Time Series Intervention Modelling Based on ESM and ARIMA Models: Daily Pakistan Rupee/Nigerian Naira Exchange Rate

Abstract

The battered Nigerian economy has slipped into its second economic recession in five years due to the fallout of the coronavirus pandemic. And this has had a remarkable effect on the value of the Nigerian Naira that are exchanged for a unit of many other currencies of the world. Intervention modelling is used to assess the impact of this external event on the Pakistan Rupee to the Nigerian Naira exchange rates. The dataset for this study is the daily Nigerian Naira exchange rate with respect to the Pakistan Rupee from January–December 2020. The intervention point is marked on April 10, 2020, as a pulse function for the PKR/NGN series. Results revealed that the economic recession due to the fallout of the coronavirus pandemic increased the value of the Nigerian Naira by 3.38% against the Pakistan Rupee, which 1PKR is exchanged for 2.2042NGN compared to the periods before and after the intervention occurred. The intervention was felt at the point of intervention itself but the effect dies immediately after the intervention. Hence, the intervention response is described as an abrupt start and abrupt decay.

Keywords: Time Series Intervention Modelling; ESM; ARIMA; Exchange Rate; Nigerian Naira; Pakistan Rupee.

1 Introduction

Since life must be understood by looking backwards and must be lived by looking forward, time series provides useful tools that help to predict the future by approximating models that use past data. Time series analysis studies the time-structure relationship in a given variable of interest, which involves the use of the basic tools to analyze a given time series variable to; construct a simple mathematical system that explains the time structure relationship in the economic and social series concisely; test hypotheses concerning social and economic details of the series; apply a reliable diagnostic tool to validate the model; use the model to explain the behaviour of the time series and make a reliable prediction for the future based on the dynamic dependence of the series on the past values and set up control mechanism that gives signals when an event goes out of bounds or by examining what might happen when the values of the parameters in the model are changed.

Time series intervention analysis is widely used in areas like finance, economics, public health, transportation, labour markets, and so on. Intervention modelling is used to assess the impact of external events on the time series of interest. Also, it may be undertaken to adjust for any unusual values in the series that might have resulted as a consequence of the special event. Thus,

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this will ensure that the results of the time series analysis of the data, such as the parameter estimates of the model, structure of the fitted model and forecasts of the future values are not distorted by the influence of these unusual values, which the classical ARIMA model cannot handle. Accordingly, there are three types of intervention viz, pulse, step and ramp function. Pulse intervention occurs only at a particular period but the effect of the intervention may exist for that particular period of time only or may exist in the subsequent period, the influence of the 2020 Nigerian economic recession due to COVID-19 which has affected some of the strongest nations of the world is an example of this type of intervention. Step function occurs at a particular period and it exists in the subsequent periods. The intervention effect may remain constant over time or it may increase or decrease over time. While ramp intervention occurs at a particular period and exists in the subsequent period with an increasing magnitude.

This study deals with investigations of time series intervention modelling based on Exponential Smoothing Methods (ESMs) and the ARIMA models aimed at studying the response of the comparative value of the Nigerian Naira with respect to the Pakistan Rupee to the 2020 economic recessions.

2 Review of Literature

Two approaches to intervention modelling are adopted here. First, the Box-Tiao (1975) approach [5] mainly conducted on the Box-Jenkins (1976) ARIMA model [2, 3] which has been widely applied by scholars since its inception. For instance, Deutsch and Alt [8], examined the Effect of Massachusetts' Gun Control Law on Gun–related Crimes in the City of Boston. Sharma and Khare [30], used an intervention analysis model to study the impact of the intervention introduced by the Indian Government to control the pollution caused by vehicular exhaust emissions. Girard [18], applied the ARIMA model with intervention to analyze the epidemiological situation of whooping – cough in England and Wales for the period of 1940 – 1990. Nelson [27], uses an ARIMA intervention analysis to estimate the impact of the Bankruptcy Act of 1978. Lai and Lu [22], use an intervention model to look at the impact of the September 11, 2001 terrorist attack on air transport passenger demand in the USA. Min [24], applied intervention analysis to assess whether two events, the 9 – 21 Earthquake in 1999 and the Severe Acute Respiratory Syndrome (SASS) outbreak in 2003, had a temporary or long–term impact on the inbound tourism demand from Japan. Lam *et al* [23], used a time series intervention ARIMA model to measure the intervention effects and the asymptotic change in the simulation results of the business process reengineering that is based on the activity model analysis. Jarrett and Kyper [21], examined the impact of the world financial crisis (WFC) on the Chinee Stock Price. Darkwah *et al* [7], use intervention time series analysis to assess the nature and impact of the establishment and operations of community policing in communities in Ghana. Mrinmoy *et al* [26], assessed the impact of Bt – Cotton variety on Cotton Yield in India. With step intervention, was the introduction of Bt–Cotton variety in 2002.

Yang [35], used ARIMA with an intervention model to analyse the impact of new product releases on revenue. Etuk and Amadi [9], analyzed the effect of the exit of Great Britain from the European Union on the GBP/USD exchange rate. Etuk and Eleki [11], modelled the daily Yuan – Naira Exchange Rates using ARIMA intervention analysis. Etuk *et al* [13], use the intervention

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analysis approach on the daily exchange rate of Yen/Naira. Etuk and Udoudo [17], use an intervention model to explain the impact of economic recession on the daily Indian Rupee and Nigerian Naira exchange rate. Etuk and Ntagu [16], examined the modelling of the intervention of daily Swiss Franc (CHF) and Nigerian Naira (NGN) exchange rates, which spanned 18th May 2016 to 16th November 2016. Etuk and Chukwukelo [10], modelled daily Moroccan Dirham (MAD) and Nigerian Naira (NGN) using intervention analysis. Shittu and Inyang [31], modelled Nigerian monthly crude oil prices using the ARIMA-Intervention model to compare the result with that of the intervention model using a lag operator. Etuk and George [12], proposed an intervention model for the exchange rate of Malaysian Ringitt (MYR) and the Liberian Dollar (LRD). Etuk *et al* [15], fitted an autoregressive integrated moving average intervention model to daily Brazilian Real (BRL)/Nigerian Naira (NGN) exchange rates. Etuk *et al* [14], investigated the impact of the Declaration of Cooperation (DoC) on Nigerian crude oil production. Moffat and Inyang [25], investigated the impact of the Nigerian government amnesty programme (GAP) on her crude oil production.

Secondly, an intervention model with exponential smoothing methods [6,19,33], has rarely been attempted. Trapero *et al* [34], investigated the accuracy of judgmental forecasting at the SKU level in the presence of promotions, based on weekly data from a manufacturing company using a simple model based on a transfer function combined with Single Exponential Smoothing (SES). Seong and Lee [29], proposed a method of intervention analysis based on exponential smoothing models through an innovational state–space model. Two applications were analyzed: the 9/11 effect on US airlines and the COVID–19 effects on the current population of Seoul, Korea. Jaganathan [20], used ARIMA Linear Transfer Function and Exponential Smoothing (ES) with intervention models to model pandemic data in the context of forecasting demand to explain its impact and build accurate predictive models.

3 Model Specification

The Box-Jenkins ARIMA model has been known as the most widely used technique for modelling and forecasting [3], denoted by ARIMA(p,d,q). But when the time series is affected by external events, the forecasting power of the ARIMA model may fail. Thus, Box-Tiao [5], proposed the intervention modelling (ARIMA-Intervention analysis) given by

$$X_t = G(L) + W_t \qquad (1)$$

Where $G(L)$ is the transfer function component and W_t is noise component

Intervention model with ARIMA

$$X_t = \frac{\omega(L)L^b}{\delta(L)} I_t + \frac{\mathrm{B}(L)}{\mathrm{A}(L)} \epsilon_t \qquad (2)$$

Since $G(L) = \frac{\omega(L)L^b}{\delta(L)} I_t$ and $W_t = \frac{\mathrm{B}(L)}{\mathrm{A}(L)} \epsilon_t$

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Where:

$\mathrm{A}(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_p L^p$ and $\mathrm{B}(L) = 1 + \beta_1 L + \beta_2 L^2 + \cdots + \beta_q L^q$
$\delta(B) = 1 - \delta_1 L - \cdots - \delta_r L^r$ and $\omega(L) = \omega_0 - \omega_1 L - \cdots - \omega_s L^s$

X_t is the response variable at t, b =delay parameter, ω=impact parameter, δ=slope parameter, α =Non-seasonal autoregressive parameter, β =Non-seasonal moving average parameter, ϵ_t =White noise.

