__Original Research Article__

PORTFOLIO OPTIMIZATION USING 0-1 KNAPSACK QUADRATIC PROGRAMMING MODEL: A CASE STUDY.

ABSTRACT

Portfolio management is critical to selecting the right mix of investments which produces the best of results for any business entity. Using the 0-1 Knapsack quadratic model together with the mean-variance approach, this study sought to determine the optimal asset mix for TCF Microfinance bank. Five asset types were evaluated at a 70% target return. After three iterations, an optimal portfolio mix constituting of three out of the five assets was achieved, which exceeded the predetermined benchmark by 49.3% and at a risk value of less than 5%.

Keywords: Portfolio optimization, Knapsack Programming, Quadratic Programming, Variance, Co-Variance

1.0 INTRODUCTION

In the industrial world, the concept of portfolio is one that refers to the combination of stocks, bonds and cash which are all financial assets. An asset is any resource held or owned by an economic entity or business that can always produce positive economic benefits to the business, Baskarada, Gao,&Koronios, (2005). Assets can be tangible or intangible. Tangible assets represent physical resources which are further sub-divided into fixed and current assets. Here, fixed assets consider long term facilities which may include equipment, buildings and landed properties. Current assets survey the short-term facilities such as accounts receivable and inventory.Intangible assets, being non-physical facilities, consist of patents, trademarks, copyright, computer programs, to mention but a few,Charlie & Edet, (2023).Every business' objective remains to make profit and maximize it while keeping the associated risk at the barest minimum. Hence, the need for proper portfolio management. The determination and use of the best asset allocationis key to maximization of returns and reduction of risks.Capitalization, price weightings, risk parity, the modern portfolio theory and more are all models through which asset allocation can be managed.According to Markowitz(1952), "A good portfolio is more than a long list of good stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies."Summarily, Portfolio management is the process of choosing the right mix of investments that would achieve a specific investment goal. Portfolio management also entails the modification of those investments over time. Portfolio management uses a myriad of optimization techniques to achieve the best risk-return balance from market assets, Mercangöz, (2021).

Markowitz(1952),regarded as the father of modern portfolio theory, constructed an investment portfolio plan called the modern portfolio theory on the rationale that investors want to continuously maximize returns, while minimizing risk. Markowitz extended the techniques of linear programming to develop the critical line algorithm. The critical line algorithm identifies all feasible portfolios that minimize risk (as measured by variance or standard deviation) for a given level of expected return and maximize expected return for a given level of risk. The plot of standard deviation versus expected return has these portfolios forming the efficient frontier. The efficient frontier represents the trade-off between risk faced by an investorand expected return when forming his portfolio. Most of the efficient frontier represents well diversified portfolios. This is because diversification is a powerful means of achieving risk reduction.Since Markowitz's 1952 work, optimizing portfolios has been a huge challenge in investment management.

Different techniques including mathematical and meta-heuristic models have emerged to address this concern.
Several studies concentrate on improving the performance of the global minimum-variance portfolio (GMVP), which provides the least possible portfolio risk and involves only the covariance matrix estimates. The classical mean-variance framework depends on the perfect knowledge of the expected returns of the assets and their variance-covariance matrix. However, these returns are unobservable and unknown. The challenge of obtaining sufficient number of data samples, unreliability of data, and differing opinions of decision makers on the future returns affect their estimation and have led to what (Mulvey & Erkan, 2003)refer to as estimation risk in portfolio selection.Scherer & Martin(2005)suggested that integrating real-world constraints into the portfolio would requireutilizing integer variables that converts the problem into a nonlinear program which is not solvable by classical methods but by specialized algorithms.Fabozzi, Kolm,Pachamanova, & Focardi,(2007)suggested a robust optimization technique of incorporating the uncertainty generated by the errors of estimation parameters used in estimation. Robust optimization offers as an advantage because modifications in the mathematical model do not change the characteristics of the problem, and so it remains a quadratic programming problem.Barro &Canestrelli (2010) proposed a multistage stochastic programming framework for a dynamic asset allocation problem which takes into account the conflicting objectives of a minimum guaranteed return and an upside capture of asset returns. Behr, Guettler, &Miebs,(2013) reported that the uniform investment strategy (naive diversification or 1/N) is rational. They confirmed this approach to be efficient in cases where high degrees of uncertainty exist about the risk/return distribution. This fact is supported by several experimental studies which demonstrated that this kind of strategy tends to be better in environments of extreme uncertainty,and is usually a good strategy for risk-averse investors.

