*Original Research Article*

# APPLICATION OF ALPHA POWER CUBIC TRANSMUTED DAGUM DISTRIBUTION TO REAL-LIFE DATA

**Abstract**:
In this article, a new model is proposed called alpha power cubic transmuted distribution. Some properties of the new distribution such as survival and hazard functions and other useful measures were obtained. The model parameters were estimated using the method of maximum likelihood. The usefulness of the proposed distribution has been shown using three real datasets. This indicates that the proposed compound distribution would be useful in the area of distribution theory in Statistics, and in modeling real life data sets relating to biology, demography, geology, economics, environmental sciences, engineering, finance, medicine, hydrology, insurance and many other fields.

**Keywords**: Alpha power transformation, Dagum distribution, Cubic transmuted Dagum distribution, Statistical properties, parameter estimation, Applications.



## 1 Introduction

The Dagum distribution was proposed by [1]. His proposals brought about the improvement of statistical distributions for fitting empirical income and wealth data, that could be adequate for both heavy tails in empirical income and wealth distributions, and also allow for interior mode. Dagum distribution has two types; the Type-I involves three parameter and Type-II deals with four parameters. Dagum in 1977 motivated his model from empirical observation that the income elasticity of the cumulative distribution function (cdf) of income into a decreasing and bounded function of F.
Recently, very useful families of probability distributions have been proposed in the literature and it has been shown that they are useful for adding skewness and flexibility to other models. A brief summary of the recently proposed families include the a new generalized Weibull-G family by [2], Logistic-X family by [3], a new Weibull-G family by [4], a Lindley-G family by [5], a Gompertz-G family by [6], an odd Lindley-G family by [7], Alpha Power transformation by [8], odd Lomax generator of distributions (Odd Lomax-G family) by [9] and Poisson-X family by [10] as so on.

A good research into these families of distributions has led to the proposition of some new distributions which include; the exponential-Lindley distribution by [11], transmuted normal distribution by [12], alpha power Weibull distribution by [13], odd Lindley-Rayleigh distribution by [14], the transmuted odd Lindley-Rayleigh distribution by [15], alpha power generalized exponential distribution by [16], odd Lindley-inverse exponential distribution by [17] and cubic transmuted Dagum distribution by [18], etc.

According to [18], “the cubic transmuted Dagum distribution based on the cubic transmuted family was judged to be better than the transmuted Dagum distribution and the conventional Dagum distribution after its application to real life data sets”.

Following the above assessment of the cubic transmuted Dagum distribution, this article will develop an extension of the distribution (cubic transmuted Dagum distribution) using the alpha power transformation proposed by [8] with the assertion that it will give better result when applied to real life datasets.

From [18], the cumulative distribution function (cdf) and probability density function (pdf) of the cubic transmuted Dagum distribution are given by:

$$F(x) = (1-\lambda)\left(1+\gamma x^{-\theta}\right)^{-\beta} + 3\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta} - 2\lambda\left(1+\gamma x^{-\theta}\right)^{-3\beta} \tag{1}$$

and

$$f(x) = \gamma\theta\beta x^{-\theta-1}\left(1+\gamma x^{-\theta}\right)^{-\beta-1}\left[1-\lambda+6\lambda\left(1+\gamma x^{-\theta}\right)^{-\beta} - 6\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta}\right] \tag{2}$$

respectively. Where, $x > 0, \gamma > 0, \theta > 0, \beta > 0, -1 \le \lambda \le 1$ and $\gamma$ is a scale parameter while $\theta$ and $\beta$ are the shape parameters and $\lambda$ is the transmuted parameter of the cubic transmuted Dagum distribution (CTDaD).

The remaining contents of this article are given in sections, the proposed distribution is defined in section two, the properties of the proposed distribution are presented in section three and the maximum likelihood approach to estimating its parameters is outlined in section four. An application of the proposed distribution to real-life datasets is provided in section five and a summary and conclusion is presented in section six.

## 2. Methods and Materials/Methodology

### 2.1 Definition of Apha Power Cubic Transmuted Dagum distribution (APCTDaD)

According to [8], the cumulative distribution function (cdf) and the probability density function (pdf) of the Alpha Power transformed family of distributions are defined as:

$$F(x) = \frac{\alpha^{G(x)} - 1}{\alpha - 1} \tag{3}$$

and

$$f(x)=\frac{\log(\alpha)}{(\alpha-1)}g(x)\alpha^{G(x)} \quad (4)$$

"respectively, where $g(x)$ and $G(x)$ stand for the pdf and cdf of the cubic transmuted Dagum distribution or any continuous distribution to be extended respectively while $\alpha>0$ $(\alpha\neq1)$ is the power/shape parameter of the alpha power family which helps induce additional skewness and flexibility in the modified model.

Substituting equation (1) and (2) into equation (3) and (4) above and simplifying, the cdf and pdf of APCTDaD are obtained respectively as:

$$F(x)=\frac{\alpha^{\left((1-\lambda)\left(1+\gamma x^{-\theta}\right)^{-\beta}+3\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta}-2\lambda\left(1+\gamma x^{-\theta}\right)^{-3\beta}\right)}-1}{\alpha-1} \quad (5)$$

and

$$f(x)=\frac{\log(\alpha)\gamma\theta\beta x^{-\theta-1}\left(1+\gamma x^{-\theta}\right)^{-\beta-1}\left[1-\lambda+6\lambda\left(1+\gamma x^{-\theta}\right)^{-\beta}-6\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta}\right]}{(\alpha-1)\alpha^{-(1-\lambda)\left(1+\gamma x^{-\theta}\right)^{-\beta}+3\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta}-2\lambda\left(1+\gamma x^{-\theta}\right)^{-3\beta}}} \quad (6)$$

Where, $x,\alpha,\theta,\gamma,\beta>0,-1\leq\lambda\leq1$ and $\alpha\neq1$ is a power parameter and $\gamma$ is a scale parameter while $\theta$ and $\beta$ are the shape parameters and $\lambda$ is the cubic transmuted parameter of the cubic transmuted Dagum distribution (CTDaD).

### 2.3 Graphical Presentation of Pdf and Cdf of APCTDaD

The plots pdf and cdf of the APCTDaD using some parameter values are displayed in **figure 1** and figure **2** as follows:

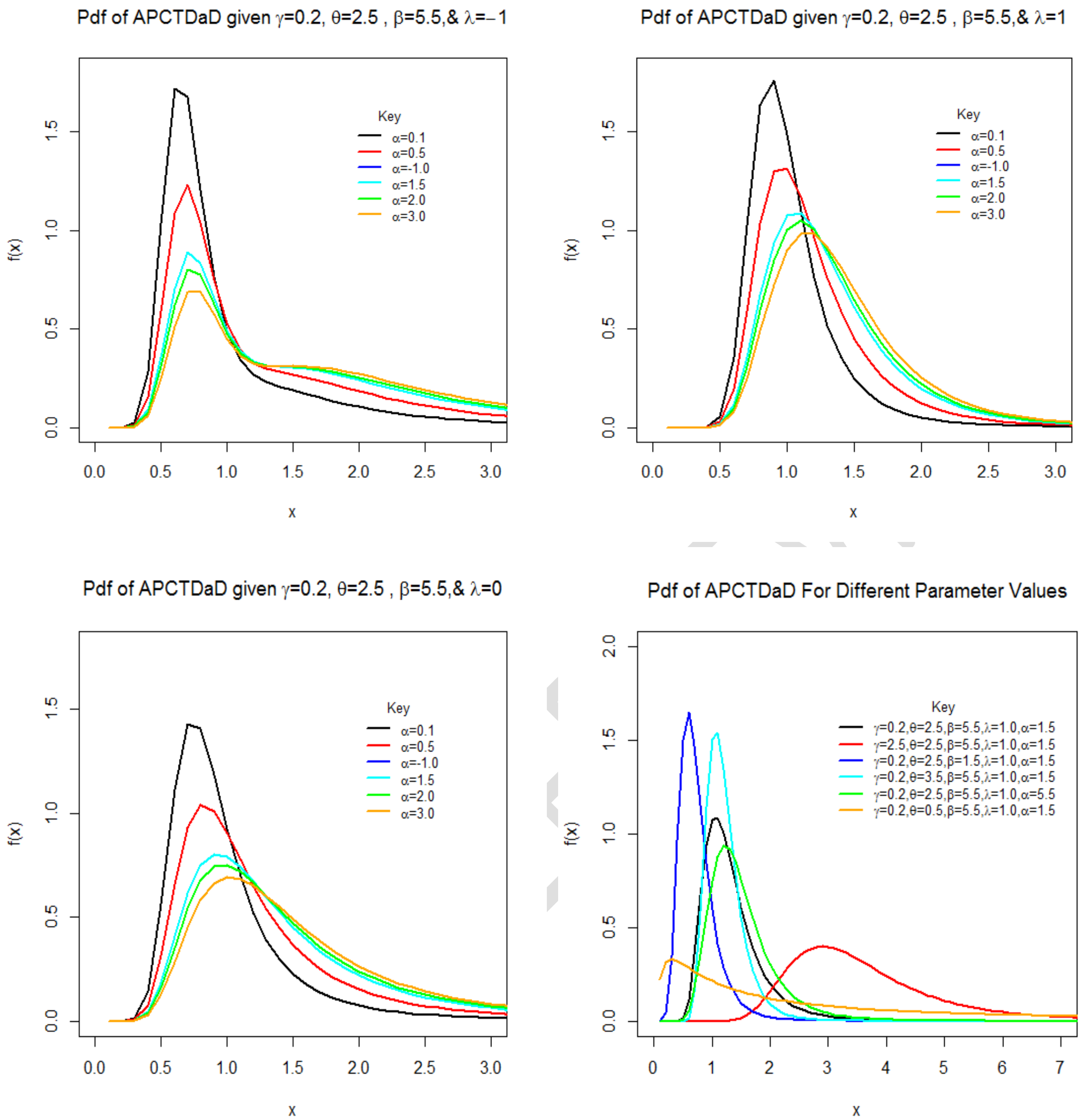


**Fig**. 1: PDF of the APCTDaD.

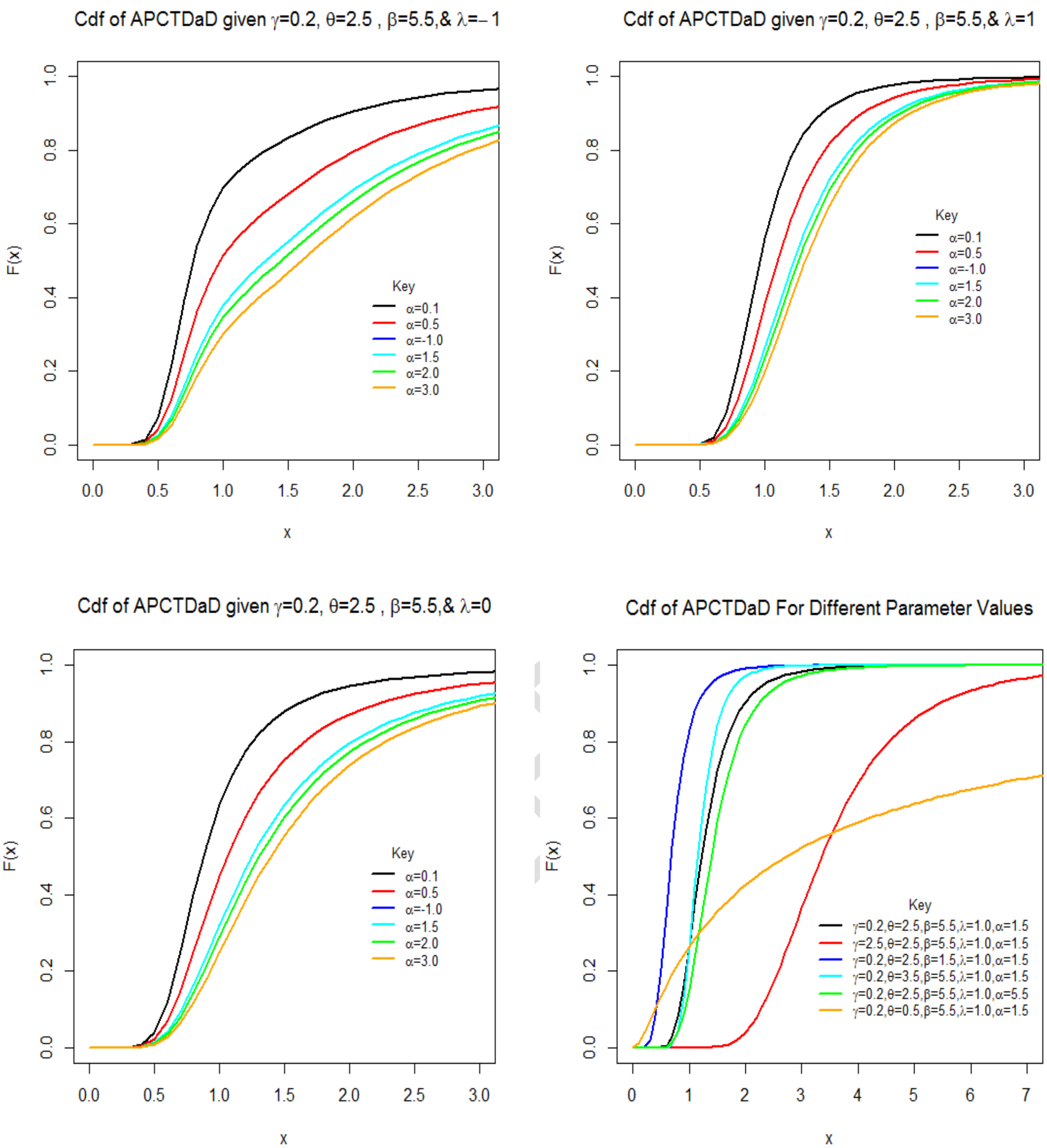


**Fig**. 2: CDF of the APCTDaD.

Considering the plot of the pdf of the ATCTDaD in figure 1 above, it is clear that the distribution is skewed and flexible and that its shape depend on the values of the parameters. It is also clear that the plots of the pdf and cdf are in line with the limiting behavior of a probability distribution.

## 2.4 Reliability analysis of the APCTDaD.

A Survival function states the chances that a system will perform its designed purpose within a specified time limit. It is defined by the following equation:

$$S(x) = 1 - F(x) \tag{7}$$

Utilizing the cdf of the proposed APCTDaD in (7), the survival function for APCTDaD is obtained as:

$$S(x) = 1 - \frac{\alpha^{\left((1-\lambda)\left(1+\gamma x^{-\theta}\right)^{-\beta} + 3\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta} - 2\lambda\left(1+\gamma x^{-\theta}\right)^{-3\beta}\right)} - 1}{\alpha - 1} \tag{8}$$

A plot of the function expressed in equation (8) is given in figure **3** below:

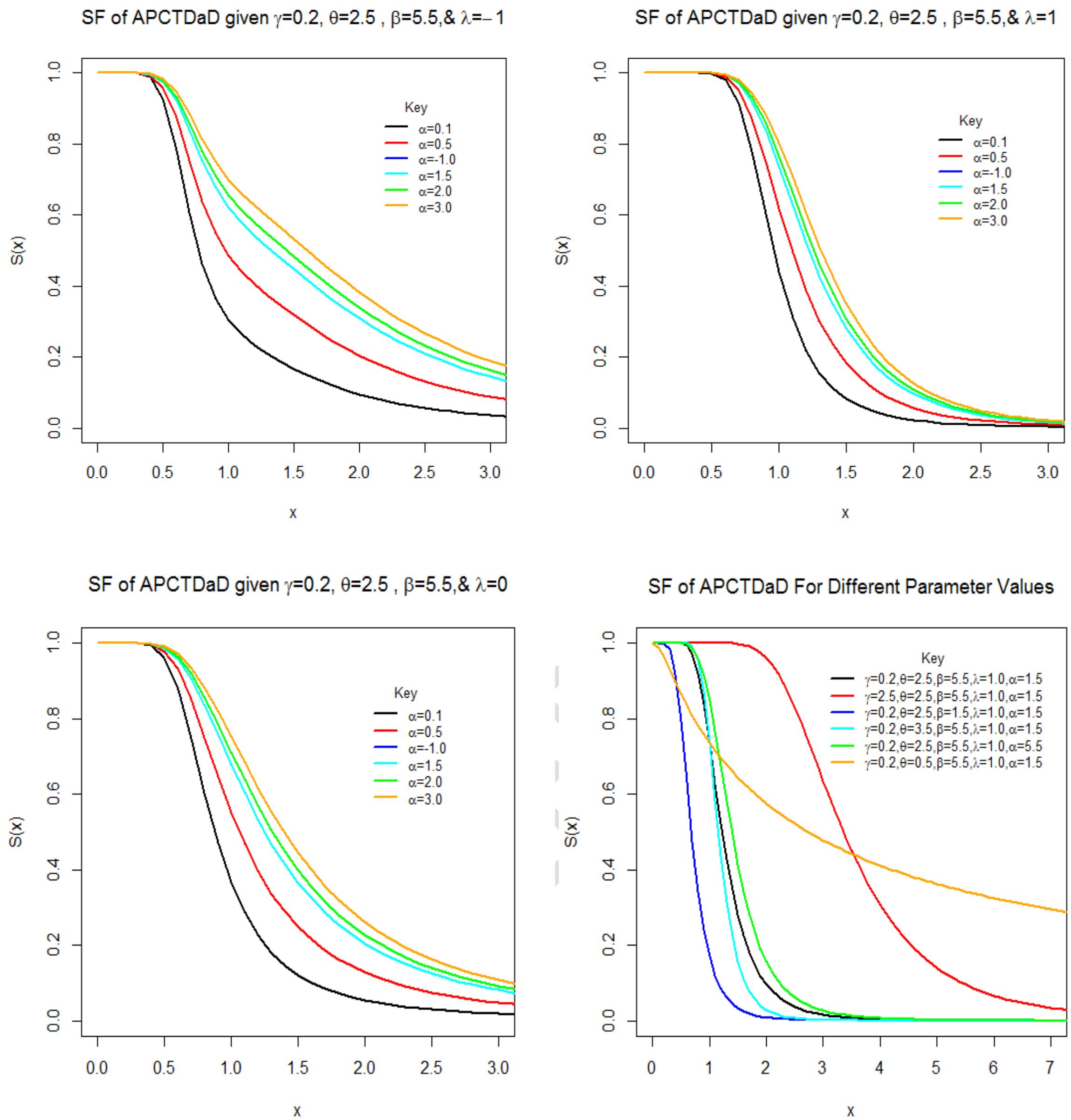


**Figure 3.**: Survival function of APCTDaD.

Based on the plots of the survival function in figure 3 above, it can be seen that the function is decreasing, that is probability of survival decreases over time or survival rate decreases with increase in age or time.

Hazard function: The hazard function describes the likelihood that a unit or component will fail within an interval of time. It is defined mathematically as:

$$h(x)=\frac{f(x)}{S(x)}=\frac{f(x)}{1-F(x)} \tag{9}$$

Hence, the hazard rate for the APCTDaD is given by:

$$h(x)=\frac{\log(\alpha)\gamma\theta\beta x^{-\theta-1}\left(1+\gamma x^{-\theta}\right)^{-\beta-1}\left[1-\lambda+6\lambda\left(1+\gamma x^{-\theta}\right)^{-\beta}-6\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta}\right]}{\dfrac{(\alpha-1)\alpha^{-(1-\lambda)\left(1+\gamma x^{-\theta}\right)^{-\beta}+3\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta}-2\lambda\left(1+\gamma x^{-\theta}\right)^{-3\beta}}}{1-\dfrac{\alpha^{\left((1-\lambda)\left(1+\gamma x^{-\theta}\right)^{-\beta}+3\lambda\left(1+\gamma x^{-\theta}\right)^{-2\beta}-2\lambda\left(1+\gamma x^{-\theta}\right)^{-3\beta}\right)}-1}{\alpha-1}}} \qquad (10)$$

The following is a plot of the hazard function for some chosen parameter values

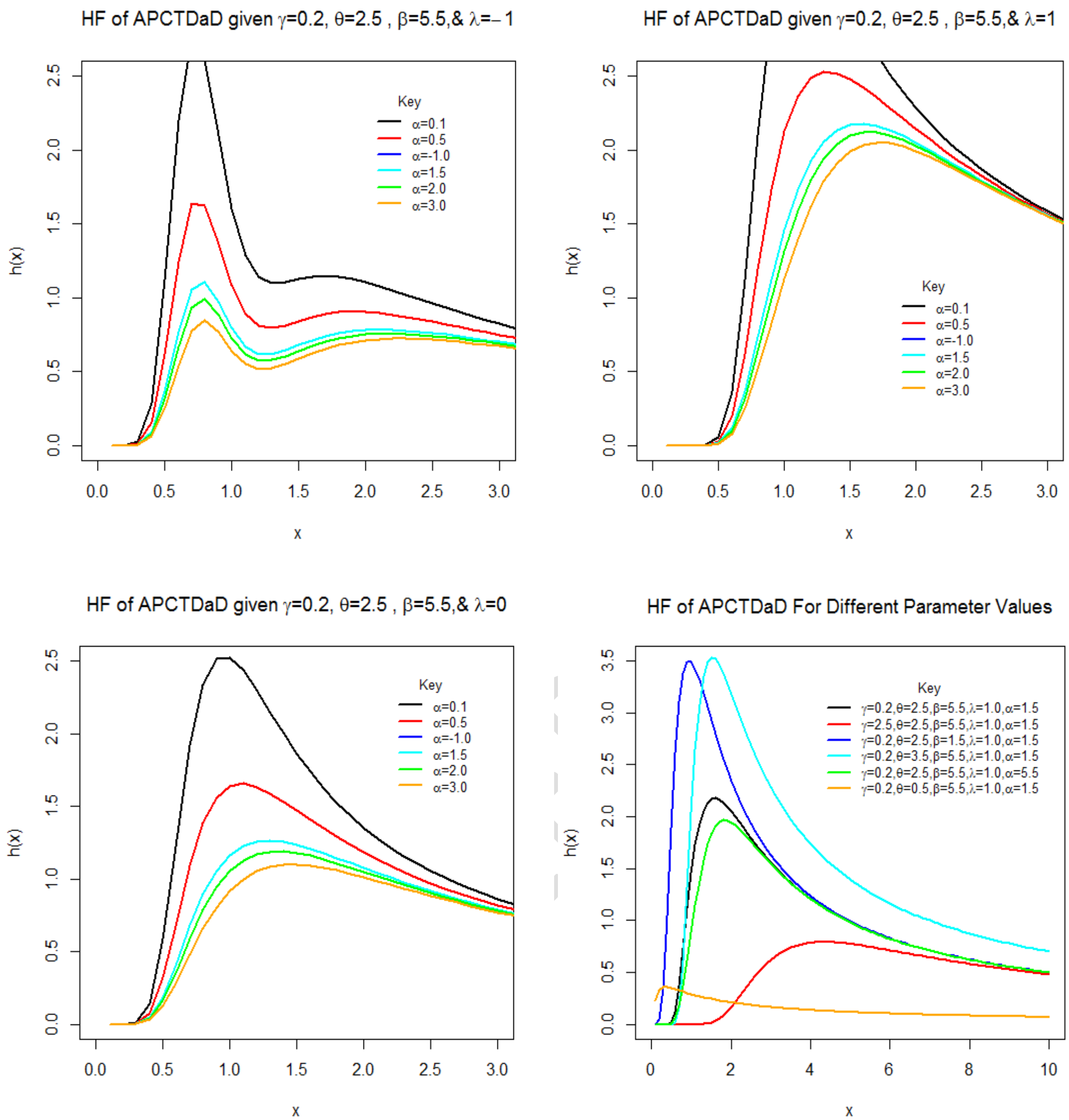


**Figure 4**: Hazard function of APCTDaD.

From figure 4 it can be seen that the plots of the hazard rate of the distribution are upside down bathtub hazard rate, meaning that the proposed model would be appropriate for analyzing random variables or events whose failure rate could be increasing and decreasing throughout the entire process.

