

# Research on Preservice Mathematics Teachers' Cognitive Degree of Space Concept Literacy in Junior High Schools in China

## ABSTRACT

At present, the research on space concept literacy in junior high school is a hot topic in the field of education in China. Many scholars have conducted research from different directions, but there is still a gap in the current research on preservice mathematics teachers' cognitive degree of space concept literacy. Therefore, this study used a semi-structured interview method to investigate 10 undergraduates and 10 graduates in a normal university in China to obtain their understanding of the contents related to space concept literacy. The results show that: 1. The cognitive breadth of space concept literacy is high, and preservice mathematics teachers can cognize all the contents, but not everyone can do this. 2. The cognitive clarity of space concept literacy is relatively high. Most preservice teachers have a clear understanding of the cognitive contents, but the cognition of space concept literacy's value is relatively vague. 3. The cognitive accuracy of space concept literacy is not high, and only a few preservice teachers can accurately master all the contents, among which the connotation is better, while the cognitive accuracy of the cultivation goals and value is low. 4. The common implementation focus is the following three aspects: preparation of space concept literacy, cultivation of space concept literacy, and intensification of space concept literacy. 5. The cognition of the implementation focus is rational, but there are also shortcomings.

**Keywords:** Space concept literacy; Preservice teachers; Cognitive degree.

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## 1. INTRODUCTION

The *Mathematics Curriculum Standards for Compulsory Education (2022 Edition)* (Hereinafter referred to as the *Standards*) states that space concept literacy is one of the main manifestations of mathematicscore literacy in compulsory education, which mainly refers to the cognition of shape, size, and positional relationship of spatial objects or graphs [1]. Space concept literacy helps students to understand the relevant knowledge of space and graphs, and it lays an empirical foundation for the formation of spatial imagination. It is closely related to the development of geometric intuition and reasoning ability, and is the basis for the development of intuitive imaginative literacy in senior high school [2], so its educational value has attracted the attention of many scholars. However, existing studies show

that some dimensions of space concept literacy of students do not develop well, and students' space concept level is constrained by teachers' mastery and implementation of space concept literacy [3]. How to develop students' space concept literacy through improving teachers' literacy urgently needs to be addressed. But at present, there is a lack of research on teachers, especially on preservice mathematics teachers' cognition. Thus this paper attempts to address this issue.

## 2. LITERATURE REVIEW

A number of scholars have studied space concept literacy in junior high school in the past two decades, and by combing through the literature, it is found that the main research directions are as follows: dimensional division of space concept literacy, current development status of junior high school students, influencing factors, cultivating ways and curriculum resources development.

### 2.1 Dimensional Division

Scholars generally categorized space concept literacy into 4-6 dimensions, with the view dimension and the graphs' motion and change dimension being common dimensions [4-17]. Some studies added the objects' position relation dimension [5,7,13-15]. A small number of studies added the projection dimension [5,7,11], the graphing dimension [5,12,14], the decomposing complex graphs dimension [12,13], or the intuitive reasoning dimension [5,14]. Yan Miao and Zou Jiaye further extracted the two dimensions of the ability to recognize graphs and the ability of spatial imagination [4,16], and Wang Wenli added the dimensions of the two-dimensional and three-dimensional conversion, and the dynamic and static conversion [9].

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### 2.2 Current Development Status of Junior High School Students

Numerous studies have shown that junior high school students have a certain degree of space concept literacy, but different dimensions have different performances. Yan Miao believed that students' performance in recognizing graphs ability was poor [4]. Yu Xiaohui considered that students' view, graphing, and intuitive reasoning skills were weak [5]. Shi Jian and Gu Jiling found that students performed better in projection and graphs' motion transformation than in view and object orientation [7]. According to Wang Wenli's investigation, students' view mapping ability and dimension conversion ability developed better and faster [9]. Zhang Dongmei concluded that students performed best in the graphs' motion and change dimension and the graphs' folding and expanding dimension, and worst in the reasoning dimension [12]. Zhao Zhixiong thought that the view and graphing dimensions, which examined students' observation skills, and the reasoning dimension, which examined students' analytical skills, performed poorly [14]. Zou Jiaye believed that students were unable to

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synthetically analyze complex graphs and their dynamic transformations [16].

