Parameter Estimated of Seasonal Auto Regressive Integrated Moving Average Model with AR(1) Error Process

ABSTRACT

This research work is focus on parameter estimate of $SARIMA(p,d,q)\times(P,D,Q)^S$ with AR(1)error process, and how auto covariance functions can be used to estimate the variances of error term that characterize the SARIMA model corrupted with AR(1) error process. This was used to developed SARIMA model corrupted with error process and can be used to estimate the true parameter of the SARIMA model. The forecast performance measurement are investigated and properties of errors with different values of parameters. Test of seasonal unit root was carried out. Simulation with R Statistical software were used to prove the finding. In addition, a monthly temperature data of Zamfara State from 1998 to 2020 was used to validate the results using iteration procedure and chi square statistic with Minitab software the studies showed that, the research finding is very significance to the process and would be useful to researchers in the prediction and handling of natural calamities.

Keywords: SARIMA, Parameter estimation, forecast performance, measurement, stationary test.

1. INTRODUCTION

1.1 Seasonal Autoregressive Integrated Moving Average Model (SARIMA)

The SARIMA model is a version of the common ARIMA model which also incorporates a seasonal part. The general SARIMA model can be expressed as Box*et al*. [1]. In the study on error process a point of contention of practical interest to researches is how to describe a relationship when concerned variable are measured with errors.

Ansley [2] worked on finite sample properties of estimators for Autoregressive Moving Average Models.He analyzed by simulation the properties of three estimators frequently used in the analysis of autoregressive moving average time series models for both non seasonal and seasonal data. The estimators considered are exact maximum likelihood, exact least squares and conditional least squares.

Komolafe *et al.,*[3] developed an integrated moving average (IMA) model with a transition matrix for error resulting in a convex combination of two ARMA errors. The basic tools they used are the auto covariance function, maximum likelihood methods, Raphson iterative method and Kolomogorov Smirnov test statistic. The result showed that the proposed model provided a generalization and more flexible specification than the existing models of AR and ARMA errors in fitting time series processes in the presence of error.

Rudelson and Zhou [4] worked with errors in variable models with dependent measurement, analyzedthe convergence rates of the gradient descent methods for solving the no convex programs and show that the composite gradient descent algorithm is guaranteed to converge at a geometric rate to a neighborhood o the global minimizes: the size of the neighborhood is bounded by the statistical error in the ℓ2 norm. The result reveals interesting connections between computational and statistical efficiency and the concentration of measure phenomenon in random matrix theory Provide simulation evidence illuminating the theoretical predictions;

Ayodeji [5] worked on three state Markov-modulated switching model for exchange rate. Examined the long swings hypothesis in exchange rates using a two-state Markov switching model. This study developed a model to investigate long swings hypothesis in currencies which may exhibit ak-state $(k \geq 2)(k \leq 2)$pattern, his model was then applied to euro's, British pounds, Japanese yen, and Nigerian naira. Specification measures such as AIC, BIC, and HIC favored a three-state pattern in Nigerian naira but a two-state one in the other three currencies. For the period January 2004 to May 2016, empirical results show the presence of asymmetric swings in naira and yen and long swings in euros and pounds. In addition, taking 0.5 as the benchmark for smoothing probabilities, choice models provided a clear reading of the cycle in a manner that is consistent with the realities of the movements in corresponding exchange rate series.

Eni [6] worked on parameter estimation of first order IMA model in the presence of ARMA (1, 1) errors using simulation method and showed that the error was uniformly AR(1) correlated. He used auto covariance functions to estimate the variances of the white noises that characterize the IMA (1) models corrupted with ARMA (1, 1) errors. He developed an iteration formula that can be used to estimate the parameters of the IMA (1) models and ARMA (1, 1) errors using simulation studies to demonstrate the findings. The results showed that the method very closely to the estimated true parameters of the process, his work demonstrates the use of autocovariance function in the isolation and measurement of correlated shocks.