### 2.5. Estimation of unknown Parameters of the APCTDaD

Let $X_1, X_2, ..., X_n$ be a sample of size "$n$" independently and identically distributed random variables from the APCTDaD with unknown parameters $\alpha, \theta, \gamma, \lambda$ and $\theta$ defined previously. The likelihood function is given by:

$$L\left(X \mid \alpha,\beta,\theta,\gamma,\lambda\right)=\frac{\left(\log\left(\alpha\right)\gamma\theta\beta\right)^{n}\prod_{i=1}^{n}\left(x_{i}^{-\theta-1}\left(1+\gamma x_{i}^{-\theta}\right)^{-\beta-1}\left[1-\lambda+6\lambda\left(1+\gamma x_{i}^{-\theta}\right)^{-\beta}-6\lambda\left(1+\gamma x_{i}^{-\theta}\right)^{-2\beta}\right]\right)}{\left(\alpha-1\right)^{n}\prod_{i=1}^{n}\left(\alpha^{-(1-\lambda)\left(1+\gamma x_{i}^{-\theta}\right)^{-\beta}+3\lambda\left(1+\gamma x_{i}^{-\theta}\right)^{-2\beta}-2\lambda\left(1+\gamma x_{i}^{-\theta}\right)^{-3\beta}}\right)}$$

(11)

Taking the partial derivative of the natural logarithm of the likelihood function equated to zero with respect to the parameters and solving for the solution of the non-linear system of equations produce the maximum likelihood estimates of parameters $\alpha$, $\beta$, $\lambda$, $\theta$ *and* $\gamma$. Understand that it is not easy to solve the equations analytically and hence Newton-Raphson's iteration method is used with available data.

## 3. Results and Discussion

In this section, three applications of the proposed model to real life datasets are provided to illustrate the flexibility of the APCTDaD introduced in Section 2. The MLEs of the model parameters are determined and some goodness-of-fit statistics for this distribution are compared with other competitive models. For all the datasets, the fits of APCTDaD is compared with those of the cubic transmuted Dagum distribution (CTDaD), transmuted Dagum distribution (TDaD) and the conventional Dagum distribution (DaD).

It is important to note that after application of the distributions to the data, the selection of the fitted probability models is based on some information criteria. These information criteria are in relation to the value of the log-likelihood function evaluated at the maximum likelihood estimates (ℓ) and include: Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC) and Hannan Quin Information Criterion (HQIC). Other model selection measures used in this research are: Anderson-Darling (A*), Cramèr-Von Mises (W*) and Kolmogorov-smirnov (K-S) statistics. For a comprehensive study of these measures the readers can check [19]. Meanwhile, the smaller the values of these statistics are for a particular model, the better the fit of the distribution model. The necessary computations in this research were carried out using the package "AdequacyModel" in R software which is freely available online from http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf.

### 3.1. Application 1: Based on Dataset I

This data represents the survival times of a group of patients suffering from head and neck cancer diseases and treated using a combination of radiotherapy and chemotherapy (RT+CT) ([20], [21], [22]).

The following table and figures present a good exploration of the dataset with some explanations.

**Table 1: Descriptive Statistics for dataset I**

| n | Minimum | $Q_1$ | Median | $Q_3$ | Mean | Maximum | Variance | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|---|---|
| **44** | 12.20 | 67.21 | 128.5 | 219.0 | 223.48 | 1776.00 | 93286.4 | 3.38382 | 13.5596 |

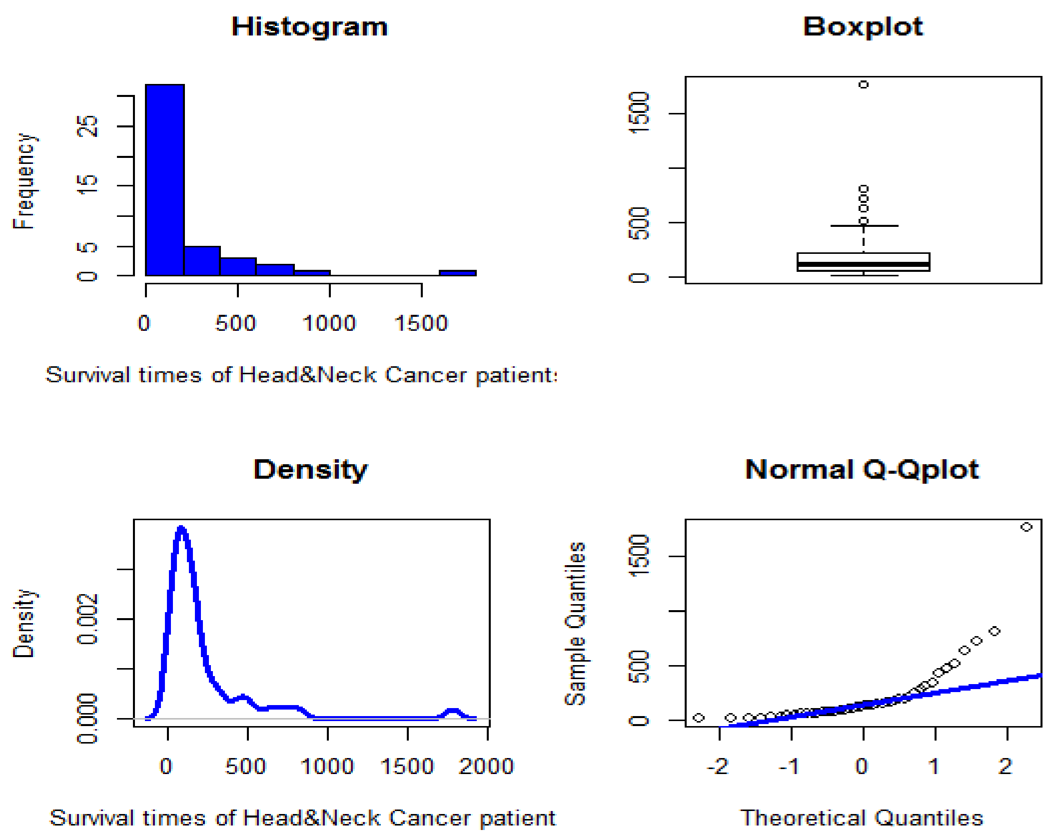


Figure 5: A graphical summary of the dataset I

Considering the dataset in table 1 and figure 5, it can be seen that data set I is skewed to the right or positively skewed and hence suitable for skewed models like the proposed APCTDaD.