### 2.3 Influencing Factors

The factors affecting the development of junior high school students' space concept literacy mentioned by scholars in their research were mainly gender, age, and school type. Through investigation, Shi Jian and GuJiling obtained that space concept ability of students had a significant difference in school type, but no significant difference in gender. In addition, their ability had a significant positive correlation with math achievement [7]. The findings of Huang Jingshu and He Xiaoya showed that there was a significant difference in space concept level of students in key schools and non-key schools, and there was no significant difference between boys and girls, nor was there a significant difference between first-year students and second-year students. [8]. Wang Wenli also investigated and found that there was no significant difference between students' space concept levels in terms of gender [9].

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### 2.4 Cultivating Ways

The research on the cultivating ways mainly focused on the design of mathematics classes. Taking the lesson "Three Views" as an example, Wu Sanyu believed that to better cultivate space concept, classroom transformation was needed. When displaying pictures of three views, it should be changed from teachers' display to students' display, and when doing examples and exercises, it should be changed from teachers' lecture to students' attempts [18]. Wang Wei designed puzzles, origami, and modeling activities to develop students' space concept literacy in the lesson "Three-dimensional Graphs and Plane Graphs" [19]. ShenYiqun and Wang Xiaofeng used the teaching of "Motion of Graphs" as an example to illustrate the important role of mathematical experimental activities in cultivating students' spatial imagination and space concept literacy [20]. XueXingying explored the role of interactive courseware in the development of students' space concept literacy by taking the lesson "Expanding and Folding a Square" as an example, and its animation demonstration, hands-on manipulation, and same-screen display functions could effectively promote the development of space concept literacy [21]. Sun Kai deemed that the teaching of "Rich Graphic World" should enable students to experience the abstraction from physical objects to geometric models, and then from geometric models to geometric figures, to develop space concept literacy [22]. Zhang Lin pointed out that in the teaching of "Congruent Triangles", it was necessary to design diversified origami activities to accumulate students' hands-on experience, strengthen their understanding of congruent triangles in complex graphs, and develop space concept literacy [23]. Liu Xiaohong analyzed that the implementation of cultivation requirements of space concept literacy required teachers to flexibly use the textbook, the Geometer's Sketchpad, and the curriculum resources in life [24].

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## 2.5 Curriculum Resources Development

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A few scholars developed curriculum resources for space concept literacy. They actively explored innovative teaching activities that can develop students' space concept literacy and verified the effectiveness of teaching activities by conducting experiments. Yang Minyi integrated Geogebra into mathematics teaching, constructed a teaching strategies model, and found that Geogebra-based geometry teaching could effectively develop students' space concept literacy, with a greater improvement in each dimension [15]. Chang Linghuan designed a teaching activity framework based on microteaching to conduct microteaching experiments, and then tested the space concept level of the students in experimental class and control class respectively, and found that the students' space concept level were significantly improved after microteaching [25]. Zou Jiaye developed an origami extension course, which focused on the five aspects of understanding patterns, memorizing steps, restoring steps, studying the mathematical value of works, and completing origami independently. After the course, space concept literacy of most students has been improved to a certain extent compared with that before the extension course [16].

It can be seen that the research on the meaning of space concept literacy and the cultivation of junior high school students' space concept literacy has been more mature, but few scholars have conducted research on the teachers' mastery of space concept literacy, especially the cognitive degree of preservice mathematics teachers. Meanwhile, the cognitive level of teachers profoundly influences the core literacy level of students, therefore, this paper aims to investigate how current preservice mathematics teachers perceive space concept literacy in junior high school and to put forward corresponding suggestions for the training of preservice mathematics teachers.

In order to get a comprehensive understanding of preservice mathematics teachers' knowledge and mastery of space concept literacy and their ideas on how to implement space concept literacy, the research questions identified in this paper are:

1. How about current preservice mathematics teachers' cognitive breadth of space concept literacy?
2. How about current preservice mathematics teachers' cognitive clarity of space concept literacy?
3. How about current preservice mathematics teachers' cognitive accuracy of space concept literacy?
4. What do preservice mathematics teachers believe is the focus of implementing space concept literacy?
5. Whether preservice mathematics teachers' cognition of the implementation focus is rational?

### 3. THEORETICAL BASIS

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Before starting the research, it is necessary to define the concept of space concept literacy.

