The Approximation of One Step Block Approach for Simulation of Second Order Oscillatory Differential equations

Abstract

The approximation of the one step block method using the Linear Block approach (LBA) for simulation of second order oscillatory differential equations was examined in this research.
The basic properties of the new method were also analyzed and satisfied. Some distinct second order oscillatory differential equations were directly applied on the new method, the results obtained were compared with those in literature and the accuracy of the new method proved to be better as it outperformed those of existing methods. One of the advantage of the new method is that it does not require much computational burden and it is also self-starting.
Keywords: One step, computational burden, oscillatory differential equations, Linear Block Approach.

1. Introduction

Differential equations in oscillatory form are used to simulate a problem with various independent variables [1]. This field of study is an interesting and important issue from mathematics (numerical analysis) to simulate various phenomena in physics, chemistry, biology, engineering, or economics [2]. For example, the problem of transport phenomena, the simple harmonic motion, mass-spring systems, highly stiff oscillators and so on. All of these cases can be simulated in the form of differential equations (oscillatory) which arise because physical phenomena in scientific studies can be expressed by the rate of change [3, 4].

The motion in which repeats after a regular interval of time is called periodic motion.

The periodic motion in which there is existence of a restoring force and the body moves along the same path to and fro about a definite point called equilibrium position/mean position, is called oscillatory motion. In all type of oscillatory motion one thing is common i.e. each body (performing oscillatory motion) is subjected to a restoring force that increases with increase in displacement from mean position.

By a single number (called its magnitude) such as volume, mass, and temperature is called a scalar. They obey all the regular rules of algebraic addition, subtraction, multiplication, division, and so on. There are also physical quantities which require a magnitude and a direction for their complete speciation. These are called vectors if their combination with each other is commutative (that is the order of addition may be changed without ejecting the result). Angular displacement, for example, may be characterized by magnitude and direction but is not a vector, for the addition of two or more angular displacements is not, in general, commutative [5, 6].

Simple harmonic motion describes backward movement and forth through an equilibrium, or central, position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. The time interval of each complete vibration is the same [7-9]. The force responsible for the motion is always directed toward the equilibrium position and is directly proportional to the distance from it. That is,

$$F = -kx \qquad (1)$$

where F is the force, x is the displacement, and k is a constant. This relation is called Hooke's law.

Mathematical models in the field mentioned above are usually developed to understand the physical phenomena. These models are always resulted to differential equations. [10] stated some of the problems that involved differential equations in oscillatory form as

i. the problem arising from determining the projectile motion, satellite, rocket or planet,
ii. the problem of how to determine the charge or current in an electric circuit,
iii. the study of chemical reactions and
iv. the study of decomposition rate of radioactive substance or population growth rate.

The problems mentioned above obey certain scientific laws that involve rates of change of one or more quantities. Mathematically, these rates of change can be expressed by derivatives. When the problems are converted to mathematical equations they will form differential equations.

We shall use the Newton's law, **t**he laws of thermodynamics and Hooke's law as regards to this research.

In this research, the simulation of second order oscillatory differential equations

$$y'' = f(x, y, y'), \; y(0) = \delta_0, \; y'(0) = \delta_1 \qquad (2)$$

shall be proposed.

equation (2) play an essential role in solving every physical or biological process for the reason that such equations occur in connection with numerous problems that are encountered in several aspects of our everyday life [11]. Several specialists such as [12-15] developed an implicit second schemes in block form solving second order oscillatory differential equation (2). While [16-19] adopt the linear block approach to derive the block method using one step.

2. Numerical Derivation of the Method

This section describe the derivation of second order hybrid method using the linear block approach (LBA) developed by [16, 17]. In order to derive the method, the following corollary was consider. Considering the general form of the block method while implementing it one-by-one to obtain the expected block method for solving second order ordinary differential equations (2).

Corollary 1

Obtain the block method from the given expression

$$y_{n+\xi} = \sum_{i=0}^{2} \frac{(\xi h)^i}{i!} y_n^{(i)} + \sum_{i=0}^{1} \varphi_{i\xi} f_{n+i},\ \xi = \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8},\ 1 \tag{3}$$

Corollary 2

Obtain the first and second derivative schemes of the block method from

$$y_{n+\xi}^{(a)} = \sum_{i=0}^{2-a} \frac{(\xi h)^i}{i!} y_n^{(i+a)} + \sum_{i=0}^{7} \Omega_{\xi i a} f_{n+i},\ a = 1_{\left(\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8},\ 1\right)},\ a = 2_{\left(\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8},\ 1\right)} \tag{4}$$