Table 2: Maximum Likelihood Parameter Estimates for dataset I

| *Distribution* | $\hat{\gamma}$ | $\hat{\theta}$ | $\hat{\beta}$ | $\hat{\lambda}$ | $\hat{\alpha}$ |
|---|---|---|---|---|---|
| APCTDaD | 2.6115052 | 0.8723578 | 9.9574230 | 0.9234020 | 8.1805877 |
| CTDaD | 9.0478552 | 0.8359463 | 4.4704835 | 0.8879190 | - |
| TDaD | 9.3003812 | 0.8012132 | 6.9245910 | 0.9839051 | - |

| | | | | | |
|---|---|---|---|---|---|
| DaD | 8.4264614 | 0.9986907 | 8.9957360 | - | - |

**Table 3:** The statistics $\ell$, AIC, CAIC, BIC and HQIC for dataset I

| Distribution | $\hat{\ell}$ | *AIC* | *CAIC* | *BIC* | *HQIC* | Ranks |
|---|---|---|---|---|---|---|
| APCTDaD | 277.9149 | 563.8297 | 564.8554 | 570.9665 | 566.4764 | 1st |
| CTDaD | 277.8219 | 563.6439 | 564.6695 | 570.7806 | 566.2905 | 2nd |
| TDaD | 277.8373 | 565.6745 | 567.2535 | 574.5955 | 568.9828 | 3rd |
| DaD | 279.5193 | 565.0386 | 565.6386 | 570.3912 | 567.0236 | 4th |

**Table 4**: The $A^*$, $W^*$, K-S statistic and P-values for dataset I

| *Distribution* | $A^*$ | $W^*$ | *K-S* | *P-Value (K-S)* | Ranks |
|---|---|---|---|---|---|
| APCTDaD | 0.1708722 | 0.02519533 | 0.063566 | 0.9893 | 1st |
| CTDaD | 0.1479777 | 0.02205189 | 0.076078 | 0.944 | 2nd |
| TDaD | 0.1489243 | 0.02100251 | 0.08537 | 0.8785 | 3rd |
| DaD | 0.3116772 | 0.04943709 | 0.10655 | 0.6606 | 4th |



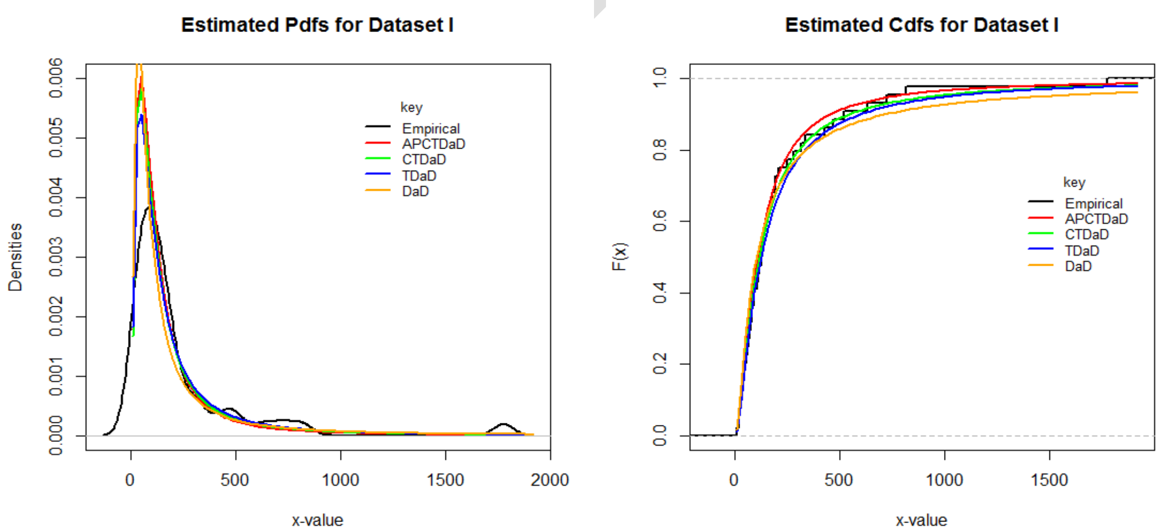


**Figure 6:** Plots of the estimated densities and cdfs of the fitted distributions to dataset I.

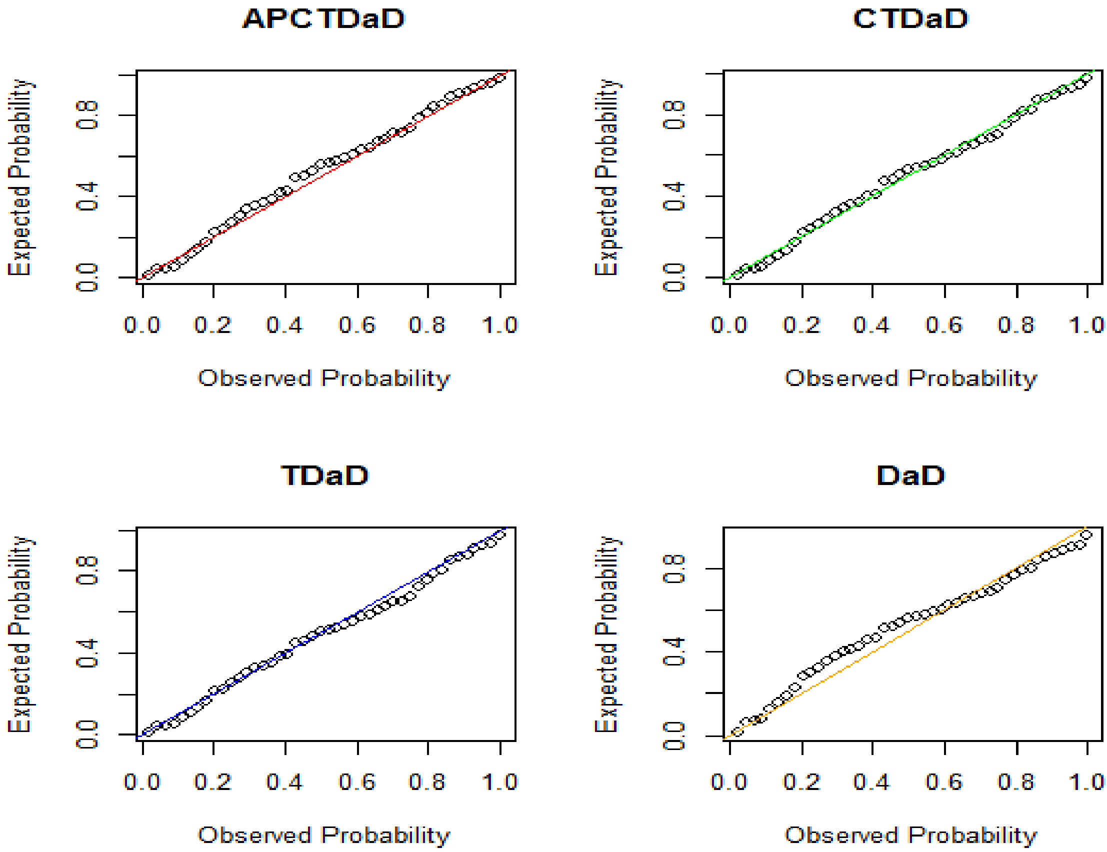


**Figure 7**: Probability plots for the fit of the APCTDaD, CTDaD, TDaD & DaD based on dataset I.

### 3.2. Application 2: Based on Dataset II

Dataset II is on the remission times of a random sample of 128 bladder cancer patients in months. This particular data has been used previously by [23], [24], [25] and [26]. Its values and summary statistics is as follows:

0.080, 0.200, 0.400, 0.500, 0.510, 0.810, 0.900, 1.050, 1.190, 1.260, 1.350, 1.400, 1.460, 1.760, 2.020, 2.020, 2.070, 2.090, 2.230, 2.260, 2.460, 2.540, 2.620, 2.640, 2.690, 2.690, 2.750, 2.830, 2.870, 3.020, 3.250, 3.310, 3.360, 3.360, 3.480, 3.520, 3.570, 3.640, 3.700, 3.820, 3.880, 4.180, 4.230, 4.260, 4.330, 4.340, 4.400, 4.500, 4.510, 4.870, 4.980, 5.060, 5.090, 5.170, 5.320, 5.320, 5.340, 5.410, 5.410, 5.490, 5.620, 5.710, 5.850, 6.250, 6.540, 6.760, 6.930, 6.940, 6.970, 7.090, 7.260, 7.280, 7.320, 7.390, 7.590, 7.620, 7.630, 7.660, 7.870, 7.930, 8.260, 8.370, 8.530, 8.650, 8.660, 9.020, 9.220, 9.470, 9.740, 10.06, 10.34, 10.66, 10.75, 11.25, 11.64, 11.79, 11.98, 12.02, 12.03, 12.07, 12.63, 13.11, 13.29, 13.80, 14.24, 14.76, 14.77, 14.83, 15.96, 16.62, 17.12, 17.14, 17.36, 18.10, 19.13, 20.28, 21.73, 22.69, 23.63, 25.74, 25.82, 26.31, 32.15, 34.26, 36.66, 43.01, 46.12, 79.05.