W_t is a Box – Jenkins ARIMA(p,d,q) model which represents the baseline daily Pakistan Rupee/Nigerian exchange rate in pre–intervention period.
$I_t = P^T{}_t$, the indicator variable. Mathematically, they are written as

$$P^T{}_t = \begin{cases} 1, & t = T \\ 0, & t \neq T \end{cases} \qquad (3)$$

$P^T{}_t$ is called the pulse function.

Hence, (2) becomes $X_t = \frac{\omega(L)}{\delta(L)} P^T{}_{t-b} + \frac{\mathrm{B}(L)}{\mathrm{A}(L)} \epsilon_t$ (4)

Intervention model with ESM
The underlying model is given by

$$Y_t = \frac{\omega(L)}{\delta(L)} P^T{}_{t-b} + ETS(A, N, N) \qquad (5)$$

The form in (5) is similar to the intervention model with ARIMA models in (2). And with a temporary change of intervention effect with ESM with additive error, no trend, no seasonal component.

Model Validation

Diagnostic test is an important step in time series model building and this consists of scrutinizing a variety of diagnostics to determine whether the selected model is healthy and hence ready to forecast. We consider here;

(a) Plot of the residual ACF
Once an appropriate ARIMA model is fitted, one can examine the goodness of fit by means of plotting the ACF of the residuals of the fitted model. If most of the sample autocorrelations coefficients of the residuals are within the bound of $\pm \frac{2}{\sqrt{T}}$, where T is the series length then the residuals are white noise indicating that the model is a good fit.

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(b) Akaike Information Criterion (AIC)
The AIC [1], is formulated as

$$\text{AIC}= V_T\left[1+\frac{2P}{T-P}\right] \quad (6)$$

Where:
V_T = Index related to production error (known as residual sum of squares)
p = No of parameters in the model, T= No. of data points.

(c) Bayesian Information Criterion (BIC)
The BIC is a criterion for model selection among a finite set of models. Given any two or more estimated models, the model with the lowest value BIC is the one to be preferred. It is given by:

$$\textbf{BIC}= \boldsymbol{n ln\hat{\sigma}_e^2} + \boldsymbol{k ln}(\boldsymbol{n}) \qquad \textbf{(7)}$$

Where $\hat{\sigma}_e^2$ is the estimated error variance defined by
$\hat{\sigma}_e^2 = \frac{1}{T}\sum_i^T (x_i - \bar{x})^2$
x = Observed data, T =Number of observations, k = Number of free parameters to be estimated.

Measuring Forecast Error
The forecasting errors represented by MAE and RMSE, which are Mean Absolute Error and Root Mean Squared Error respectively, are chosen as the performance metric.

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|e_t| \quad (8)$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} {e^2}_t} \qquad (9)$$

Where e_t is residual at time t and T is the total number of the period. If a method fits the past time series data very well, MAE is near zero, whereas if a method fits the past time series data poorly, MAE is large. Therefore, when two or more forecasting methods are compared, the one with the minimum MAE is selected as the most accurate. RMSE is the square root of MSE.

4 Data Description

The data used in this study are the daily Pakistan Rupee/Nigerian Naira (PKR/NGN) exchange rates spanning from January - December 2020, which were obtained from the Central Bank of Nigeria Website. The dataset was divided into observations belonging to pre-intervention (Jan. 1 – April 9, 2020) and post-intervention periods (April 10 – Dec. 2020). The statistical package used for the analysis of this work is the R language (R-4.1.2-win & R-4.1.3-win).

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5 Results and Discussion

The time plot showing the daily Pakistan Rupee to Nigerian Naira exchange rate from January to December 2020 is given in Figure 1 of Appendix 1. The graph of the series rises and falls at random perhaps due to the mechanism that generated the dataset. From the plot, we can observe the lowest exchange rate of 2.2042NGN on Friday 10, April 2020 which was marked as a point of intervention while the highest exchange rate of 1PKR to 2.4635NGN was witnessed on Saturday 26, December 2020 and with an average exchange rate of 2.3503NGN.

Intervention Modelling

The suspected point where the intervention took place on the exchange rate dataset is labelled by indicator functions as:

$$P_t{}^T = \begin{cases} 1, & t = April\ 10,\ 2020 \\ 0, & t \neq April\ 10,\ 2020 \end{cases} \qquad (10)$$

Where: T= April 10, 2020 and $P_t{}^T$ is the Pulse function type.

ARIMA-Intervention Model

Data on the daily Pakistan Rupee to Nigerian Naira exchange rate from January 01, 2020, to April 9, 2020, is used for pre-intervention ARIMA model fitting and data from April 10, 2020 – December 31, 2020 have been used to know the intervention component form.

Pre-intervention time plot shown in Figure 2, the graph of the series exhibits the characteristics of a non-stationary which is confirmed by the unit root test in Table 1 of Appendix 2.

To attain stationarity, the series was differenced once. The graph of the differenced series in Figure 6 indicates that the series is stationary. A unit root test at first difference further confirmed that the series is stationary since the p-value of the Augmented Dickey-Fuller Test is less than the alpha level (p-value=0.01 < 0.05), Table 2. Based on correlogram of the differenced series in Figures 7 and 8, the ARIMA(1,1,1) model is fitted with their statistics summarized in Table 3. The adequacy of ARIMA(1,1,1) model is not in doubt since the coefficients of ACF of the residuals are within the significance bounds of $\pm\frac{2}{\sqrt{100}} = \pm 0.20$, Figure 9. With the least BIC and AIC values of -477.3847 and -485.17 respectively, Table 4. Therefore, the model is statistically significant, appropriate and adequate for the dataset.

Forecasts generated from the ARIMA(1,1,1) model were very close to the actual value of the post-intervention series when compared, Table 5. The forecasting is done to first five post-intervention observations to compute the impulse response function. From the impulse response function in Figure 10, it's seen that b=0, which implies that the effect of the intervention was felt at the point of the intervention itself, April 10, 2020.

The parameter ω with value -0.0344 was significant with a p-value of 0.0000. The negative sign of the intervention parameter indicates that there was a decrement in the exchange rate. That is, the Pakistan Rupee depreciated in favour of the Nigerian naira in the exchange rate market on

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Friday 10 April 2020 at Rs1 PKR=2.2042NGN, compared to the periods before the intervention occurred. Thus, the intervention increased the value of the Nigerian Naira by 3.38%=$\{1-\exp(-0.0344)\}\times 100\%]$ against the Pakistan Rupees in the exchange rate market.
Therefore, the ARIMA-Intervention model is represented mathematically as

$$Y_t = -0.0344 P_t^{\,T} + \frac{(1+.5934B)}{(1-0.2347B)}\varepsilon_t \qquad (11)$$

The model in (11) was found to be statistically significant and adequate for the dataset when diagnosed, Figure 11. This was confirmed by the plot of the fitted ARIMA-Intervention model with the actual values since the fitted values mimic the actual values, Figure 12.

ESM-Intervention Model

The Simple Exponential Smoothing model with additive error is tentatively specified, since there is no presence of trend and seasonality in the PKR/NGN series. The model is labelled ETS(A,N,N), the abbreviation indicating error, trend and seasonal components. The A,N,N between the parentheses indicate Additive error, No trend and No seasonality respectively. The ETS(A,N,N) model with intervention components are estimated in Table 7. The smoothing constant α, controls the flexibility of the level and is estimated as $\alpha = 0.6342$. Corresponding intervention parameters are $\omega = 0.0658$. The estimates are all significant at 5% level. The impact parameter implies a positive change, that is, the intervention causes a rise in the value of the Nigerian Naira against the Pakistan Rupee (1 PKR = 2.2042NGN) compared to the period before the intervention occurred.
The ETS-Intervention model is represented mathematically as

$$Y_t = 0.0658 P_t^{\,T} + ETS(A,N,N) \qquad (12)$$

The model in (12) is statistically significant and adequate, Figure 14. The plot of the fitted ETS-Intervention model with the actual values confirmed that (12) is a good fit, since the fitted values mimic the actual values, Figure 15.

6 Conclusion

The Pakistan Rupee/Nigeran Naira have been considered with a pulse intervention being the introduction of the 2020 economic recession due to the global pandemic.
The intervention effect was never persistent, confirmed by the plot of the total intervention effect in Figure 13. With b=0 implies that the effect of the intervention was felt at the point of intervention, April 10, 2020. Comparing the two models, it is evident that the ARIMA – Intervention model shows a better fit than the ESM – Intervention model, since the estimated error variance, AIC and forecast accuracy measures, MAE and RMSE, of ARIMA – Intervention model are smaller than those of the ESM – Intervention model, Table 8. The fitted plots from the two models confirmed that the models are a good fit since the fitted values mimic the actual values, in Figure 12 and 15. Hence, when forecasting the future values of PKR/NGN using the ARIMA – Intervention model, the forecasted values were very close to the actual values when compared, Table 9.

