The Expected Values of Buys-Ballot Estimates for Multiplicative Model with the Error Terms

Abstract: The expected values of Buys-Ballot estimates with the error terms for multiplicative model in time series decomposition when trend-cycle component is linear is discussed. The expected values of Buys-Ballot estimates of row, column and overall means with the error terms are derived. A real example for the linear case is applied to determine the estimation of trend parameters, seasonal indices, choice of appropriate transformation and choice of model of the Buys-Ballot table. Results show that, the expected values of periodic and seasonal means of the Buys-Ballot table are; (1) periodic mean mimic the shape of the trending curves of the original series and contains seasonal indices (2) seasonal mean is a function of the trending parameters of the original series and contain seasonal indices

Keywords: Descriptive Time Series, Multiplicative Model, Linear Trend, Buys-Ballot Estimates, Expected Values, Choice of Appropriate Model

1. Introduction

One of the objectives of time series analysis is description. Descriptive time series analysis involves the examination of trend, seasonality and cycles, it is usual described as having trends, seasonal indices, cyclic pattern and the error terms. Since the emphasis in descriptive time series analysis is on model building, the following additive, multiplicative and mixed models are taken into consideration.

Additive Model: $X_t = T_t + S_t + C_t + I_t$ (1)

Multiplicative Model: $X_t = T_t \times S_t \times C_t \times I_t$ (2)

Mixed Model: $X_t = T_t \times S_t \times C_t + I_t$ (3)

Cyclical component refers to the long term oscillation or swing about the trend appears to an appreciable magnitude only in long period sets of time series data. "However, for short period of time, the cyclical component is jointly estimated into the trend and the observed time series $\left(X_t,\ t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component (S_t) and the irregular/residual component (e_t)". [1] Therefore, the decomposition models are

Additive Model

$$X_t = M_t + S_t + e_t \tag{4}$$

Multiplicative Model

$$X_t = M_t \times S_t \times e_t \tag{5}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \tag{6}$$

[1] stated that, "the successive periodic mean $(\bar{X}_i)$ gives a simple description of the underlying trend from the methods of monthly or quarterly means. It was given that the estimates of the seasonal indices can be obtained from the column means $\bar{X}_{.j}$. Hence, while the periodic means estimate the trend, the column means gives estimates of seasonal indices. It has been noted that a time series theoretically contains four components, However, for short series, the trend components is estimated into the cyclical and trend cycle component is obtained and denoted by m_t. Under these conditions, it can be stated that estimates of trend-cycle and seasonal components can be obtained from the row and column means, respectively, of the Buys-Ballot table. These estimates have been designated "Buys-Ballot" estimates in this study the details of the procedure for estimation of the trend-cycle component (m_t) are presented in section 2.1 for the multiplicative model".

Iwueze, *et al,* [2] proposed "the use of joint plot of seasonal means and standard deviations for choice between additive and multiplicative models in descriptive in time analysis".

Dozie [3] discussed "the expression of the parameters of linear trend cycle and seasonal components with emphasis on the periodic, seasonal and overall means for the mixed model. In his summary, he demonstrated that, the linear trend cycle and seasonal components when there is zero trend and b = 0 in time series decomposition". Oladugba, *et al*, [4] stated that "when the seasonal fluctuation displays constant amplitude with regards to the trend, then the seasonal indices is said to be additive model and Equation (1) may be applied. If the seasonal fluctuation is a product of the trend, then it is said to be multiplicative".

2. Methodology

An important aspect of the descriptive time series analysis using Buys-Ballot approach is the arrangement of the observed seasonal time series data in a Buys-Ballot table as given in Table 1. This method is based on the row, column and overall means of the Buys-Ballot table with m rows and s columns. For details of this method see Wei [5], Iweze *et.al* [2], Nwogu *et.al* [6], Dozie and Ihekuna [7], Dozie *et.al* [8], Dozie and Nwanya [9], Dozie [3], Dozie and Ijeomah [10], Dozie and Ibebuogu [11], Dozie and Uwaezuoke [12], Dozie and Ihekuna [13], Dozie and Ibebuogu [14], Akpanta and Iwueze [15], Dozie and Uwaezuoke [16], Dozie [17].

