

Original Research Article

Time Series Intervention Modelling Based on ESM and ARIMA Models: Daily Pakistan Rupee/Nigerian Naira Exchange Rates

Abstract

The battered Nigerian economy has slipped into its second economic recessions in five years due to the fallout of the coronavirus pandemics. And this has had a remarkable effect on the values of the Nigerian Naira that are exchanged for a unit of many other currencies of the world. Intervention modelling is used to assess the impact of this external event on the Pakistan Rupee to the Nigerian Naira exchange rates. The dataset for this study is the daily Nigerian Naira exchange rate with respect to the Pakistan Rupee from January–December 2020. The intervention point is marked in April 10, 2020 as a pulse function for the PKR/NGN series. Result revealed that the economic recession due to fallout of the coronavirus pandemic increased the value of the Nigerian Naira by 3.38% against the Pakistan Rupee, which 1PKR is exchanged for 2.2042NGN compared to period before and after the intervention occurred. The intervention was felt at the point of intervention itself but the effect dies immediately after the intervention. Hence, the intervention response is described as an abrupt start and abrupt decay.

Keywords: Time Series Intervention Modelling; ESM; ARIMA; Exchange Rate; Nigerian Naira; Pakistan Rupee.

1 Introduction

Since life must be understood looking backwards and must be lived by looking forward, time series provides useful tools that helps to predict the future by approximating models that use past data. Time series analysis studies the time-structure relationship in a given variable of interest, which involve the use of the basic tools to analyze a given time series variable with a view to; construct a simple mathematical system that explain the time structure relationship in the economic and social series in a concise way; test hypotheses concerning social and economic details of the series; apply reliable diagnostic tool to validate the model; use the model to explain the behavior of the time series and make reliable prediction for the future on the basis of the dynamic dependence of the series on the past values and set up control mechanism that gives signals when an event goes out of bounds or by examining what might happen when the values of the parameters in the model are changed.

Time series intervention analysis is widely used in areas like finance, economics, public health, transportation, labor markets, and so on. Intervention modelling is used to assess the impact of external events on the time series of interest. Also, it may be undertaken to adjust for any unusual values in the series that might have resulted as a consequence of the special event. Thus,

this will ensure that the results of the time series analysis of the data, such as the parameter estimates of the model, structure of the fitted model and forecasts of the future values are not distorted by the influence of these unusual values, which the classical ARIMA model cannot handle. Accordingly, there are three types of intervention viz, pulse, step and ramp function. Pulse intervention occurs only at a particular period of time but the effect of the intervention may exist for that particular time period of time only or may exist in the subsequent time period, the influence of the 2020 Nigerian economic recession due to COVID-19 which has affected some of the strongest nations of the world is an example of this type of intervention. Step function occurs at a particular period of time and it exists in the subsequent time periods. The intervention effect may remain constant over time or it may increase or decrease over time. While, ramp intervention occurs at a particular period of time and exists in the subsequent time period with an increasing magnitude.

This study deals with investigations of time series intervention modelling based on Exponential Smoothing Methods (ESMs) and the ARIMA models aimed at studying the response of the comparative value of the Nigerian Naira with respect to the Pakistan Rupee to the 2020 economic recessions.

2 Review of Literature

Two approaches to intervention modelling are adopted here. First, the Box-Tiao (1975) approach [5] mainly conducted on the Box-Jenkins (1976) ARIMA model [2, 3] which has been widely applied by scholars since inception. For instance, Deutsch and Alt [8], examined the Effect of Massachusetts' Gun Control Law on Gun – related Crimes in the City of Boston. Sharma and Khare [30], used intervention analysis model to study the impact of the intervention introduced by the Indian Government to control the pollution caused by the vehicular exhaust emissions. Girard [18], applied the ARIMA model with intervention in order to analyze the epidemiological situation of whooping – cough in England and Wales for the period of 1940 – 1990. Nelson [27], uses an ARIMA intervention analysis to estimate the impact of the Bankruptcy Act of 1978. Lai and Lu [22], uses intervention model to look at the impact of the September 11, 2001 terrorist attack on air transport passenger demand in the USA. Min [24], applied intervention analysis to assess whether two events, the 9 – 21 Earthquake in 1999 and the Severe Acute Respiratory Syndrome (SARS) outbreak in 2003, had a temporary or long – term impact on the inbound tourism demand from Japan. Lam *et al* [23], used a time series intervention ARIMA model to measure the intervention effects and the asymptotic change in the simulation results of the business process reengineering that is based on the activity model analysis. Jarrett and Kyper[21], examined the impact of world financial crisis (WFC) on the Chinese Stock Price. Darkwah *et al* [7], uses intervention time series analysis to assess the nature and impact of the establishment and operations of community policing in communities in Ghana. Mrinmoy *et al*[26], assessed the impact of Bt – Cotton variety on Cotton Yield in Indian. With step intervention, being the introduction of Bt – Cotton variety in 2002.

Yang[35],used ARIMA with intervention model analyzed the impact of new product releases on revenue. Etuk and Amadi [9], analyzed the effect of the exit of Great Britain from European Union on GBP/USD exchange rate. Etuk and Eleki [11], modelled the daily Yuan – Naira

Exchange Rates using ARIMA intervention analysis. Etuk *et al* [13], use the intervention analysis approach on the daily exchange rate of Yen/Naira. Etuk and Udoudo [17], uses intervention model to explain the impact of economic recession on the daily Indian Rupee and Nigerian Naira exchange rate. Etuk and Ntagu [16], examined modelling of the intervention of daily Swiss Franc (CHF) and Nigerian Naira (NGN) exchange rates, which spanned 18th May 2016 to 16th November 2016. Etuk and Chukwukelo [10], modelled daily Moroccan Dirham (MAD) and Nigerian Naira (NGN) using intervention analysis. Shittu and Inyang [31], modelled Nigerian monthly crude oil prices using ARIMA-Intervention model with a view to comparing the result with that of the intervention model using lag operator. Etuk and George [12], proposed an intervention model for the exchange rate of Malaysian Ringitt (MYR) and the Liberian Dollar (LRD). Etuk *et al* [15], fitted an autoregressive integrated moving average intervention model to daily Brazilian Real (BRL)/Nigerian Naira (NGN) exchange rates. Etuk *et al* [14], investigated the impact of declaration of cooperation (DoC) on the Nigerian crude oil production. Moffat and Inyang [25], investigated the impact of the Nigerian government amnesty programme (GAP) on her crude oil production.

Secondly, intervention model with exponential smoothing methods [6,19,33], which has rarely been attempted. Trapero *et al* [34], investigated the accuracy of judgmental forecasting at SKU level in the presence of promotions, based on weekly data from a manufacturing company using simple model based on a transfer function combined with Single Exponential Smoothing (SES). Seong and Lee [29], proposed a method of intervention analysis based on exponential smoothing models through an innovational state – space model. Two applications were analyzed: the 9/11 effect on US airlines and the COVID – 19 effects on the current population of Seoul, Korea. Jaganathan [20], used ARIMA Linear Transfer Function and Exponential Smoothing (ES) with intervention models to model pandemic data in the context of forecasting demand to explain its impact and build accurate predictive models.