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7 Recommendations

Exchange rate prediction is one of the most demanding applications of modern time series forecasting. Therefore, reliable forecasts from the developed model would help financial authorities and experts enhance their planning and decision-making processes to improve the value of the Naira.

References

[1] Akaike, H. (1974). A New Look at the Statistical Model Identification. I. E. E. E. Transactions of Automatic Control, AC, 19, 716 – 723.

[2] Box, G. and Jenkins, G. (1970). Time Series Analysis: Forecasting and Control. Holden – Day, San Francisco.

[3] Box, G. E. P. and Jenkins, G. M. (1976). Time Series Analysis: Forecasting and Control, Revised Edition, San Francisco: Holden Day.

[4] Box, G. E. P., Reinsel, G. C., and Jenkins, G. M. (1994). Time Series Analysis: Forecasting and Control. 3rd Ed. Prentice – Hall, England Cliffs, N. J.

[5] Box, G. E. P. and Tiao, G. C. (1975). Intervention Analysis with Application to Economic and Environmental Problems. Journal of American Statistical Association, Vol,. 70, No. 349. Pp. 70-79.

[6] Brown, R. G. (1959). Statistical Forecasting for Inventory Control. McGraw-Hill, New York.

[7] Darkwah, K. F., Okyere, G. A., and Boakye, A. (2012). Intervention Analysis of serious Crimes in the Eastern Region of Ghana. International Journal of Business and Social Research (IJBSR), Vol. 2, No. 7, December 2012.

[8] Deutsch, S. J. and Alt, F. B. (1977). The Effect of Massachusetts' Gun Control Law on Gun –related Crimes in the City of Boston. Evaluation Quarterly, Vol. 1, No. 4, pp. 543 – 568, November 1977.

[9] Etuk, E. H. and Amadi, E. H. (2016). Intervention Analysis of Daily GBP – USA Exchange Rates Occassioned by BREXIT. International Journal of Management, Accounting and Economics, 3(12), 797 – 805.

[10] Etuk, E. H. and Chukwukelo, V. N. (2018). Intervention Analysis of Daily Moroccan Dirham/Nigerian Naira Exchange Rates. International Journal of Management Studies, Business & Entrepreneurship Research. Vol. 3, No. 1, March 2018.

[11] Etuk, H. E. and Eleki, A. G. (2016). Intervention Analysis of Daily Yuan – Naira Exchange Rates.CARD International Journal of Science and Advanced Innovative Research (IJSAIR), Vol. 1, No. 3, December 2016.

UNDER PEER REVIEW

[12] Etuk, E. H. and George, D. S. (2020). Interrupted Time Series Modelling of Daily Malaysian Ringitt MYR/Liberian Dollar LRD Exchange Rates. International Journal of Science and Advance Innovative Research, Vol. 5, No. 2, June 2020.

[13] Etuk, E. H., Dimkpa, M., Sibeate, P., and Onyeka, N. G. (2017). Intervention Analysis of Daily Yen/Naira Exchange Rates. Management and Administrative Sciences Review, Vol. 6, Issue 1., January 2017.

[14] Etuk, E.H.,Inyang, E. J. and Udoudo, U. P. (2022). Impact of Declaration of Cooperation on the Nigerian Crude Oil Production. International Journal of Statistics and Applied Mathematics 2022; 7(2): 165 – 169.

[15] Etuk, E. H., Onyeka, G. N., and Leesie, L. (2021). An Autoregressive Integrated Moving Average Intervention Model of 2016 Brazilian Real and Nigerian Naira Exchange Rates. Journal of Basic and Applied Research International, 27 (9): 1 – 7, 2021.

[16] Etuk, E. H. and Ntagu, O. K. (2018). Modelling of the Intervention of Daily Swiss Franc (CHF)/Nigerain Naira (NGN) Exchange Rates. International Journal of Science and Advanced Innovative Research, Vol. 3, No. 1, March 2018.

[17] Etuk, E. H. and Udoudo, U. P. (2018). Intervention Analysis of Daily Indian Rupee/Nigerian Naira Exchange Rates. Noble International Journal of Business and Management Research. Vol. 02, No. 06, pp: 47 – 52, 2018.

[18] Girard, D. Z. (2000). Intervention Time Series Analysis of Pertussis Vaccination in England and Wales. Health Policy, 54, 13 – 25.

[19] Holt, C. C. (1957). Forecasting Trends and Seasonals by Exponentially Weighted Averages. O.N.R. Memorandum 52/1957, Camegies Instute of Technology.

[20] Jaganathan, S. (2021). Modeling and Predicting Demand During Pandemics Using Time Series Models.https://www.linkedin.com/pulse/modelling-predicting-demand-during-pandemics-using-time-series-jaganathan.

[21] Jarrett, J. E. and Kyper, E. (2011). ARIMA Modelling with Intervention to Forecast and Analyze Chinese Stock Prices. Int. j. eng. bus. Manag., Vol. 3, 53 – 58.

[22] Lai, S. L. and Lu, W. L. (2005). Impact Analysis of September 11 on Air Travel Demand in the USA. Journal of Air Transport Management, 11(6), 455 – 458.

[23] Lam, C. Y., Ip, W. H., and Lau, C. W. (2009). A Business Process Activity Model and Performance Measuremen Using a Time Series ARIMA Intervention Analysis. Expert Systems with Aplications, 36, 925 – 932.

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[24] Min, J. C. H. (2008). Forecasting Japanese Tourism Demad in Taiwan using an Intervention Analysis. International Journal of Culture, Tourism and Hospital Research, Vol. 2, Iss 3, pp. 197 – 216.

[25] Moffat, I. U. and Inyang, E. J. (2022). Impact Assessment of Gap on Nigerian Crude Oil Production: A Box-Tiao Intervention Approach. Asian Journal of Probability and Statistics, 17(2), 52 – 60. https://doi.org/10.9734/ajpas/2022/v17i230419

[26] Mrinmoy, R., Ramasubramanian, V., Amrender, K. and Anil, R. (2014). Application of Time Series Intervention Modelling for Modelling and Forcasting Cotton Yield. Statistics and Applications. Vol. 12, No. 1& 2, pp. 61 – 70

[27] Nelson, J. P. (2000). Consumer Bankruptcies and the Bankruptcy Reform Act: A Time – Series Intervention Analysi, 1960 – 1997. Journal of Financial Services Research, 17:2, 181 – 200, 2000.

[28] PKR/NGN (2020). https://www.exchangerates.org.uk/PKR-NGN-spot-exchange-rates-history-2020.html

[29] Seong, B. and Lee, K. (2020). Intervention Analysis Based on Exponential Smoothing Methods: Application to 9/11 and COVID – 19 Effects. Economic Modelling, https: //doi.org/10.1016/j.econmod.2020.11.014.

[30] Sharma, P. and Khare, M. (1999). Application of Intervention Analysis for Assessing the Effectiveness of CO Pollution Control Legislation in India. Transportation Research Part D 4 (1999), 427 – 432.

[31] Shittu, O. I. and Inyang, E. J. (2019). Statistical Assessment of Government's Interventions on Nigerian Crude Oil Prices. A Publication of Professional Statisticians Society of Nigeria, Proceedings of 3rd International Conference. 2019; 3:519-524.

[32] Shittu, O. I. and Yaya, O. S. (2016). Introduction to Time Series Analysis. Department of Statistics, University of Ibadan, Ibadan, Nigeria.

[33] Winters, P. R. (1960). Forecasting Sales by Exponentially Weighted Averages. Management Sciences, 6, 324 – 342.

[34] Trapero, J. R., Pedregal, D. J., Fildes, R., and Kourentzes, N. (2013). Analysis of Judgmental Adjustments in the Presence of Promotions. International Journal of Forecasting, 29(2), 234 – 243.

[35] Yang, L. (2014). Pricing Virtual Goods: Using Intervention Analysis and Products' Usage Data. A Thesis Presented to the University of Waterloo in Fulfillment of the

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Thesis Requirement of the Degree of Master of Applied Science in Management Sciences, 2014.

