Research on the Cognitive Degree of Pre-service Mathematics Teachers for Mathematical Reasoning Ability in Junior High School in China

Abstract:
Currently, theeducation of junior high school mathematical reasoning ability has been widely concerned by all walks of life. Many relevant problems about it have been studied except the cognitive degree of pre-service mathematics teachers for the mathematical reasoning ability in junior high school. To address this gap, this study investigates 25 pre-service mathematics teachers' cognitive degree of junior high school mathematical reasoning ability through open-ended interviews. After analyzing, it can be found that: (1) The cognitive scope of the current pre-service mathematics teachers' mathematical reasoning ability is not wide, more than half of the people do not recognize half of the content; (2) The cognition of many pre-service teachers is not deep, and their understanding of ability performances is simple; (3) The cognition of many pre-service teachers for many aspects of mathematical reasoning ability is not clear. Therefore, it is suggested that: (1) Experts and teachers who are responsible for educating teachers should pay more attention to mathematical reasoning ability in junior high school and add relevant courses; (2) Pre-service mathematics teachers should understand the content of junior high school mathematical reasoning ability comprehensively and deeply, and clarify relevant statements.

Keywords: Pre-service teachers, Junior high school, Mathematical reasoning ability, Cognitive degree

1. Introduction

Mathematical reasoning ability is an essential ability for students to learn mathematics, cultivating students' mathematical reasoning ability has always been among the goals of mathematics compulsory education in China [1]. *The Compulsory Education Mathematics Curriculum Standards(2022 Edition)* (hereinafter referred to as the *Curriculum Standards (2022 Edition)*) promulgated by the Ministry of Education of the People's Republic of China in 2022 clearly gives the meaning of mathematical reasoning ability, that is, mathematical reasoning ability refers to the ability that it starts from some facts and propositions and other propositions or conclusions are deduced according to the rules, and points out that this ability helps students develop a habit of thinking that emphasizes arguments and logic, and form a scientific attitude and rational spirit of seeking truth from facts [2]. Therefore, the education of mathematical reasoning ability in junior high school has been widely concerned by all walks of life. However, after extensive discussion and research by many scholars and teachers on the current situation of junior high school students' mathematical reasoning ability, it is found that the current level of junior high school students' mathematical reasoning ability is not high, and the cultivation of mathematical

reasoning ability has not been well implemented in practical teaching [3]. What are the reasons for this? How to cultivate the mathematical reasoning ability of junior high school students? These questions are worth studying.

2. Literature Review

Currently, there have been several studies on this issue of implementing mathematical reasoning ability in junior high school mathematics teaching.

2.1. The Situation of Junior High School Students' Mathematical Reasoning AbilityLevel

Before the promulgation of the *Curriculum Standards (2022 Edition)*, Zhou pointed out that the mathematical reasoning ability of junior high school students was at a medium level based on the results of the 2016 academic quality monitoring in Jiangsu Province, and there was a problem of low level of plausible reasoning [4]. Wang SL tested the students and found that the overall level of mathematical reasoning ability of junior high school students was at a medium level, their ability to find mathematical problems in related situations was weak, and their ability to understand mathematical propositions and solve complex problems in related mathematical propositions was weak [5]. According to Wang SN, she found that the level of mathematical reasoning ability of junior high school students was above average, and there was no significant difference in mathematical reasoning ability between genders, but there were significant differences between schools, students in key junior high schools had stronger mathematical reasoning ability than those in ordinary junior high schools. In addition, she pointed out that the development of the mathematical reasoning ability of junior high school students depends on the mastery of reasoning rules and the understanding of mathematical knowledge [6].

After the promulgation of the *Curriculum Standards (2022 Edition)*, Xiong tested grade 8 students, and found similar research results to Wang SN, he pointed out that grade 8 students had problems of over-reliance on intuitive thinking and weak logical reasoning completeness [6,7]. Liu found that students' level of mathematical reasoning ability was mainly at a medium level, and there was a problem of not thinking rigorously [3].