Zanjirdar (2020) in their study attempted to offer a valuable tool for portfolio selection theory by reviewing literature, recent developments, and optimization methods in the field. Batrancea, Rus,Masca, & Morar, (2021) investigated how fiscal pressure influenced the financial performance of 88 publicly listed energy companies over 16 years (2005Q1–2020Q3) by analyzing financial data using panel data techniques for the oil, gas, and electricity sectors. It was found that fiscal pressure significantly affected company financial performance, as measured by return on assets, return on equity, and return on investment. Mercangöz (2021) delved into the Markowitz mean-variance model, a bedrock of modern portfolio theory, focusing on Particle Swarm Optimization (PSO), which is a key method in financial portfolio optimization. Using four portfolio techniques (mean-variance, robust portfolio, minimum-variance, and equi-risk budgeting), and four covariance estimators (sample covariance, ordinary least squares (OLS) covariance, cross-validated eigenvalue shrinkage covariance, and eigenvalue clipping), Bitar, De Carvalho, &Gatignol, (2023) combined different portfolio allocation techniques with covariance matrix estimators to meet two types of clients' requirements, one who is risk-averse and another who has some degree of risk appetite. Bevilaqua, da Silva, & De Mattos, (2020) introduced a portfolio composition using the Knapsack problem and compared its performance to an investment website's share portfolio. Other studies in this regard include those of Ledoit& Wolf (2003),Logubayom&Victor (2019), Liu, Xi, & Wang,(2021).

Indeed, as long as business and financial operations continue, research and project works on portfolio optimization won't seize. TCF microfinance bank (MFB) is a limited liability company and part of the finance and insurance industry. TCF MFB, like all other financial industry players, is faced with the problem of matching their available assets with the right investment in the money, capital and stock markets. This challenge can be alluded to a lack of proper study of their risks and returns ratio, which is key to proper portfolio allocation. This research proposes a good portfolio management system in this financial institution, by developing a mathematical model which sees to proper fund allocation for the right mix of their assets with the highest returns and at a minimized risk. Just as portfolios can be held by individual investors, financial institutions, brokers, banks and other financial institutions, this work would be helpful to all who for the reason of making profits stake their funds, as it presents an effective way of asset distribution to different investment opportunities in use at any given time. This research work used the 0-1 Knapsack quadratic portfolio optimization model to dissect assets and dividends allocation of TCF Microfinance bank limited without consideration of their business activities, their products or the type of services they render.

2.0 MATERIALS

The data in use was extracted from the audited financial statement dated 31st December 2021 of the firm presented as the case study.The compiled data for this work are the five-year financial summary, the annual rate of return(%) 2016-2019, and the assets average rate of return. This is displayed in Table 1.

ASSET	YR. 2020	YR. 2019	YR. 2018	YR. 2017	YR. 2016
A1	185,206,279	11,739,797	4,773,080	6,732,680	13,171,000
A2	192,979,271	360,538,578	361,643,953	52,696,723	36,520,000
A3	22,921,039	20,758,567	19,485,144	35,958,618	10,598,000
A4	9,636,641	11,111,228	14,258,103	18,801,530	9,807,000
A5	19,090,864	22,209,944	16,042,582	17,307,650	6,704,000

Table 1: Assets Five-Year Financial Summary

Cash & bank balance, Investments, Loans & advances, Other assets, and Fixed assets are represented with A1, A2, A3, A4 and A5 respectively.