All the derivations made in this section are based on the systematic component of the multiplicative model of the Buys-Ballot table. The results of the Buys-Ballot estimates for multiplicative model with the error terms are given in Table 2. From Table 2, we observed that, the Buys-Ballot estimates of the intercept and seasonal indices $\left(S_{j,} i = 1, 2, \ldots, m\right)$ depend on the estimate of the slope. The derivations of expected values of row, column and overall means for multiplicative model with the error terms are given in Table 3

Table 1: Buys - Ballot Tabular Arrangement of Time Series Data

Rows/ Period (i)	Columns (season) j								
	1	2	$\cdots$	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	$\cdots$	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	$\cdots$	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	$\cdots$	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$\cdots$	$X_{(i-1)s+j}$	$\cdots$	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	$\cdots$	$X_{(m-1)s+j}$	$\cdots$	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	$\cdots$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\cdots$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	$\cdots$	$\hat{\sigma}_{.j}$	$\cdots$	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

The study will therefore concentrate on the derivations of the Buys-Ballot Estimates with the error terms and the expected values for multiplicative model.

2.1: Buys-Ballot Procedure for the Multiplicative Model

From equation (2), we say that

$$X_{(i-1)s+j} = X_{ij} = M_{(i-1)s+j} \times S_j \times e_{(i-j)s+j} \tag{7}$$

So that,

$$M_t = a + bt = M_{(i-1)s+j} = a + b\left[(i-1)s + j\right]$$

$$X_{ij} = a + b\ \left[(i-1)s + j\right] S_j \times e_{(i-1)s+j} \tag{8}$$

Where $t = (i-1)s + j,\ S_{t+j} = S_j,\ \sum_{j=1}^{s} S_j = S,\ i = 1, 2...m,\quad j = 1, 2..s,\quad n = ms$

From table 1, the ith row total is given as

$$T_i = \sum_{j=1}^{s} X_{ij} = \sum_{j=1}^{s} \left\{a + b\left[(i-1)s + j\right]\right\} S_j \times e_{(i-1)s+j} \tag{9}$$

$$= \sum_{j=1}^{s} \left\{\left[a + b(i-1)s + j\right] S_j e_{(i-1)s+j} + b_j S_j e_{(i-1)s+j}\right\}$$

$$T_i = \left[a + b\left(i-1\right)s\right]\sum_{j=1}^{s} S_j e_{(i-1)s+j} + b_j \sum_{j=1}^{s} S_j e_{(i-1)s+j} \qquad (10)$$

In deriving equation (10), we made use of the assumption $\sum_{j=1}^{s} S_j = 0$. Now, the *ith* row average is therefore, $\bar{X}_{i.} = \frac{T_{i.}}{s}$

$$\bar{X}_{i.} = \frac{1}{s}T_{i.} = \left[a + b\left(i-1\right)s\right]\frac{\sum_{j=1}^{s} S_j e_{(i-1)s+j}}{s} + \frac{b}{s}\sum_{j=1}^{s} jS_j e_{(i-1)s+j} \qquad (11)$$

Next, we derive an expression for each of the *jth* column total and average.

With $\sum_{i=1}^{m} S_j = mS_j, the\ jth$ column total becomes

$$T_{\cdot j} = \sum_{i=1}^{m} X_{ij} = \sum_{i=1}^{m}\left\{a = b\left[\left(i-1\right)s + j\right]\right\}S_j e_{\cdot j} \qquad (12)$$

$$= \sum_{i-1}^{m}\left\{aS_j e_{ij} + bs\left(i-1\right)S_j e_{\cdot j} + b_j S_j e_{\cdot j}\right\}$$

$$= aS_j \sum_{i=1}^{m} e_{\cdot j} + bsS_j \sum_{i=1}^{m}\left(i-1\right) + bjS_j \sum_{i-1}^{m} e_{\cdot j}$$

$$\left(aS_j + b_j S_j\right)\sum_{i-1}^{m} e_{\cdot j} + bsS_j \sum_{i-1}^{m}\left(i-1\right)e_{ij}$$

$$\left[\left(a + b_j\right)\sum_{i-1}^{m} e_{\cdot j} + bs\sum_{i-1}^{m}\left(i-1\right)e_{\cdot j}\right]S_j$$

$$\bar{X}_{.j} = \frac{1}{m}T_{\cdot j} = \left[\left(a + b_j\right)\bar{e}_{\cdot j} + \frac{bs\sum_{i-1}^{m}\left(i-1\right)e_{\cdot j}}{m}\right]S_j \qquad (13)$$