It can be seen that before and after the promulgation of the *Curriculum Standards (2022 Edition)*, the level of mathematical reasoning ability of junior high school students is generally at a medium level. Many problems in junior high school mathematics teaching have not been solved, and the level of students' mathematical reasoning ability has not been improved.

2.2. The Factors Affecting Students' Mathematical Reasoning Ability

Through data analysis and interviews, Wang SL concluded that junior high school students' mathematics learning strategies, mathematics cognitive structure, teachers' teaching strategies, and learning motivation had a direct positive impact on their mathematical reasoning ability, and the effect decreased in turn [5]. Through interviews, Lan concluded that the direct factors affecting the mathematical reasoning

ability of grade 8 students included mathematical reading, knowledge base, thinking mode, level requirements, other core mathematical literacy and ability, and reflection level. The indirect factors included the psychological process of individual students (cognitive process, emotional process, will process), personality psychology (ability, interest, personality, habits, etc.), teaching resources, teachers, and classroom atmosphere of schools and classes [8]. Through questionnaires and interviews, Han obtained the influencing factors of junior high school students' mathematical inductive reasoning ability, including two aspects of teachers and students. Teachers' factors included the understanding of the importance of inductive reasoning, the degree of mining of teaching materials, and teaching methods. Students' factors included interest in mathematics learning, cognition of mathematical inductive reasoning, mathematics learning habits, and mathematical thinking in the process of reasoning [9].

Therefore, it can be seen that the current influencing factors on junior high school students' mathematical reasoning ability are mainly concentrated in two aspects: teachers and students. Students' mathematical cognitive structure, learning strategies, personality psychology, teachers' cognition of the importance of mathematical reasoning ability, and teaching strategies all have an important impact on junior high school students' mathematical reasoning ability.

2.3. Strategies for Developing Junior High School Students' Mathematical Reasoning Ability

According to Wang XF, he pointed out that mathematical experiments could provide students with rich problem situations for their mathematical learning so that students could experience the complete reasoning process of plausible reasoning to find conclusions and deductive reasoning to prove conclusions. It was an important learning method to cultivate students' mathematical reasoning ability [10]. Lan stressed that schools need to strengthen the training of teachers and explore new teaching resources; teachers should pay attention to the cultivation of students' mathematical reasoning ability and improve students' reflective ability [8]. Han believed that teachers themselves should: (1) Improve their mathematical literacy; (2) Explore the content of teaching materials, deepen the understanding of inductive reasoning, and consciously infiltrate inductive reasoning into the classroom; (3) Pay attention to the creation of problem situations and cultivate students' mathematical inductive reasoning ability in situational teaching. She also believed that in terms of students, teachers should: (1) Deepen students' understanding of inductive reasoning and standardize the problem-solving process; (2) Establish students' confidence in learning and improve their interest in learning mathematics in the learning process; (3) Cultivate students' good learning habits in the learning process [9]. Xiong pointed out that teachers need to pay attention to the cultivation of reasoning ability, combine the students' cognitive developing rule, stimulate students' interest in learning, and guide students to learn to think independently and think from multiple perspectives [7].

It can be seen that the strategies proposed by scholars need to be implemented by teachers. Therefore, teachers play an important role in the cultivation of junior high

school students' mathematical reasoning ability.

From the above studies, it can be seen that many scholars have studied mathematical reasoning ability in terms of its situation, influencing factors, and cultivation strategies. However, few people have studied teachers' mathematical reasoning ability, and the research on teachers' cognitive degree of mathematical reasoning ability in junior high school is in a gap.The improvement of students' mathematical reasoning ability is mainly realized in classroom teaching, and teachers play a key role in cultivating students' mathematical reasoning ability. Only when teachers' cognition of mathematical reasoning ability is scientific, can the implementation of their teaching behavior achieve better teaching results [11].Therefore, the purpose of this study is to find out the current pre-service mathematics teachers' cognitive degree of mathematical reasoning abilityin junior high school through investigation. The cognitive degree generally includes cognitive breadth, cognitive depth, and cognitive clarity. Therefore, the main issues of this study are:

1. How wide is the cognitive degree of junior high school mathematical reasoning ability of pre-service mathematics teachers?