The annual rate of return for a given year is deduced from Table 1 as:[(The closing balance of the subsequent year) - (The closing balance of the given year)] / The closing balance of the given year × 100/1.The results are shown in table 2 below.

ASSET	2016 (%)	2017 (%)	2018 (%)	2019 (%)
A1	-48.88	-29.11	145.96	1477.59
A2	44.30	587.27	-0.31	46.47
A3	239.30	-45.81	6.54	10.42
A4	91.72	-24.17	-22.07	-13.27
A5	158.17	-7.31	38.44	-14.04

Table 2: Annual rates of return

ASSET	Av. Rate of Return	Variance	Std. Deviation
A1	3.8639	0.5369	0.7327
A2	1.6918	0.0778	0.2789

A3	0.5261	0.0162	0.1273
A4	0.0805	0.0029	0.0539
A5	0.4382	0.0064	0.0800

Table 3: Average Rates of Return, Variance and Standard Deviation of the Assets

The mathematical expression, Cov.(X,Y) = $\frac{\sum_{i=1}^{n} \mathrm{E}(\mathrm{X}-\square)*\mathrm{E}(\mathrm{Y}-\mathrm{V})}{n-1}$ is used to compute the variance and co-variance matrix of the assets where x, y are random variables, E(μ) is the expected value of the random variable X,E(Y) = V is the expected value of the random variable Y, n = number of items in the data set.Here, X and Y represents any of the pairs of A1, A2, A3, A4, A5 in consideration.

	A1	A2	A3	A4	A5
A1	0.5369	-0.0707	0.0291	-0.0130	-0.0301
A2	-0.0707	0.0778	0.0195	-0.0054	0.0003
A3	0.0291	0.0195	0.0162	0.0070	0.0097
A4	-0.0130	-0.0054	0.0070	0.0029	0.0042
A5	-0.0301	0.0003	0.0097	0.0042	0.0064

Table 4: Variance and co-variance matrix of the assets

Notice in table 4, that the leading diagonal of the covariance matrix are variances of A1, A2, A3, A4, A5 respectively because; Cov.(A1, A1) = Var.(A1),Cov.(A2, A2) = Var.(A2),etc. Also, it is worthy of note that this variance co-variance matrix is symmetric i.eCov.(A1, A2) = Cov.(A2, A1), Cov.(A1, A3) = Cov.(A3, A1),etc.Let Z=70%,be the expected target return for this study. A nominal portfolio is to be constructed for this expected rate of return.

Since investors are faced with minimizing risk and getting high returns, the objective function thus seeks the portfolio's risk minimization as follows:

Min. $\delta^2{}_{\mathrm{p}} = \sum_{i=1}^{n} . \sum_{j=1}^{n} x_i x_j \; \delta_{ij}^2$ where x_i is the decision variable; i = 1,2,...5, x_j is the 0-1 variable representing the assets; x_j = 1 if selected and equals 0 otherwise.δ_{ij}^2 represents corresponding assets' variance.

Also, since the expected portfolio return is given by the sum of the expected returns on individual assets, we have that E[R(x)] = $\sum_{j=1}^{n} r_j x_j$, where R(x) is the portfolio return andr_j denotes individual returns on assets.To avoid fractional value of x_j we set$\sum_{j=1}^{n} r_j x_j \;\; \geq Z$ where Z is the expected target return. The set value of Z is allowed to be exceeded as it is a credit to the investment.

Thus, we have a single constraint 0-1 KQPP as follows;

Min. $\delta^2{}_{\mathrm{p}} = \; \sum_{i=1}^{n} . \sum_{j=1}^{n} x_i x_j \; \delta_{ij}^2$