3 Model Specification

The Box-Jenkins ARIMA model has been known as the most widely used technique for modelling and forecasting [3], denoted by ARIMA(p,d,q). But when the time series is affected by external events, the forecasting power of ARIMA model may fail. Thus, Box-Tiao [5], proposed the intervention modelling (ARIMA-Intervention analysis) given by

$$X_t = G(L) + W_t \quad (1)$$

Where $G(L)$ is the transfer function component and W_t is noise component

Intervention model with ARIMA

$$X_t = \frac{\omega(L)L^b}{\delta(L)} I_t + \frac{B(L)}{A(L)} \epsilon_t \quad (2)$$

Since $G(L) = \frac{\omega(L)L^b}{\delta(L)} I_t$ and $W_t = \frac{B(L)}{A(L)} \epsilon_t$

Where:

$$A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p \text{ and } B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$$

$$\delta(B) = 1 - \delta_1 L - \dots - \delta_r L^r \text{ and } \omega(L) = \omega_0 - \omega_1 L - \dots - \omega_s L^s$$

X_t is the response variable at t , b =delay parameter, ω =impact parameter, δ =slope parameter, α =Non-seasonal autoregressive parameter, β =Non-seasonal moving average parameter, ϵ_t =White noise.

W_t is a Box – Jenkins ARIMA(p,d,q) model which represents the baseline daily Pakistan Rupee/Nigerian exchange rate in pre – intervention period.

$I_t = P^T_t$, the indicator variable. Mathematically, they are written as

$$P^T_t = \begin{cases} 1, & t = T \\ 0, & t \neq T \end{cases} \quad (3)$$

P^T_t is called the pulse function.

$$\text{Hence, (2) becomes } X_t = \frac{\omega(L)}{\delta(L)} P^T_{t-b} + \frac{B(L)}{A(L)} \epsilon_t \quad (4)$$

Intervention model with ESM

The underlying model is given by

$$Y_t = \frac{\omega(L)}{\delta(L)} P^T_{t-b} + ETS(A, N, N) \quad (5)$$

The form in (5) is similar to the intervention model with ARIMA models in (2). And with a temporary change of intervention effect with ESM with additive error, no trend, no seasonal component.

Model Validation

Diagnostic test is an important step in time series model building and this consist of scrutinizing a variety of diagnostics to determine whether the selected model is healthy and hence ready to forecast. We consider here;

(a) Plot of the residual ACF

Once an appropriate ARIMA model is fitted, one can examine the goodness of fit by means of plotting the ACF of the residuals of the fitted model. If most of the sample autocorrelations coefficients of the residuals are within the bound of $\pm \frac{2}{\sqrt{T}}$, where T is the series length then the residuals are white noise indicating that the model is a good fit.

(b) Akaike Information Criterion (AIC)

The AIC [1], is formulated as

$$AIC = V_T \left[1 + \frac{2P}{T-P} \right] \quad (6)$$

Where:

V_T = Index related to production error (known as residual sum of squares)

p = No of parameters in the model, T = No. of data points.

(c) Bayesian Information Criterion (BIC)

The BIC is a criterion for model selection among a finite set of models. Given any two or more estimated models, the model with the lowest value BIC is the one to be preferred. It is given by:

$$BIC = n \ln \hat{\sigma}_e^2 + k \ln(n) \quad (7)$$

Where $\hat{\sigma}_e^2$ is the estimated error variance defined by

$$\hat{\sigma}_e^2 = \frac{1}{T} \sum_i^T (x_i - \bar{x})^2$$

x = Observed data, T = Number of observations, k = Number of free parameters to be estimated.

Measuring Forecast Error

The forecasting errors represented by MAE and RMSE, which are Mean Absolute Error and Root Mean Squared Error respectively, are chosen as the performance metric.

$$MAE = \frac{1}{T} \sum_{t=1}^T |e_t| \quad (8)$$

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T e^2_t} \quad (9)$$

Where e_t is residual at time t and T is the total number of the time period. If a method fits the past time series data very good, MAE is near zero, whereas if a method fits the past time series data poorly, MAE is large. Therefore, when two or more forecasting methods are compared, the one with the minimum MAE is selected as most accurate. RMSE is the square root of MSE.

4 Data Description

The data used in this study are the daily Pakistan Rupee/Nigerian Naira (PKR/NGN) exchange rates spanning from January - December 2020, were obtained from the central Bank of Nigeria

Website. The dataset was divided into observations belonging to pre-intervention (Jan. 1 – April 9, 2020) and post-intervention periods (April 10 – Dec., 2020). The statistical package used for the analysis of this work is the R language (R-4.1.2-win & R-4.1.3-win).

5 Results and Discussion

The time plot showing the daily Pakistan Rupee to Nigerian Naira exchange rate from January to December 2020 is given in Figure 1 of Appendix I. The graph of the series rises and falls at random perhaps due to the mechanism that generated the dataset. From the plot, we can observe the lowest exchange rate of 2.2042NGN on Friday 10, April 2020 which was marked as point of intervention while the highest exchange rate of 1PKR to 2.4635NGN was witnessed on Saturday 26, December 2020 and with an average exchange rate of 2.3503NGN.

Intervention Modelling

The suspected point where the intervention took place on the exchange rate dataset is labelled by indicator functions as:

$$P_t^T = \begin{cases} 1, & t = \text{April 10, 2020} \\ 0, & t \neq \text{April 10, 2020} \end{cases} \quad (10)$$

Where: T= April 10, 2020 and P_t^T is the Pulse function type.

ARIMA-Intervention Model

Data on daily Pakistan Rupee to Nigerian Naira exchange rate from January 01, 2020 to April 9, 2020 is used for pre-intervention ARIMA model fitting and data during April 10, 2020 – December 31, 2020 have been used to know the intervention component form.

Pre-intervention time plot shown in Figure 2, the graph of the series exhibits the characteristics of a non-stationary which is confirmed by the unit root test in Table 1 of Appendix II.

To attain stationarity, the series was differenced once. The graph of the differenced series in figure 6 indicates that the series is stationary. A unit root test at first difference further confirmed that the series is stationary, since the p-value of the Augmented Dickey-Fuller Test is less than the alpha level (p-value=0.01 < 0.05), Table 2. On the basis of correlogram of the differenced series in Figure 7 and 8, ARIMA(1,1,1) model is fitted with their statistics summarized in Table 3. Adequacy of ARIMA(1,1,1) model is not in doubt since coefficients of ACF of the residuals are within significance bounds of $\pm \frac{2}{\sqrt{100}} = \pm 0.20$, Figure 9. With the least BIC and AIC values of -477.3847 and -485.17 respectively, Table 4. Therefore, the model is statistically significant, appropriate and adequate for the dataset.

Forecasts generated from ARIMA(1,1,1) model was very close to the actual value of the post-intervention series when compared, Table 5. The forecasting is done to first five post-intervention observations in order to compute the impulse response function. From the impulse response function in Figure 10, it's seen that b=0, which implies that the effect of intervention was felt at the point of the intervention itself, April 10, 2020.

The parameter ω with value -0.0344 was significant with p-value of 0.0000. The negative sign of the intervention parameter indicates that there was decrement in the exchange rate. That is, the Pakistan Rupees depreciated in favor of the Nigerian naira in the exchange rate market on Friday 10 April, 2020 at Rs1 PKR=2.2042NGN, compared to the periods before the intervention occurred. Thus, the intervention increased the value of the Nigerian Naira by 3.38%= $\{1 - \exp[-0.0344]\} \times 100\%$ against the Pakistan Rupees in exchange rate market. Therefore, the ARIMA-Intervention model is represented mathematically as

$$Y_t = -0.0344P_t^T + \frac{(1+0.5934B)}{(1-0.2347B)} \varepsilon_t \quad (11)$$

The model in (11) was found to be statistically significant and adequate for the dataset when diagnosed, Figure 11. This was confirmed by the plot of the fitted ARIMA-Intervention model with the actual values since the fitted values mimic the actual values, Figure 12.