2. How deep is the cognitive degree of junior high school mathematical reasoning ability of pre-service mathematics teachers?

3. Is the current pre-service mathematics teachers' cognition of junior high school mathematical reasoning ability clear?

At present, several studies have shown that the level of mathematical reasoning ability of junior high school students is not high, and teachers' cognition of mathematical reasoning ability is an important factor affecting students' mathematical reasoning ability. Therefore, the hypotheses of this study are:

Hypothesis 1: The current pre-service mathematics teachers' cognitive degree of junior high school mathematical reasoning ability is not wide;

Hypothesis 2: The current pre-service mathematics teachers' cognitive degree of junior high school mathematical reasoning ability is not deep;

Hypothesis 3: The current pre-service mathematics teachers' cognitive degree of junior high school mathematical reasoning ability is not clear.

3. Theoretical Basis

There have been many previous studies on mathematical reasoning ability and its main components.

The famous mathematician Polya points out that mathematical reasoning includes proof reasoning and plausible reasoning in his book *Mathematics and Conjecture* [12]. In 2011, the Ministry of Education of the People's Republic of China promulgated the *Compulsory Education Mathematics Curriculum Standards (2011 Edition)*, which points out that the development of reasoning ability should run through the whole process of mathematics learning. Reasoning is the basic way of thinking in mathematics, and it is also a common way of thinking in people's learning and life. Reasoning generally includes plausible reasoning and deductive reasoning. Plausible reasoning is based on the existing facts, relying on experience and intuition, through

induction and analogy to infer some results; deductive reasoning starts from the existing facts (including definitions, axioms, theorems, etc.) and certain rules (including the definition, rules, order, etc.), and proves and calculates according to the rules of logical reasoning [13]. Subsequently, more and more scholars and experts have studied the connotation and structure of mathematical reasoning ability. Wu believes that reasoning ability is the synthesis of relatively stable individual psychological characteristics formed, embodied, and developed in reasoning activities, which affects the effect of reasoning activities. Through the method of theoretical speculation, he analyzes the structural elements of reasoning ability from the perspective of the five different requirements of reasoning effectiveness, clarity and orderliness, flexibility, creativity, and introspection in the process of problem-solving [14]. Cao believes that the essential function of reasoning is to draw conclusions and generate new knowledge [15]. *The Curriculum Standards for General High School Mathematics (2017 Edition)* points out that the core literacy of mathematics is a comprehensive embodiment of the thinking quality, key ability, emotion, attitude, and values with the basic characteristics of mathematics, and lists logical reasoning as one of the core literacy of mathematics. It also points out that logical reasoning refers to the literacy that it starts from some facts and propositions and other propositions are deduced according to the rules, including reasoning from special to general and reasoning from general to special. The former's reasoning forms mainly include induction and analogy, and the latter's reasoning forms mainly include deduction [16]. The Ministry of Education of the People's Republic of China has comprehensively summarized the above viewpoints. *Curriculum Standards (2022 Edition)*, the latest edition, points out that students should form and develop the core literacy needed for social and personal development through mathematics learning, and the main manifestations of core literacy are accurately defined. Among them, mathematical reasoning ability is one of the main manifestations of core literacy in junior high school. Its meaning refers to the ability that it starts from some facts and propositions and other propositions or conclusions are deduced according to the rules [2]. Furthermore, *Curriculum Standards (2022 Edition)* makes a very clear statement on the performances of mathematical reasoning ability, including two aspects: key ability performances and character and value performances. The key ability performances include understanding the importance of logical reasoning in the formation of mathematical concepts, rules, theorems, and problem-solving, and mastering the basic forms and rules of reasoning. For some simple problems, general conclusions can be inferred from special results. To understand the structure and connection of propositions, explore and express the process of demonstration. To understand the rigor of mathematics and form the habit of logical expression and communication. Character and values performances include gradually developing a habit of thinking that emphasizes arguments and logic, forming a scientific attitude, and a rational spirit of seeking truth from facts [2,17].