Cao Caihan defines space concept as the ability to extract spatial graphs from physical objects, reflect physical objects from spatial graphs, decompose simple and basic graphs from complex graphs, find out basic elements and their relationships from basic graphs, and make or draw graphs from words or symbols [26].

Wang Linqun also defines space concept as an ability: space concept is a comprehensive ability containing the ability to cognize and understand graphs, the ability to decompose and combine graphs, the ability to construct and explore graphs, the ability to appreciate graphs' movements and transformations, and the ability to solve problems using geometric intuition [27].

The *Dictionary of Practical Education*, edited by Wang Huanxun, states that space concept is a representation of shape and size of objects and their mutual positional relations (orientation and distance) formed based on space perception [28].

Han et al. add to the above definition that the representation includes the generalized geometric image of an object left in the human brain, the synergy of thinking and external manipulative skills, figurative thinking accompanied by abstract thinking, and abstract thinking accompanied by figurative thinking [29].

Sun et al. believe that space concept is the understanding and grasping of the interrelationship between space and plane based on the direct perception of the surrounding environment, and it is a kind of ability of students to "blur" the boundaries between two-dimensional and three-dimensional space actively, consciously, or automatically [30].

Bao and Zhang suggest that space concept is an individual's understanding of space and graphs, as well as their concepts and relationships. In particular, space concept in junior high school is more integrated into geometric reasoning. The specific manifestations are as follows. (1) Be able to describe the shape, position, and size characteristics of objects, abstract geometric figures, and form representations of the actual objects based on the geometric figures. (2) Be able to perceive, visualize, and express the spatial orientation of objects and their positional relationships to each other. (3) Be able to perceive and describe the patterns of motion and change of graphs, and use them to solve related problems [2].

On the basis of a comprehensive overview of the above statement, the *Standards* propose that space concept is one of the main manifestations of the mathematics literacy in compulsory education, and that space concept literacy mainly refers to the cognition of shape, size, and positional relationship of spatial objects or graphs. Be able to abstract geometric figures according to the characteristics of objects, and imagine the actual objects described

according to geometric figures. Be able to imagine and express the spatial orientation of objects and their position relations with each other. Be able to sense and describe the graphs' motion and change. Space concept literacy helps to understand the form and structure of real-life spatial objects and is the empirical basis for the formation of spatial imagination [1]. This is the most authoritative and popular explanation of space concept in China.

## 4. RESEARCH METHODS

### 4.1 Instrument

In order to get results closer to reality, this study used a qualitative research method, semi-structured interview method, to conduct the investigation. The interview outline was compiled around the concept and implementation of space concept literacy, which was divided into the following two types of questions:

Question 1:

(1) Please talk about what space concept literacy is.

(2) Please talk about what are the goals of junior high school mathematics for the development of space concept literacy for students.

(3) What do you think is the value of space concept literacy for junior high school students' future development?

Question 2:

(1) How do you think space concept literacy can be implemented in general junior high school teaching (including pre-school preparation, classroom teaching, after-school activities, etc.)?

(2) Can you elaborate on how space concept literacy should be implemented in classroom teaching?

### 4.2 Participants

Since undergraduates majoring in Mathematics and Applied Mathematics (Teacher Training) and graduates majoring in Subject Teaching (Mathematics) will form the vast majority of mathematics teacher population in elementary and high schools, these two categories of students are defined as preservice mathematics teachers. This study randomly selected 10 senior students and 10 first-year graduate students from the School of Mathematics and Statistics of Shandong Normal University as the research objects. All of them hold senior high school mathematics teacher qualification certificates, but through communication, they all indicated that they would also consider teaching in junior high schools, so they have certain representativeness and can be used as research objects.

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### 4.3 Data Collection

After coordinating the interview time, the semi-structured interview method was used to interview the research subjects and the whole process was recorded after obtaining consent. For those who were unable to conduct offline interviews, QQ voice telephone was used for telephone interviews, and the whole process was recorded after obtaining consent.

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### 4.4 Data Processing

Removing meaningless ums and ahs and other intonations, the contents of all recorded interviews were converted into text, and organized strictly according to the original responses. In order to better analyze the contents of interview material, each sentence was coded, and the numbering contained the following meanings: interviewee's grade level - interviewee's number - question number - sentence number of the question, e.g., No. G-1-1-1 indicated the first sentence of the first question answered by the first interviewed graduate student, and No. U-2-2-2 indicated the second sentence of the second question answered by the second interviewed undergraduate student.