ESM-Intervention Model

The Simple Exponential Smoothing model with additive error is tentatively specified, since there is no presence of trend and seasonality in the PKR/NGN series. The model is labeled ETS(A,N,N), the abbreviation indicating error, trend and seasonal components. The A,N,N between the parentheses indicates Additive error, No trend and No seasonality respectively. The ETS(A,N,N) model with intervention component are estimated in Table 7. The smoothing constant α , controls the flexibility of the level and is estimated as $\alpha = 0.6342$. Corresponding intervention parameters are $\omega = 0.0658$. The estimates are all significant at 5% level. The impact parameter implies a positive change, that is, the intervention causes a rise in the value of the Nigerian Naira against the Pakistan Rupee (1 PKR = 2.2042NGN) compared to period before the intervention occurred.

The ETS-Intervention model is represented mathematically as

$$Y_t = 0.0658P_t^T + ETS(A, N, N) \quad (12)$$

The model in (12) is statistically significant and adequate, Figure 14. The plot of the fitted ETS-Intervention model with the actual values confirmed that (12) is good fit, since the fitted values mimic the actual values, Figure 15.

6 Conclusion

The Pakistan Rupee/Nigeran Naira have been considered with a pulse intervention being the introduction of 2020 economic recession due to the global pandemic.

The intervention effect was never persistent, confirmed by the plot of total intervention effect in Figure 13. With $b=0$ implies that the effect of the intervention was felt at the point of intervention, April 10, 2020. Comparing the two models, it is evident that ARIMA – Intervention model show a better fit than the ESM – Intervention model, since the estimated error variance, AIC and forecast accuracy measures, MAE and RMSE, of ARIMA – Intervention model are smaller than those of the ESM – Intervention model, Table 8. The fitted plots from the two models confirmed that the models are good fit, since the fitted values mimic the actual values, Figure 12 and 15. Hence, forecasting the future values of PKR/NGN using the ARIMA –

Intervention model, the forecasted values were very close to the actual values when compared, Table 9.

7 Recommendations

Exchange rate prediction is one the demanding applications of modern time series forecasting. Therefore, accurate predictions from the model arrived at can aid financial authorities and professionals improve their planning and decision-making processes in bettering the value of the Naira.

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Appendix I

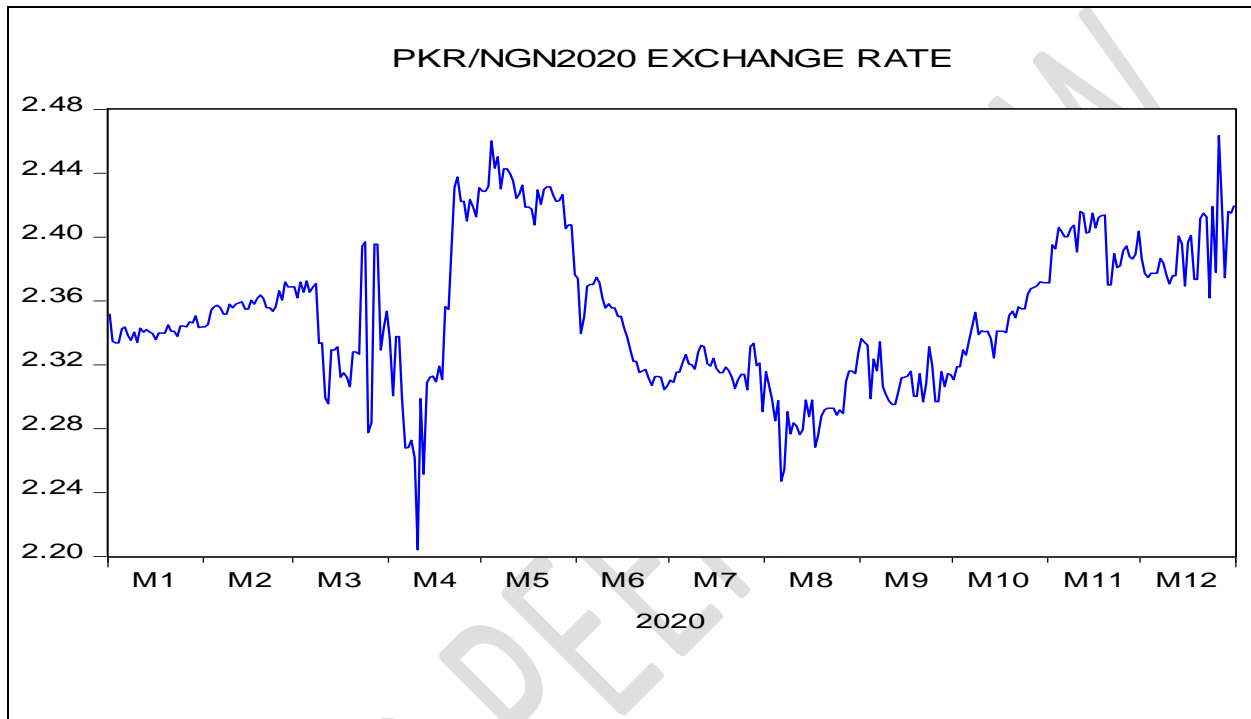


Figure 1: Time Series Plot of PKR/NGN2020 Exchange Rate.