Therefore, this study adopts the definition of mathematical reasoning ability by *Curriculum Standards (2022 Edition)* to research the cognitive degree of pre-service mathematics teachers for mathematical reasoning ability in junior high school from

three aspects: the meaning of mathematical reasoning ability, the key ability performances, and the character and value performances.

4. Research Method

4.1. Participants

In this study, 10 undergraduate graduates and 15 master of education graduates from the School of Mathematics and Statistics of Shandong Normal University in China are selected as the survey objects. They all hold mathematics teacher qualification certificates and have the intention to go to junior high school teaching. Taking them as the survey objects can truly reflect the current pre-service mathematics teachers' cognitive degree of junior high school mathematical reasoning ability.

4.2. Instrument

In this study, the open-ended interview method is used to investigate, and the interview outline is designed with three questions, "How do you understand mathematical reasoning ability in junior high school? What are the key ability performances of mathematical reasoning ability in junior high school? What are the character and value performances of mathematical reasoning ability in junior high school?". Then invite the survey objects to say their understanding of these three issues in detail and comprehensively. The open-ended interview method is adopted because itis fast, convenient, flexible, and not restricted by written language, and it facilitates in-depth investigations to obtain the most direct information.

4.3.Data Collection

To ensure the reliability of the research, this study uses an open-ended interview method to interview 25 pre-service mathematics teachers one by one individually. In addition, the whole interview content is recorded during the whole process after the consent of the survey object is sought.

4.4.Data Processing

Firstly, divide and code the content related to the junior high school mathematical reasoning ability in the *Curriculum Standards (2022 Edition)*.A, B, and C are used to represent the meaning, key ability performances, character, and value performances of mathematical reasoning ability, and the content of each aspect is represented by numbers. A total of 12 items of A1-C2 are divided. The specific coding and its meaning are shown in Table 1.

Subsequently, the survey objects are coded, with 15 graduates of Master of Education corresponding to codes a1 to a15, and 10 graduates of undergraduate corresponding to codes b1 to b15. Then convert the interview recording content into text form and compare it with the encoded content one by one. If the content meaning is similar, the survey object is considered to be able to recognize this item. In addition, cognitive clarity is judged based on the completeness and accuracy of the survey objects' expressions. Finally, count the number of items recognized by each survey object and the number of people mentioned in each item, calculate the corresponding percentage,

and make a statistical table.

Table 1: Content Coding

Category	Label	Content
A Meaning	A1	Starts from some facts and propositions
	A2	According to the rules
	A3	Deduce other propositions or conclusions
B Key Ability Performances	B1	Understand the importance of logical reasoning in the formation of mathematical concepts, rules, theorems, and problem-solving
	B2	Master the basic forms and rules of reasoning
	B3	General conclusions can be inferred from special results
	B4	Understand the structure and connection of propositions
	B5	Explore and express the process of demonstration
	B6	Understand the rigor of mathematics
	B7	Form the habit of logical expression and communication
C Character and Value Performances	C1	Develop a habit of thinking that emphasizes arguments and logic
	C2	Form a scientific attitude and rational spirit of seeking truth from facts

5. Results

5.1.Cognitive Breadth

In the *Curriculum Standards (2022 Edition)*, the content of junior high school mathematical reasoning ability is divided into 12 items. From the perspective of the number of personal cognitive items, the survey objects recognize at most 11 items and at least 1 item. Among them, only 11 people recognize 6 or more items, and the other 14 people recognize only 2 items. The details are shown in Figure 1.