Madansky[7], worked on fitting of straight lines when both variables are subject to error**.** Heconsider the situation where X and Y are related by $Y = \alpha + \beta X$, where α and β are unknown and observe X and Y with error, i.e., observed $x = X + u$ and $y = Y + v$. Assume that $Eu = Ev = 0$ and that the errors (u and v) are uncorrelated with the true values (X and Y) he survey and comment on the solutions to the problem of obtaining consistent estimates of

α and β from a sample of (*x, y*)'s, when one makes various economic applications, the estimators are compared in terms of bias, mean squared error, and predictive ability. The reliability of the usually calculated confidence intervals is assessed for the maximum likelihood estimator.
Lindley [8] worked on regression lines and linear functional relationship. Using least square method and maximum likelihood estimation method for fitting a straight line, Y =α + βX. All these methods led to the same result, a quadratic equation which can be solved to give an estimate of β.
Dent and Min [9] worked on a Monte Carlo study of autoregressive integrated moving average processes. Six of the simpler ARMA type models are examined with respect to properties of a variety of proposed estimators of unknown parameters. The results show that only one estimation method was available to a work the choice should probably be maximum likelihood. Stationarity and inevitability restricted estimation would appear appropriate when parameters are thought to be within 5 percent of constraint boundaries.
Schnelweiss and Shalesh[10] worked on the estimation of linear relation, when error variances are known, using maximum likelihood method. The results are linearly correlated.
Eni and Mahmud [11] worked on the parameter estimation of first order IMA model corrupted with AR(1) error using maximum likelihood method, the result showed that error pattern varied between AR and ARMA process within a specified period arising from the varying dynamic process to be observed.

The aim of this research work is to estimate the parameter of SARIMA model corrupted with AR(1) error process. Estimate of the parameters for the formulated model using iteration procedure. To examine the properties of error pattern and variation with different values of the parameters. To test the seasonal, unite root on the data and finally to investigate the forecast performance measures. This research paper tends to add to existing literature on time series model with corrupted error process and its application tend to help researchers and government official in making decision on crucial area under similar studies.

2. MATERIAL AND METHODS

$SARIMA(p,d,q)\times(P,D,Q)^S$ with error process,

$$(1-L)X_t = e_t + (\theta-1)e_{t-1} \quad (1)$$

$$(1-\emptyset L)(1-\emptyset_s L)(1-L)X_t = e_t + (\theta-1)e_{t-1} \quad (2)$$

$$Z_t = (1+\emptyset_s L)(1+\emptyset L)e_t + (1-L) \quad (3)$$

Suppose the error b_t is a Markov modulated mixture of AR (1) and b_t is AR (1) correlated.

$$b_t = \frac{e_t}{1-\alpha_1 L} \quad (4)$$

2.1 Stationarity

In this research work, stationarity refers to weak-stationarity. A time series is said to be weakly stationary if the following are true:

i. $E(y_t) = \mu$ for all t,
(5)

ii. $\gamma_{t,t-k} = \gamma_{0,k}$ for all time t and lag k,
(6)

where μ is the mean and γ_k the auto covariance at lag k (Cryer and Chan [11]).

When testing for stationarity, the alternative (or null hypothesis, depending on the test), is that the series has a unit root. If the series has a unit root it is non-stationary. A unit root process can be described in the following way considering an ARMA process

$$(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)y_t = (1+\theta_1 B+\theta_2 B^2+\cdots+\theta_q B^q)\epsilon_t \quad (7)$$

Where the moving average polynomial is invertible. The autoregressive polynomial in equation (7) is then factored as:
$(1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p) = (1 - \lambda_1 B)(1 - \lambda_2 B)\cdots(1 - \lambda_p B)$(8)

The process has a unit root if any of the Eigen values λ lies outside of the unit circle. By testing both the null hypothesis of a unit root and null hypothesis of stationarity, one can differentiate between series that are stationary, series that have a unit root and series where the data are not informative enough to determine if the series is stationary or integrated (Kwiatkowski *et al.*, [13])

