t-test and the results are presented in Table 3 and Table 4.

Table 3 Independent t-test of the comparison of b-values between least squares regression method and maximum likelihood estimate method

	Levene's Test for Equality of Variances		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	F	Sig.	14.413	18	0.000	0.43900	0.03046
Equal variances not assumed	1.649	1.649	14.413	14.926	0.000	0.43900	0.03046

Table 4 Independent t-test of the comparison a-values between least squares regression method and maximum likelihood estimate method

	Levene's Test for Equality of Variances		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	F	Sig.	6.924	18	0.000	2.89200	0.41767
Equal variances not assumed	0.549	0.468	6.924	16.530	0.000	2.89200	0.41767

Table 3 indicates that the p-value or significant value of 0.000 with degrees of freedom (df =18) is less than 0.05 level of significance. This implies that the null hypothesis is rejected and the alternative hypothesis is retained. Hence, there is a significant difference between the use of the least squares regression method and the maximum likelihood estimate method in the determination of b- and a-value of the earthquake in a given region. This indicates that the b-and a-values obtained using both methods are not the same and there is variation.

Conclusion

The two methods have been employed by researchers in seismicity studies over the years till now, but the analysis and findings of this study revealed that the maximum likelihood estimate gives a better estimate of b- and-a values since it is robust and has a smaller standard error than the least squares regression method. the implication of this is that the maximum likelihood estimate should be in the determination of b- and a-values of earthquakes in a given region.

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