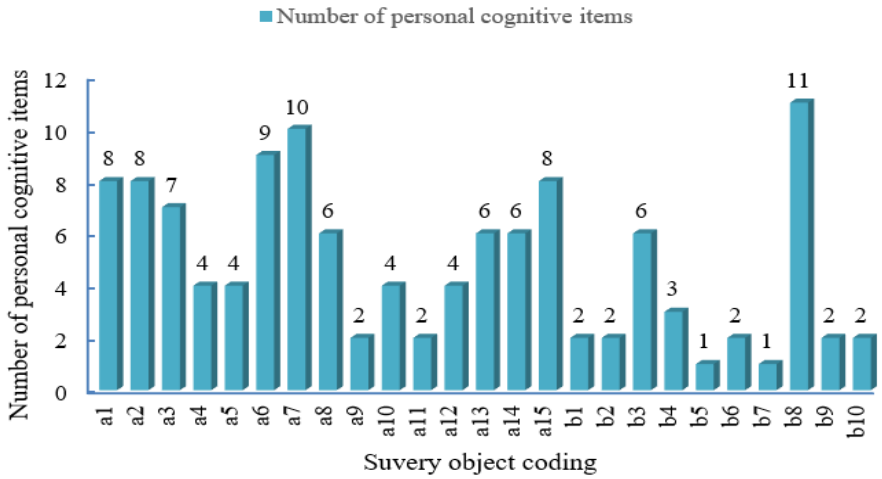

Figure 1: Number of Personal Cognitive Items of Survey Objects

From the number of people mentioned in each item, all survey objects in the 12 items can recognize 11 of them to the greatest extent. The meaning is divided into 3 items, and survey objects can recognize 3 items to the greatest extent. The key ability performances are divided into 7 items, and survey objects can recognize 6 items to the greatest extent. Character and value performances are divided into 2 items, and survey objects can recognize two items to the greatest extent. However, there is a large gap in the number of people recognized for each item.The number of people who recognize that mathematical reasoning ability should be based on rules is the largest, with 15 people, accounting for 60% of the total number. The number of people who have recognized the three items of "understand the structure and connection of propositions", "form the habit of logical expression and communication", and "understand the rigor of mathematics" is small. There are 0 people, 4 people, and 5 people, respectively, accounting for 0%, 16%, and 20% of the total number.

Therefore, it can be seen that the current pre-service mathematics teachers do not have a wide range of cognition about mathematical reasoning ability, and more than half of them do not recognize half of the content of mathematical reasoning ability.In terms of specific content, the cognitive degree of the stronger generality of mathematical reasoning ability is higher, such as its meaning, while the cognitive degree of key ability performances injunior high school is lower. The details are shown in Table 2.

Table 2: Statistics of Cognitive Breadth and Depth Results

Category	Label	Number	Percentage	Cognitive points	Total points	Percentage
A Meaning	A1	14	56%	3	3	100%
	A2	15	60%			
	A3	13	52%			
B Key Ability Performances	B1	10	40%	6	7	85.71%
	B2	13	52%			
	B3	14	56%			
	B4	0	0%			
	B5	12	48%			
	B6	5	20%			
	B7	4	16%			
C Character and Value Performances	C1	14	56%	2	2	100%
	C2	6	24%			

5.2.Cognitive Depth

From Table 2, it can be seen that survey objects have a deeper understanding of the three items of the meaning of mathematical reasoning ability, and the number of people who recognize each item is more than half of the total number. Among the seven items of the key ability performances of mathematical reasoning ability, survey objects have a deeper understanding of the three items of "master the basic forms and rules of reasoning", "general conclusions can be inferred from special results", and

"explore and express the process of demonstration", while the cognitive depth of the four items of "understand the importance of logical reasoning in the formation of mathematical concepts, rules, theorems and problem-solving", "understand the rigor of mathematics", "form the habit of logical expression and communication", "understand the structure and connection of propositions" is gradually reduced, the number of people who recognized accounted for 40%, 20%, 16% and 0% of the total number, respectively.In terms of the character and value performances of mathematical reasoning ability, 56 % of the survey objects recognized the item "develop a habit of thinking that emphasizes arguments and logic", and only 6 people recognized the content of "form a scientific attitude and a rational spirit of seeking truth from facts".

Therefore, it can be seen that the current pre-service mathematics teachers have a deep and comprehensive understanding of the meaning of mathematical reasoning ability, while the two aspects of junior high school mathematical reasoning ability are relatively simple and incomplete. On the whole, the pre-service mathematics teachers' cognition of mathematical reasoning ability in junior high school is relatively shallow and not deep enough.