**Table 5: Summary Statistics for the data set II**

| parameters | n | Minimum | $Q_1$ | Median | $Q_3$ | Mean | Maximum | Variance | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|---|---|---|
| **Values** | 128 | 0.0800 | 3.348 | 6.395 | 11.840 | 9.366 | 79.05 | 110.425 | 3.3257 | 19.1537 |

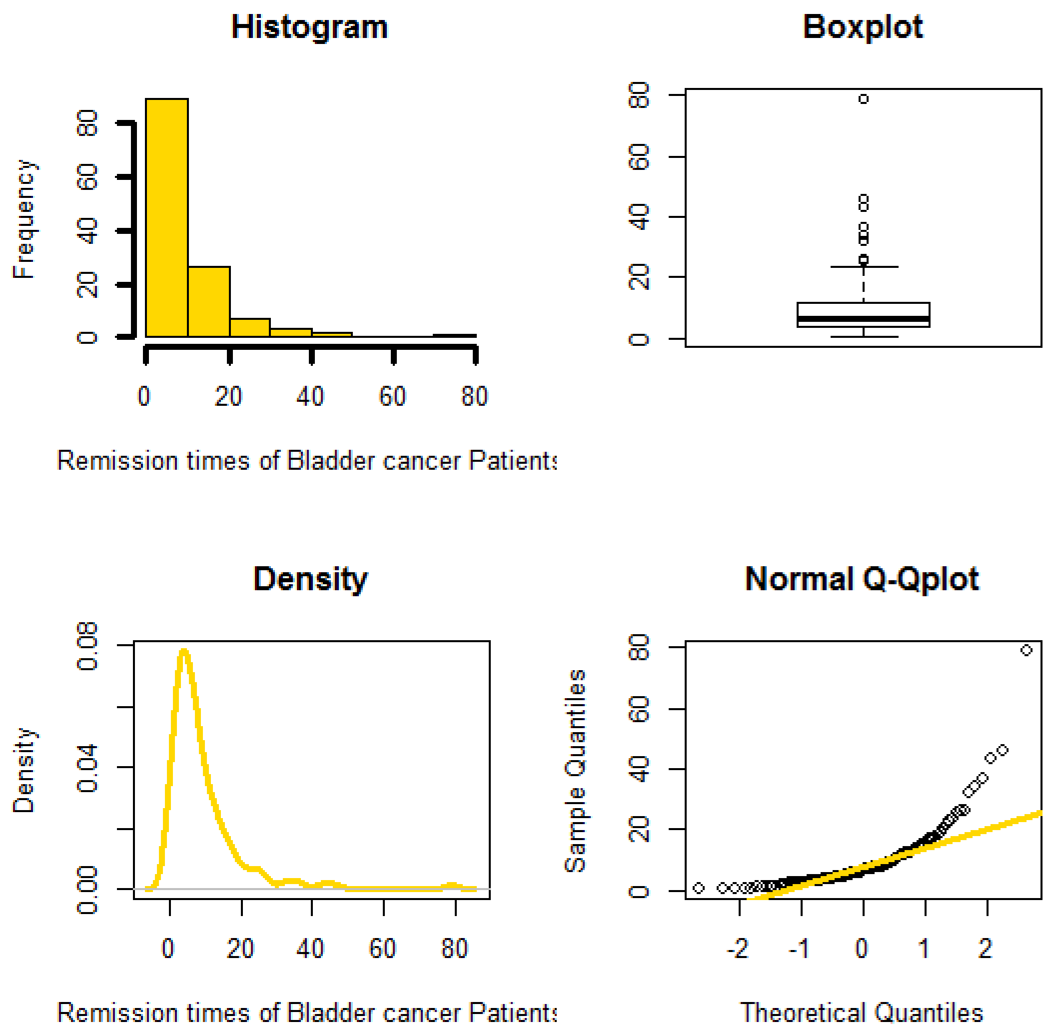


**Figure 8**: A graphical summary of the dataset II.

Also, looking at the summary statistics and graphical presentation of dataset II in table 5 and figure 8 respectively, it still clear that data set II is skewed to the right or positively skewed and hence good for skewed models like the proposed APCTDaD.

**Table 6:** Maximum Likelihood Parameter Estimates for dataset II

| *Distribution* | $\hat{\gamma}$ | $\hat{\theta}$ | $\hat{\beta}$ | $\hat{\lambda}$ | $\hat{\alpha}$ |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| APCTDaD | 9.2007268 | 1.3511256 | 0.6522685 | 0.5313419 | 7.7493862 |
| CTDaD | 9.8212178 | 1.2807240 | 1.0175680 | 0.6221916 | - |
| TDaD | 6.1177948 | 1.5123881 | 1.1742541 | -0.6486219 | - |
| DaD | 8.235246 | 1.712200 | 1.574800 | - | - |

**Table 7:** The statistics $\ell$, AIC, CAIC, BIC and HQIC for dataset II

| Distribution | $\hat{\ell}$ | *AIC* | *CAIC* | *BIC* | *HQIC* | Ranks |
|---|---|---|---|---|---|---|
| APCTDaD | 411.3755 | 830.7511 | 831.0763 | 842.1592 | 835.3863 | 1st |
| CTDaD | 411.5118 | 833.0236 | 833.5154 | 847.2838 | 838.8176 | 2nd |
| TDaD | 413.2724 | 834.5448 | 834.87 | 845.9529 | 839.18 | 3rd |
| DaD | 420.3174 | 846.6349 | 846.8284 | 855.191 | 850.1113 | 4th |

**Table 8:** The $A^*$, $W^*$, K-S statistic and P-values for dataset II

| *Distribution* | $A^*$ | $W^*$ | *K-S* | *P-Value (K-S)* | Ranks |
|---|---|---|---|---|---|
| APCTDaD | 0.2838934 | 0.03962499 | 0.040366 | 0.9852 | 1st |
| CTDaD | 0.2021853 | 0.02865787 | 0.040712 | 0.9838 | 2nd |
| TDaD | 0.6238288 | 0.09281483 | 0.069858 | 0.5599 | 3rd |
| DaD | 1.088383 | 0.1649073 | 0.14645 | 0.008252 | 4th |

