

Modelling of Earthquake b-and a-values using Least Squares and Maximum Likelihood Estimate Methods in Different Tectonic Regions of the World

ABSTRACT

Aims: This study modelled a- and b-values of earthquakes employing the least squares regression and maximum likelihood estimate methods.

Methodology: Data used in the study were obtained from International Seismological Centre (ISC), an earthquake catalogue of United Kingdom. The time window was from 1st January, 1988 to 31st December, 2010 (30 years) with earthquake focal depth of 0-700km and magnitude $M_b \geq 1.3$. Ten different locations were selected and a total of 149,965 events were used. The acquired data were processed and analysed using Microsoft Excel and the hypothesis was tested using independent t-test statistic with the aid of Statistical Software for Social Sciences (SPSS) version 23.0.

Results: The findings of the study revealed that the b- and a-values calculated using the least squares regression method was higher than the ones obtained using the maximum likelihood estimate method. Also the hypothesis revealed that there is a significant difference between the use of least squares regression method and maximum likelihood estimate method in the determination of b- and a-value of earthquake in a given region.

Conclusion: The maximum likelihood estimate gives better estimate of b- and a values than the least squares regression method.

Keywords: Modelling; a- and b-values; least squares regression; maximum likelihood; tectonic region

INTRODUCTION

One of the commonest methods of dealing with problems in seismology that involves statistical of earthquakes is the Gutenberg- Richter's relation. It was developed by Richter and Gutenberg in 1944 in California. Before now, similar formula was developed in Japan in 1939 using amplitude of earthquakes by lida and Ishimoto. Later Utsu (1965) developed the maximum likelihood estimate method. The b- and a-values which appear in the two methods in seismology are referred to as the properties of the seismic medium and level of seismicity or productivity respectively.

[1] and [2] reported that the b- and a-values can be estimated either by linear least square regression or by maximum likelihood method. But some researchers are of the opinion that the maximum likelihood is the most robust and generally accepted method for estimating b-values ([1]; [3]).

The two constants b- and a-values are very important parameters in seismological studies, but b-value is the most studied. Experimental rock effects have proven that the b - value is inversely proportional to the stress magnitude and that low b - value regions have excessive strain accumulations ([4]; [5]; [6]). In precise, there's a clear pattern of decreasing b -values before a rock rupture or an earthquake tremor.

Many seismologists have studied the physical significance of the b - value extensive, and lots of case researches have corroborated the phenomenon of decrease b - value frequently going on around and adjacent areas before earthquakes. For instance, [7] studied the adjustments within the b-values before an M s-6.0 earthquake in Changning, China, and observed that low b -value anomalies (≤ 0.85) had been present inside the epicenter area and adjoining regions before the Changning earthquake, with a lower inside the b -values close to the epicenter fewer months earlier than the earthquake. [7] examined the fluctuations within the b -value in and around the source vicinity of magnitude of 6.9 and 6.8 earthquakes off the coast of Miyagi Prefecture, Japan. The authors determined that the earthquakes occurred close to an area with very small b - values, even after the earthquake. Also [9] examined the pre-seismic b-value variation of a 6.0 magnitude earthquake in Luxian, China, and observed that anomalous low b -value features came about in and across the source region.

[10] studied the characteristics of the pre-earthquake b - value anomaly in Jiuzhaigou, China, for a 7.0-magnitude earthquake and found that this area had extensively low b -value anomalies before the earthquake. Also [11] examined spatial and temporal b -value for large- and small-scale regions and found that b -values in the large-scale region ranged from 0.689 to 1.169, with a mean value of 0.928, while the b -values in the small-scale region ranged from 0.694 to 1.223, with a mean value of 0.925. The b -values in the study area were below the mean value prior to the medium and strong earthquake occurrence, and all associated with the unusual feature of a sudden drop-low peak rise.

Hypothesis

HO: There is no significant difference between the use of least squares regression method and maximum likelihood estimate method in the determination of b- and a-value of earthquake in a given region.

HA: There is a significant difference between the use of least squares regression method and maximum likelihood estimate method in the determination of b- and a-value of earthquake in a given region.