5.3.Cognitive Clarity

In this study, according to the completeness and accuracy of the survey objects' expressions, the cognitive clarity is judged, and the percentage of people with high and low clarity in the number of people who recognize each item is calculated respectively, as well as the percentage of people with high cognitive clarity in the total number of people. The details are shown in Table 3.

Table 3: Statistics of Cognitive Clarity Results

Category	Label	Number of people recognized	Number of people with clear cognition	Percentage of cognitive clarity (high degree: low degree)	Percentage
A Meaning	A1	14	2	14.29%:85.71%	8%
	A2	15	7	46.67%:53.33%	28%
	A3	13	3	23.08%:76.92%	12%
B Key Ability Performances	B1	10	2	20.00%:80.00%	8%
	B2	13	7	53.85%:46.15%	28%
	B3	14	3	21.43%:78.57%	12%
	B4	0	0	0.00%:100.00%	0%
	B5	12	4	33.33%:66.67%	16%
	B6	5	4	80.00%:20.00%	16%
	B7	4	1	25.00%:75.00%	4%
C Character and Value Performances	C1	14	6	42.86%:57.14%	24%
	C2	6	1	16.67%:83.33%	4%

From Table 3, it can be seen that the clearest cognition of survey objects is the two items of “according to the rules” in the meaning and “master the basic forms and rules of reasoning” in the performances, accounting for 28% of the total number.The survey objects had the lowest cognitive clarity on “understand the structure and connection of propositions”, “form the habit of logical expression and communication”, and “form a scientific attitude and rational spirit of seeking truth from facts”, accounting for 0%, 4% and 4% of the total number, respectively.

From the perspective of the clarity of the survey objects recognized by each item, there is no 100 % clarity in the 12 items. Most people only know these items but lack completeness and accuracy in their expression.

Therefore, it can be seen that the current pre-service mathematics teachers’ cognitive clarity is low, and their expression of mathematical reasoning ability is vague.

6. Discussion

6.1. Cognitive Breadth

According to the above data analysis, it can be seen that the current pre-service mathematics teachers do not have a wide range of cognition about junior high school mathematical reasoning ability, and more than half of them do not recognize half of the content of mathematical reasoning ability. In terms of specific content, the cognitive degree of the stronger generality of mathematical reasoning ability is higher, such as its meaning, while the cognitive degree of key ability performances is lower.From this, we can see that the current pre-service mathematics teachers do not have a wide range of cognition of mathematical reasoning ability in junior high school. Regarding this question, when studying the influencing factors of junior high school students’ mathematical reasoning ability, Jia finds that some junior high school mathematics teachers are not clear about the meaning and classification of mathematical reasoning ability, teachers’ understanding of mathematical reasoning ability is not in place [18]. Thus hypothesis 1 is confirmed.

6.2. Cognitive Depth

According to the above data analysis, it can be seen that the current pre-service mathematics teachers have a deep and comprehensive understanding of the meaning of mathematical reasoning ability, while the two aspects of mathematical reasoning ability are relatively simple and incomplete. On the whole, the pre-service mathematics teachers’ cognition of junior high school mathematical reasoning ability is relatively shallow and not deep enough. From this, we can see that the current pre-service mathematics teachers’ cognition is not deep. Regarding this issue, Wang SL finds that junior high school teachers can realize the importance of mathematical reasoning ability, but their understanding of mathematical reasoning ability only stays in its meaning part, and their understanding of its performance is simple [5]. Thus hypothesis 2 is confirmed.

6.3. Cognitive Clarity

According to the above data analysis, it can be seen that current pre-service

mathematics teachers' cognitive clarity is low, and their expression of junior high school mathematical reasoning ability is vague. From this, we can see that the current pre-service mathematics teachers' cognitive clarity is low.Regarding this question, Song interviews junior high school mathematical teachers and finds that although they have an understanding of mathematical reasoning ability, they can not accurately express the meaning and performances of mathematical reasoning ability [19]. Thus hypothesis 3 is confirmed.