Subject to:$\sum_{j=1}^{n} r_j x_j \;\; \geq Z$

By this, the model thus formulated herein is;

$$\left.\begin{aligned}
\text{Min.}\,\delta^2_p = {} & 0.5369x^2_1 + 0.0778x^2_2 + 0.0162x^2_3 + 0.0029x^2_4 + 0.0064x^2_5 \\
& - 0.0707x_1x_2 + 0.0291x_1x_3 - 0.0130x_1x_4 - 0.0301x_1x_5 + 0.0195x_2x_3 \\
& - 0.0054x_2x_4 + 0.0003x_2x_5 + 0.0070x_3x_4 + 0.0097x_3x_5 + 0.0042x_4x_5 \\
\text{Subject to: } & 3.8639x_1 + 1.6918x_2 + 0.5261x_3 + 0.08054x_4 + 0.4382x_5 \geq 0.7 \\
& x_j = 0 \text{ or } 1;\ i = 1, 2, 3, 4, 5.
\end{aligned}\right\} \quad (1)$$

3.0 METHODOLOGY

Mathematical programming problems are generally of the form:

Max. or min. $f = f(X)$ (2)

Subject to: $g_i(x) \leq$ or $\geq b_i$

Where $X = (x_1, x_2, x_3, \dots x_n)$, $i = 1, 2, \dots n$

If $g_i = 0$, and $b_i = 0$, $\forall i$; 1,2, ..., m.

Quadratic Programming Problem (QPP) is one of the forms of nonlinear programming, which have their application in the optimization of financial portfolios, performance of least squares method of regression, control of scheduling in chemical plants and in sequential quadratic programming. Its' first use in portfolio optimization was carried out by Markowitz with the general programming model stated as;

Max.(Min.) $f(x) = (1/2)x^TQx + q^Tx$ (3)

Such that; $Ax = a$

$Bx \leq b$

$x \geq 0$

Knapsack Quadratic Programming Problem (KQPP) was first introduced by Gallo, Hammer& Simeone, (1980). Given a bag, and a set of items to be picked into the bag, Knapsack problems seeks to pick the most important of the given items as the volume of the bag is fixed or constrained and cannot be exceeded. The Knapsack quadratic problem can be stated as;

Max.(Min.) $f(x) = (1/2)x^TQx + x^TK$ (4)

Such that; $a_ix_i = b_i i \in I$

$a_ix_i \leq b_i$ (or $\geq$)$b_i i \in J$

$x_i \geq 0$

Where Q is a symmetric matrix, K is a constant factor, I and J are index sets of equality and inequality constraintsrespectively.

0-1 Knapsack Quadratic Programming Problems are special cases of equation (4), where K = 0, I and J are singleton sets. Thus, the expression;

Max.(Min.) $f(x) = (1/2)x^TQx$ (5)

Subject to; $a_ix_i = b_i i \in I$

$a_ix_i \leq b_i$ (or $\geq$)$b_i i \in J$

$x_i \geq 0$

x = 1(If selected), x=0(Otherwise).

Mean or the expected value is the average of any given set of data while variance, δ^2, measures the degree of spread in any given data set. It is calculated by taking the average of squared deviations from the mean. Further, the square root of the varianceis the standard deviation(δ), which helps determine the consistency of an investment's return over a period of time.Variance is useful to investors in portfolio management in order to ascertain how much risk an investment carries, whether a given venture would be profitable or not, and to compare the relative performance of each asset in a portfolio to achieve the best asset allocation.Here, variance is expressed as risk and the mean as the expected return. Thus, Mean-Variance analysis is weighing risk against the expected return, thereby gifting firms and investors the opportunity of adopting securities or the assets with the biggest reward to the attached risk.

To achieve risk reduction i.e. variance reduction, Kacker (1985) developed the performance measure statistic, by making two assumptions on how the variance is functionally related to the mean. For cases where the variance increases linearly with the mean, the bias [A - μ] can be reduced by the use of coefficient of variation (δ^2/μ) with the efficient performance measure as a monotone function given as $\emptyset = 10\log\mu^2/s^2$, where s is the unbiased estimator of the mean μ.On the other hand, when the mean and variance are independent of each other, the bias (the difference between the asset to be invested and the expected return) can be reduced independently with the performance measure given as the monotone function $\emptyset = -\log(\delta^2)$ where $\delta^2$is the corresponding variance. The monotone function is known as the efficient or reasonable performance measure, where $\emptyset$, $\emptyset^{(2)}$, $\emptyset^{(3)}$ are the performance statistics for the first, second and third iterations respectively.