2.2 Hylleberg-Engle-Granger-Yoo(HEGY) Test

The Hylleberg-Engle-Granger-Yoo test (HEGY-test) was proposed by Hylleberg *et al*., [14] to test for seasonal unit roots on quarterly data. Factorizing.
the quarterly seasonal difference operator as:

$$\Delta_4 = (1 - B^4) = (1 - L)(1 + L)(1 + iL)(1 - iL), \qquad (9)$$

shows that it will have four unit roots on the unit circle: $1, -1 and + i,$ where 1 is non seasonal. The HEGY test uses the following auxiliary regression to test for the unit roots:

$$\psi(\mathrm{L})\, \mathrm{y}_{4,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_3 y_{4,t-1} + \mu_t + \varepsilon_t \qquad (10)$$

where

$y_{1,t} = (1 + L + L^2 + L^3) y_t,$

$y_{2,t} = -(1 - L + L^2 - L^3) y_t$

$y_{3,t} = -(1 - L^2) y_t,$

$y_{4,t} = (1 - L^4) y_t,$

$y_t =$ Deterministic components which can be an intercept, seasonal dummies and/or trend,

$\psi =$ is a polynomial of L.

2.3 Forecast Performance Measures

The forecast performance measures or forecast performance metrics.

Now, in applying a particular model to some real or simulated time series to generate forecasts, we first divided the raw data into two parts:

Y_t: is the actual value

$\hat{Y}_t$: is the forecasted value

$\varepsilon_t = Y_t - \hat{Y}_t$: is the forecast error

n: is the size of the test set

: is the test variation

2.3.1 The Mean Forecast Error (MFE)

This measure is defined as:

$$MFE = \frac{1}{n}\sum_{t=1}^{n} \varepsilon_t \qquad (11)$$

2.3.2 The Mean Absolute Error (MAE)

The mean absolute error is defined as:

$$MAE = \frac{1}{n}\sum_{t=1}^{n} |\varepsilon_t| \qquad (12)$$

2.3.3 The Mean Absolute Percentage Error (MAPE)

This measure is given by

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} \left|\frac{\varepsilon_t}{Y_t}\right| \times 100 \qquad (13)$$

2.3.4 The Mean Percentage Error (MPE)

It is defined as:

$$MPE = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{\varepsilon_t}{Y_t}\right) \times 100 \quad (14)$$

2.3.5 The Mean Squared Error (MSE)

Mathematical definition of this measure is:

$$MSE = \frac{1}{n}\sum_{t=1}^{n} {\varepsilon_t}^2 \quad (15)$$

2.3.6 The Sum of Squared Error (SSE)

It is mathematically defined as:

$$SSE = \frac{1}{n}\sum_{t=1}^{n} {\varepsilon_t}^2 \quad (16)$$

2.3.7 The Signed Mean Squared Error (SMSE)

This measure is defined as:

$$SMSE = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{\varepsilon_t}{|\varepsilon_t|}\right) {\varepsilon_t}^2 \quad (17)$$

2.3.8 The Root Mean Squared Error (RMSE)

Mathematically,

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} {\varepsilon_t}^2} \quad (18)$$

2.3.9 The Normalized Mean Squared Error (NMSE)

This measure is defined as:

$$NMSE = \frac{MSE}{\delta^2} = \frac{1}{\delta^2 n}\sum_{t=1}^{n} {\varepsilon_t}^2 \quad (19)$$

The Theil's U-statistics this important measure is defined as:

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n} {\varepsilon_t}^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n} {\hat{Y}_t}^2}\sqrt{\frac{1}{n}\sum_{t=1}^{n} {Y_t}^2}} \quad (20)$$

3. RESULTS AND DISCUSSION

Consider the SARIMA model $(1-\mathrm{L})(1-\mathrm{L}^{12})\,\mathrm{y}_t = (1-\theta L)(1-\theta L^{12})\varepsilon_t - (1)$ where y_t is an output variable ε_t is a white noise with a constant mean of 0 and variance of σ^2, $\theta\, and\, \theta$ are weight parameter (Box and Jenkins [15]).