7. Conclusion

In this study, through the investigation and analysis of the cognitive degree of junior high school mathematical reasoning ability of 25 pre-service mathematics teachers, it is found that the current pre-service mathematics teachers: 1. The cognitive breadth of junior high school mathematical reasoning ability is not wide, more than half of the people do not recognize half of the content, and the understanding of mathematical reasoning ability mainly focuses on the content with strong generality; 2. The cognitive depth of junior high school mathematical reasoning ability is generally low, the understanding of the meaning of mathematical reasoning ability is relatively deep, and the cognition of the performances of mathematical reasoning ability in junior high school is relatively simple. 3. The cognitive clarity of junior high school mathematical reasoning ability is low, and the expression is relatively vague.

Based on the above conclusions, it is suggested that teachers and experts responsible for educating teachers should pay more attention to junior high school mathematical reasoning ability and strengthen the training of pre-service mathematics teachers by adding relevant courses. Pre-service mathematics teachers themselves should take the initiative to study and research, improve their understanding of junior high school mathematical reasoning ability, carefully study the *Curriculum Standards (2022 Edition)*, fully understand the relevant content of mathematical reasoning ability, and clarify the relevant statements.

References

1. Yan Q, Yu P. A survey on the current situation of logical reasoning ability of junior high school students. Journal of Mathematics Education. 2021;30(01):49-53+78.
2. Ministry of Education of the People's Republic of China. Compulsory education mathematics curriculum standards (2022 Edition). Beijing:Beijing Normal University Press; 2022.
3. Liu HN. A survey on the reasoning ability of grade 9 students.Master's thesis, Minnan Normal University, Zhangzhou; 2023.
4. Zhou XB. Investigation on the development of logical reasoning of junior high school students based on quality monitoring. Journal of Mathematics Education. 2017;26(01):16-18.
5. Wang SL. Research on the current situation and influencing factors of junior high school students' mathematical logical reasoning ability. Master's thesis, Tianjin

Normal University, Tianjin; 2020.
6. Wang SN. Investigation and research on mathematical logical reasoning ability of junior high school students. Master's thesis, Yangzhou University, Yangzhou; 2022.
7. Xiong CH. Investigation and research on the Situation of Mathematical Logical Reasoning Ability of Grade 8 Students. Master's thesis, Yangzhou University, Yangzhou; 2023.
8. Lan J. A survey of eighth-grade students' mathematical reasoning ability and its influencing factors. Master's thesis, Gannan Normal University, Gannan; 2020.
9. Han Q. Investigation on the current situation of mathematics inductive reasoning ability of grade 9 students. Master's thesis, Northwest Normal University, Lanzhou; 2022.
10. Wang XF. Mathematical experiment and logical reasoning. Mathematical Bulletin. 2021;60(03):13-17.
11. Du XF, Zhao JF, Zhou D. A case analysis of junior high school mathematics teachers' belief in reasoning teaching. Teaching & Administration. 2020;(21):23-26.
12. George Polya. Mathematics and conjecture. Beijing:Beijing Normal University Press; 1984.
13. Ministry of Education of the People's Republic of China. Compulsory education mathematics curriculum standards (2011 Edition). Beijing:Beijing Science Press; 2012.
14. Wu H. Reasoning performance: elements, levels, and evaluation indicators. Educational Research and Experiment. 2014;(01):47-51.
15. Cao PY. Crossing the fault and getting out of the misunderstanding: the seventh practical interpretation of the core words of "mathematics curriculum standard" -reasoning ability (I). Primary School Mathematics Teacher. 2014;(Z1):87-94.
16. Ministry of Education of the People's Republic of China. General high school mathematics curriculum standards (2017 edition). Beijing: People's Education Press; 2018.
17. Yu P. "Compulsory Education Mathematics Curriculum Standards (2022 Edition)" academic quality interpretation and teaching thinking. Curriculum, Teaching Material and Method. 2023;43(01):123-130.
18. Jia YY. Investigation on the current situation of junior high school students' mathematical reasoning ability. Master's thesis, Northwest Normal University, Lanzhou; 2023.
19. Song BS. Investigation on mathematical reasoning ability of junior high school students. Master's thesis, Ludong University, Yantai; 2023.