$\varphi_{\xi i} = A^{-1}B$ and $\Omega_{\xi i a} = A^{-1}D$ where

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{h}{8} & \frac{3h}{8} & \frac{5h}{8} & \frac{7h}{8} & h \\ 0 & \frac{\left(\frac{h}{8}\right)^2}{2!} & \frac{\left(\frac{3h}{8}\right)^2}{2!} & \frac{\left(\frac{5h}{8}\right)^2}{2!} & \frac{\left(\frac{7h}{8}\right)^2}{2!} & \frac{(h)^2}{2!} \\ 0 & \frac{\left(\frac{h}{8}\right)^3}{3!} & \frac{\left(\frac{3h}{8}\right)^3}{3!} & \frac{\left(\frac{5h}{8}\right)^3}{3!} & \frac{\left(\frac{7h}{8}\right)^3}{3!} & \frac{(h)^3}{3!} \\ 0 & \frac{\left(\frac{h}{8}\right)^4}{4!} & \frac{\left(\frac{3h}{8}\right)^4}{4!} & \frac{\left(\frac{5h}{8}\right)^4}{4!} & \frac{\left(\frac{7h}{8}\right)^4}{4!} & \frac{(h)^4}{4!} \\ 0 & \frac{\left(\frac{h}{8}\right)^5}{5!} & \frac{\left(\frac{3h}{8}\right)^5}{5!} & \frac{\left(\frac{5h}{8}\right)^5}{5!} & \frac{\left(\frac{7h}{8}\right)^5}{5!} & \frac{(h)^5}{5!} \end{pmatrix},\ B = \begin{pmatrix} \frac{(\xi h)^2}{2!} \\ \frac{(\xi h)^3}{3!} \\ \frac{(\xi h)^4}{4!} \\ \frac{(\xi h)^5}{5!} \\ \frac{(\xi h)^6}{6!} \\ \frac{(\xi h)^7}{7!} \end{pmatrix},\ D = \begin{pmatrix} \frac{(\xi h)^{2-a}}{(2-a)!} \\ \frac{(\xi h)^{3-a}}{(3-a)!} \\ \frac{(\xi h)^{4-a}}{(4-a)!} \\ \frac{(\xi h)^{5-a}}{(5-a)!} \\ \frac{(\xi h)^{6-a}}{(6-a)!} \\ \frac{(\xi h)^{7-a}}{(7-a)!} \end{pmatrix}$$

The implementation of corollary 1 and equation (3), yield the following form

$$\left.\begin{aligned}
y_{n+\frac{1}{8}} &= y_n + \frac{h}{8}y'_n + \left[\varphi_{10} f_n + \varphi_{11} f_{n+\frac{1}{8}} + \varphi_{12} f_{n+\frac{3}{8}} + \varphi_{13} f_{n+\frac{5}{8}} + \varphi_{14} f_{n+\frac{7}{8}} + \varphi_{15} f_{n+1}\right] \\
y_{n+\frac{3}{8}} &= y_n + \frac{3h}{8}y'_n + \left[\varphi_{20} f_n + \varphi_{21} f_{n+\frac{1}{8}} + \varphi_{22} f_{n+\frac{3}{8}} + \varphi_{23} f_{n+\frac{5}{8}} + \varphi_{24} f_{n+\frac{7}{8}} + \varphi_{25} f_{n+1}\right] \\
y_{n+\frac{5}{8}} &= y_n + \frac{5h}{8}y'_n + \left[\varphi_{30} f_n + \varphi_{31} f_{n+\frac{1}{8}} + \varphi_{32} f_{n+\frac{3}{8}} + \varphi_{33} f_{n+\frac{5}{8}} + \varphi_{34} f_{n+\frac{7}{8}} + \varphi_{35} f_{n+1}\right] \\
y_{n+\frac{7}{8}} &= y_n + \frac{7h}{8}y'_n + \left[\varphi_{40} f_n + \varphi_{41} f_{n+\frac{1}{8}} + \varphi_{42} f_{n+\frac{3}{8}} + \varphi_{43} f_{n+\frac{5}{8}} + \varphi_{44} f_{n+\frac{7}{8}} + \varphi_{45} f_{n+1}\right] \\
y_{n+1} &= y_n + hy'_n + \left[\varphi_{50} f_n + \varphi_{51} f_{n+\frac{1}{8}} + \varphi_{52} f_{n+\frac{3}{8}} + \varphi_{53} f_{n+\frac{5}{8}} + \varphi_{54} f_{n+\frac{7}{8}} + \varphi_{55} f_{n+1}\right]
\end{aligned}\right\} \quad (5)$$