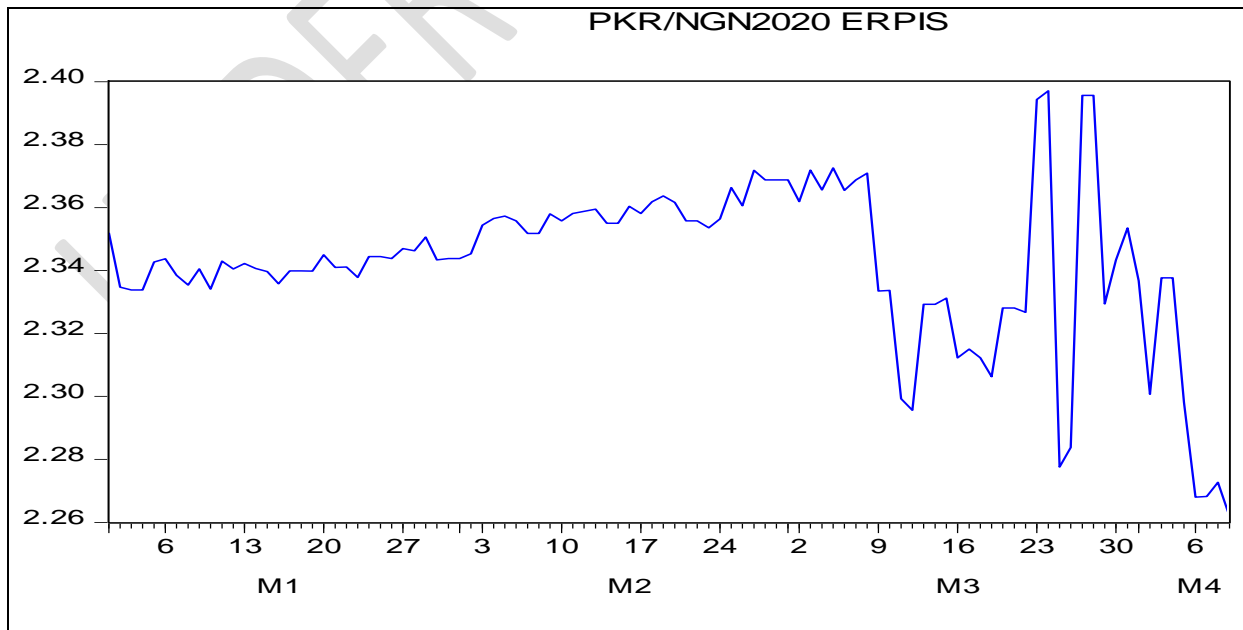


Figure 2: Time Series Plot of PKR/NGN2020 Exchange Rate PIS

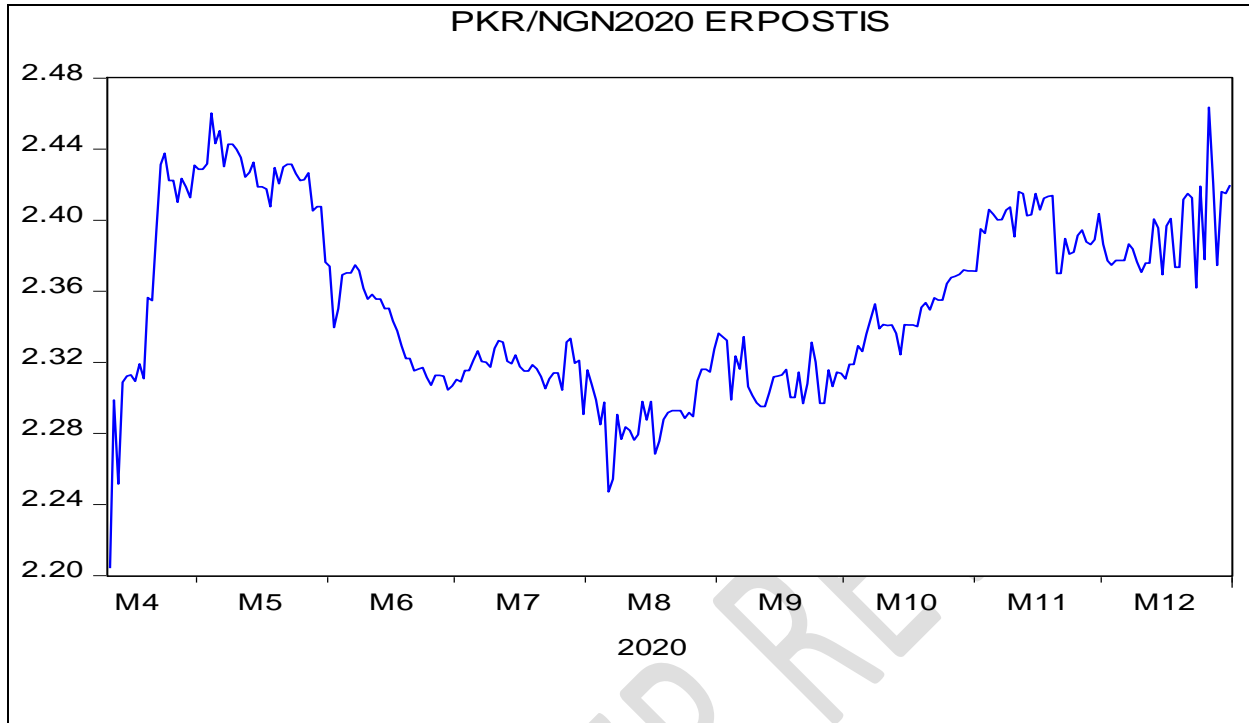


Figure 3: Time Series Plot of PKR/NGN2016 Exchange Rate POSTIS

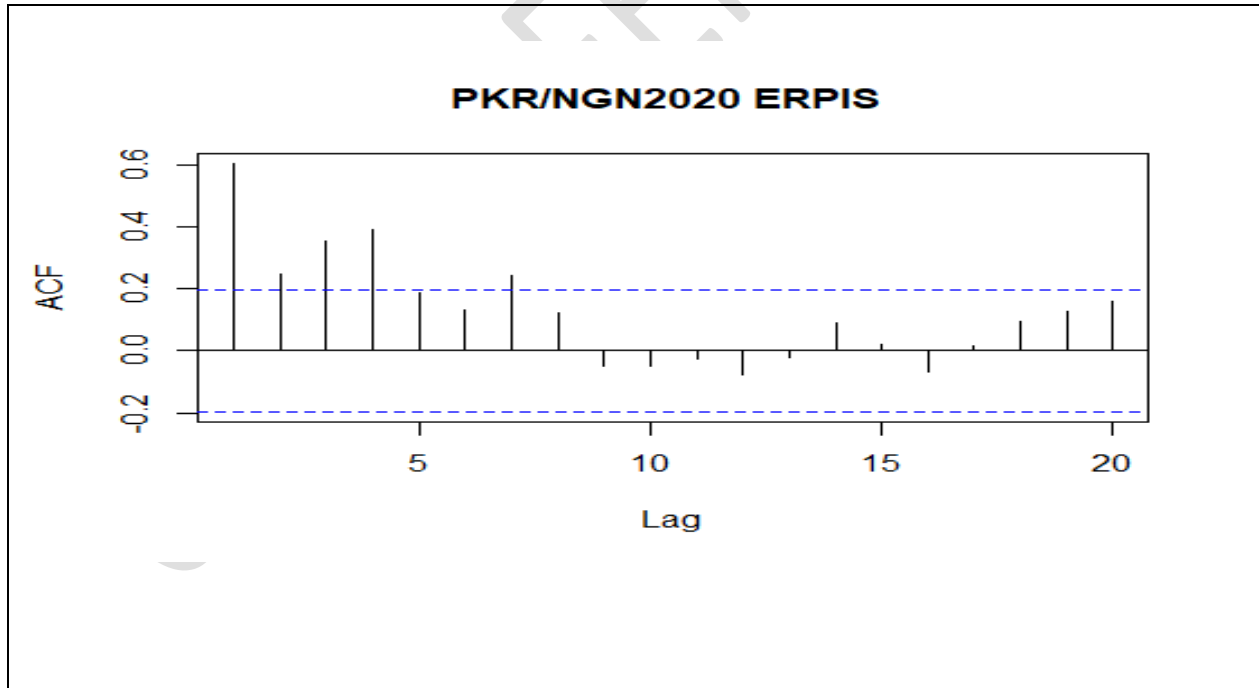
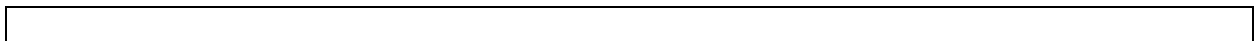