Furthermore, the grand total of the observations is give as

$$T_{..} = \sum_{i-1}^{m} T_{\cdot i} = \sum_{j-1}^{s} T_{\cdot j} \qquad (14)$$

$$= T_{..} = \sum_{j-1}^{s} T_{\cdot j} = \quad \left[\left(a + b_j \right) \sum_{i-1}^{m} e_{\cdot j} + bs \sum_{i-1}^{m} \left(i - 1 \right) e_{\cdot j} \right] S_j$$

For $ms = n,$ therefore the grand mean is obtained by dividing equation (13) by n. Hence,

$$= \sum_{i=1}^{m} \left\{ s \left[a - b \left(s - \frac{1}{s} \sum_{j=1}^{s} jS_j \right) + (bs)i \right] \right\}$$

$$= n \left[a + \frac{b}{2} \left(n - s \right) + \frac{b}{s} \sum_{j=1}^{s} jS_j \right]$$

Thus, the grand mean is

$$= a + \frac{b}{2} \left(n - s \right) + \frac{b}{2} \sum_{j=1}^{s} jS_j \qquad (15)$$

$$\bar{X}_{..} = a + b \left(\frac{n - s}{2} \right) + bc_1 + \bar{e}_{..} \qquad (16)$$

2.2: Expected Values of Row, Column and Overall Averages with Error Terms

2.2.1: Expected Value of Row Average with Error Term

From equation (12), the expected value of row average is given as thus

$$E\left(\bar{X}_{i.} \right) = \left[a + b \left(i - 1 \right) s \right] \frac{\sum_{j=1}^{s} S_j}{s} + \frac{b}{s} \sum_{j=1}^{s} jS_j e_{(i-1)s+j} \qquad (17)$$

$$= \; a + b \left(i - 1 \right) s + bc_1, c_1 = \frac{1}{s} \sum_{j=1}^{s} jS_j$$

$$= \; a + bc_1 - bs + \left(bs \right) i$$

$$= \; a - b \left(s - c_1 \right) + \left(bs \right) i = \alpha + \beta_i, i = 1, 2 \ldots m$$

Where $\alpha = a - b\left(s - c_1 \right),\ \beta = \left(bs \right) = b$

2.2.2: Expected Value Column Average with Error Term

$$E\left(\bar{X}_{.j}\right)=\left[a+b_j+\frac{bs}{m}\sum_{i-1}^{m}(i-1)\right]S_j \quad (18)$$

$$=\left[a+b_j+\frac{bs}{m}+\frac{m(m-1)}{2}\right]S_j$$

$$=\left[a+b\left(\frac{n-s}{2}\right)+b_j\right]S_j$$

This can be written as $\bar{X}_{.j}=(\mu+\gamma j)S_j$

Where $\mu=a+b\left(\frac{n-s}{2}\right)$, $\gamma=b$, $\hat{b}=\gamma$, $\hat{a}=\mu-\hat{b}\left(\frac{n-s}{2}\right)$ and $S_j=\frac{\bar{X}_{.j}}{\mu+\gamma j}$

2.2.3: Expected Value of Overall Average with Error Term

$$E\left(\bar{X}_{..}\right)=E\left[a+b\left(\frac{n-s}{2}\right)+bc_1\right]+E\left(\bar{e}_{..}\right) \quad (19)$$

Hence , the expected value for the overall mean is

$$E\left(\bar{X}_{..}\right)=a+b\left(\frac{n-s}{2}\right)+bc_1 \quad (20)$$

where $E\left(\bar{e}_{..}\right)=1$

Table 2: Estimates of Row, Column and Overall Means with the Error Terms

Measures	Multiplicative Model $(M_t=a+bt)$
$\bar{X}_{i.}$	$\left[a-bs+\frac{b}{s}\sum_{j=1}^{s}jS_j+bsi\right]*\bar{e}_{i.}$
$\bar{X}_{.j}$	$\left[a\bar{e}_{.j}+\frac{bs}{m}\sum_{i=1}^{m}ie_{ij}-bs\bar{e}_{,j}+bj\bar{e}_{.j}\right]*S_j$
$\bar{X}_{..}$	$a+b\left(\frac{n-s}{2}\right)+bC_1$

Source: Nwogu *et al* (2019) and Dozie, *et al*, (2020)

$$C_1 = \frac{1}{s}\sum_{j=1}^{s} jS_j$$

Table 3: Expected values of Row, Column and Overall Means for Multiplicative Model with the Error Terms