The implementation of corollary 2 and equation (4), yield the following form

$$\left.\begin{aligned}
y'_{n+\frac{1}{8}} &= y'_n + \left[\Omega_{101} f_n + \Omega_{111} f_{n+\frac{1}{8}} + \Omega_{121} f_{n+\frac{3}{8}} + \Omega_{131} f_{n+\frac{5}{8}} + \Omega_{141} f_{n+\frac{7}{8}} + \Omega_{151} f_{n+1}\right] \\
y'_{n+\frac{3}{8}} &= y'_n + \left[\Omega_{201} f_n + \Omega_{211} f_{n+\frac{1}{8}} + \Omega_{221} f_{n+\frac{3}{8}} + \Omega_{231} f_{n+\frac{5}{8}} + \Omega_{241} f_{n+\frac{7}{8}} + \Omega_{251} f_{n+1}\right] \\
y'_{n+\frac{5}{8}} &= y'_n + \left[\Omega_{301} f_n + \Omega_{311} f_{n+\frac{1}{8}} + \Omega_{321} f_{n+\frac{3}{8}} + \Omega_{331} f_{n+\frac{5}{8}} + \Omega_{341} f_{n+\frac{7}{8}} + \Omega_{351} f_{n+1}\right] \\
y'_{n+\frac{7}{8}} &= y'_n + \left[\Omega_{401} f_n + \Omega_{411} f_{n+\frac{1}{8}} + \Omega_{421} f_{n+\frac{3}{8}} + \Omega_{431} f_{n+\frac{5}{8}} + \Omega_{441} f_{n+\frac{7}{8}} + \Omega_{451} f_{n+1}\right] \\
y'_{n+1} &= y'_n + \left[\Omega_{501} f_n + \Omega_{511} f_{n+\frac{1}{8}} + \Omega_{521} f_{n+\frac{3}{8}} + \Omega_{531} f_{n+\frac{5}{8}} + \Omega_{541} f_{n+\frac{7}{8}} + \Omega_{551} f_{n+1}\right]
\end{aligned}\right\} \quad (6)$$

To get the unknown coefficients φ, it is defined that $\varphi_{\xi i} = A^{-1}B$ were A and B are defined above. Therefore, using equation (5) and corollary 1,

$$\left.\begin{aligned}
(\varphi_{10},\varphi_{11},\varphi_{12},\varphi_{13},\varphi_{14},\varphi_{15})^T &= \left(\frac{48281h^2}{11289600},\frac{1217h^2}{282240},-\frac{44051h^2}{3225600},\frac{1147h^2}{1612800},-\frac{1601h^2}{4515840},\frac{1391h^2}{11289600}\right) \\
(\varphi_{20},\varphi_{21},\varphi_{22},\varphi_{23},\varphi_{24},\varphi_{25})^T &= \left(\frac{17139h^2}{1254400},\frac{4905h^2}{100352},\frac{201h^2}{22400},-\frac{549h^2}{358400},\frac{117h^2}{250880},-\frac{171h^2}{1254400}\right) \\
(\varphi_{30},\varphi_{31},\varphi_{32},\varphi_{33},\varphi_{34},\varphi_{35})^T &= \left(\frac{9925h^2}{451584},\frac{45625h^2}{451584},\frac{8875h^2}{129024},\frac{25h^2}{8064},\frac{625h^2}{903168},-\frac{125h^2}{451584}\right) \\
(\varphi_{40},\varphi_{41},\varphi_{42},\varphi_{43},\varphi_{44},\varphi_{45})^T &= \left(\frac{7007h^2}{230400},\frac{14063h^2}{92160},\frac{31213h^2}{230400},\frac{26411h^2}{460800},\frac{49h^2}{5760},-\frac{343h^2}{230400}\right) \\
(\varphi_{50},\varphi_{51},\varphi_{52},\varphi_{53},\varphi_{54},\varphi_{55})^T &= \left(\frac{375h^2}{11025},\frac{79h^2}{441},\frac{263h^2}{1575},\frac{143h^2}{1575},\frac{67h^2}{2205},-\frac{37h^2}{2205}\right)
\end{aligned}\right\} \quad (7)$$

Similarly, to obtain the unknown coefficients Ω, it is defined that $\Omega_{\xi ia} = A^{-1}D$ were A and D are defined above. Therefore, using equation (6) and corollary 2,

$$\left.\begin{aligned}
(\Omega_{101},\Omega_{111},\Omega_{121},\Omega_{131},\Omega_{141},\Omega_{151})^T &= \left(\frac{9679h}{201600},\frac{14339h}{161280},-\frac{2203h}{115200},\frac{409h}{38400},-\frac{851h}{161280},\frac{41h}{22400}\right)\\
(\Omega_{201},\Omega_{211},\Omega_{221},\Omega_{231},\Omega_{241},\Omega_{251})^T &= \left(\frac{663h}{22400},\frac{3963h}{17920},\frac{1869h}{12800},-\frac{381h}{12800},\frac{213h}{17920},-\frac{87h}{22400}\right)\\
(\Omega_{301},\Omega_{311},\Omega_{321},\Omega_{331},\Omega_{341},\Omega_{351})^T &= \left(\frac{295h}{8064},\frac{2125h}{10752},\frac{1325h}{4608},\frac{515h}{4608},-\frac{125h}{10752},\frac{25h}{8064}\right)\\
(\Omega_{401},\Omega_{411},\Omega_{421},\Omega_{431},\Omega_{441},\Omega_{451})^T &= \left(\frac{889h}{28800},\frac{4949h}{23040},\frac{28469h}{115200},\frac{10633h}{38400},\frac{2779h}{23040},-\frac{49h}{3200}\right)\\
(\Omega_{501},\Omega_{511},\Omega_{521},\Omega_{531},\Omega_{541},\Omega_{551})^T &= \left(\frac{103h}{3150},\frac{22h}{105},\frac{58h}{225},\frac{58h}{225},\frac{22h}{105},\frac{103h}{3150}\right)
\end{aligned}\right\} \quad (8)$$

3 Properties of the Block Method

The properties of the block method shall be investigated to insure the convergence of the block method when solving the equation (2). The properties includes the order, error constant, consistency, zero stability and convergence [20].