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A, B, and C are used to represent the three indicators of space concept literacy connotation, cultivation goals, and value. Secondly, 1, 2, and 3 are used to represent the contents of different entries. Thus, there are nine items, namely, A1, A2, A3, B1, B2, B3, B4, C1, and C2, and the specific coding items are shown in Table 1.

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**Table 1: Question 1 coding table**

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Indicator	Label	contents
A	A1	Space concept literacy is the cognition of spatial objects or graphs
	A2	Space concept literacy studies shape, size characteristics
	A3	Space concept literacy studies positional relation
B	B1	Be able to abstract geometric figures according to object features
	B2	Be able to visualize the actual objects based on geometric figures
	B3	Be able to imagine and express the spatial orientation of objects and their positional relations with each other
	B4	Be able to perceive and describe the motion and change of graphs
C	C1	It is helpful to understand the morphology and structure of spatial objects in real life
	C2	It lays the foundation for the formation of spatial imagination

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## 4.5 Data Analysis

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For research questions 1-3, the interview material was compared with the question 1 coding table item by item. We counted the number of people who mentioned each point and the points that were mentioned of each indicator, and then the percentage was calculated to obtain the cognitive breadth.

Next, we analyzed the people who can cognize a certain point. The cognition degree was determined based on the completeness and accuracy of the interview material. Cognitive clarity was obtained by calculating the percentage of people with high or low levels of cognition to the total number of people after an item-by-item comparison.

Further, those who could correctly express all the contents of the three indicators were defined as preservice mathematics teachers with accurate cognition, and the percentage of this group to the total number of people was calculated to obtain cognitive accuracy.

For research question 4, the data was analyzed according to a three-tier coding procedure to obtain the common implementation focus of preservice mathematics teachers. The interviews for question 2(1) were analyzed as shown in Table 2. Firstly, open coding was used to enter the coded sentences into a newly created Excel database to obtain concept clusters consisting of 82 initial concepts, and 19 primary categories were obtained by merging concept clusters with the same attributes. Secondly, in the spindle coding stage, further merge the primary categories and discover the interrelationships among these primary categories to get 7 thematic categories. Finally, selective coding was used to think deeply about the relationships among the thematic categories, and then identify three core categories to summarize the implementation focus for developing space concept literacy.

**Table 2: Question 2(1) coding table**

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Open coding	Spindle coding	Selective coding
Guide students to make their own geometric models	Before class: Students engage in making, observing, and collecting activities	Space concept preparation
Guide students to observe graphs and geometry in life		
Guide students to gather information for a class		
Conduct space concept level tests		
Do good instructional design	Before class: Teachers prepare teaching procedures, teaching activities, and teaching tools	
Design appropriate teaching activities, such as observation activities, manipulative activities		

Choose appropriate teaching tools, such as physical teaching aids, multimedia courseware, geometry software		
Use material objects to assist in teaching		
Use multimedia to assist in teaching	In class:	
Use pictures or videos to assist in teaching	Use multiple teaching tools	
Use drawing tools to assist in teaching		
Create situations that relate to real-life		
Be good at asking questions to guide		Space concept
Lead students to conduct observation, operation activities	In class: Design a variety of teaching sessions	cultivation
Focus on group teaching		
Conduct student-centered teaching		
Stimulate students' interest in learning	In class:	
Flexibly adjust the teaching contents	Overall control class	
Assign homework that relates to real-life		
Assign tasks related to measurement, observation, or operation activities	After class:	
Test students' space concept level	Consolidate knowledge	Space concept enhancement
Guide students to observe life and dig math problems	After class:	
Conduct extracurricular activities	Conduct deeper expansion	

For research question 5, the research results of question 4 were discussed with the requirements in the *Standards* and the views put forward in authoritative studies to judge the rationality of the cognition of the implementation focus.

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## 5. RESULTS

### 5.1 Cognitive Breadth

As shown in Table 3, it is found that the current preservice mathematics teachers can cognize all the points related to space concept literacy in the *Standards*, and for each point, the number of those who can cognize it is more than half. Among these, "Space concept literacy is the cognition of spatial objects or graphs", "Space concept literacy studies shape,

size characteristics" and "Be able to abstract geometric figures according to object features" are cognized by the most people, with 17 people each able to cognize them, accounting for 85% of the total. "Be able to imagine and express the spatial orientation of objects and their positional relations with each other" and "Be able to perceive and describe the motion and change of graphs" are the least cognized by 11 people, accounting for 55% of the total.