L is an operator with can be forward or backward. Most of time y_t may be necessary obtain by transformation as $y_t = x_t - b_t, \phi = 1-\theta\, and\, \lambda = (1-\theta)$

Substituting $y_t = x_t - b_t$ in to equation (1) where b_t is an error component introduce by faulty measurement or observation

$$(1-L)(1-L^{12})(x_t - b_t) = (1-\phi L)(1-\lambda\,\mathrm{L}^{12})\varepsilon_t \quad (22)$$

$$(1-L)(1-L^{12})x_t = (1-\phi L)(1-\lambda\,\mathrm{L}^{12})\varepsilon_t - (1-L)(1-L^{12})b_t \quad (23)$$

$$\text{Let}\ \omega_t = (1-L)(1-L^{12})x_t$$

Equation (27) become

$$\omega_t = (1-\phi L)(1-\lambda L^{12})\varepsilon_t + (1-L)(1-L^{12})b_t \quad (24)$$

$$Since\, b_t\ is\ AR(1)$$

$$b_t = \frac{e_t}{(1-\alpha L)} \quad (Hamilton\ [16]) \quad (25)$$

and substituting into equation (24) we have

$$\omega_t = \alpha\omega_{t-1} + \varepsilon_t + (\phi\alpha - \alpha - \phi)\,\varepsilon_t - \lambda\varepsilon_t - 12 + (\phi\lambda + \lambda\alpha - \phi\alpha\lambda)\varepsilon_t - 13 + e_t - e_{t-1} - e_{t-12} + e_{1-13} \qquad (26)$$

$$Let\ z_t = \omega_t - \alpha\omega_{t-1}\ and\ \beta_1 = (\phi\alpha - \alpha - \phi), \beta_2 = (\phi\lambda + \lambda\alpha - \phi\alpha\lambda)$$

$$z_t = \varepsilon_t + \beta_1\varepsilon_{\varepsilon-1} - \lambda\varepsilon_t - 12 + \beta_2\varepsilon_{t-13} + e_t - e_{t-1} - e_{t-12} + e_{1-13} \qquad (27)$$

Multiply equation (27) by z_t and take expectation

$$V_0 = \left(1 + \overset{2}{\underset{1}{\beta}} - \lambda^2 + \overset{2}{\underset{2}{\beta}}\right)\sigma_\varepsilon^2 + 4\sigma_\ell^2 \qquad (28)$$

Multiply equation (27) by z_{t-1} and take expectation

$$V_1 = \lambda^2 \overset{2}{\underset{2}{\beta}} \sigma_\varepsilon^2 + 2\sigma_\ell^2 \qquad (29)$$

$$\sigma_e^2 = \frac{V_1 - \lambda^2\beta_2^2\sigma_\varepsilon^2}{2} \qquad (30)$$

$$\sigma_\varepsilon^2 = V_0 - 2V_1 \left|\left(1 - \beta_1^2 - \lambda^2 + \beta_2^2 - 2\lambda^2\beta_2^2\right)\right. \qquad (31)$$

Where e_t is white noise uncorrelated with ε_t

Groping the white Noise we get

$$u_t = \varepsilon_t + e_t$$

$$\Omega_1 u_{t-1} = \beta_1\varepsilon_{t-1} - e_{t-1}$$

$$\Omega_2 u_{t-12} = -\lambda\varepsilon_{t-13} + e_{t-12}$$

$$\Omega_3 u_{t-13} = \beta_2\varepsilon_{t-13} + e_{t-13}$$

$$Z_t = u_t + \Omega\mu_{t-1} - \Omega_2 u_{t-12} + \Omega_3 u_{t-13} \qquad (32)$$

The model Developed in equation (32) was $SARIMA(0,0,1)(0,0,1)^{12}$

Our interest is to estimate x_t through $y_t = x_{t-bt}$ we define the following known facts Hamilton [14] for white noise processes

$$E\left(\varepsilon_t\varepsilon_{t-i}\right) = \begin{cases} \sigma_\varepsilon^2 & for\ \ i=0 \\ 0 & for\ \ i\neq 0 \end{cases}$$

$$E\left(e_t e_{t-i}\right) = \begin{cases} \sigma_\ell^2 & for\ \ i=0 \\ 0 & for\ \ i\neq 0 \end{cases}$$

$$E\left(u_t u_{t-j}\right) = \begin{cases} \sigma_u^2 & for\ \ j=0 \\ 0 & for\ \ j\neq 0 \end{cases}$$