For n-risky assets of expected returns (μ) and variance (δ_i^2), Z as the expected target return, $A_j$represents assets, E, $E^{(2)}$, $E^{(3)}$ represents the expected values of assets in the first, second and third iterations respectively, $\emptyset$, $\emptyset^{(2)}$,$\emptyset^{(3)}$are the performance statistics for the first, second and third iterations respectively, A_k is the asset with the highest performance measure, i, j = 1, 2,...n, the following steps are adopted;

1. Determine the performance statistics for each asset, A_j; j = 1, 2, 3, ... k, ... n, where the largest of the performance statistic is denoted as the k^{th} asset and the knapsack containing only the k^{th} asset is the one with the least variance and the expected return of the k^{th} asset is equal to or less than the target return Z.
2. If expected return equals or exceeds Z, then stop, as optimum portfolio is achieved. Otherwise, go to step 3.
3. Increase variance and expected return as follows;

$(\text{Var.})^2 = \delta^2 (A_k + A_j),\ k \neq j;\ k, j = 1, 2, 3, ..., n.$

$E^{(2)} = E(A_k + A_j),\ k \neq j;\ j \neq 1, 2, 3, ...,n$

$\emptyset^{(2)} = -\log\delta^2$

With the next choice portfolio as that satisfying $\text{Max.}_{j\neq k} \{\emptyset^{(2)} : E^{(2)} \leq Z\}$ Then see step 2. Otherwise, see step 4.

4. Increase variance and expected return as follows;

$(\text{Var.})^3 = \delta^2 (A_k + A_j + A_i),\ k \neq j \neq i;\quad k, j, i = 1, 2, 3, ...,n.$

$E^{(3)} = E(A_k + A_j + A_i),\ k \neq j \neq i;\quad k, j, i = 1, 2, 3, ...,n$

$\emptyset^{(3)} = -\log(\text{var.})^3$

With the next choice portfolio as that satisfying, $\text{Max.}_{j\neq k}\{\emptyset^{(3)} : E^{(3)} \leq Z\}$. Then see step 2, otherwise, continue the process of increasing the variance and expected mean until the t^{th} asset enters the portfolio having, $t \leq n$ and $E^{(t)} \geq Z$.

4.0 RESULTS AND DISCUSSION

For the analysis of the optimization problem in this work, the variance is considered independent of the mean for a specific target value. Hence the algorithm developed by Kacker (1985) which utilizes the performance measure described as $\emptyset = -\log\delta^2$ is adopted.

Recall model (1),

$$\text{Min.}\delta^2_p = 0.5369x^2_1 + 0.0778x^2_2 + 0.0162x^2_3 + 0.0029x^2_4 + 0.0064x^2_5 - 0.0707x_1x_2 + 0.0291X_1X_3 - 0.0130x_1x_4 - 0.0301x_1x_5 + 0.0195x_2x_3 - 0.0054x_2x_4 + 0.0003x_2x_5 + 0.0070x_3x_4 + 0.0097x_3x_5 + 0.0042x_4x_5.$$

Subject to; $3.8639x_1 + 1.6918x_2 + 0.5261x_3 + 0.08054x_4 + 0.4382x_5 \geq 0.7$

$x_j = 0$ or 1; i = 1, 2, 3, 4, 5.

FIRST ITERATION

By Kacker's algorithm, the performance measure, $\emptyset = -\log\delta^2$, is determined as follows:

For A1: $\delta^2 = 0.5369$, E = 3.8639, $\emptyset = -\log 0.5369 = 0.2701$

For A2: $\delta^2 = 0.0778$, E = 1.6918, $\emptyset = -\log 0.0778 = 1.1090$

For A3: $\delta^2 = 0.0162$, E = 0.5261, $\emptyset = -\log 0.0162 = 1.7905$

For A4: $\delta^2 = 0.0029$, E = 0.0805, $\emptyset = -\log 0\ 0.0029 = 2.5376$

For A5: $\delta^2 = 0.0064$, E = 0.4382, $\emptyset = -\log 0.0064 = 2.1938$

ASSSETS	EXPECTED RETURN (E)	VARIANCE (δ^2)	PERFORMANCE MEASURE (Ø)
A1	3.8639	0.5369	0.2701
A2	1.6918	0.0778	1.1090
A3	0.5261	0.0162	1.7905
A4	0.0805	0.0029	2.5376
A5	0.4382	0.0064	2.1938

Table 5: Iteration 1 results

Notice from table 5 that the highest performance measure recorded from our computation is 2.5376, which is associated with the asset denoted as A4 (i.e. Other Assets). This performance measure, which indicates the asset's potential to generate returns, places A4 as the most promising asset of choice among the range of assets under consideration, thus making it the first feasible asset solution. With A4 as the asset with the highest performance measure, we consider another very important metric, the expected or anticipated returns of the asset, denoted by E. This value is calculated to be 0.0805 (i.e. 8.05%), and is clearly very much less than the predetermined threshold, E = 0.0805 < 0.7, (E < Z, where Z = 70%). This signifies that the expected performance of A4 asset is not up to our desired target. Hence, whilst A4 demonstrates great potential and is the feasible solution for this first iteration, it is not optimal.

Following this, further investigations are made to explore other possible options that will yield higher returns. Subsequent iterations will build on a refinement of A4 which is the front-runner choice, by strategically choosing the next asset to add to the portfolio mix. Hence the second iteration would see to a mixing of A4 singly with other assets. With the goal of attaining or exceeding the predetermined anticipated returns threshold, A4 was compared with other assets to ascertain the next asset to go into the portfolio. The next iteration seeks to unveil a more optimal solution that better aligns with the desired returns.

SECOND ITERATION

Still by Kacker's algorithm, the performance measure such that $\emptyset = -\log\delta^2$ is determined:

For A4 + A1:

[Var.(A4,A1)] = Var.(A4) + Var.(A1) + Cov.(A4+A1) = 0.0029 + 0.5369 - 0.0130 = 0.5268

$E^2 = 0.0805 + 3.8639 = 3.9444$

$\emptyset^2$= -log0.5268 = 0.2784

For A4 + A2:
[Var.(A4,A2)] = Var.(A4) + Var.(A2) + Cov.(A4+A2) = 0.0029 + 0.0778 - 0.0054 = 0.0753
E^2 = 0.0805 + 1.6918 = 1.7723
$\emptyset^2$ = -log0.0753 = 1.1232

For A4 + A3:
[Var.(A4,A3)] = Var.(A4) + Var.(A3) + Cov.(A4+A3)= 0.0029 + 0.0162 + 0.0070 = 0.0261
E^2 = 0.0805 + 0.5261 = 0.6066
$\emptyset^2$ = -log0.0261 = 1.5834

For A4 + A5:
[Var.(A4,A5)] = Var.(A4) + Var.(A5) + Cov.(A4+A5) = 0.0029 + 0.0064 + 0.0042 = 0.0135
E^2 = 0.0805 + 0.4382 = 0.5187
$\emptyset^2$ = -log0.0135 = 1.8697

ASSETS	EXPECTED RETURN ($E^{(2)}$)	VARIANCE (δ^2)	PERFORMANCE MEASURE ($\emptyset^{(2)}$)
A4 + A1	3.9444	0.5268	0.2784
A4 + A2	1.7723	0.0753	1.1232
A4 + A3	0.6066	0.0261	1.5834
A4 + A5	0.5187	0.0135	1.8697

Table 6: Iteration 2 results

The combination of the pair of A4(Other assets) andA5(Fixed assets) yields the highest performance measure, $\emptyset^{(2)}$ = 1.8697, as can be seen in Table 6. Upon further consideration of the results of the pair, a remarkable leap in the expected returns from the duo is observed. Moving away from the 0.0805 expected returns for the single A4 asset, the duo of A4 and A5 is anticipated to give a 51.87% return ($E^{(2)}$= 0.5187). This is a remarkable increase and a progression towards the desired 70% target.