Appendix 1

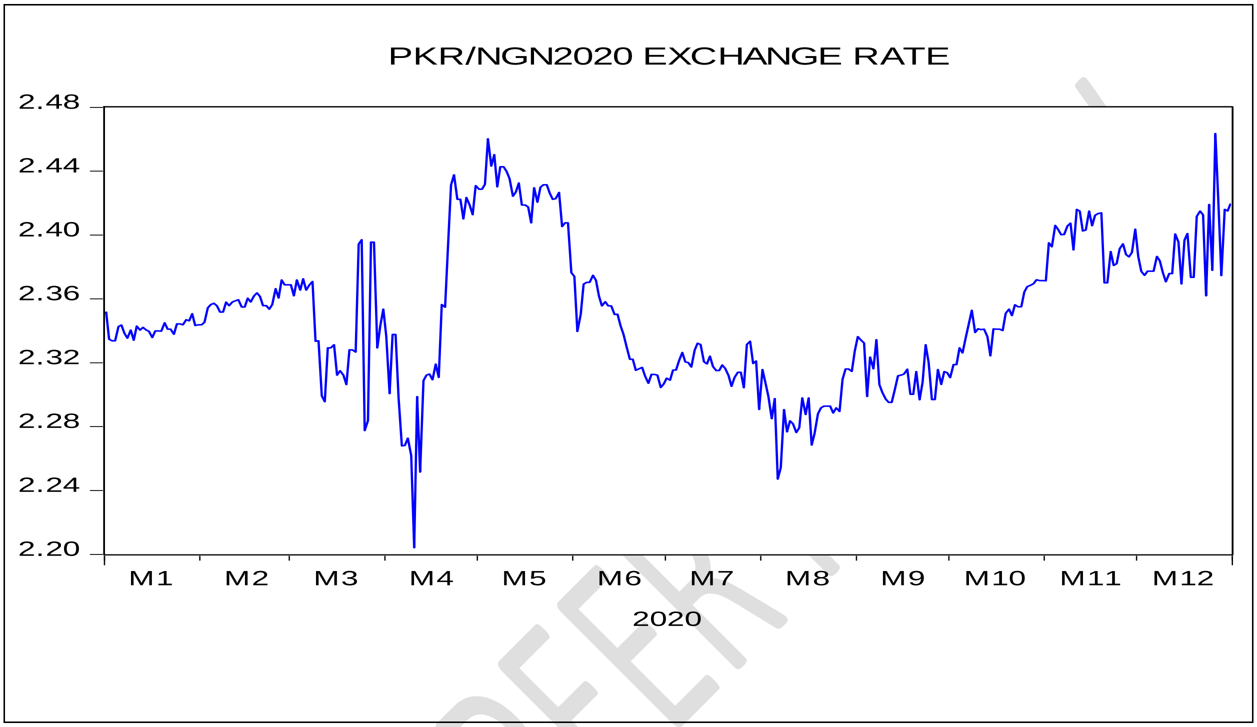

Figure 1: Time Series Plot of PKR/NGN2020 Exchange Rate.