3.1. Order and Error constant of the Method

Corollary 3 [21, 22]

Let the linear operator

$$l[y(x_n);h] \quad (9)$$

compared with the new method (5) and (6), with the truncation error $C_{06}h^{06}y^{06}(x_n)+0(h^{07})$.

Proof

We compared the linear difference operators (9) with the new method (5) and (6) as

$$\left.\begin{aligned}
l_{\frac{1}{8}}[y(x_n);h] &= y\left(x_n+\frac{1}{8}h\right)-\left(\alpha_r\left(x_n+\frac{1}{8}h\right)+\alpha_{\frac{3}{8}}\left(x_n+\frac{3}{8}h\right)+h^2\sum_{j=0}^{1}\left(\beta_j(x)f_{n+j}+\beta_\tau(x)f_{n+\tau}\right)\right)\\
l_{\frac{3}{8}}[y(x_n);h] &= y\left(x_n+\frac{3}{8}h\right)-\left(\alpha_r\left(x_n+\frac{1}{8}h\right)+\alpha_{\frac{3}{8}}\left(x_n+\frac{3}{8}h\right)+h^2\sum_{j=0}^{1}\left(\beta_j(x)f_{n+j}+\beta_\tau(x)f_{n+\tau}\right)\right)\\
l_{\frac{5}{8}}[y(x_n);h] &= y\left(x_n+\frac{5}{8}h\right)-\left(\alpha_r\left(x_n+\frac{1}{8}h\right)+\alpha_{\frac{3}{8}}\left(x_n+\frac{3}{8}h\right)+h^2\sum_{j=0}^{1}\left(\beta_j(x)f_{n+j}+\beta_\tau(x)f_{n+\tau}\right)\right)\\
l_{\frac{7}{8}}[y(x_n);h] &= y\left(x_n+\frac{7}{8}h\right)-\left(\alpha_r\left(x_n+\frac{1}{8}h\right)+\alpha_{\frac{3}{8}}\left(x_n+\frac{3}{8}h\right)+h^2\sum_{j=0}^{1}\left(\beta_j(x)f_{n+j}+\beta_\tau(x)f_{n+\tau}\right)\right)\\
l_1[y(x_n);h] &= y(x_n+h)-\left(\alpha_r\left(x_n+\frac{1}{8}h\right)+\alpha_{\frac{3}{8}}\left(x_n+\frac{3}{8}h\right)+h^2\sum_{j=0}^{1}\left(\beta_j(x)f_{n+j}+\beta_\tau(x)f_{n+\tau}\right)\right)
\end{aligned}\right\} \quad (10)$$

$$\tau=\frac{1}{8},\frac{3}{8},\frac{5}{8},\frac{7}{8}$$

Corollary 4 [22]

To find the local truncation error of (5) and (6), we assume $y(x)$ to be sufficiently differentiable and expanding $y(x_n + qh)$ and $y(x_n + jh)$ about x_n using Taylor series. Collect the like terms (the coefficient of h) to obtain the expressions for the local truncation error of (10) as

$$l_{\frac{1}{8}}[y(x_n);h] = -\frac{1217}{19818086400}h^{06}y^{(06)}(x_n) + 0(h^{07})$$
$$l_{\frac{3}{8}}[y(x_n);h] = \frac{67}{734003200}h^{06}y^{(06)}(x_n) + 0(h^{07})$$
$$l_{\frac{5}{8}}[y(x_n);h] = -\frac{25}{792723456}h^{06}y^{(06)}(x_n) + 0(h^{07})$$
$$l_{\frac{7}{8}}[y(x_n);h] = \frac{343}{28315200}h^{06}y^{(06)}(x_n) + 0(h^{07})$$
$$l_1[y(x_n);h] = \frac{37}{619315200}h^{06}y^{(06)}(x_n) + 0(h^{07})$$

Thus, from the above results, the order of the method (5) and (6) is 6, and the error constants is

$$C = \left[-\frac{1217}{19818086400} \quad \frac{67}{734003200} \quad -\frac{25}{792723456} \quad \frac{343}{283155200} \quad \frac{37}{619315200}\right]^T.$$

3.2 Consistency of the Method

Definition: According to [21], a block method is said to be consistent if its order is greater than or equal to one. From the above analysis, it is obvious that our method is consistent.