$\varepsilon_t, \mathrm{e}_t$ are uncorrelated where u_t is also a white noise process Moran [17] has shown that if the ratio $\lambda = \frac{\sigma_\varepsilon{}^2\varepsilon_t}{\sigma_e{}^2}$ is known, The maximum likelihood estimation for the parameter set can be found by directly solving likelihood equation Chen and mark [18] obtained the maximum likelihood estimates for the case where both $\sigma^2_{\varepsilon_t} and\, \sigma^2_{e_t}$ are known and where the observations are replicated. Eni *et al.,*[19], have used the same method to isolate errors of AR(1) corrupted with MA(1) process. Eni and Mahmud[11], have considered the case of IMA(1) with white noise in a similar cases, Eni [20], has considered the case of GARCH (1, 1) model with white noise errors using the proposed method.

3.1 The Auto Covariance 0f SARIMA (0,0,1) (0,0,1)

$$\gamma_0 = \left(1+\phi^2\right)\left(1+\lambda^2\right)\sigma^2$$

$$\gamma_1 = \phi\left(1+\lambda\right)\sigma^2$$

$$\gamma_{11} = \phi\lambda\sigma^2$$

$$\gamma_{12} = -\lambda\left(1+\phi^2\right)\sigma^2$$

$$\gamma_{13} = \phi\lambda\sigma^2$$

3.2 The Analysis in R software

Table 1: SARIMA (0,01) $(0,0,1)_{12}$

AR(1)	Estimate	Stand. Error	Z-value	P-value
MA (1)	0.2962	0.0108	27.4259	$4.22712e^{-154}$
SMA(1)	-0.0275	0.0142	-1.9366	0.0537
MEAN	-0.2003	0.0221	-9.0634	$2.147207e^{-21}$

Table 1 reveal the results of parameter estimated of $SARIMA(1,0,0)(0,0,1)^{12}$ corrupted with AR(1) error process and is significance considering the p- values.

Sigma^2 estimated as 0.8464 log likelihood = -6676.31

AIC = 13360.61 AICC = 13360.62 BIC = 13386.68

Table 2: Forecast measurements

	ME	RMSE	MAE	MPE	MAPE	MAE	MASE	ACFI
Training Set	$1.4039e^{-05}$	0.9197	0.7320	127.5667	158	42.47	0.6928	0.00622

Table2 Indicated the results of forecast performance measurement and properties of errors with different values and is significance.

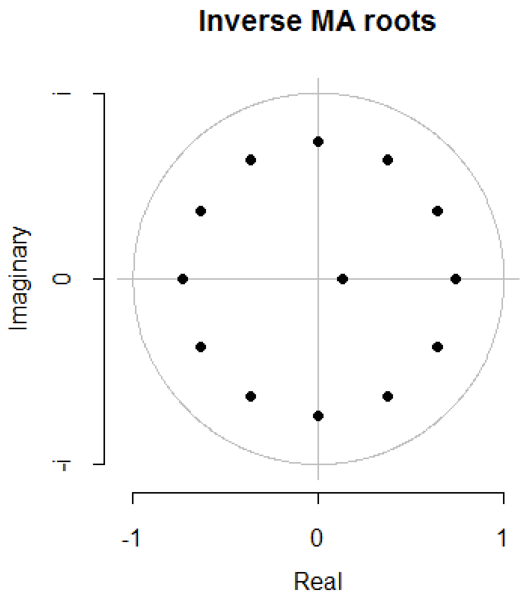

Figure 1: Plot of inverse of MA(1) root

The plot in figure 1 is pictorial of SARIMA model for the period of twelve (12) months with a single MA root i.e. SARIMA(0.0.1)$(0,0,1)^{s=12}$ the twelve circle point represent the seasonal effects while the single inner point represent the MA(1) root