METHODOLOGY

Source of data

The data employed in this study was acquired from an earthquake catalogue called International Seismological Centre (ISC) hosted by Piper Lane, Thatchman, Berkshire, United Kingdom on the website

(<http://www.isc.ac.uk/>). The time window was from 1st January, 1988 to 31st December, 2010 (30 years) with earthquake focal depth of 0-700km and magnitude $M_b \geq 1.3$.

Study Areas

The study areas covered Mediterranean, South Africa, Western Europe, Western Pacific, Southern Australia, Southern Pacific, West South America, West of North America, Arctic and Japan. The coordinates of the respective regions are as shown in Table 1 and Fig.1

Table1: Study areas

S/N	Location	Symbol	Min. Lat	Max. Lat	Min Long.	Max. Long
1	Mediterranean	R1	20 ⁰ N	47.5 ⁰ N	17.5 ⁰ W	45.5 ⁰ E
2	Southern Africa	R2	35 ⁰ S	5 ⁰ S	5 ⁰ E	55 ⁰ E
3	Western Europe	R3	55 ⁰ N	70 ⁰ N	0 ⁰	49 ⁰ E
4	Western Pacific	R4	20 ⁰ S	5 ⁰ N	99 ⁰ E	150 ⁰ E
5	Southern Australia	R5	45 ⁰ S	20 ⁰ S	110 ⁰ E	160 ⁰ E
6	Southern Pacific	R6	45 ⁰ S	20 ⁰ S	160 ⁰ E	180 ⁰ E
7	West of South America	R7	45 ⁰ S	5 ⁰ N	90 ⁰ W	60 ⁰ W
8	West of North America	R8	20 ⁰ N	50 ⁰ N	140 ⁰ W	10 ⁰ W
9	Arctic	R9	50 ⁰ N	68 ⁰ N	160 ⁰ W	130 ⁰ W
10	Japan	R10	30 ⁰ N	55 ⁰ N	130 ⁰ E	170 ⁰ E