3.3 Zero Stability of the Method

Definition: The numerical method is said to be zero-stable, if the roots $m_s,\ s = 1, 2, \cdots, k$ of the first characteristics polynomial $\rho(m)$ defined by $\rho(m) = \det\left(mA^{(0)} - E\right)$ satisfies $|m_s| \le 1$ and every root satisfies $|z_s| = 1$ have multiplicity not exceeding the order of the differential equation. The first characteristic polynomial is given by,

$$\rho(m) = \left| m\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right| = \left\| \begin{bmatrix} m & 0 & 0 & 0 & -1 \\ 0 & m & 0 & 0 & -1 \\ 0 & 0 & m & 0 & -1 \\ 0 & 0 & 0 & m & -1 \\ 0 & 0 & 0 & 0 & m-1 \end{bmatrix} \right\| = m^4(m-1)$$

Thus, solving for q in $m^7 - m^6$ gives $m = 0, 0, 0, 0, 1$. Hence the method is said to be zero stable [21].

3.4 Convergence of the Block Method

Theorem: the necessary and sufficient conditions for linear multistep method to be convergent are that it must be consistent and zero-stable. Hence our method is convergent according to [22].

Region of Absolute Stability of our Method

Definition: the region of absolute stability is the region of the complex z plane, where $z = \lambda h$ for which the method is absolute stable. To determine the region of absolute stability of the block method, the methods that compare neither the computation of roots of a polynomial nor solving of simultaneous inequalities was adopted. Thus, the method according to [17] is called the boundary locus method. Applying the method we obtain the region of absolute stability in as

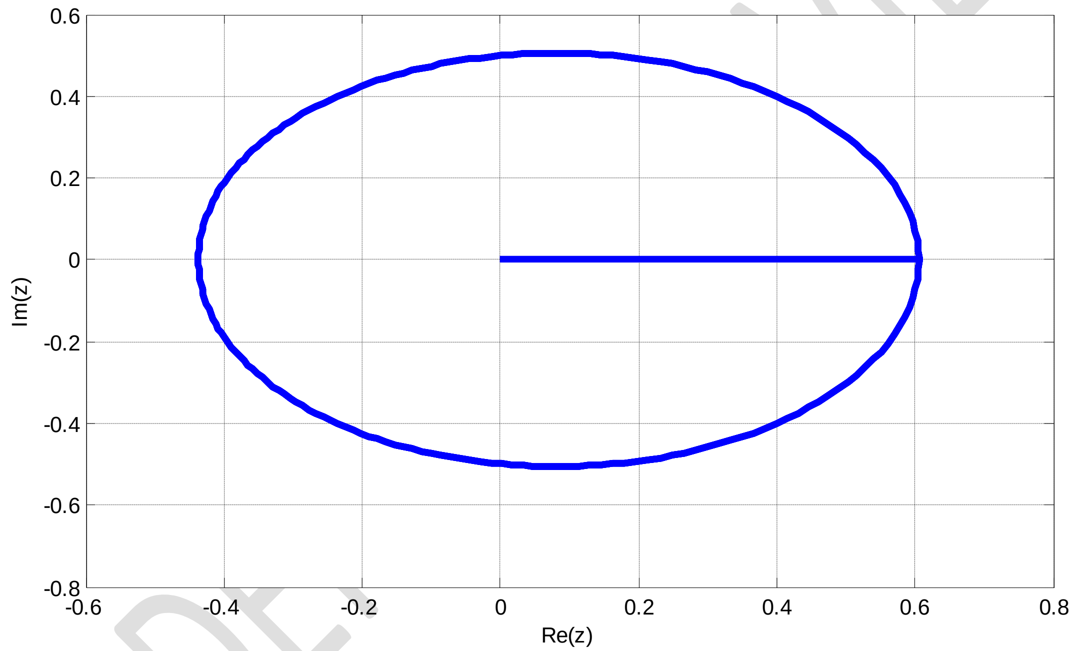

Figure 1: Region of Absolute Stability of our method

4 Simulation of the Method

In this section, the newly derived method is used to simulate some oscillatory differential equation of the form (2). The first oscillatory differential equation would be carried out to determine the type of spring in a motion. Secondly, nonlinear oscillatory differential equation and lastly, the highly stiff oscillatory differential equation is been simulated. The class of oscillatory differential equation is used to validate the accuracy and convergence of the newly derived method.

The following notation is use in the tables and figures.

ES: Exact Solution

CS: Computed Solution

ENM: Error in New Method

EAR23: Error in [23]

EAO24: Error in [24]

EAO25: Error in [25]

EOM26: Error in [26]

EJL27: Error in [27]

EAO28: Error in [28]

4.1 Numerical Examples

Example 1:

Consider the second order oscillatory differential equation that was set up in motion.