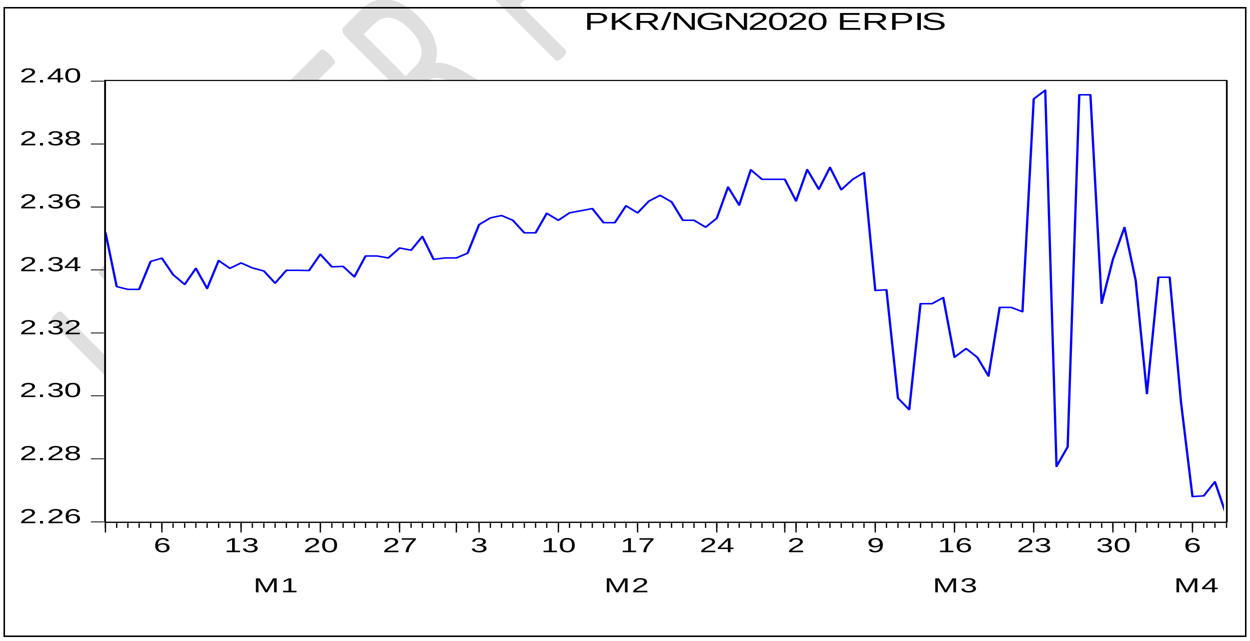

Figure 2: Time Series Plot of PKR/NGN2020 Exchange Rate PIS

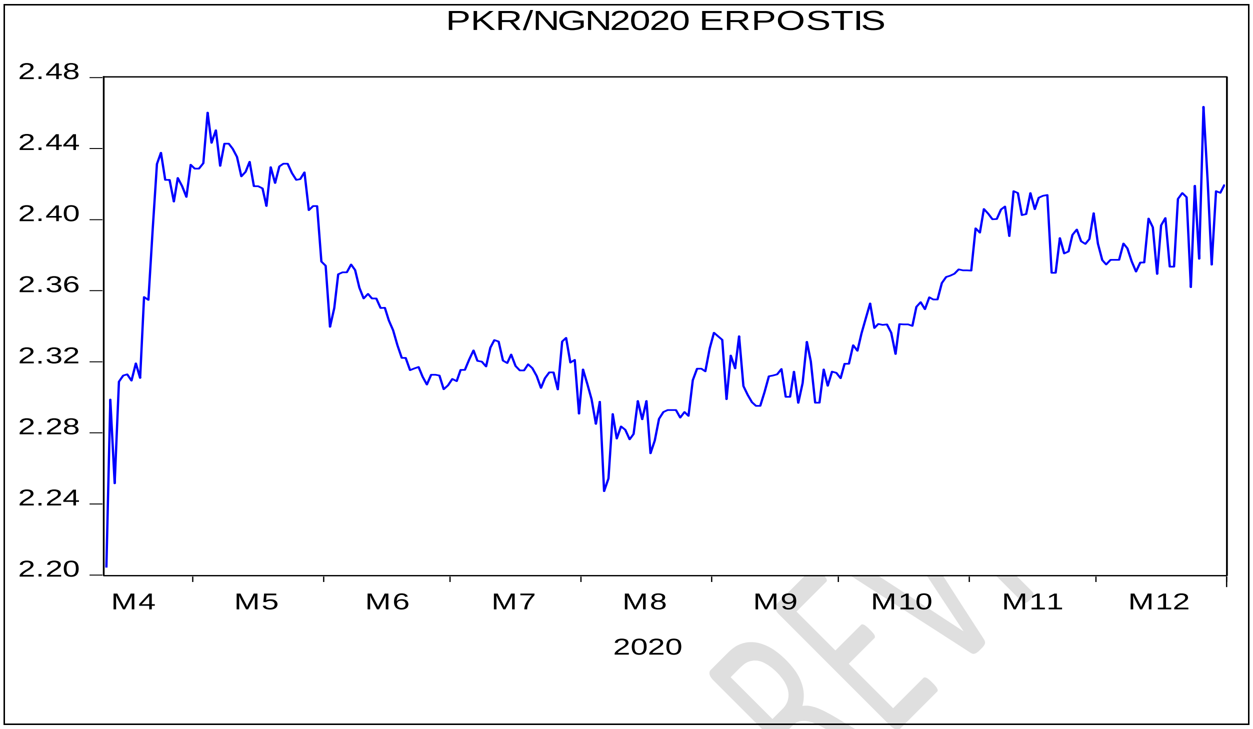

Figure 3: Time Series Plot of PKR/NGN2016 Exchange Rate POSTIS

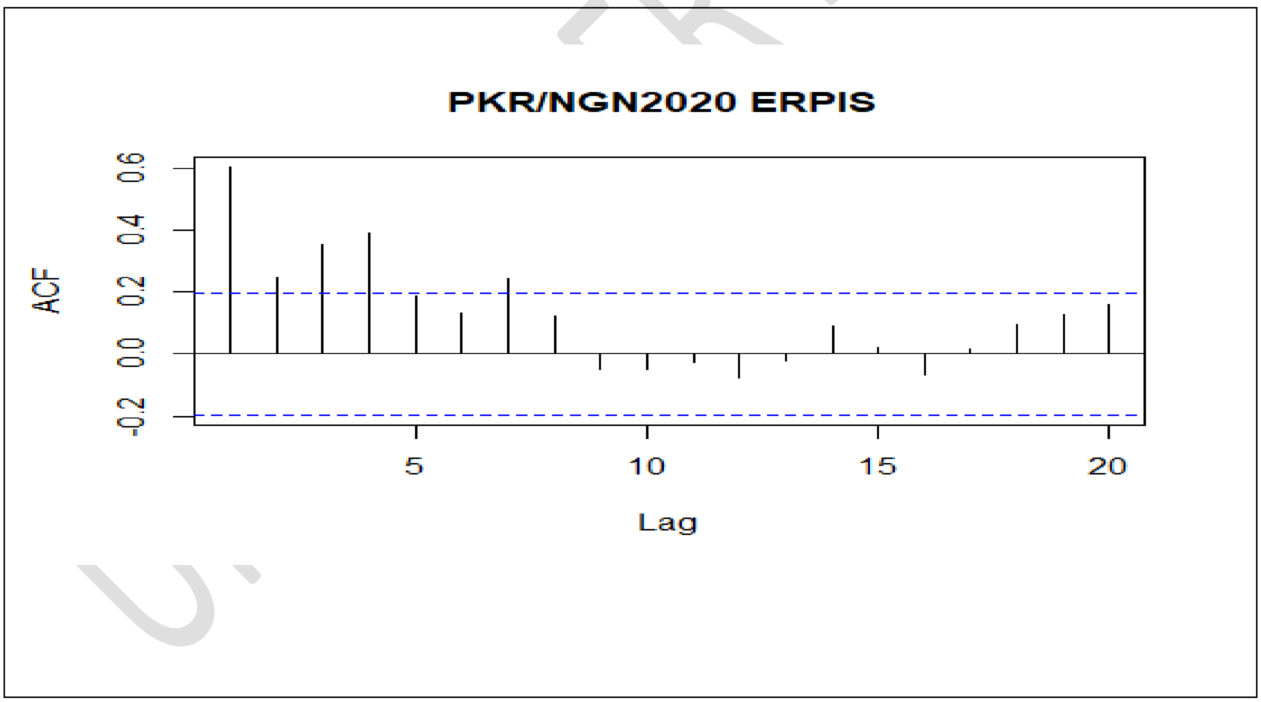

Figure 4: ACF of PKR/NGN2020 Exchange Rate PIS

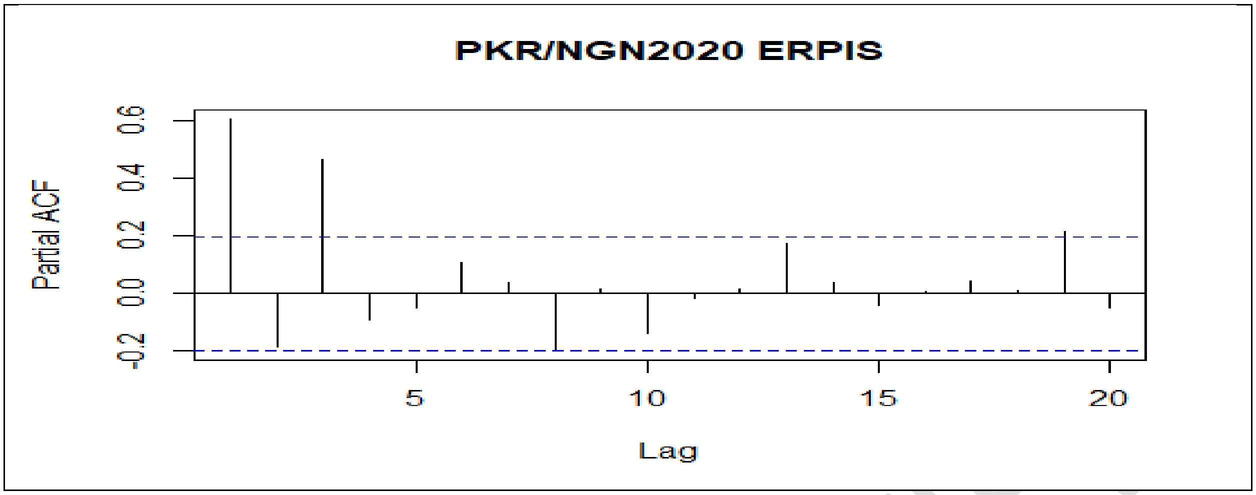

Figure 5: PACF of PKR/NGN2020 Exchange Rate PIS

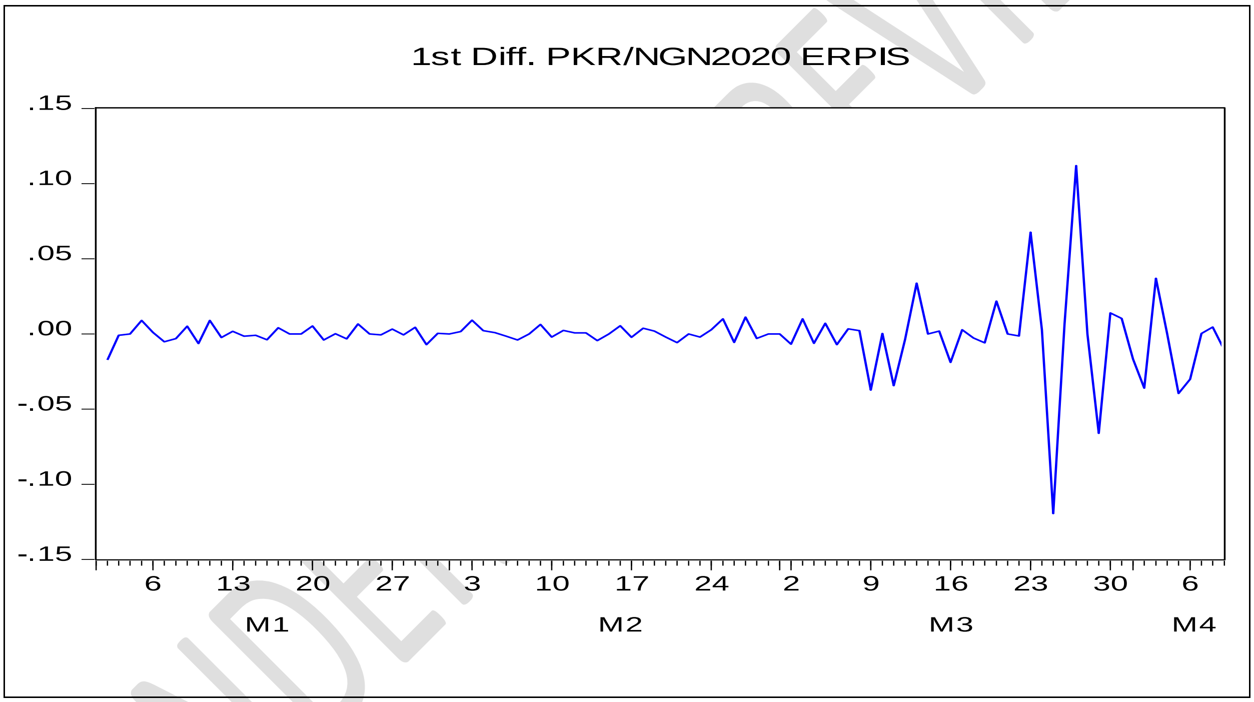

Figure 6: First Difference of PKR/NGN2020 Exchange Rate PIS

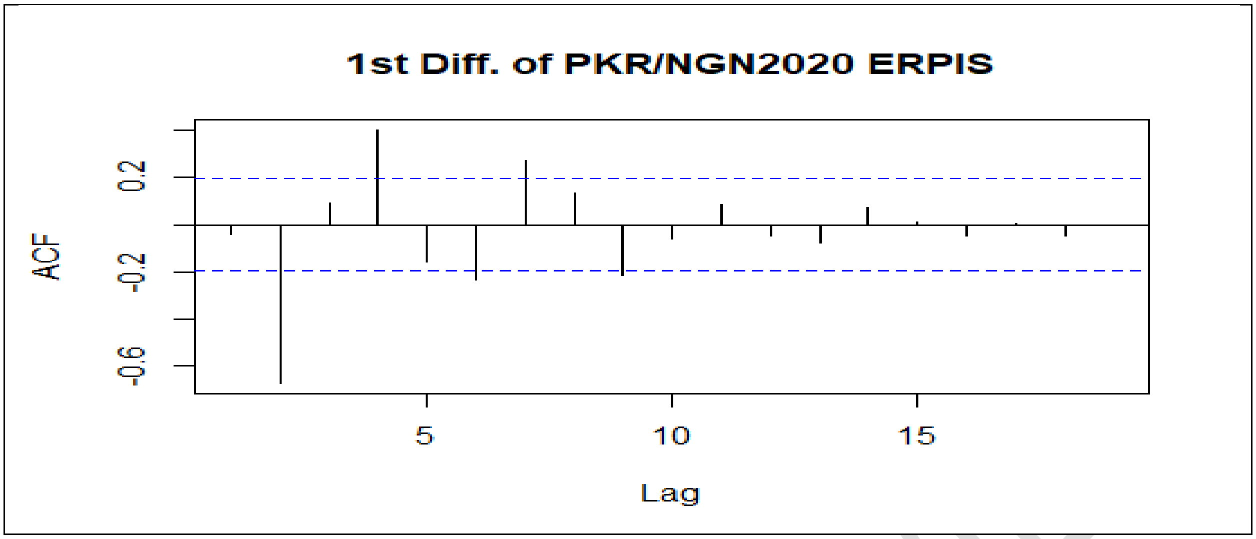

Figure 7: ACF of First Difference of PKR/NGN2020 Exchange Rate PIS

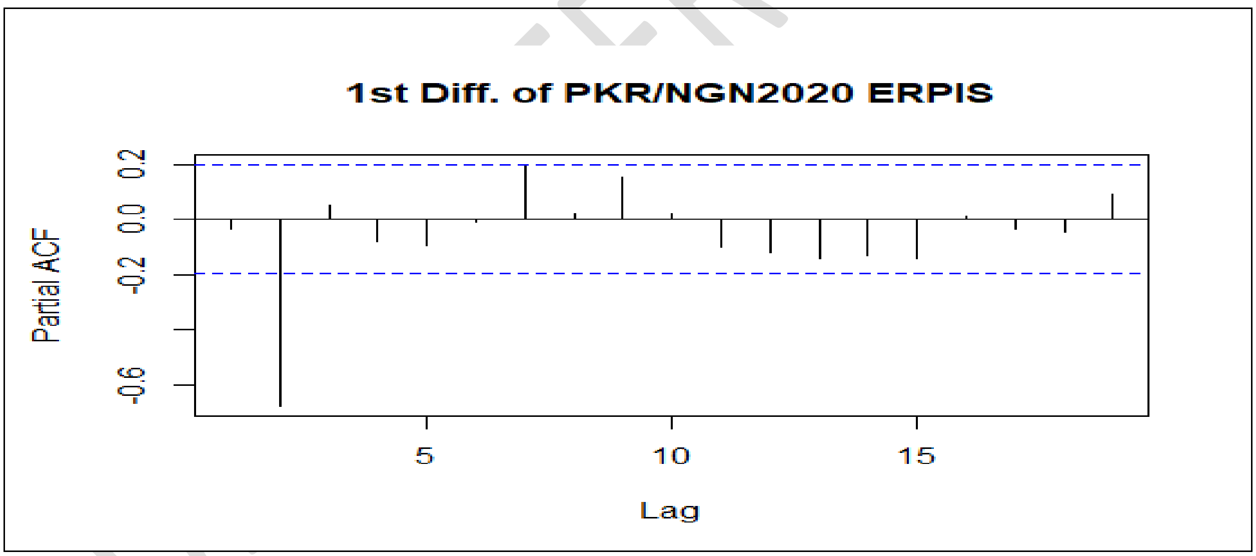

Figure 8: PACF of First Difference of PKR/NGN2020 Exchange Rate PIS

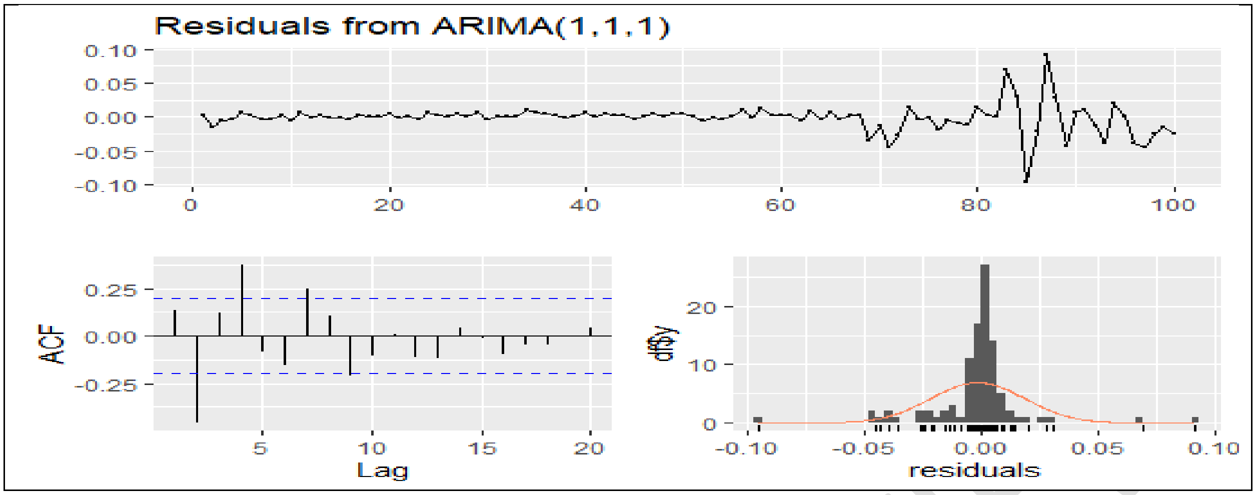

Figure 9: Residuals of the Fitted ARIMA(1,1,1) Model for PKR/NGN2020 ER PIS

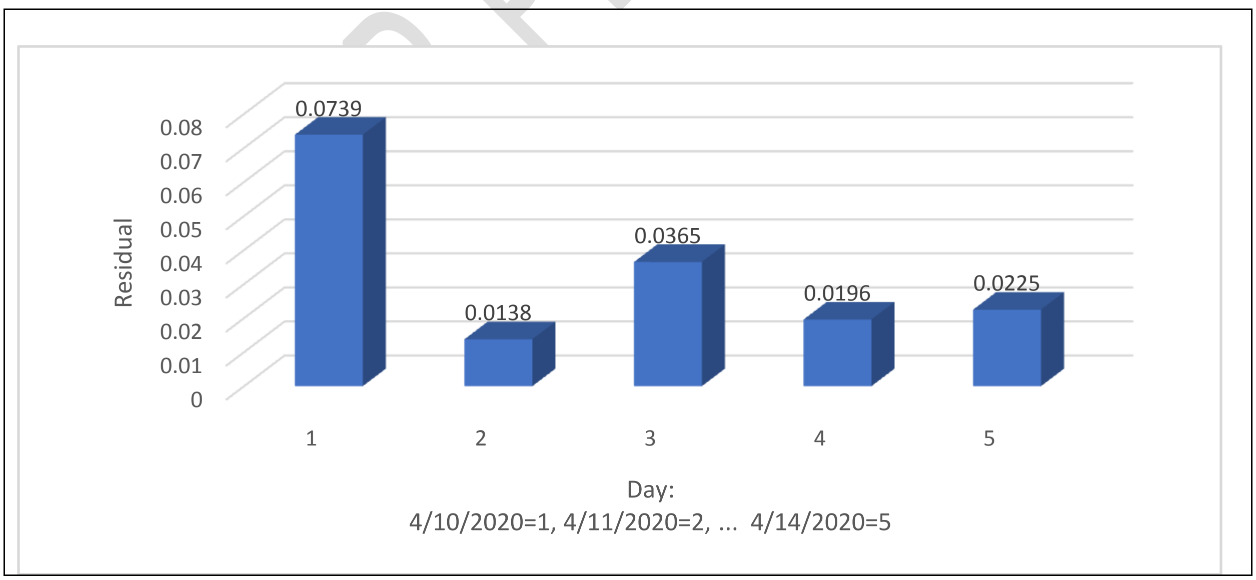

Figure 10: Impulse Response Function for PKR/NGN2020 ER

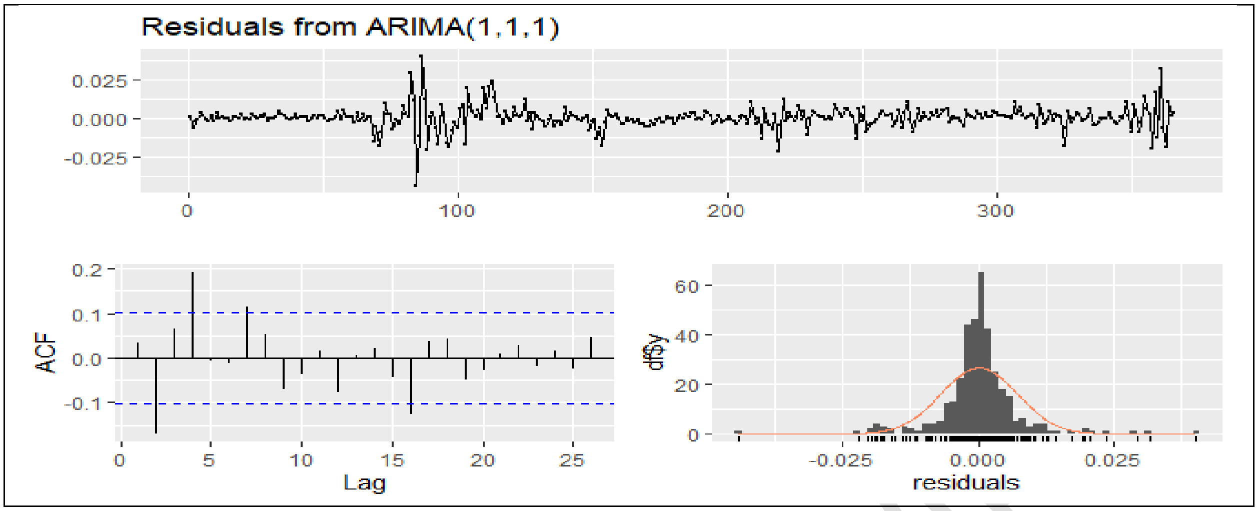