Linear trend-cycle component: $M_t = a + b_t, \quad t = 1,2,...,n = ms$

Expected Value	Multiplicative model
$E(\bar{X}_{i.})$	$[a - bs + bsi] + \frac{b}{s}\sum_{j=1}^{s} jS_j$
$E(\bar{X}_{.j})$	$\left[a + b\left(\frac{n-s}{2}\right) + bj\right]S_j$
$E(\bar{X}_{..})$	$a + b\left(\frac{n-s}{2}\right) + bc_1$

Where, $c_1 = \frac{1}{s}\sum_{j=1}^{s} jS_j$

2.2 Cochran's Test for Constant Variance

To test the null hypothesis that the variances are equal, that is

$H_0 = \sigma_1^2 = \sigma_2^2 = ... = \sigma_k^2$

Against the alternative

$H_1 \neq \sigma_1^2 \neq \sigma_2^2 \neq ... \neq \sigma_k^2$ $(Atleast\ one\ \text{var}iance\ is\ different\ \ from\ others)$

The test statistic is given as

$$C = \frac{\max\left(S_j^2\right)}{\sum_{j=1}^{k} S_j^2} \qquad (21)$$

Where, $\max\left(S_j^2\right)$ is the maximum variance among all column variances

$\sum_{j=1}^{k} S_j^2$ is the sum of the variances

S_j has the range j = 1, 2, …, s, which are the variances of the j[th] sub-group.

Using the parameters of the Buys-Ballot table: $S_j^2 = \hat{\sigma}_j^2$, the statistic in (21) is then given as;

$$C = \frac{\max\left(\hat{\sigma}_j^2\right)}{\sum_{j=1}^{k} \hat{\sigma}_j^2} \qquad (22)$$

Where, $\sigma_j^2 = (j = 1,\ 2,\ \ldots\ s)$ is the column variance of the Buys-Ballot table.

2.3 Choice of Transformation

For series arranged in Buys-Ballot table Akpanta and Iwueze [15] proposed the slope of the regression equation of log of group standard deviation on log of group mean as given in equation (23) is what is required for choosing appropriate transformation. Some of the values of slope β and their implied transformation are stated in Table 4

$$\log_e \hat{\sigma}_i = a + \beta \log_e \hat{X}_{i.} \qquad (23)$$

Table 4: Bartlett's Transformation for Some Values of β

S/No	1	2	3	4	5	6	7
β	**0**	$\frac{1}{2}$	**1**	$\frac{3}{2}$	**2**	**3**	**-1**
Transformation	**No transformation**	$\sqrt{X_t}$	$\log_e X_i$	$\frac{1}{\sqrt{X_t}}$	$\frac{1}{X_t}$	$\frac{1}{X_t^2}$	X_t^2

Akpanta and Iwueze [15] proposed a method in selecting appropriate transformation, the natural logarithm of standard deviation will be applied to regress against the natural logarithm of periodic means and the result of the β - value will determine the type of transformation.

3. **Empirical Example**

Table 5 shows the data based on short series in which the trend cycle component is jointly estimated. The time series data presented in Table 5 is analysed using the Buys-Ballot method. The time series data for the period 2009 to 2019 is used to determine the suitable model for decomposition of the study series. The corresponding graphs of the actual series of registered birth rate collected from Salvation Hospital Owerri are given in Figures 1, and 2. The time plots of actual and periodic series shown Figures 1 and 2 and Table 5 indicate that, the time plot of $\bar{X}_{i.}$ for time series data with linear trend curve mimic the time plot of the entire time series and the periodic means ($\bar{X}_{i.}$) can therefore be employed to estimate trend. The periodic standard deviations are stable while the seasonal standard deviations are different, suggesting that the data requires transformation to stabilize the variance. The time series data was transformed by taking the inverse square root of the one hundred and thirty two (132) observed values given in Appendix B. From the transformed series, the periodic and seasonal totals, means and standard deviations are obtained in Tables 3 and 4. The seasonal mean of the transformed series was plotted against the seasonal standard deviation in Figure 3 and the deviation of seasonal means for overall means $X_t - \hat{M}_t$ (for the additive model) was plotted in Figure 4 and the ratio $X_t / \hat{M}_t$ (for the multiplicative model). Figures 3, 4 and Table 5 show that, the graphs suggest multiplicative model as the seasonal standard deviations show no significant increase or decrease relative to any increase or decrease in the seasonal means.