It can be seen that the current preservice mathematics teachers have a wide range of cognition about space concept literacy. Each point of the three indicators can be cognized, but the cognition of the majority focuses on individual points, such as the shape of objects and graphs and the transformation of each other. Many people ignore the positional relation of objects and the movement of graphs.

**Table 3: Cognitive breadth statistics**

Indicator	Label	Number	Percentage	Points	Percentage
A	A1	17	85%	3	100%
	A2	17	85%		
	A3	16	80%		
B	B1	17	85%	4	100%
	B2	12	60%		
	B3	11	55%		
	B4	11	55%		
C	C1	14	70%	2	100%
	C2	13	65%		

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## 5.2 Cognitive Clarity

The cognitive clarity of preservice mathematics teachers who can cognize a certain point is further analyzed, and the results are shown in Table 6. As a whole, people with a high level of cognition per point are more than those with a low level of cognition, and the clarity percentage of all points reaches 72.6%: 27.4%. Among them, the clarity percentage of A2 and B1 is the highest, reaching 94%: 6%, and the label with the lowest clarity percentage is C1, which reaches 50%: 50%.

It can be seen that preservice mathematics teachers have a clear cognition of space concept literacy. The points "Space concept literacy studies shape, size characteristics" and "Be able to abstract geometric figures according to object features" are most clearly cognized, while "It is helpful to understand the form of spatial objects in real life" is the most ambiguous.

**Table 4: Cognitive clarity statistics**

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Indicator	Label	Percentage (High: Low)
A	A1	88%: 12%
	A2	94%: 6%
	A3	81%: 19%
B	B1	94%: 6%
	B2	67%: 33%
	B3	55%: 45%
	B4	55%: 45%
C	C1	50%: 50%
	C2	69%: 31%

### 5.3 Cognitive Accuracy

As shown in Table 5, the number of people who can accurately cognize the three indicators of space concept literacy are 16, 9, and 10 respectively, accounting for 80%, 45%, and 50% of the total, while only 6 people can accurately cognize all three indicators, among which 4 are graduates and 2 are undergraduates.

It can be seen that most people can accurately cognize the connotation of space concept literacy, while only about half of them can accurately cognize the cultivation goals and value, indicating that half of them are not clear or can not understand the contents of space concept literacy in the *Standards*. Even fewer can cognize all three, and undergraduates perform worse.

**Table 5: Cognitive Accuracy Statistics**

Indicator	Number	Percentage
A	16	80%
B	9	45%
C	10	50%
A、B、C	6	30%

### 5.4 Implementation Focus

As can be seen from Table 6, preservice mathematics teachers generally believe that the implementation of space concept literacy in junior high school focuses on three aspects: preparation of space concept literacy, cultivation of space concept literacy, and intensification of space concept literacy. The statistical results are shown in the following table.

**Table 6: Implementation Focus Data Statistics**

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Implementation Focus	Concrete Measures	Number	Percentage
Preparation	Students engage in making, observing, and collecting activities	18	90%
	Teachers prepare teaching procedures, teaching activities, and teaching tools	14	70%
Cultivation	Use multiple teaching tools	15	75%
	Design a variety of teaching sessions	14	70%
Intensification	Overall control class	8	40%
	Consolidate knowledge	16	80%
	Conduct in-depth expansion	8	40%

In terms of preparation of space concept literacy, the vast majority of preservice mathematics teachers believe that implementation should be carried out from two directions of students and teachers, which are mentioned 18 times and 14 times respectively, accounting for 90% and 70%. For students, before class, they need to make preparations related to class contents, such as making their own geometric models or geometric images, observing the geometric properties of objects in real life, searching relevant geometric material, etc., to initially cultivate students' space concept literacy. Teachers can also test students' space concept levels to prepare for classroom teaching. For teachers, it is necessary to design teaching procedures, teaching activities, and teaching tools before class. Interviewee U-10 believed that teachers should set spatial situations closely related to real life, prepare for problems and exploratory tasks, and carefully select examples when designing teaching programs. Interviewee G-2 said: "Teachers should design a variety of teaching activities before class, such as activities for students to operate." Interviewees U-7 and U-9 considered that teachers should select appropriate teaching tools according to the teaching contents before class, such as geometric models, ruler and compass, original objects, multimedia courseware, mathematical geometry software, etc.