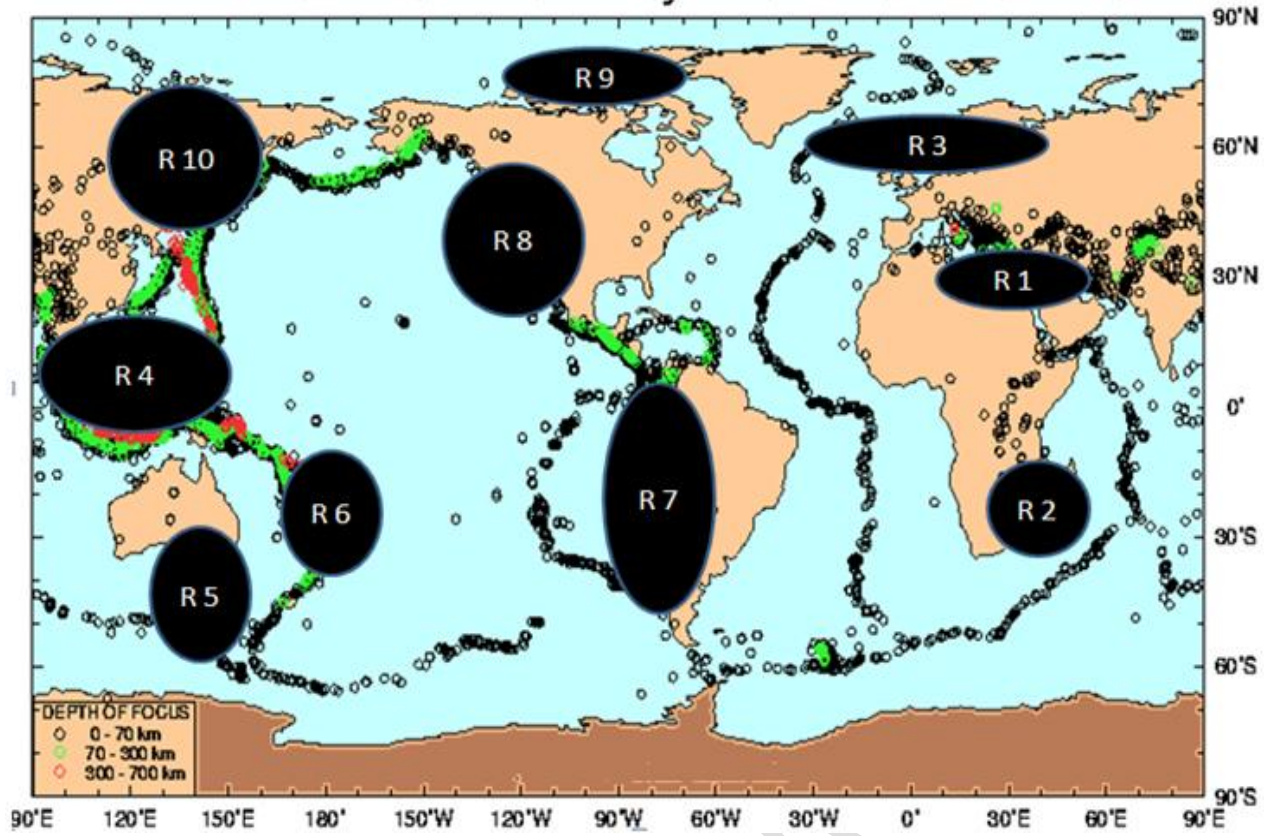


Fig.1. Map showing the study areas across the world ([12])

Theoretical background

- (i) Maximum likelihood estimate (MLE)

Maximum likelihood estimate (MLE) according to [13] is given by:

$$b^{(u)} = \frac{\text{Log}_e}{\left[\left(\bar{M} \right) - \left(M_c - \frac{\Delta M}{2} \right) \right]} = \frac{0.4343}{\left[\bar{M} - (M_c - 0.05) \right]} \quad 1$$

Where \bar{M} is the average magnitude for a particular region, M_c is the magnitude of completeness

The error or uncertainty in b is given by:

$$\sigma_b^{(u)} = \frac{b^{(u)}}{\sqrt{N}} \quad 2$$

The a-value is given by [14] as:

$$a = \text{Log} \left(\frac{N}{T} \right) + b.M_c \quad 3$$

Where T = time interval employed in the study.

The error or uncertainty is given by:

$$\sigma_a = \left[(M_c \sigma_b)^2 + (b \Delta M)^2 \right]^{\frac{1}{2}} \quad 4$$

Where $\Delta M = 0.1$

σ_b = error in the b-value

(ii) Least squares regression (LSR) method

Least squares regression method was developed by Gutenberg and Richter. It expresses the relationship between number of earthquakes and magnitude in the form:

$$\text{Log } N = a - bM \quad 5$$

Where N is the cumulative number of earthquakes with magnitudes ($M \geq 0$)

a and b are constants

a = the level of seismicity or productivity

b = the tectonic parameter that describes the properties of the seismic medium.

Results and discussion

Table 2 Summary of b- and a- values of earthquakes by Least squares method and Maximum likelihood method in the study area

S/N	Location	Label	Number of earthquakes	Least squares regression method (LSR)				Maximum likelihood estimate method (MLE)			
				b	σ_b	a	σ_a	b	σ_b	a	σ_a
1	Mediterranean	R1	16613	1.00	0.05	7.60	0.23	0.41	0.03	4.22	0.22
2	South Africa	R2	1230	0.79	0.06	5.85	0.27	0.40	0.05	3.09	0.25
3	Western Europe	R3	191	0.77	0.03	4.64	0.12	0.49	0.02	2.37	0.11
4	Western Pacific	R4	53907	0.92	0.06	7.98	0.28	0.39	0.04	4.70	0.22
5	Southern Australia	R5	257	0.72	0.05	5.03	0.21	0.40	0.04	2.33	0.20
6	Southern Pacific	R6	10676	0.81	0.05	6.62	0.24	0.38	0.04	3.84	0.22
7	West of South America	R7	11096	0.82	0.05	6.67	0.24	0.35	0.03	3.72	0.19
8	West of North America	R8	8297	0.76	0.05	6.34	0.23	0.43	0.04	3.90	0.22
9	Arctic	R9	4613	0.86	0.03	6.63	0.15	0.34	0.02	3.41	0.14
10	Japan	R10	43085	0.84	0.05	7.32	0.25	0.31	0.03	4.18	0.17
	Total		149965								