Table 5: Estimates of Means and Standard Deviations

Year	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec	$\bar{X}_{i.}$	$\sigma_{i.}$
2009	8	15	12	17	16	9	9	8	10	14	9	11	11.50	3.23
2010	23	11	10	10	18	15	17	13	16	14	19	18	15.33	3.96
2011	12	9	14	9	5	7	9	11	10	12	15	16	10.75	3.25
2012	13	15	18	18	14	14	16	9	8	14	8	9	13.00	3.67
2013	8	12	2	6	6	8	5	9	8	15	10	17	8.83	4.22
2014	20	13	12	16	12	10	7	14	9	12	15	13	12.75	3.39
2015	14	18	9	12	7	10	15	10	8	11	14	9	11.42	3.26
2016	20	17	21	15	13	23	7	6	12	11	7	7	13.25	5.99
2017	11	13	24	17	12	16	11	17	17	16	13	9	15.42	3.70
2018	9	14	18	18	12	19	14	9	18	15	17	14	14.75	3.44
2019	21	11	15	15	21	12	8	9	8	19	16	12	13.92	4.70

$\bar{X}_{.j}$	14.45	13.46	14.09	13.91	12.36	13.00	10.73	10.55	11.27	14.00	13.27	12.64		
$\sigma_{.j}$	5.57	2.70	6.12	4.06	4.97	4.96	4.13	3.33	3.90	2.49	4.05	3.50		

Table 6**: Row totals, means and standard deviations**

Periods i		Linear trend cycle		
	r_i	T_i	$\bar{X}_i$	$\sigma_{i.}$
1	11	26.51	2.41	0.27
2	11	29.70	2.70	0.27
3	11	25.63	2.33	0.33
4	11	27.72	2.52	0.30
5	11	22.55	2.05	0.56
6	11	27.61	2.51	0.27
7	11	26.40	2.40	0.28
8	11	27.28	2.48	0.48
9	11	29.81	2.71	0.23
10	11	29.26	2.66	0.26
11	11	28.38	2.58	0.35

$$n = \sum_{j=1}^{r} c_j = \sum_{i=1}^{c} r_i = total \;\; number \;\; of \;\; observation$$

Where,

r_i = Number of observation in the rth row

c_j = Number of observation in the jth column.

Table 7: **Seasonal totals, means and standard deviations**

Seasons j		Linear trend cycle		
	c_j	$T._j$	$\bar{X}_{.j}$	$\sigma_{.j}$
1	12	31.2	2.60	0.40
2	12	30.96	2.58	0.20
3	12	30.00	2.50	0.67
4	12	30.96	2.58	0.35
5	12	29.16	2.43	0.35
6	12	30.00	2.50	0.37
7	12	27.60	2.30	0.40
8	12	27.72	2.31	0.30
9	12	28.44	2.37	0.32
10	12	31.56	2.63	0.17
11	12	30.48	2.54	0.34
12	12	30.00	2.50	0.29

Table 8: **Estimates of Seasonal Indices**

j	$\bar{X}_{.j}$	$\bar{X}_{.j} - \bar{X}_{..}$
1	2.6010	0.1140
2	2.5806	0.0936
3	2.5010	0.0140
4	2.5830	0.0960
5	2.4280	-0.0590
6	2.5010	0.0140
7	2.3020	-0.1850
8	2.3141	-0.1729
9	2.3728	-0.1142
10	2.6251	0.1381
11	2.5370	0.0500
12	2.4989	0.0119

3.1 Choice of Appropriate Model

The test statistic shown in (22) is used to determine the choice of appropriate model. The null hypothesis that the data accepts additive model is rejected, if calculated value C is greater than the tabulated value $C_{tab}\{k, V, 1-\alpha\}$. level of significance, or do not reject null hypothesis otherwise

From Appendix A and Table 7, m=1 $\max \hat{\sigma}_j^2 = 37.490$, $\sum_{j=1}^{k} \hat{\sigma}_j^2 = 219.753$

Table 9: **Estimates of Seasonal Indices**

j	$\bar{X}_{.j}$	$\frac{\bar{X}_{.j}}{\bar{X}_{..}}$
1	2.6010	1.0458
2	2.5806	1.0376

3	2.5010	1.0056
4	2.5830	1.0386
5	2.4280	0.9763
6	2.5010	1.0056
7	2.3020	0.9256
8	2.3141	0.9305
9	2.3728	0.9541
10	2.6251	1.0555
11	2.5370	1.0201
12	2.4989	1.0048