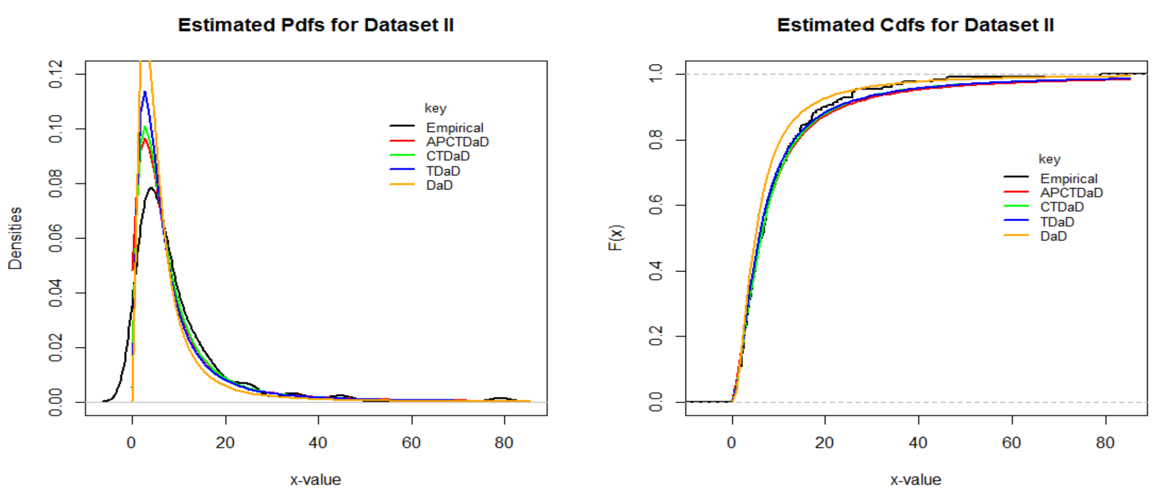


**Figure 9:** Plots of the estimated densities and cdfs of the fitted distributions to dataset II.

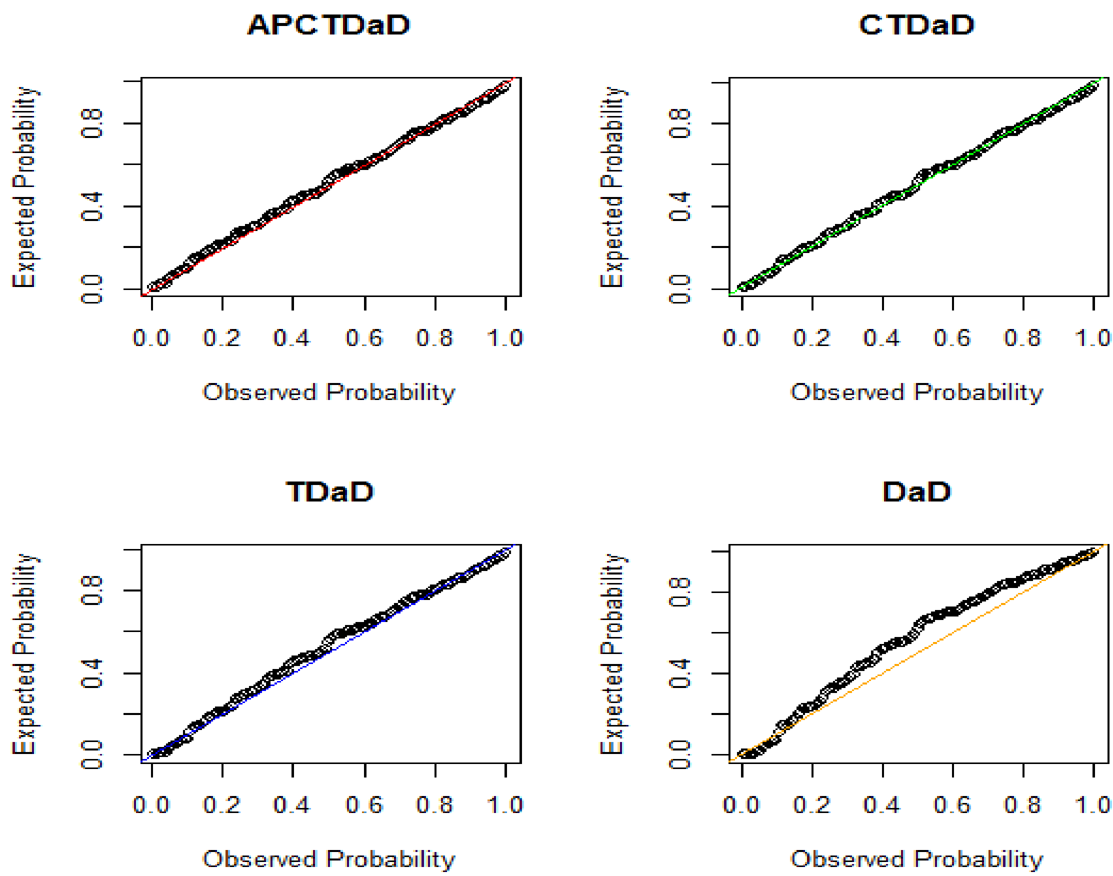


**Figure 10:** Probability plots for the fit of the APCTDaD, CTDaD, TDaD & DaD based on dataset II.

### 3.3. Application 3: Based on Dataset III

Dataset III is on the survival times of 72 guinea pigs infected with virulenttubercle bacilli in days used previously by [27]. They are the Regiment 4.3, Study M.: 10, 33, 44,56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116,120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176,183, 195, 196, 197, 202, 213, 215, 216, 222, 230,231, 240, 245, 251, 253, 254, 255, 278, 293, 327,342, 347, 361, 402, 432, 458, 555.

The following table and figures present a good exploration of the dataset with some explanations.

**Table 9: Descriptive Statistics for dataset III**

| parameters | n | Minimum | $Q_1$ | Median | $Q_3$ | Mean | Maximum | Variance | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Dataset A** | 72 | 10.0 | 108.0 | 149.5 | 224.0 | 176.8 | 555.0 | 10705.1 | 1.34128 | 1.98852 |

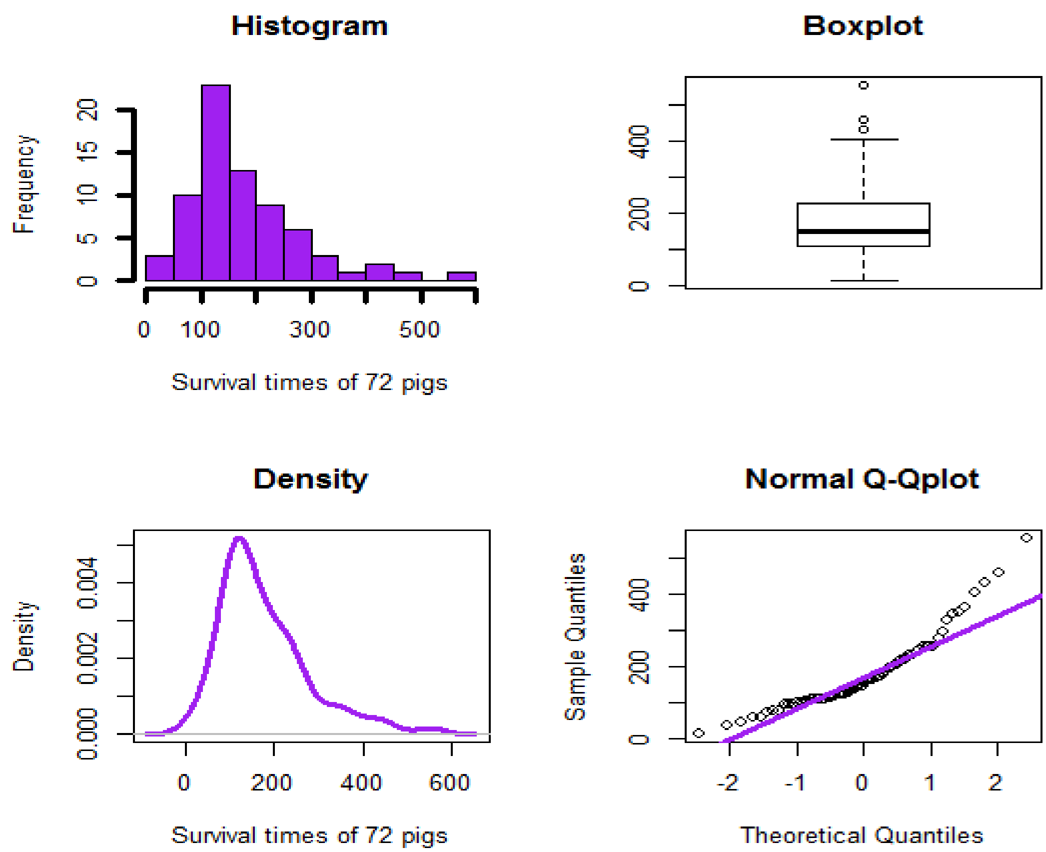


**Figure 11:** A graphical summary of the dataset III

A summary of dataset III in table 9 and figure 11 has shown that it is a skewed data and therefore requires a skewed and flexible probability distribution for its proper analysis.