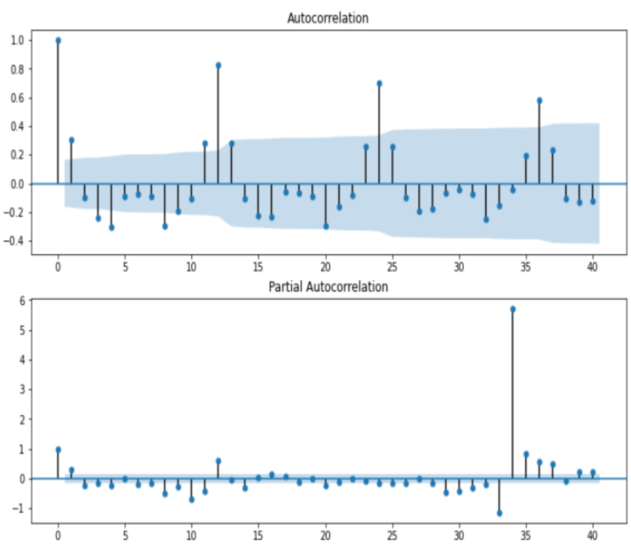

Figure 2: Plot of ACF and PACF s at lag 40

The ACF plot in figure 2 shows the correlation of the series with its lagged values .it describes how present value of the series related with its past and consider seasonal effect with upper confidence interval. PACF plot shows the correlation of the residuals in the series and lags

Table 3: $SARIMAMODEL(001)(001)_{12}$ for Monthly Temperature of Zamfara State from1998 to 2020

Estimates at each iteration

Iteration	SSE	Parameters		
0	13601.0	0.100	0.100	65.774
1	10524.3	-0.050	0.083	65.763
2	8497.8	-0.200	0.057	65.749
3	7133.5	-0.350	0.013	65.728
4	6247.7	-0.500	-0.068	65.696
5	5906.5	-0.605	-0.218	65.648
6	5885.7	-0.572	-0.259	65.658
7	5885.4	-0.577	-0.261	65.661
8	5885.4	-0.575	-0.261	65.662
9	5885.4	-0.576	-0.261	65.662
10	5885.4	-0.576	-0.261	65.662

Relative change in each estimate less than 0.0010

The results in Table 3. Showed the estimate at each iteration for the monthly temperature when the SARIMA model switch to AR (1) error process.

Table. 4: Final Estimates of Parameters

Type	Coef	SE Coef	T	P
MA (1)	-0.5756	0.0691	-8.33	0.000
SMA (12)	-0.2607	0.0824	-3.16	0.002
Constant	65.662	1.070	6135	0.000
Mean	65.662	1.070		

Number of observations: 144

Residuals: SS = 5853.52 (back forecasts excluded)

MS = 41.51 DF = 141

The results in Table 4. Showed the final parameter estimate for SARIMA model corrupted with AR(1) error process.

Table 5: Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	142.9	151.5	165.8	203.9
DF	9	21	33	45
P-Value	0.000	0.000	0.000	0.000

The results in the Table 5. is the Chi-square statistic for the SARIMA model corrupted with AR(1) error process at lag 12, 24, 36 and 48 respectively.

4. CONCLUSION

This research work deal with modification of SARIMA models corrupted with AR(1) error process using Autocovariance function and maximum likelihood to estimate the parameter of the models using R Software. An investigation of forecast performance measurement error on simulated data are obtained. The test for seasonal unit root on data shows that all the unit root fall within unit circle, properties of error pattern and variation with different values of parameter are also investigated his study formulated a credible, flexible and versatile models capable of accounting for errors from different sources and has been able to apply basic tools such as the autocovariance function, maximum likelihood method, and R Software to estimate the parameter of the models that fit the formulated specification to data. Based on the theoretical results and data application, we concluded that the proposed models issignificances and is a generalization of the existing models on MA(1) corrupted with AR(1) error and IMA(1) corrupted with AR(1) error.

5. RECOMMENDATION

Finally, this study discussed SARIMA models corrupted with AR(1) it will certainly enhance research if other version of time series like Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) are considered for volatility. More data will be required for better results.

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