Figure 11: Residuals for ARIMA-INTERVENTION Model for PKR/NGN2020 ER

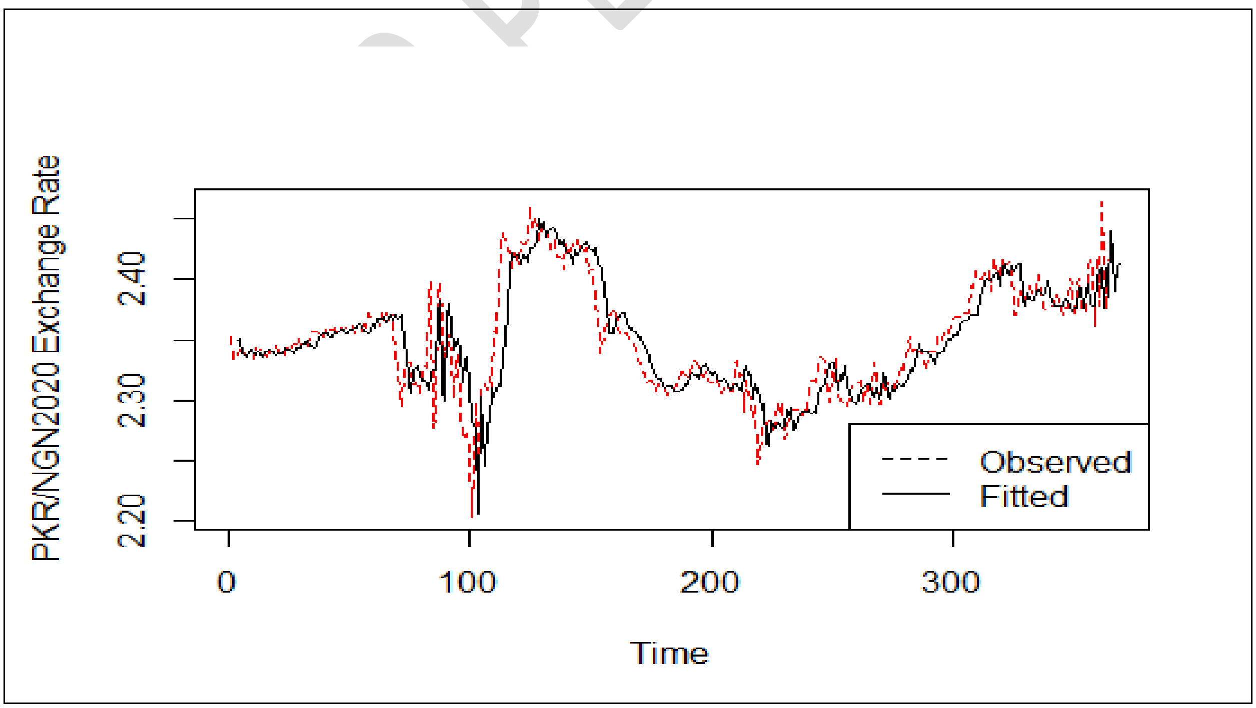

Figure 12: Fitted ARIMA-INTERVENTION Model for PKR/NGN2020 ER

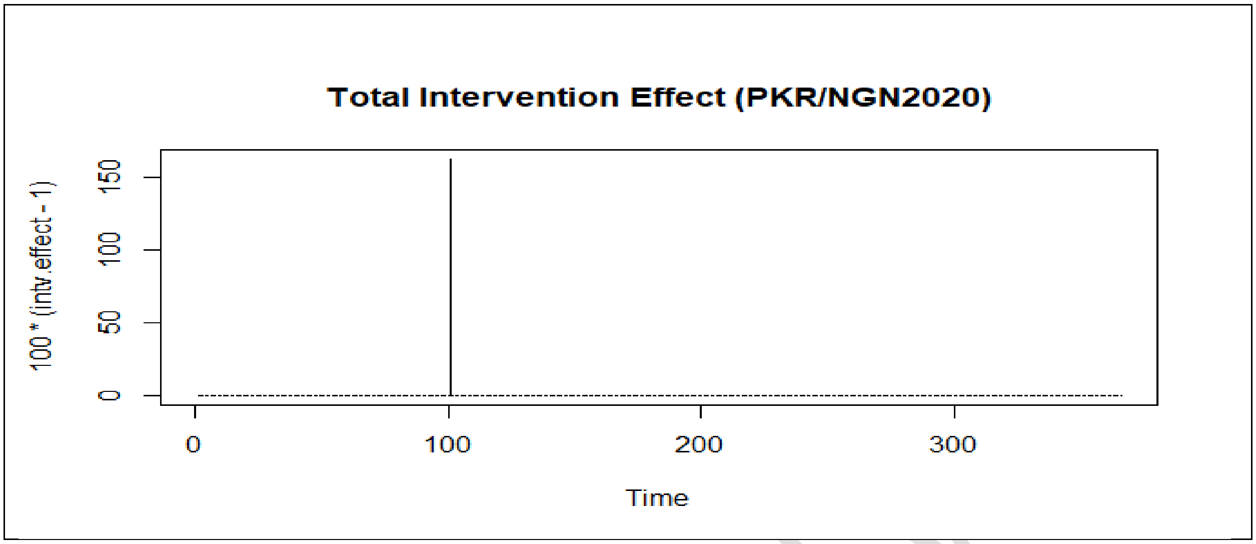

Figure 13: Total Intervention Effect for PKR/NGN2020 ER

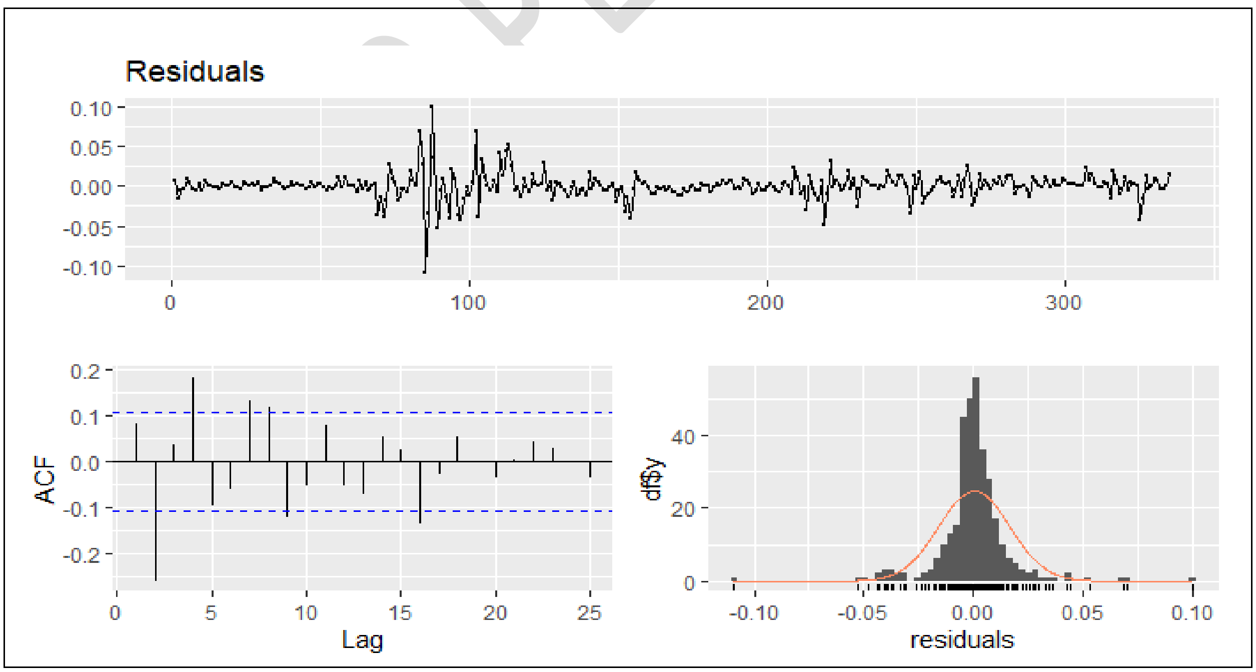

Figure 14: Residuals for ETS(A,N,N) Model

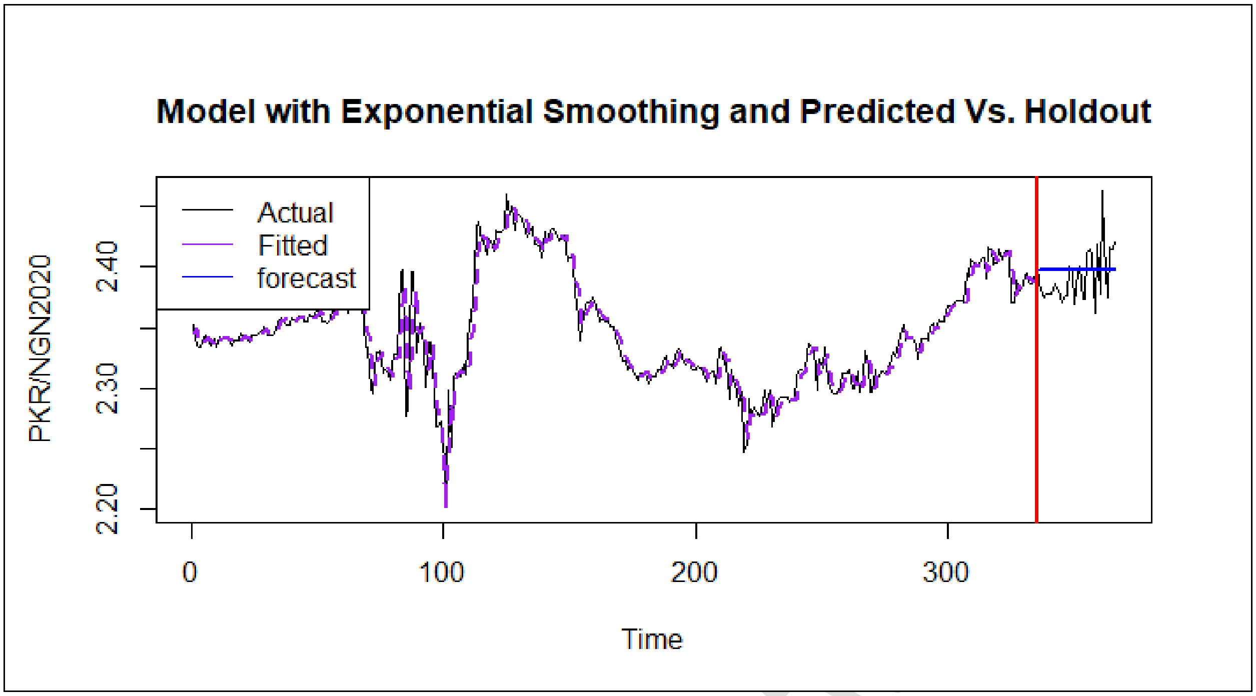

Figure 15: Fitted ETS(A,N,N)-INTERVENTION Model for PKR/NGN2020 ER

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Appendix 2

TABLE 1: UNIT ROOT TEST AT LEVEL (PKR/NGN2020 PIS)

Test	Augmented Dickey-Fuller
Data	PKR/NG2020
Dickey-Fuller	-0.96745
Lag order	4
P-value	0.9394
Alternative hypothesis	Stationary

TABLE 2: UNIT ROOT TEST AT FIRST DIFFERENCE (PKR/NGN2020 PIS)

Test	Augmented Dickey-Fuller
Data	PKR/NG2020
Dickey-Fuller	-6.0194
Lag order	4
P-value	0.01
Alternative hypothesis	Stationary

TABLE 3: PARAMETERS OF THE ESTIMATED MODELS (PKR/NGN2020 PIS)