Figure 4: ACF of PKR/NGN2020 Exchange Rate PIS



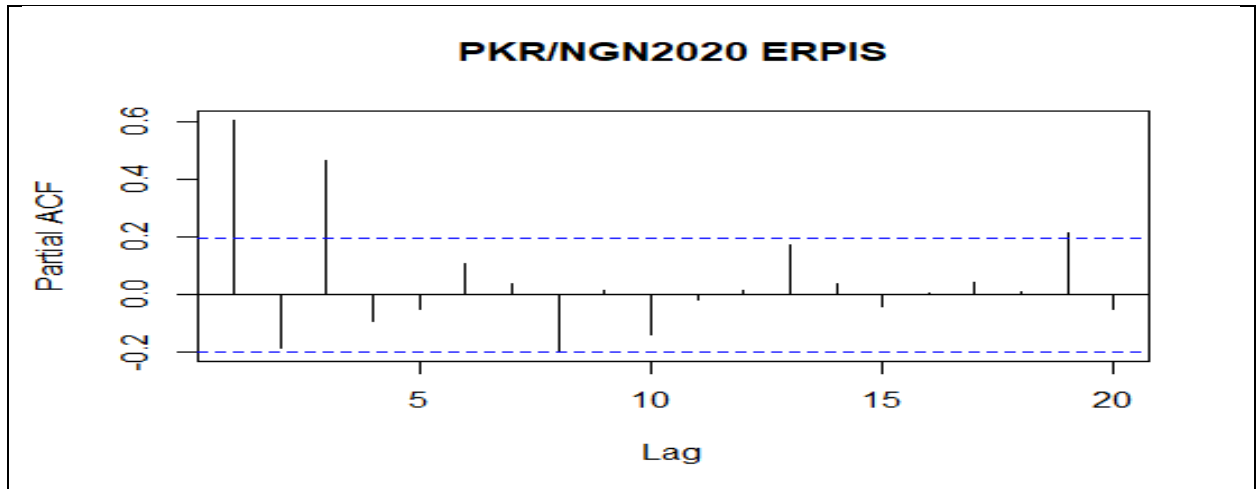


Figure 5: PACF of PKR/NGN2020 Exchange Rate PIS

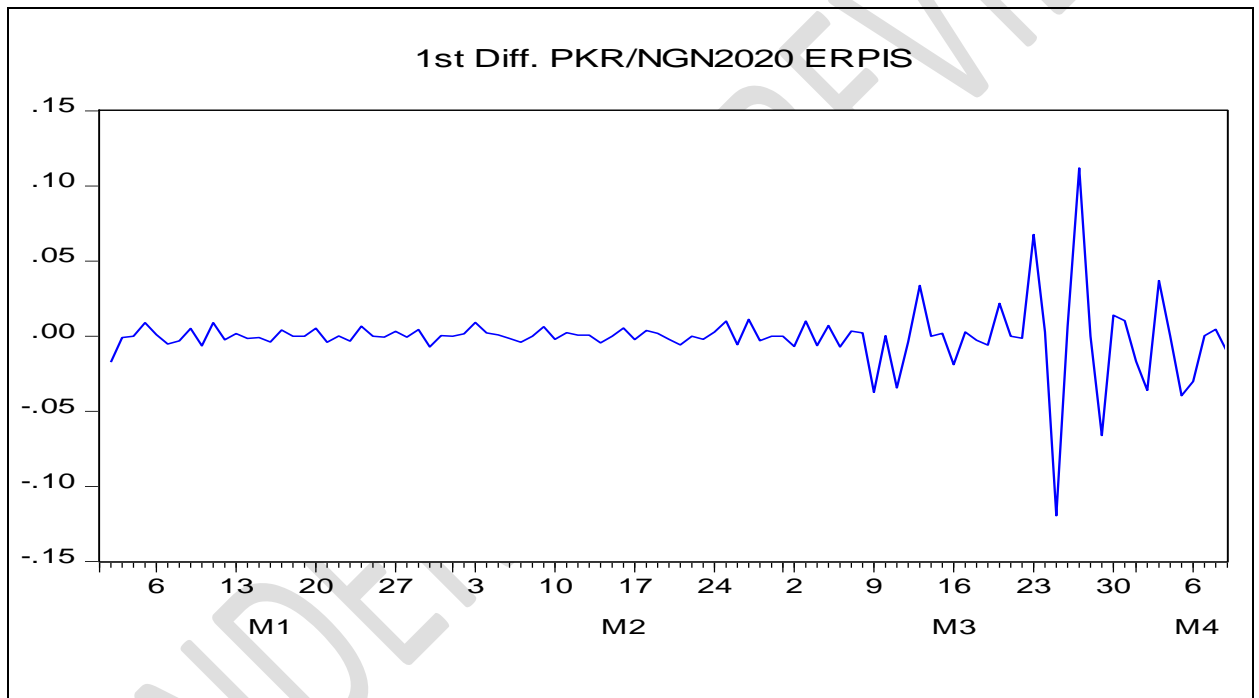
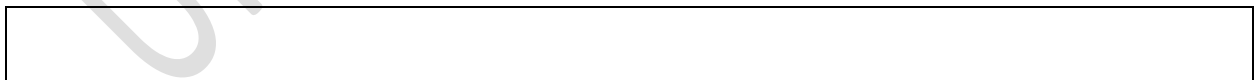