In terms of cultivation of space concept literacy, the focus of preservice mathematics teachers can be divided into three categories: using multiple teaching tools, designing a variety of teaching sessions, and overall control class, accounting for 75%, 70%, and 40% respectively. Several interviewees cited the use of various teaching tools: in the teaching of special geometric figures and spatial orientation of objects, physical objects are used to demonstrate; in the teaching of function image and graph motion change, Geogebra, Matlab, Flash and other mathematical software are used to realize dynamic display; in the teaching of

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knowledge closely related to life, such as translation, rotation, projection, use pictures, images, etc., for visual display; the drawing tools should be used to assist in the drawing with ruler and compass. When further explaining the teaching process, most interviewees mentioned the creation of situations related to actual life, teachers' questions to guide thinking, independent exploration of observation images, hands-on operation to draw conclusions, group cooperation, etc. At the same time, many interviewees also paid attention to taking students as the main body in class, mobilizing students' learning interests and exploration desires, flexibly adjusting contents according to students' actual situation, and constantly monitoring students' class status.

In terms of intensification of space concept literacy, the main views of preservice mathematics teachers can be roughly divided into knowledge consolidation and in-depth expansion, and the proportion of these two views is 80% and 40% respectively. In order to consolidate graphics and geometry knowledge and improve space concept literacy, most teachers will assign homework after class, including exercises, observation and measurement tasks, and drawing operation tasks. Interviewee G-4 also said that students' space concept level should be tested after class to help teachers reflect on the effectiveness of their teaching. The interviewee who advocated in-depth expansion believed that students should be further allowed to explore math problems in their own lives or to carry out project-based learning in group cooperation.

### **5.5 Cognitive Rationality of Implementation Focus**

The views of preservice mathematics teachers are basically consistent with the teaching suggestions put forward in the *Standards*. The *Standards* proposes to choose the teaching methods that can trigger students' thinking, pay attention to the teaching methods such as heuristic teaching, exploratory teaching, and interactive teaching, and select the appropriate combination of teaching methods according to the different learning tasks and learning objects, so that students can experience the learning process of practice, exploration, experience, reflection, cooperation, and communication, and promote the development of core literacy. Preservice mathematics teachers also believe that different teaching tools and teaching sessions should be used to guide students to conduct independent thinking, group cooperation, and experimental exploration.

The *Standards* proposes to strengthen situation design and problems posing to promote students' active participation in teaching activities. The preservice mathematics teachers also put forward the problem exploration activities related to the life situation in three aspects: preparation of the space concept, cultivation of the space concept, and intensification of the space concept.

In addition, the *Standards* emphasizes the integration of information technology and

mathematics teaching, such as the use of information technology to comprehensively process text, images, sounds, etc., and the use of mathematical software to carry out mathematical experiments. Similar points have been made by preservice mathematics teachers.

Comparing the implementation focus cognized by preservice mathematics teachers with the five teaching stages in Van Hiele's Theory, it can be found that the preparation of space concept literacy, the cultivation of space concept literacy, and the intensification of space concept literacy are just in line with the teaching sequence of information, guidance orientation, explication, free orientation, and integration, which can promote the development of students' geometric thinking level and further promote the development of space concept literacy.

Bao et al. believe that representation manipulation, measurement activities in real life, and transformation and construction activities of graphs play an important role in the development of students' space concept [2]. Similar views have been put forward by preservice mathematics teachers, who always pay attention to students' observation, measurement, operation, drawing, cooperation, and thinking.

However, preservice mathematics teachers rarely mention the structuration of teaching contents, and they have little understanding of the instructional design of units and lack the ability to capture emerging technologies, such as AR technology. It can be seen that preservice teachers' cognition of the implementation focus is basically rational, but there are also shortcomings.

## 6. DISCUSSION

Through the analysis of interview material, the research results of preservice mathematics teachers' cognitive breadth, clarity, accuracy, implementation focus, and cognitive rationality of the focus are obtained.

In terms of the cognitive breadth, it can be seen from the data that preservice mathematics teachers can cognize all the points of space concept literacy connotation, cultivation goals, and value, and more than half can cognize each point, indicating that teachers have a wide range of cognition. In terms of the specific contents, the cognition degree of the nature and transformation of objects and graphs is high, while the cognition degree of spatial orientation and movement change is low. It can be seen that teachers are not proficient in every point.