However, even though a more robust solution is herein attained, it still falls short of the predetermined threshold because $E^{(2)}$= 0.5187 < 0.7. Consequently, another iteration is imminent, to enhance the portfolio performance. Leveraging on the concept of ‘mixing’ assets, the next iteration will see to the mixing of ‘A4 and A5’doubly with other assets to amplify their performance and subsequently, the returns. Further exploration is based on the promise of finer modification to achieve a more effective portfolio solution.

THIRD ITERATION
For A4 + A5 + A1:
[Var.(A4,A5 + A1)] = Var.(A4) + Var.(A5) + Var.(A1) + Cov.(A4,A5) + Cov.(A4,A1) + Cov.(A5,A1) = 0.0029 + 0.0064 + 0.5369 + 0.0042 - 0.0130 - 0.0301 = 0.5073
E^3 = 0.0805 + 0.4382 + 3.8639= 4.3826
$\emptyset^3$ = -log0.5073 = 0.2947

For A4 + A5 + A2:

[Var.(A4,A5 + A2)] = Var.(A4) + Var.(A5) + Var.(A2) + Cov.(A4,A5) + Cov.(A4,A2) + Cov.(A5,A2) = 0.0029 + 0.0064 + 0.0778 + 0.0042 - 0.0054 - 0.0003 = 0.0856
E^3 = 0.0805 + 0.4382 + 1.6918 = 2.2105
$\emptyset^3$ = -log0.0856 = 1.0675

For A4 + A5 + A3:
[Var.(A4,A5 + A3)] = Var.(A4) + Var.(A5) + Var.(A3) + Cov.(A4,A5) + Cov.(A4,A3) + Cov.(A5,A3)= 0.0029 + 0.0064 + 0.0162 + 0.0042 + 0.0070 +0.0097 = 0.0464
E^3 = 0.0805 + 0.4382 + 0.5261 = 1.0448
$\emptyset^3$ = -log0.0464 = 1.3335

ASSETS	EXPECTED RETURN ($E^{(3)}$)	VARIANCE (δ^2)	PERFORMANCE MEASURE ($\emptyset^{(3)}$)
A4 + A5 + A1	4.3826	0.5073	0.2947
A4 + A5 + A2	2.2105	0.0856	1.0675
A4 + A5 + A3	1.0448	0.0464	1.3335

Table 7: Iteration 3 results

By careful consideration of our results as displayed in Table 7, it is apparent that the combination of A4 (Other assets), A5 (Fixed assets) and A3(Loans & Advances) is the asset mix with the highest performance measure, $\emptyset^{(3)}$ = 1.3335. This quantity is an indicator of the potential of this asset mix to produce substantial returns while adequately minimizing risks. A closer look at the anticipated return of this mix shows that the threshold target return is exceeded, as $E^{(3)} = 1.0448 > 0.7$. Having surpassed the set benchmark for determining the acceptable minimum level of expected return,which in this work is 70%, the above stated asset mix of A4 (Other assets), A5 (Fixed assets) and A3(Loans & Advances)is an optimal portfolio. This achievement signals the termination of the iteration and optimization process.

From the analysis of the iterations, it is important to understand the significance of having unexpected sharp changes in an asset. These abrupt changes in balance, as the type noticed in asset A1, do not necessarily depict that the asset would be an ideal part of the optimal portfolio. If anything, it portrays the assets' risk profile and instability which could stem from various factors. Such an asset may be unreliable and an inconsistent performer in the market place, thereby resulting in possible losses due to heightened risk from market fluctuations in the negative direction.Such erratic behaviour is not recommended for a balanced risk-reward investment.