Table 2 indicates that the **b**- and **a**-values evaluated using the least squares regression method was higher the ones obtained using the maximum likelihood estimate method. The distribution of the results is as shown in Fig. 2 and Fig. 3. It was also observed that the standard error obtained for b-and a-values using maximum likelihood estimate method was smaller than the ones obtained with least squares regression method. This implies that the smaller the standard error the better the accuracy. The findings in this study are in line with [3] and [15] concluded after their comparative analysis of b- and a- values that the maximum likelihood method is more robust as compared to the least squares regression method in the estimation of b-and –a values in a given region.

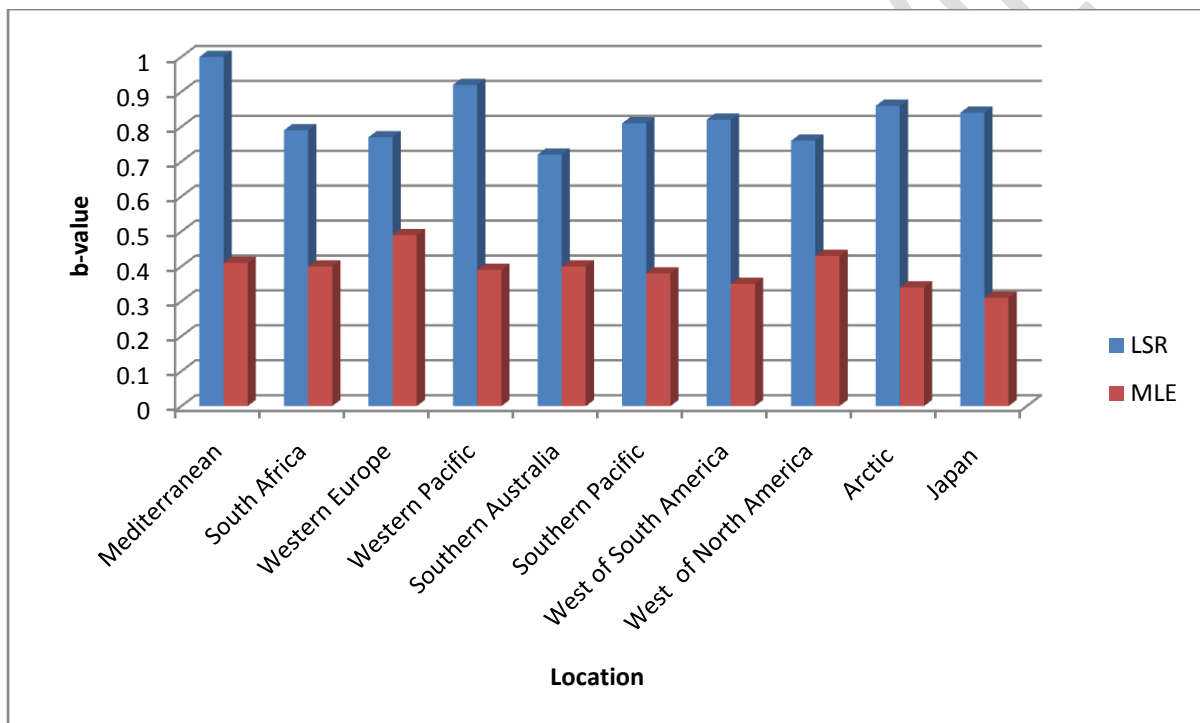


Fig. 2 Bar chart showing the b-values of the least squares regression method and maximum likelihood estimate methods