ARIMA(p,d,q) Model		Estimate	Std. Error	z value	Prob. Value
(1,1,0)	AR1	-0.035065	0.100373	-0.3493	0.7268
(0,1,1)	MA1	-0.569663	0.097428	-5.847	5.004e-09 ***
(1,1,1)	AR1	0.42644	0.18392	2.3186	0.02042 *
	MA1	-0.81998	0.13677	-5.9953	2.031e-09 ***

TABLE 4: MODEL EVALUATION (PKR/NGN2020 PIS)

Model	BIC	AIC
ARIMA (1,1,0)	-465.7488	-470.939
ARIMA (0,1,1)	-474.2164	-479.4066
ARIMA (1,1,1)	-477.3847	-485.17

Table 5: Forecasts with ARIMA(1,1,1) Model (PKR/NGN2020 PIS)

DATE	Actual value (POSTIS)	Forecast	|Residuals|
2020/04/10	2.2042	2.278107	0.0739
2020/04/11	2.2988	2.285018	0.0138
2020/04/12	2.2515	2.287966	0.0365
2020/04/13	2.3088	2.289222	0.0196
2020/04/14	2.3123	2.289758	0.0225

Table 6: Parameter Estimation for the Full Intervention Model (PKR/NGN2020)

Parameter	Estamte	Std.Error	Z-value	P-value
AR(1)	0.2346545	0.0923854	2.5400	0.01109 *
MA(1)	-0.5934268	0.0703192	-8.4390	2.2e-16 ***
ω	-0.0344186	0.0059053	-5.8284	5.596e-09 ***
b	0			