Figure 6: First Difference of PKR/NGN2020 Exchange Rate PIS



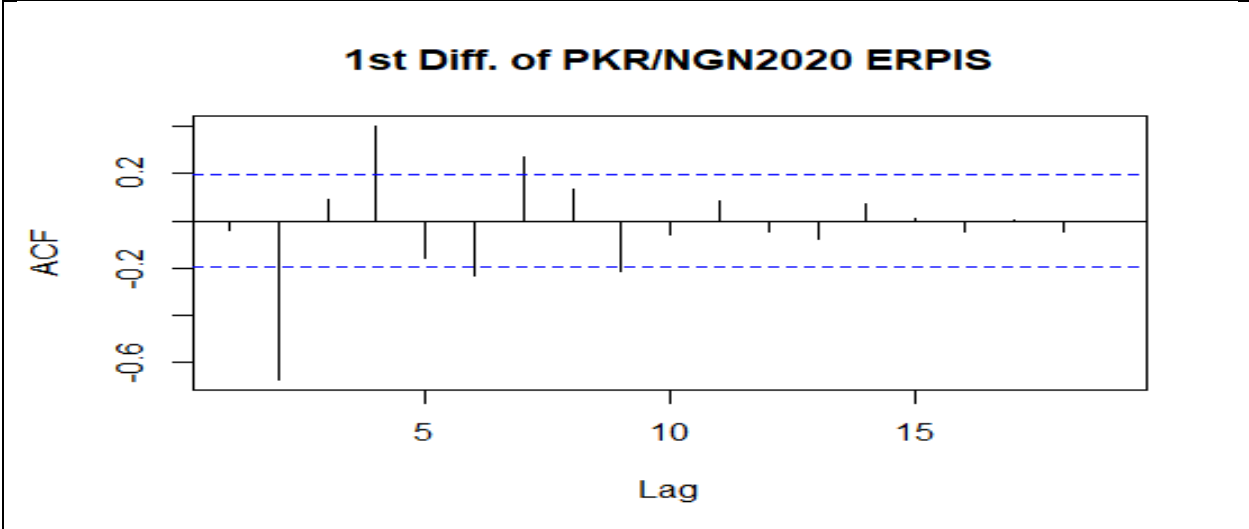


Figure 7: ACF of First Difference of PKR/NGN2020 Exchange Rate PIS

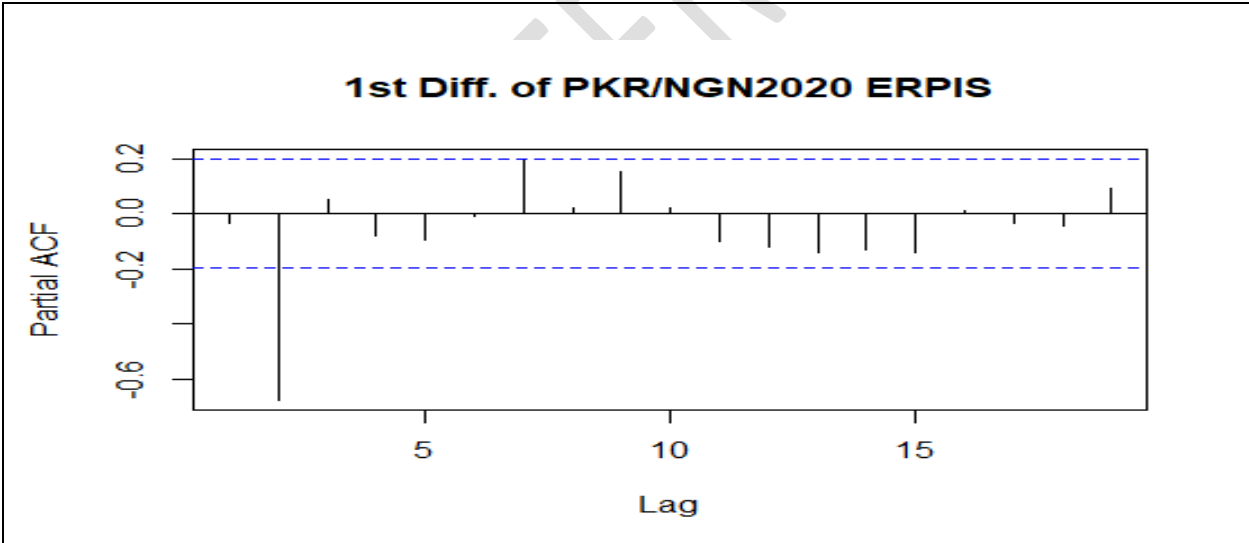
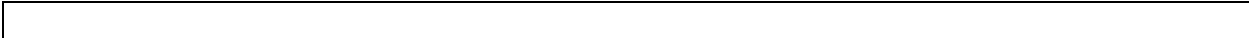