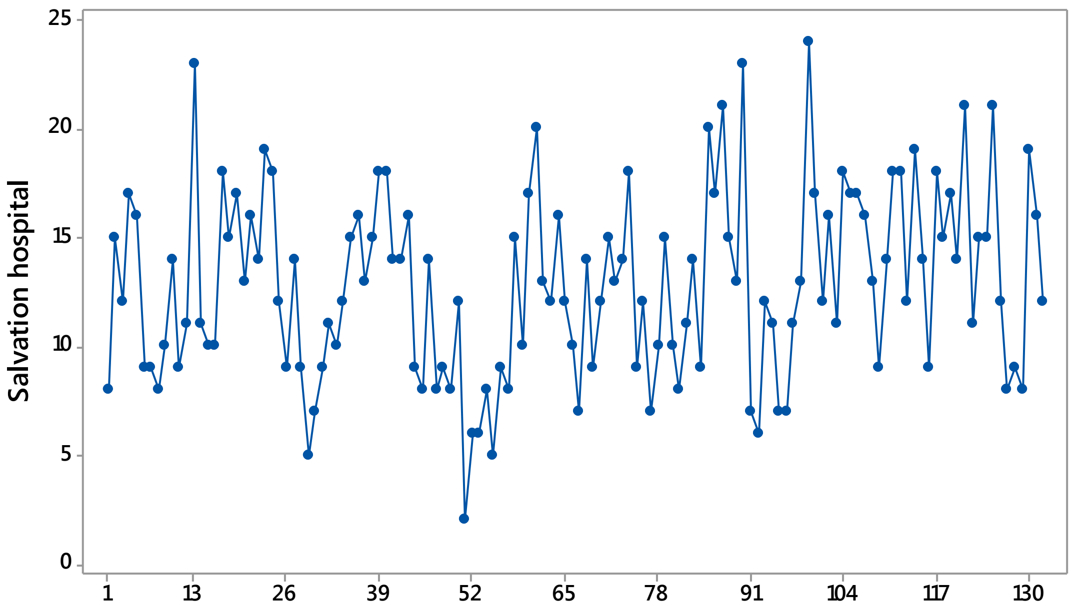

Fig 1 : time plot of actual series , between 2009 to 2019

$$C = \frac{37.490}{219.753} = 0.1706$$

Reject H_0 if $C > C_{tab}$
{11,12:0.05}

The test statistic C is less than, when compared with the tabulated value (0.2353), suggesting that the data accepts additive time series model. Secondly, the study data requires transformation to stabilize the variance in the distribution. When the seasonal variances of the transformed time series data listed in Table 10 are subjected to test for constant variance, the calculated test statistic (0.2647) is greater than the tabulated value (0.2353) at C_{tab} {k, V, 1 – α} level significant. The study data indicates that the variance is not constant and the transformed series does not accepts additive model.

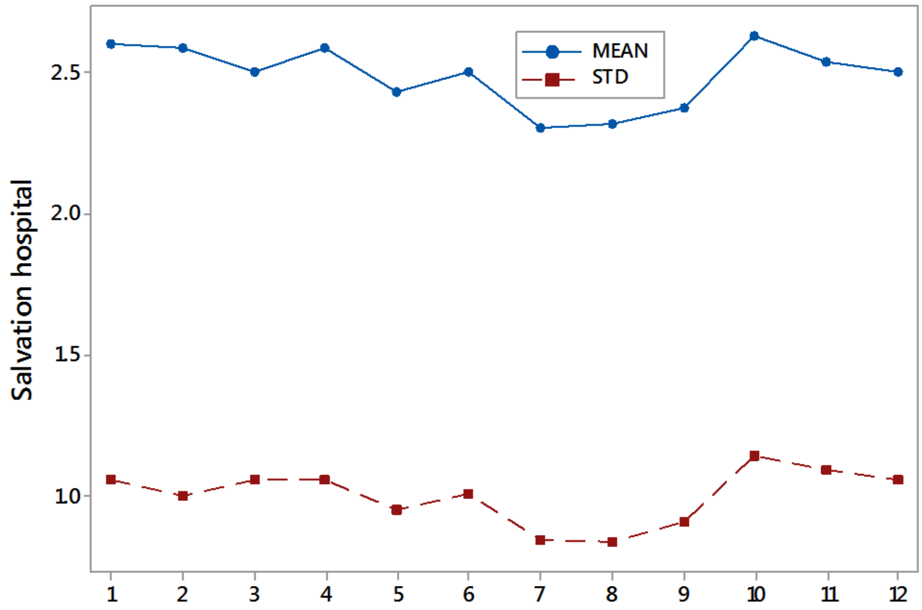

Fig 2 : Means and standard Deviation

Table 10: Seasonal means ($\bar{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$)