In terms of the cognitive clarity, according to the data, nearly 70% of preservice mathematics teachers have a clear understanding of the cognitive contents, so the cognitive clarity is relatively high. In terms of the specific contents, the cognition of "Space concept literacy studies shape, size characteristics" in connotation and "Be able to abstract geometric figures according to object features" in cultivation goals is the clearest, while the cognition of the practical significance of space concept literacy is the vaguest, which shows that the value

cognition is low.

In terms of the cognitive accuracy, according to the data, the number of people who can accurately cognize the three indicators of connotation, cultivation goals, and value only accounts for 30%, indicating that the cognitive accuracy of preservice mathematics teachers is not high. Looking at the three indicators separately, most teachers can accurately cognize the connotation, but less than half of them can accurately cognize the cultivation goals and value, indicating that teachers do not have a good grasp of the latter two indicators. At the same time, the data shows that the performance of postgraduates is slightly better than that of undergraduates, which is probably because postgraduates have received more professional teacher education and have a better understanding of mathematics core literacy.

Regarding the implementation focus, preservice mathematics teachers believe that the focus should be on three aspects, namely preparation, cultivation, and intensification, which correspond to the three periods of time, namely, before, during, and after class. In terms of preparation, Most agree that it should be implemented from both student and teacher entry points. Students should carry out preparatory activities such as making models, observing objects, and collecting information before class, and teachers should do a good job in the preparation of teaching procedures, teaching activities, and teaching tools. In terms of cultivation, most suggest that a variety of teaching tools and teaching sessions should be flexibly utilized, and a few are concerned that the class should be reasonably adjusted according to students' status, and be student-oriented. In terms of intensification, preservice teachers consider that the implementation focus should not only be on consolidating knowledge at the end of the class but also on expanding it further. In addition to assigning homework, students should be allowed to experience more observation, experimentation, and other activities to actively explore mathematical problems related to graphics and geometry.

In view of the cognitive rationality of the implementation focus, the common implementation focus of preservice mathematics teachers is basically consistent with the requirements of the *Standards* and the views of Van Hiele's Theory and Bao Jiansheng et al., but there are also some places that preservice teachers have not noticed, so it can be considered that their cognition of implementation focus is comparatively rational.

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## 7. CONCLUSION

What is the current preservice mathematics teachers' cognitive degree of space concept literacy in junior high school? Through investigation and analysis, this paper draws the following conclusions:

1. The current preservice mathematics teachers' cognitive breadth of space concept literacy is high, and teachers can cognize all the contents, but not everyone can do this.
2. The current preservice mathematics teachers' cognitive clarity of space concept literacy

is relatively high. Most teachers have a clear understanding of the cognitive contents, however, as a whole, teachers' perceptions of the value are more ambiguous.

3. The current preservice mathematics teachers' cognitive accuracy of space concept literacy is not high, and only a few teachers can accurately master all the contents. To be more specific, teachers perform better in the connotation, but the cognitive accuracy of the cultivation goals and value is low.

4. Currently, it is cognized that the focus of implementing space concept literacy lies in three aspects: preparation of space concept literacy, cultivation of space concept literacy, and intensification of space concept literacy, which are embodied in students' pre-class activities, teachers' pre-class preparation, flexible use of various teaching tools and teaching sessions in class, overall control of class, knowledge consolidation and in-depth development after class.

5. The current preservice mathematics teachers have a rational cognition of the implementation focus, but there are also some neglectful places.

Based on the above conclusions, the following suggestions are put forward:

1. In the training of preservice mathematics teachers, colleges and universities should attach importance to the penetration of relevant contents of space concept literacy, add matched training courses, and emphasize the integration of space concept literacy in practical training classes.

2. Preservice mathematics teachers should take the initiative to study and research and practice their teaching skills in school. They should not only master the relevant contents comprehensively, clearly, and accurately, but also truly implement space concept literacy after becoming teachers.

The limitation of this study is the small number of objects selected for the survey. Therefore, the sample scope should be expanded in the future, and preservice mathematics teachers of different grades in different universities should be selected for research. In addition, only qualitative research methods were used in this study, and different research methods can be explored to enrich research conclusions in the future.

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