Figure 8: PACF of First Difference of PKR/NGN2020 Exchange Rate PIS



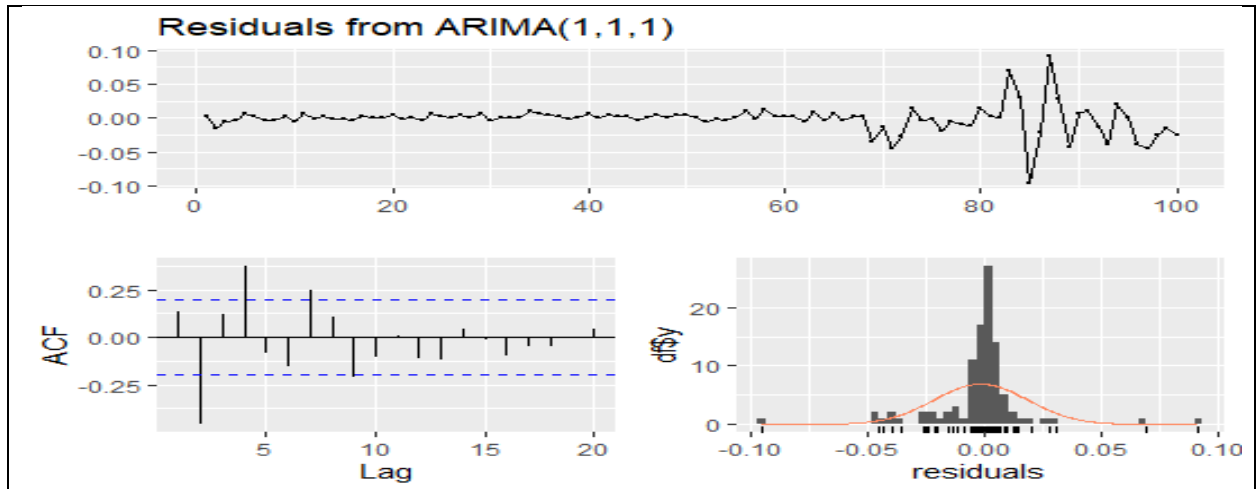


Figure 9: Residuals of the Fitted ARIMA(1,1,1) Model for PKR/NGN2020 ER PIS

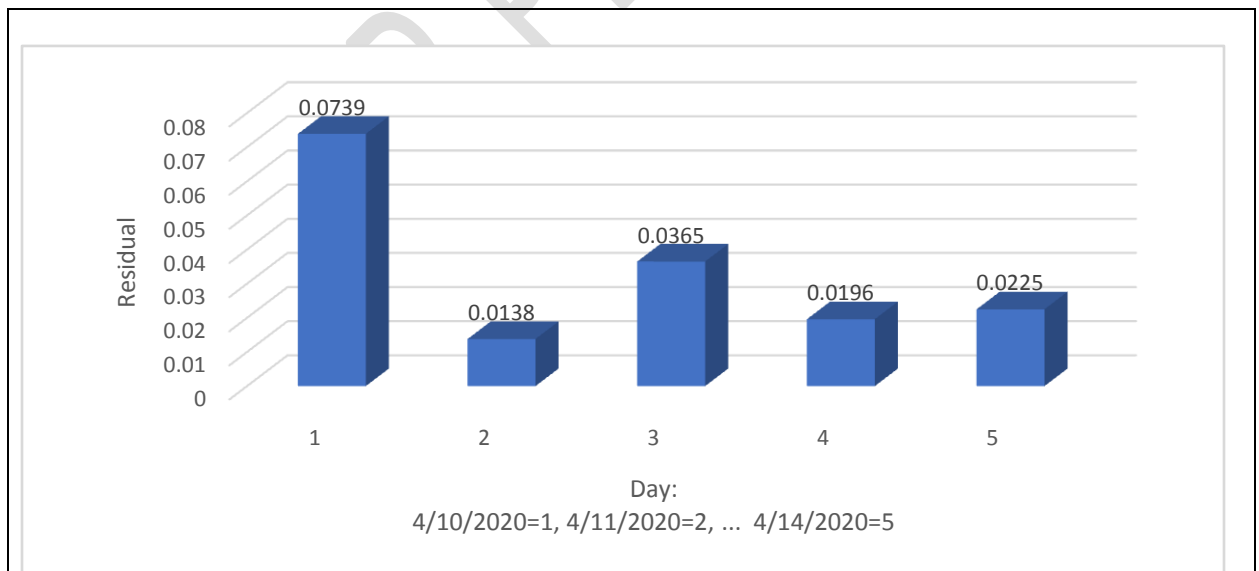


Figure 10: Impulse Response Function for PKR/NGN2020 ER

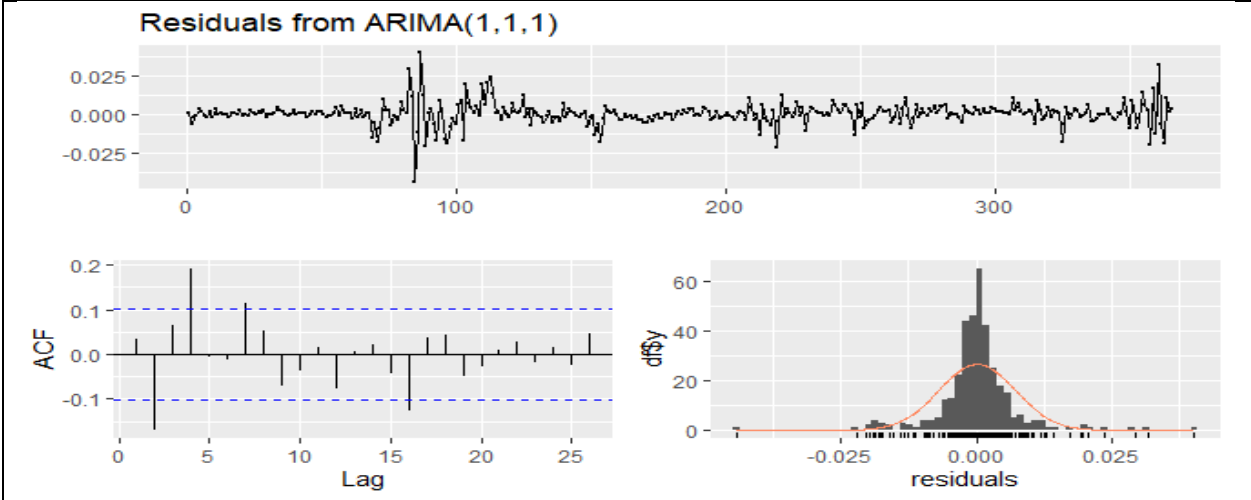


Figure 11: Residuals for ARIMA-INTERVENTION Model for PKR/NGN2020 ER

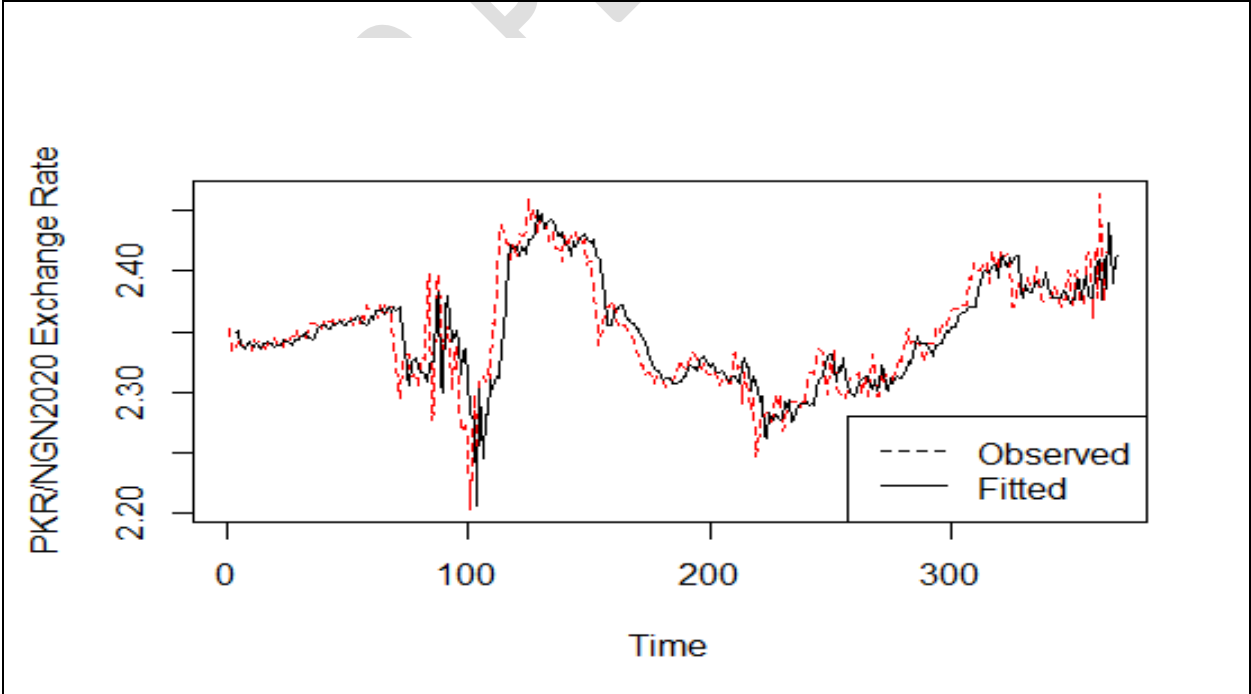


Figure 12: Fitted ARIMA-INTERVENTION Model for PKR/NGN2020 ER

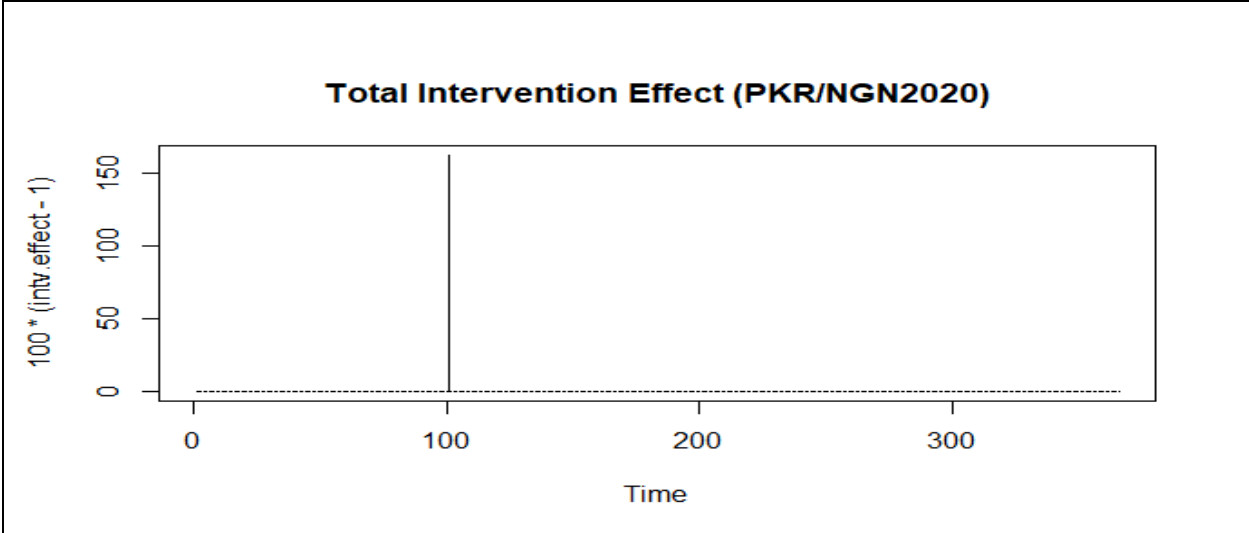


Figure 13: Total Intervention Effect for PKR/NGN2020 ER

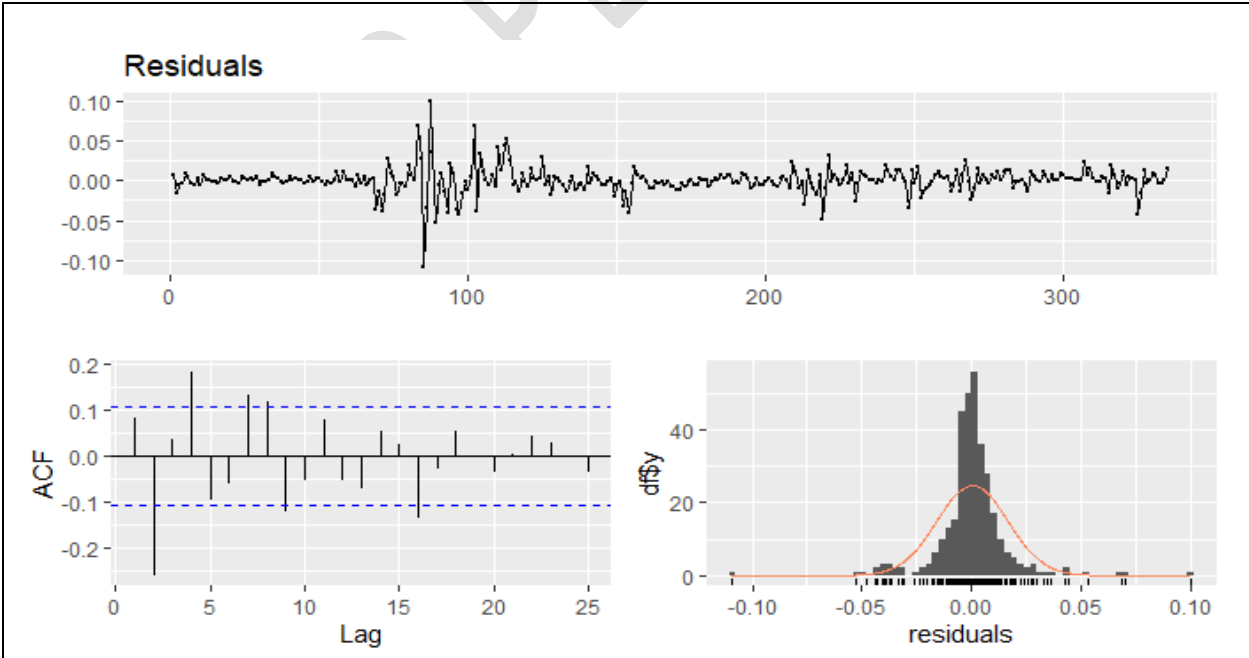


Figure 14: Residuals for ETS(A,N,N) Model

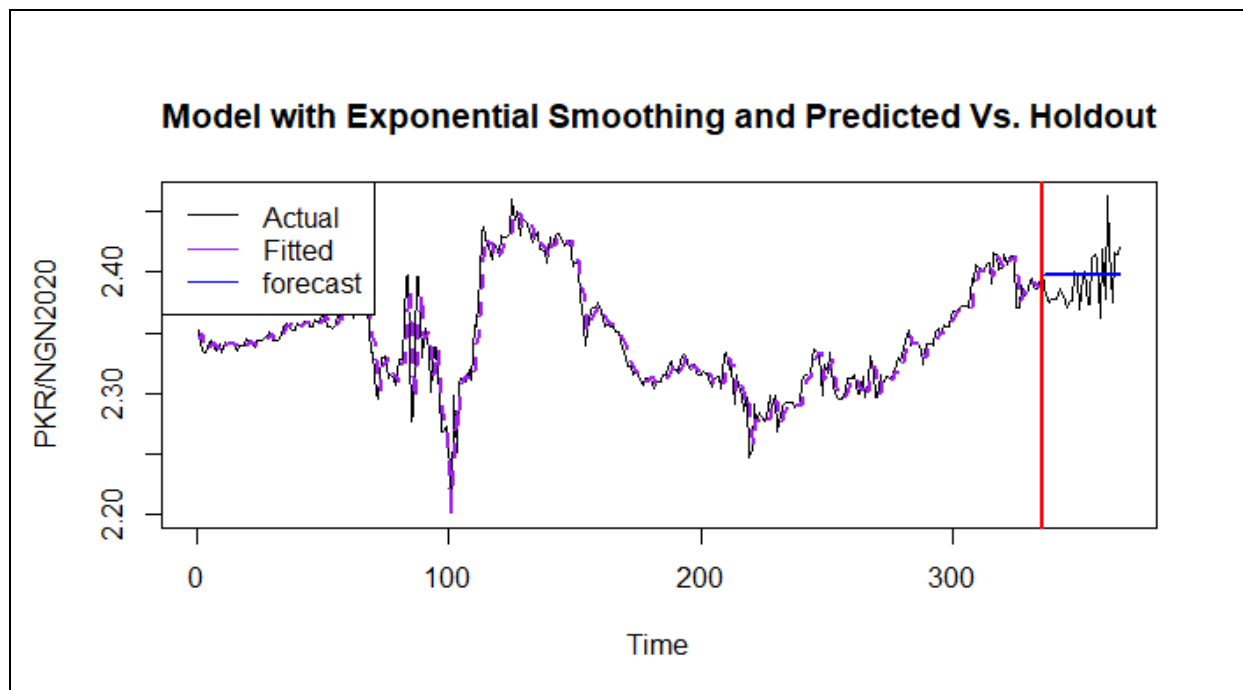


Figure 15: Fitted ETS(A,N,N)-INTERVENTION Model for PKR/NGN2020 ER

Appendix II

TABLE 1: UNIT ROOT TEST AT LEVEL (PKR/NGN2020 PIS)

Test	Augmented Dickey-Fuller
Data	PKR/NG2020
Dickey-Fuller	-0.96745
Lag order	4
P-value	0.9394
Alternative hypothesis	Stationary

TABLE 2: UNIT ROOT TEST AT FIRST DIFFERENCE (PKR/NGN2020 PIS)

Test	Augmented Dickey-Fuller
Data	PKR/NG2020
Dickey-Fuller	-6.0194
Lag order	4
P-value	0.01
Alternative hypothesis	Stationary

TABLE 3: PARAMETERS OF THE ESTIMATED MODELS (PKR/NGN2020 PIS)

ARIMA(p,d,q) Model		Estimate	Std. Error	z value	Prob. Value
(1,1,0)	AR1	-0.035065	0.100373	-0.3493	0.7268
(0,1,1)	MA1	-0.569663	0.097428	-5.847	5.004e-09 ***
(1,1,1)	AR1	0.42644	0.18392	2.3186	0.02042 *
	MA1	-0.81998	0.13677	-5.9953	2.031e-09 ***

TABLE 4: MODEL EVALUATION (PKR/NGN2020 PIS)