Given that the identified optimal asset mix is A4 + A5 + A3, their corresponding values, x_4, x_5, $x_3$are set to 1 because they are chosen (i.e. part of the optimal mix). However, since assets A1 and A2 are not chosen, they are assigned zero values, indicative of their absence from the optimal portfolio mix. Substituting these in our model (1), the following outcome is derived:

Min.δ^2_p= 0.5369(0)+ 0.0778(0) + 0.0162x^2_3 + 0.0029(1)+ 0.0064(1) - 0.0707(0) + 0.0291(0) - 0.0130(0)- 0.0301(0) + 0.0195(0)- 0.0054(0) + 0.0003(0)+ 0.0070(1)+ 0.0097(1)+ 0.0042(1) = 0.0162 + 0.0029 + 0.0064 + 0.0070 + 0.0097 + 0.0042 = 0.0464

The goal of portfolio optimization is as much to minimize risk as it is to maximize returns. One cannot be adversely sacrificed on the altar of the other. The computed risk value here is 0.0464 which is a confirmation of the variance of the best assets mix. Variance is a statistical measure that quantifies how data points are dispersed around the mean value. As regards portfolio management, a lower variance connotes lower risk and volatility of the asset, signifying that the asset returns in the portfolio are closer to the mean (expected) return. This value asserts the degree of variability in the chosen assets return within the portfolio. Thus, 0.0464 suggests that the returns on the portfolio is not subject to any extreme fluctuations and hence sufficiently stable. This very minimal risk level has been achieved by a strategic combination of assets A3, A4 and A5. Just as a properly-structured portfolio seeks to balance the reward-risk trade off, quantifying this risk value and comparing it with the expected returns will enable investors and fund managers to make well-informed decisions. This is crucial in portfolio recommendation as it underscores the potency of the chosen asset mix to yield the desired reward-risk outcome.

One of thechallenges investors and institutions constantly grapple with is what combination of their assetsthat would yield reasonable return at a minimum risk. From the results here, TCF Microfinance bank is advised to combine their Other assets(which may depict their short-term assets), their fixed assets and their loans and advances, as it would yield highest returns for them and at the least risk.The essence of this recommendation lies in the synergy achieved through combining these different asset types. "Other assets" offer liquidity and short-term benefits like quickly navigating market changes and capitalizing on opportunities, while fixed assets provide stability and potential appreciation over time. Loans and advances, on the other hand, provide the possibility of earnings through interest payments. TCF Microfinance bank can therefore fully utilize the unique benefits of each asset type when these distinct asset categories are combined.

Increment yield beyond the 70% target return is given as;

$$\frac{1.0448 - 0.7000}{0.7000} \times \frac{100}{1} = 49.3\%$$

Thus, this asset mix of A4(Other assets) + A5(Fixed assets) + A3(Loans & advances)is their optimal portfolio mix and it assures even more than a 100% return.

5.0 CONCLUSION

In this study, portfolio optimization using 0-1 Knapsack quadratic model(TCF MFB, as a case study) was investigated to enhance and advance the financial operations of the above named financial institution. Kacker's algorithm which involves the use of mathematical and statistical tools such as mean, variance, co-variance and performance measure, was employed.A combination of assets A4, A5 and A3 yielded the best return of 1.0448 at a minimum risk of 0.0464 (< 5%).The methodology used is efficient and can always be used to determine a better way of maximizing returns and minimizing risk for every investor who is very considerate of the associated risks in investments.

For more risk-averse effect, TCF Microfinance bank should consider a reduction of their expected target return to 50%. This would see them combining only assets A4 and A5 with an expected target return of 0.5187 at a more reduced risk of 0.0135 (1.35%). This implies a 3.7% surplus on the 50% target return;

$$\frac{0.5187 - 0.5}{0.5} \times \frac{100}{1} = 3.7\%$$

By scale of preference, other assets(which maybe suggestive of short term assets) is the foremost on the list of assets to be selected, followed by fixed assets and then loans and advances.This order of selection is a pointer to the magnitude of the contribution potential of each asset, so that stakeholders can strategically allocate their resources. The stakeholders/shareholders can divest the other two assets which are not part of the optimal portfolio, and invest more on the 3 ascertained assets that make up the optimal portfolio, for higher returns at a minimized risk level.

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