An object stretches a spring 6 inches in equilibrium.

i. Set up the equation of motion and find its general solution.

ii. Find the displacement of the object for $t > 0$, if it's initially displaced 18 inches above equilibrium and given a downward velocity of $3\frac{ft}{s}$.

From Newton's second law of motion, we have

$$my'' + cy' + ky = F \tag{11}$$

By setting $c = 0$ and $F = 0$, we get

$$my'' + ky = 0 \Rightarrow y'' + \frac{k}{m}y = 0 \tag{12}$$

The equation of the weight of the object is given as follow:

$$mg = k\Delta l \Rightarrow \frac{k}{m} = \frac{g}{\Delta l} \tag{13}$$

Substituting $g = 32\frac{ft}{s^2}$, $\Delta l = \frac{6}{12}ft$ into (13) we obtain

$$\frac{k}{m} = \frac{32}{\frac{6}{12}} = 64 \tag{14}$$

Substituting equation (14) into the equation (12) we get

$$y'' + 64y = 0 \tag{15}$$

The initial upward displacement of 18 inches is positive and must be expressed in feet. The initial downward velocity is negative; thus, $y(0)=\frac{3}{2}$, $y'(0)=-3$ and $h=0.1$. We make use of (15) as

$$dsolver\left(\left\{y''(x)+64y(x)=0,\ y(0)=\frac{3}{2},\ y'(0)=-3\right\}\right) \quad (16)$$

We obtain the exact solution (16) as

$$y(x)=-\frac{3}{8}\sin(8x)+\frac{3}{2}\cos(8x) \quad (17)$$

Source [23]

Example 2:

Consider the oscillatory differential equation

$$y''+1001y+1000y=0,\ y(0)=1,\ y'(0)=-1, \quad (18)$$

Whose exact solution is

$$y(x)=\exp(-x) \quad (19)$$

See [24, 25]

Example 3:

Consider the nonlinear oscillatory differential equation

$$y''-4y'+8y=x^3,\ y(0)=2,\ y'(0)=4, \quad (20)$$

Whose exact solution is

$$y(x)=\exp(2x)\left(2\cos(2x)-\frac{3}{64}\sin(2x)\right)+\frac{3x}{32}+\frac{3x^2}{16}+\frac{x^2}{8} \quad (21)$$

Source: [26, 27].

Example 4:

Consider the nonlinear oscillatory differential equation

$$y''=-y,\ y(0)=y'(0)=1, \quad (22)$$

Whose exact solution is

$$y(x)=\cos x+\sin x \quad (23)$$

Source: [28]

5. Results and Discussion

Table 1: Comparison between the new method and existing method for problem 1

x	ES	CS	ENM	EAR 23
0.1	0.77605152993342709579	0.77605152389579373913	6.0376(-09)	3.3496(-07)
0.2	- 0.41863938459249752594	- 0.41863940002060177343	1.5428(-08)	1.6371(-06)
0.3	- 1.3593892660185498469	- 1.35938927769431688740	1.1678(-08)	3.2716(-06)
0.4	- 1.4755518599067871611	- 1.47555184845953636160	1.1447(-08)	3.5979(-06)
0.5	- 0.69666449555494477770	- 0.69666445459004623494	4.0965(-08)	1.3589(-06)
0.6	0.50481020347261010590	0.50481025540395725921	5.1931(-08)	2.9143(-06)
0.7	1.4000738069674951883	1.40007383380115662890	2.6834(-08)	6.7226(-06)
0.8	1.4460714263183540043	1.44607139912130218950	2.7197(-08)	7.0589(-06)
0.9	0.61490152285494961183	0.61490144505266366544	7.7802(-08)	2.6543(-06)
1.0	- 0.58925939319668845548	- 0.58925947996884071070	8.6772(-08)	4.6056(-06)

Table 2: Comparison between the new method and existing methods for problem 2

x	ES	CS	ENM	EAO 24	EAO 25
0.1	0.90483741803595957316	0.90483741803595952927	4.3890(-17)	1.0547(-14)	2.0500(-11)
0.2	0.81873075307798185867	0.81873075307798182897	2.9700(-17)	1.7764(-14)	4.3900(-11)
0.3	0.74081822068171786607	0.74081822068171781989	4.6180(-17)	2.3426(-14)	6.5500(-11)
0.4	0.67032004603563930074	0.67032004603563925767	4.3070(-17)	2.7978(-14)	8.3800(-11)
0.5	0.60653065971263342360	0.60653065971263337424	4.9360(-17)	3.1308(-14)	9.8600(-11)
0.6	0.54881163609402643263	0.54881163609402638382	4.8810(-17)	3.3973(-14)	1.1000(-10)
0.7	0.49658530379140951470	0.49658530379140946379	5.0910(-17)	3.5638(-14)	1.1900(-10)
0.8	0.44932896411722159143	0.44932896411722154097	5.0460(-17)	3.6748(-14)	1.2400(-10)
0.9	0.40656965974059911188	0.40656965974059906128	5.0600(-17)	3.7304(-14)	1.2800(-10)
1.0	0.36787944117144232160	0.36787944117144227189	4.9710(17)	3.7415(-14)	1.3000(-10)