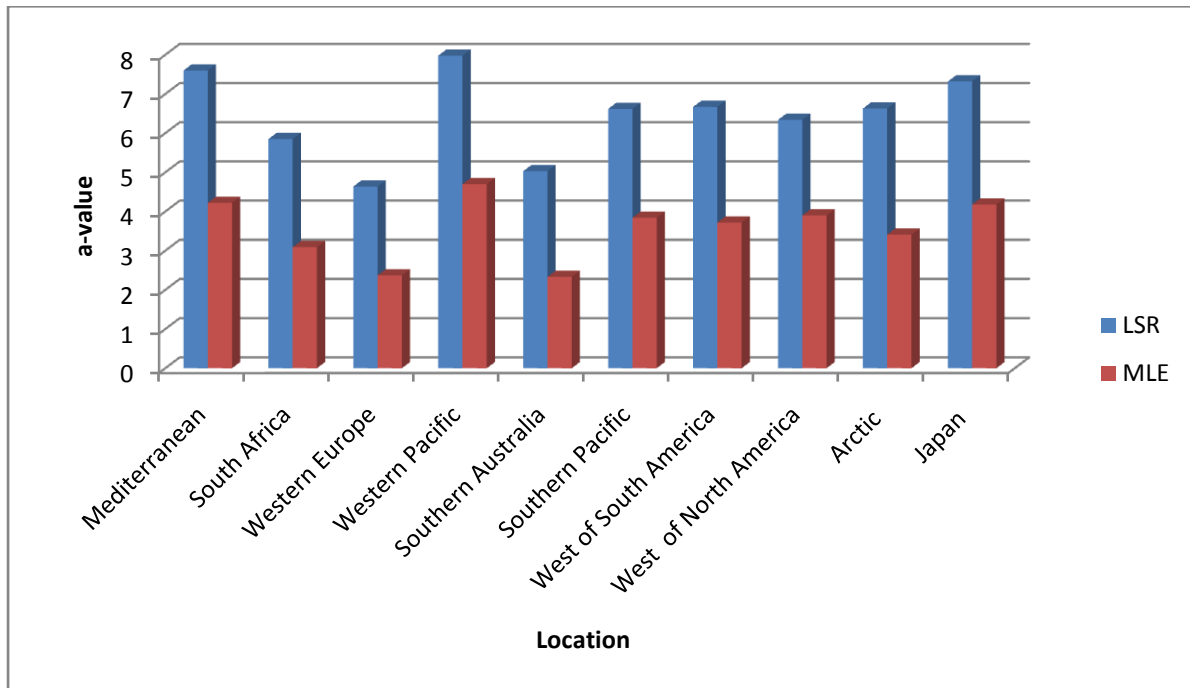


Fig.3 Bar chart showing the **a**-values of the least squares regression method and maximum likelihood estimate methods

Test of hypothesis

There is no significant difference between the use of least squares regression method and maximum likelihood estimate method in the determination of **b**- and **a**-value of earthquake in a given region.

This hypothesis was tested using independent t-test and the results are presented in Table 3 and Table 4.

Table 3 Independent t-test of the comparison of **b**-values between least squares regression method and maximum likelihood estimate method

	Levene's Test for Equality of Variances		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
	F	Sig.					
Equal variances assumed			14.413	18	0.000	0.43900	0.03046
Equal variances not assumed	1.649	1.649	14.413	14.926	0.000	0.43900	0.03046

Table 4 Independent t-test of the comparison **a**-values between least squares regression method and maximum likelihood estimate method

	Levene's Test for Equality of Variances		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
	F	Sig.					
Equal variances assumed			6.924	18	0.000	2.89200	0.41767
Equal variances not assumed	0.549	0.468	6.924	16.530	0.000	2.89200	0.41767

Table 3 indicates that the p-value or significant value of 0.000 with degrees of freedom (df =18) is less than 0.05 level of significance. This implies that the null hypothesis is rejected and the alternative hypothesis retained. Hence, there is a significant difference between the use of least squares regression method and maximum likelihood estimate method in the determination of b- and a-value of earthquake in a given region. This indicates that the b-and a-values obtained using both methods are not the same and there is variation.

Conclusion

The two methods have been employed by researchers in seismicity studies over the years till now, but the analysis and findings of this study revealed that the maximum likelihood estimate gives better estimate of b- and-a values since it is robust and has smaller standard error than the least squares regression method. the implication of this is that the maximum likelihood estimate should be in the determination of b-and a-values of earthquakes in a given region.

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