Table 7: Estimated Intervention Parameters for $\boldsymbol{ETS(A,N,N) + Intervention\,Type}$

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Parameters	α	l	ω
Estimates	0.6342	2.3457	0.0658
Standard Errors	0.0558	0.0167	0.0210

Table 8: Model Evaluation for the Two Techniques

Model +Intervention Type	$\hat{\sigma}^2$	AIC	MAE	RMSE
$ARIMA(1,1,1)+\omega(B)B^b {P_t}^T$	0.0000499	-2571.076	0.004260986	0.007058458
$ETS(A,N,N)+\omega(B)B^b {P_t}^T$	0.0002657	-1805.494	0.02	0.023

Table 9: Forecasting with ARIMA(1,1,1)-INTERVENTION Model (PKR/NGN2020)

S/N	DATE	ACTUAL VALUE	FORECAST	95% PREDICTION INTERVAL	
				LOWER	UPPER
336	1-Dec-2020	2.3864	2.398713	2.365515	2.432376
337	2-Dec-2020	2.3773	2.397569	2.357521	2.438298
338	3-Dec-2020	2.3748	2.397302	2.352584	2.442869
339	4-Dec-2020	2.3774	2.397239	2.348537	2.446950
340	5-Dec-2020	2.3774	2.397224	2.344898	2.450718
341	6-Dec-2020	2.3774	2.397221	2.341523	2.454243
342	7-Dec-2020	2.3866	2.397220	2.338353	2.457569

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