Table 3: Comparison between the new method and existing methods for problem 3

x	ES	CS	ENM	EOM 26	EJL 27
0.1	2.3941125769963956181	2.39411257699352383890	2.8718(-12)	7.1426(-08)	5.1070(-06)
0.2	2.7481413324264235256	2.74814133241247383060	1.3949(-11)	1.7491(-07)	1.4959(-05)
0.3	3.0078669405110678859	3.00786694047387329760	3.7195(-11)	3.6449(-07)	2.7853(-05)

0.4	3.1017624057742078185	3.10176240569717560410	7.7032(-11)	6.1898(-07)	4.2891(-05)
0.5	2.9395431007452620774	2.93954310060732702720	1.3794(-10)	6.9889(-07)	6.7031(-05)
0.6	2.4118365344157147255	2.41183653419197661580	2.2374(-10)	1.4794(-06)	1.0264(-04)
0.7	1.3915548304898433104	1.39155483015321951580	3.3662(-10)	2.1022(-06)	1.4491(-04)
0.8	- 0.262326758334357631	- 0.26232675881007383216	4.7572(-10)	2.8409(-06)	1.9091(-04)
0.9	- 2.697771160773070925	- 2.69777116140831225640	6.3524(-10)	3.6689(-06)	2.3973(-04)
1.0	- 6.058560720845666951	- 6.05856072164790713340	8.0224(-10)	4.5617(-06)	2.9467(-04)

Table 4: Comparison between the new method and existing method for problem 4

x	**ES**	**CS**	**ENM**	**EAO 28**
0.1	1.09483758192485391840	1.09483758192485360470	3.1370(-16)	1.1570(-07)
0.2	1.17873590863630284660	1.17873590863630156920	1.2774(-15)	3.0990(-07)
0.3	1.25085669578694559480	1.25085669578694266340	2.9314(-15)	5.0550(-07)
0.4	1.31047933631153557450	1.31047933631153027220	5.3023(-15)	6.9570(-07)
0.5	1.35700810049457571640	1.35700810049456731460	8.4018(-15)	8.7890(-07)
0.6	1.38997808830471365440	1.38997808830470142760	1.2227(-14)	1.0540(-06)
0.7	1.40905987452217947990	1.40905987452216272100	1.6759(-14)	1.0080(-06)
0.8	1.41406280024668818260	1.41406280024666621860	2.1964(-140	9.2260(-06)
0.9	1.40493687789814784490	1.40493687789812005180	2.7793(-14)	8.2610(-06)
1.0	1.38177329067603622400	1.38177329067600204150	3.4183(-14)	7.2160(-06)

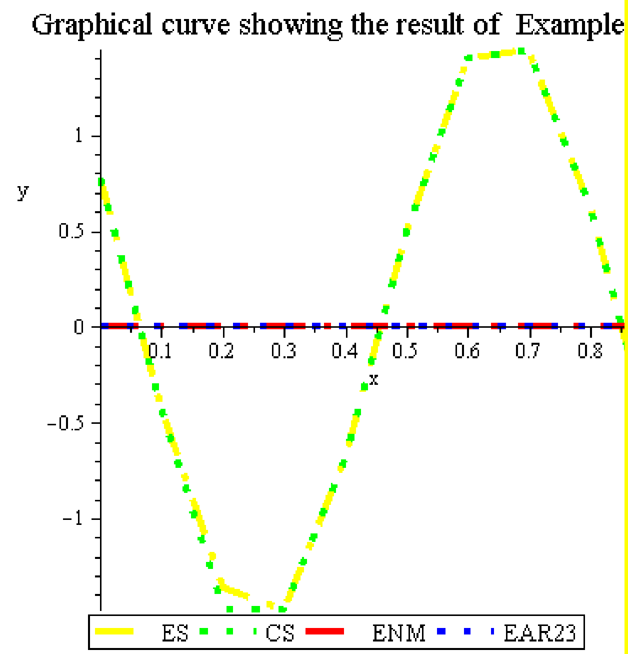

Figure 2: Comparison between the new method and [23]