Model	BIC	AIC
ARIMA (1,1,0)	-465.7488	-470.939
ARIMA (0,1,1)	-474.2164	-479.4066
ARIMA (1,1,1)	-477.3847	-485.17

Table 5: Forecasts with ARIMA(1,1,1)Model (PKR/NGN2020 PIS)

DATE	Actual value (POSTIS)	Forecast	Residuals
2020/04/10	2.2042	2.278107	0.0739
2020/04/11	2.2988	2.285018	0.0138
2020/04/12	2.2515	2.287966	0.0365
2020/04/13	2.3088	2.289222	0.0196
2020/04/14	2.3123	2.289758	0.0225

Table 6: Parameter Estimation for the Full Intervention Model (PKR/NGN2020)

Parameter	Estimate	Std. Error	Z-value	P-value
AR(1)	0.2346545	0.0923854	2.5400	0.01109 *
MA(1)	-0.5934268	0.0703192	-8.4390	2.2e-16 ***
ω	-0.0344186	0.0059053	-5.8284	5.596e-09 ***
b	0			

Table 7: Estimated Intervention Parameters for $ETS(A, N, N) + Intervention Type$

Parameters	α	l	ω
Estimates	0.6342	2.3457	0.0658
Standard Errors	0.0558	0.0167	0.0210

Table 8: Model Evaluation for the Two Techniques

Model +Intervention Type	$\hat{\sigma}^2$	AIC	MAE	RMSE
$ARIMA(1,1,1)+\omega(B)B^bP_t^T$	0.0000499	-2571.076	0.004260986	0.007058458
$ETS(A,N,N)+\omega(B)B^bP_t^T$	0.0002657	-1805.494	0.02	0.023

Table 9: Forecasting with ARIMA(1,1,1)-INTERVENTION Model (PKR/NGN2020)

S/N	DATE	ACTUAL VALUE	FORECAST	95% PREDICTION INTERVAL	
				LOWER	UPPER
336	1-Dec-2020	2.3864	2.398713	2.365515	2.432376
337	2-Dec-2020	2.3773	2.397569	2.357521	2.438298
338	3-Dec-2020	2.3748	2.397302	2.352584	2.442869
339	4-Dec-2020	2.3774	2.397239	2.348537	2.446950
340	5-Dec-2020	2.3774	2.397224	2.344898	2.450718
341	6-Dec-2020	2.3774	2.397221	2.341523	2.454243
342	7-Dec-2020	2.3866	2.397220	2.338353	2.457569