j	$\bar{X}_j$	$\hat{\sigma}_j^2$
1	14.45	31.07
2	13.46	7.27
3	14.09	37.49
4	13.91	16.49
5	12.36	24.65
6	13.00	24.60
7	10.73	17.02
8	10.55	11.07
9	11.27	15.22

10	14.00	6.20
11	13.27	16.42
12	12.64	12.25

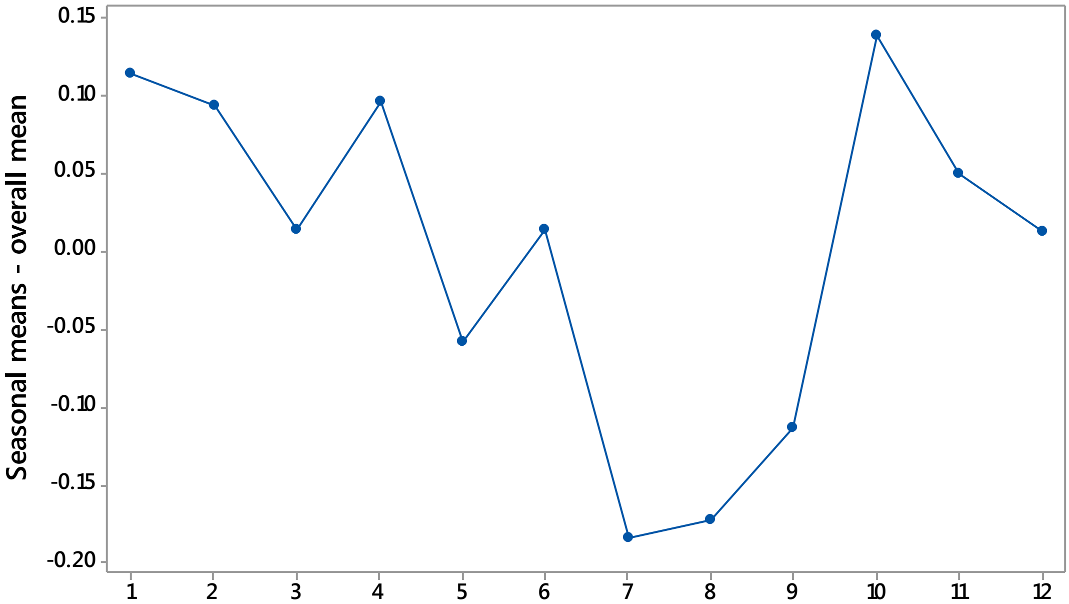

Fig 3 : Seasons of the transformed series

Table 11: Seasonal means ($\overline{X}_j$) and estimate of the column variance ($\hat{\sigma}_j^2$)

j	$\overline{X}_j$	$\hat{\sigma}_j^2$
1	2.60	0.16
2	2.58	0.04
3	2.50	0.45
4	2.58	0.12
5	2.43	0.21
6	2.50	0.14
7	2.30	0.16
8	2.31	0.09
9	2.37	0.10
10	2.62	0.03
11	2.53	0.12

12	2.50	0.09

$$m=12,\ \max \hat{\sigma}_j^2 = 0.4520,\quad \sum_{j=1}^{k} \hat{\sigma}_j^2 = 1.7075$$