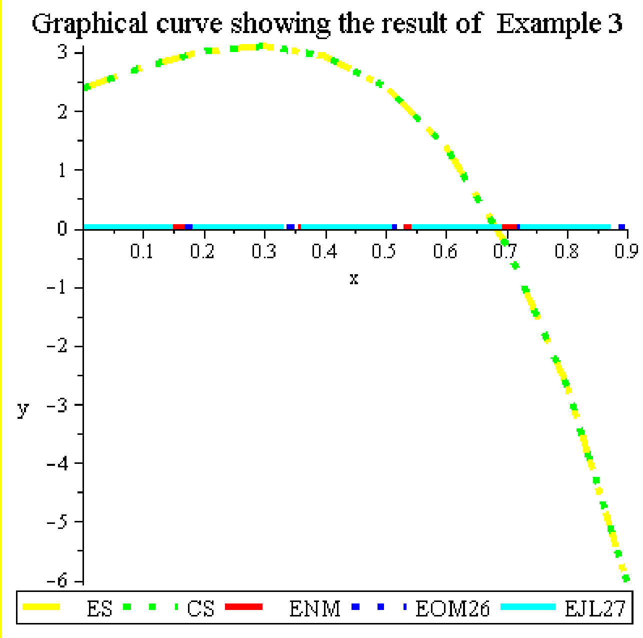

Figure 4: Comparison between the new method and [26, 27]

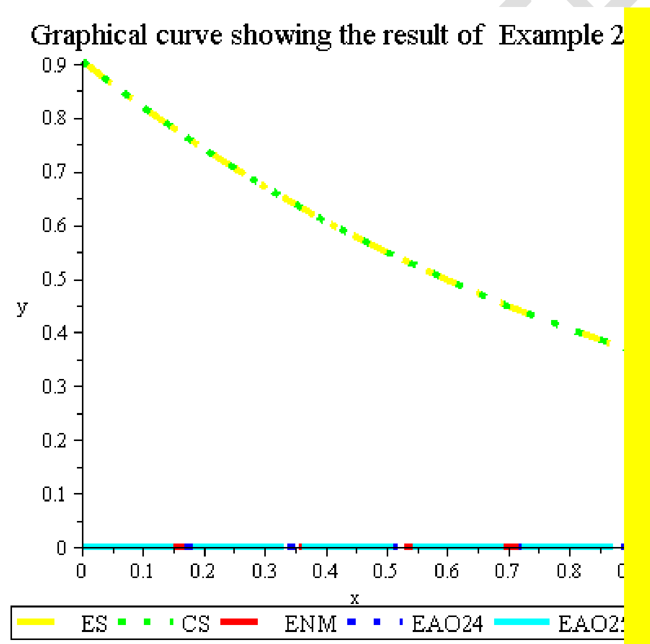

Figure 3: Comparison between the new method and [24, 25]

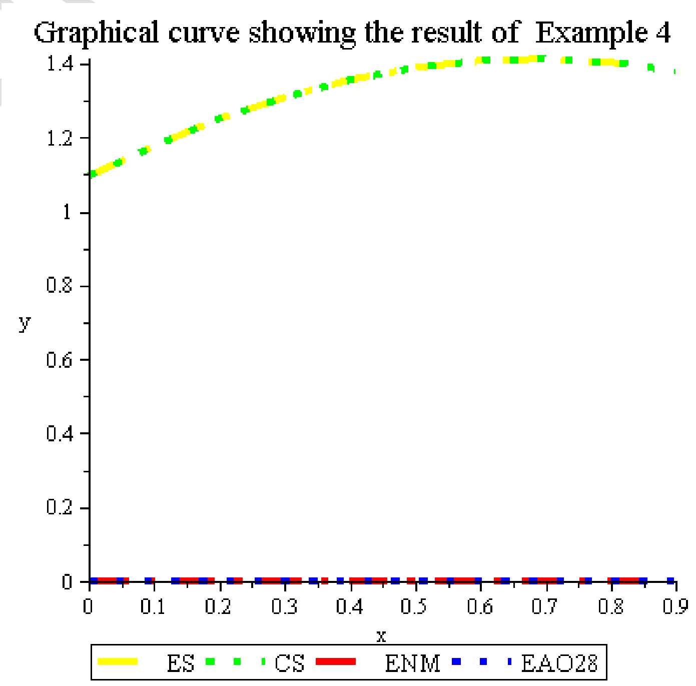

Figure 5: Comparison between the new method and [28]

The new method was applied to solve some special second order oscillatory differential equation (2). [23] proposed an efficient one-eight step hybrid block method for solving (16, 17), the new method shows better than convergence than [23] (see table 1 and figure 2). In literature, [24, 25] proposed a methods that solved (18, 19). When comparing, the new method is converges faster (see table 2 and figure 3). Also, the hybrid multistep methods with Legendre basic function developed by [26] and a self-starting linear multistep method proposed by [27] for solving (20, 21), their methods diverges while the new method converges (see table 3 and figure 4). Finally, example 4 was applied on the new method and compared with the hybrid block method developed by [28] using Taylor series expansions (see table 4 and figure 5). Therefore, the new method performs better than the existing methods we considered.

6. Conclusion

The new method was derived using the linear block approach (LBA) proposed by [16, 17] which is quite straight forward to adopt. The analysis of the basic properties of the new method, viz. order, error constant, consistency, zero-stable, convergence, and region of absolute stability were analyzed and satisfied. The new methods was applied on some special second order oscillatory differential equation and compared with existing once in literature. Hence, the new method proved better accuracy and faster convergence than the existing methods considered.

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