$$C = \frac{0.4520}{1.7075} = 0.2647$$

Reject H_0 if $C > C_{tab}$
{11,12 : 0.05}

4 Summary, Conclusion and Recommendation

This study presented the expected values of Buys-Ballot estimates of row, column and overall means for multiplicative model with the error terms when trend-cycle component is linear. In this study, we derived the expected values of Buys-Ballot estimates with the error terms of row, column and overall means for multiplicative model. The method of estimation is based on the row, column and overall means of time series data arranged in a Buys-Ballot table. A real example for the linear case is applied to illustrate the estimation of trend parameters, seasonal indices, choice of appropriate transformation and choice of model of the Buys-Ballot table. The study shows that, the time plot of $\bar{X}_{i.}$ for time series data with linear trend curve mimic the time plot of the entire time series and the periodic mean ($\bar{X}_{i.}$) and can be employed to estimate trend. Successful transformation is carried to stabilize the variance and make the distribution normal. Results show that, the expected values of periodic and seasonal means of the Buys-Ballot table are; (1) periodic mean mimic the shape of the trending curves of the original series and contains seasonal indices (2) seasonal mean is a function of the trending parameters of the original series and contain seasonal indices (3) the appropriate model that best describe the pattern of the study data in the transformed series listed in the summary table (Table 10) is multiplicative. No attempt has been made to discuss this method when the trend-cycle component is not linear or when the cyclical component is separated from the trend. Therefore, further research in these directions are recommended

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Appendix A: Original Series of Salvation Hospital, Owerri, Imo State

Year	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	8	15	12	17	16	9	9	8	10	14	9	11	11.50	10.46
2010	23	11	10	10	18	15	17	13	16	14	19	18	15.33	15.70
2011	12	9	14	9	5	7	9	11	10	12	15	16	10.75	10.57
2012	13	15	18	18	14	14	16	9	8	14	8	9	13.00	13.45
2013	8	12	2	6	6	8	5	9	8	15	10	17	8.83	17.79
2014	20	13	12	16	12	10	7	14	9	12	15	13	12.75	11.48
2015	14	18	9	12	7	10	15	10	8	11	14	9	11.42	10.63
2016	20	17	21	15	13	23	7	6	12	11	7	7	13.25	35.84
2017	11	13	24	17	12	16	11	17	17	16	13	9	15.42	13.72
2018	9	14	18	18	12	19	14	9	18	15	17	14	14.75	11.84
2019	21	11	15	15	21	12	8	9	8	19	16	12	13.92	22.08
$\bar{X}_{.j}$	14.45	13.46	14.09	13.91	12.36	13.00	10.73	10.55	11.27	14.00	13.27	12.64		
$\sigma^2_{.j}$	31.07	7.27	37.49	16.49	24.65	24.60	17.02	11.07	15.22	6.20	16.42	12.25		

Source: Salvation Hospital Owerri, Imo State Nigeria (2009-2019)

Appendix B: Transformed Series of Salvation Hospital Owerri, Imo State

Year	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	2.08	2.71	2.48	2.83	2.77	2.20	2.20	2.08	2.30	2.64	2.20	2.40	2.41	0.07
2010	3.14	2.40	2.30	2.30	2.89	2.71	2.83	2.56	2.77	2.64	2.94	2.89	2.70	0.07
2011	2.48	2.20	2.64	2.20	1.61	1.95	2.20	2.40	2.30	2.48	2.71	2.77	2.33	0.11
2012	2.56	2.71	2.89	2.89	2.64	2.64	2.77	2.20	2.08	2.64	2.08	2.20	2.52	0.09
2013	2.08	2.48	0.69	1.79	1.79	2.08	1.61	2.20	2.08	2.71	2.30	2.83	2.05	0.32
2014	3.00	2.56	2.48	2.77	2.48	2.30	1.95	2.64	2.20	2.48	2.71	2.56	2.51	0.08
2015	2.64	2.89	2.20	2.48	1.95	2.30	2.71	2.30	2.08	2.40	2.64	2.20	2.40	0.08
2016	3.00	2.83	3.04	2.71	2.56	3.14	1.95	1.79	2.48	2.40	1.95	1.95	2.48	0.23
2017	2.40	2.56	3.18	2.83	2.48	2.77	2.40	2.89	2.83	2.83	2.77	2.56	2.71	0.05
2018	2.20	2.64	2.89	2.89	2.48	2.94	2.64	2.20	2.89	2.71	2.83	2.64	2.66	0.07
2019	3.04	2.40	2.71	2.71	3.04	2.48	2.08	2.20	2.08	2.94	2.77	2.48	2.58	0.12
$\bar{X}_{.j}$	2.60	2.58	2.50	2.58	2.42	2.50	2.30	2.31	2.37	2.63	2.54	2.50		
$\sigma^2_{.j}$	0.16	0.04	0.45	0.12	0.21	0.14	0.16	0.09	0.10	0.03	0.12	0.09		

Source: Salvation Hospital Owerri, Imo State Nigeria (2009-2019)