**Table 10:** Maximum Likelihood Parameter Estimates for dataset III

| *Distribution* | $\hat{\gamma}$ | $\hat{\theta}$ | $\hat{\beta}$ | $\hat{\lambda}$ | $\hat{\alpha}$ |
|---|---|---|---|---|---|
| APCTDaD | 8.0772768 | 1.0429266 | 9.4344820 | 0.9586407 | 8.2604805 |
| CTDaD | 8.6642460 | 0.9446059 | 9.2356805 | 0.9792477 | - |
| TDaD | 8.5805922 | 0.8044233 | 8.6707749 | 0.9692952 | - |
| DaD | 9.0129861 | 0.9666121 | 9.1183652 | - | - |

**Table 11:** The statistics $\ell$, AIC, CAIC, BIC and HQIC for dataset III

| Distribution | $\hat{\ell}$ | *AIC* | *CAIC* | *BIC* | *HQIC* | Ranks |
|---|---|---|---|---|---|---|
| APCTDaD | 435.1453 | 880.2905 | 881.1996 | 891.6738 | 884.8222 | 1st |
| CTDaD | 438.1489 | 884.2979 | 884.8949 | 893.4045 | 887.9233 | 2nd |
| TDaD | 444.912 | 897.824 | 898.421 | 906.9307 | 901.4494 | 3rd |
| DaD | 454.7097 | 915.4195 | 915.7724 | 922.2495 | 918.1385 | 4th |

**Table 12:** The $A^*$, $W^*$, K-S statistic and P-values for dataset III

| *Distribution* | $A^*$ | $W^*$ | *K-S* | *P-Value (K-S)* | Ranks |
|---|---|---|---|---|---|
| APCTDaD | 1.059438 | 0.1451422 | 0.16932 | 0.03223 | 1st |
| CTDaD | 1.442234 | 0.2061458 | 0.16971 | 0.03161 | 2nd |
| TDaD | 1.193157 | 0.1675743 | 0.19743 | 0.007299 | 3rd |
| DaD | 1.775791 | 0.259787 | 0.26276 | 9.62e-05 | 4th |

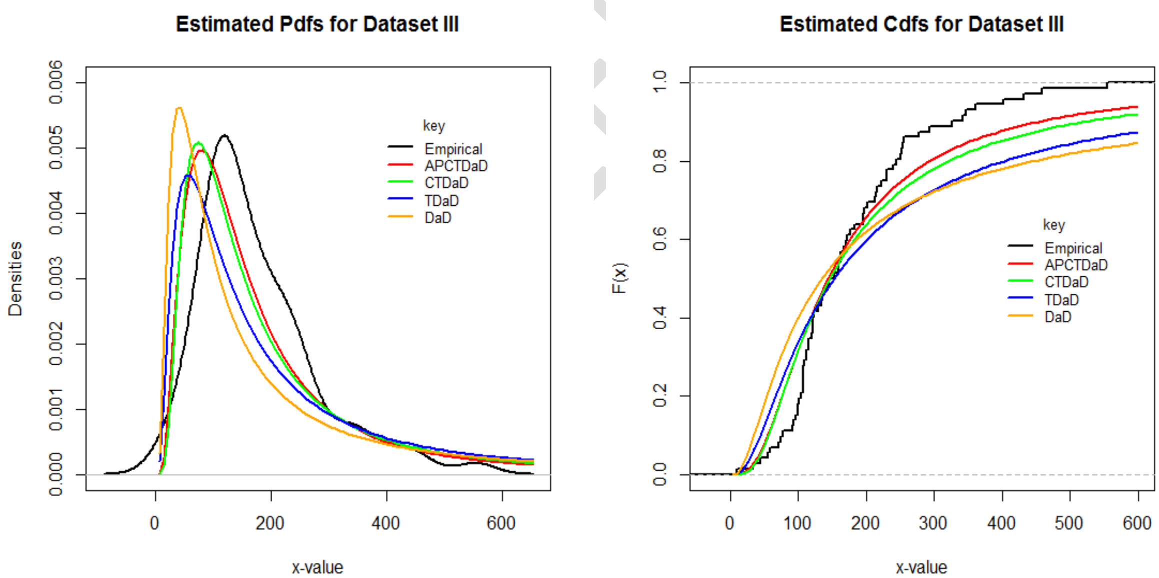


**Figure 12:** Plots of the estimated densities and cdfs of the fitted distributions to dataset III.

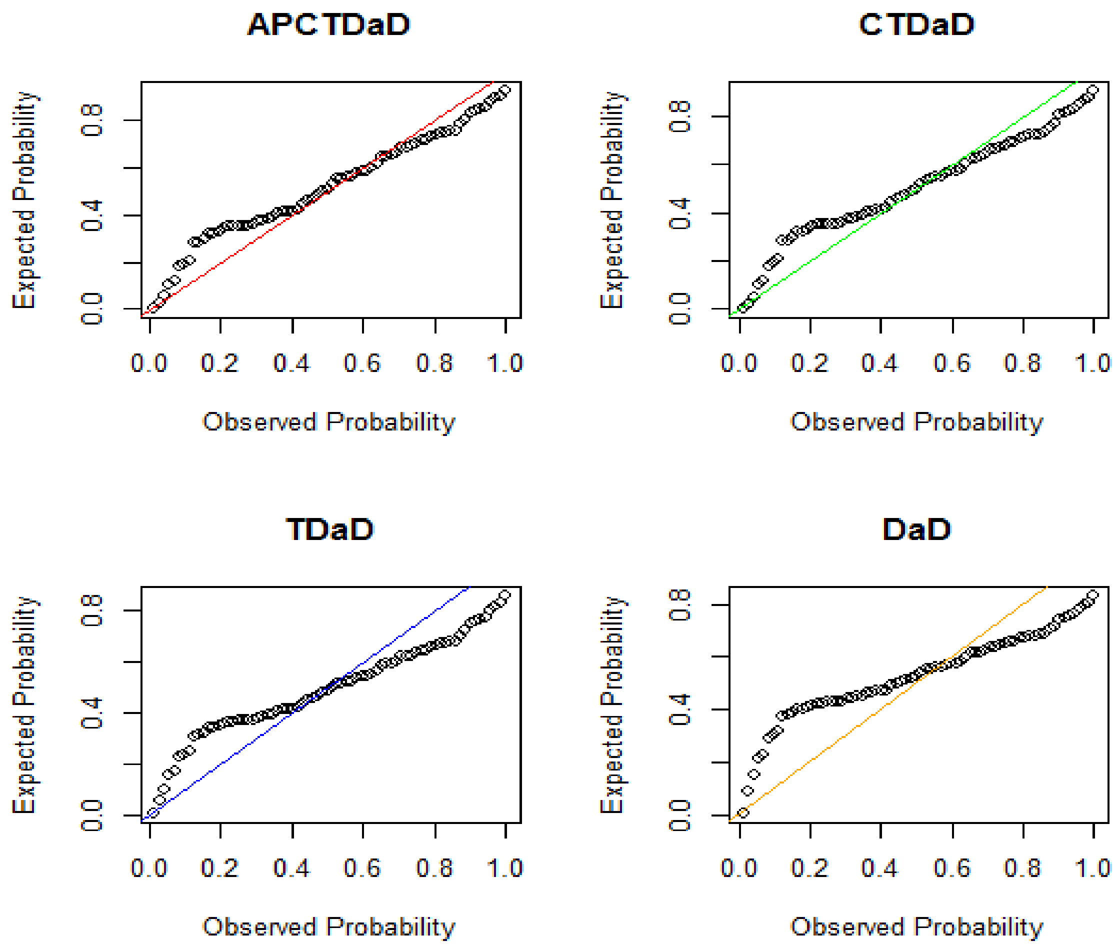


**Figure 13:** Probability plots for the fit of the APCTDaD, CTDaD, TDaD & DaD based on dataset III.

Tables 2, 6 and 10 present the MLEs of the fitted model parameters based on dataset I, II and III respectively, table 3, 7 and 11 present the AIC, CAIC, BIC and HQIC values for dataset I, II and III respectively. Relatively, the values of A*, W* and K-S for datasets I, II and III are given in Tables 4, 8 and 12 respectively.

The plots of the estimated density and cumulative distribution function of APCTDaD with those of the other fitted distributions are presented in figure 6, 9 and 12 for datasets I, II and III respectively. The probability plots of the fitted distributions are also given in Figures 7, 10 and 13 for datasets I, II and III respectively. Considering the results from all the measures based on all the three datasets above, it is observed that the alpha power cubic transmuted Dagum distribution (APCTDaD) performs much better for all the real life datasets compared to the other three fitted distributions (cubic transmuted Dagum distribution (CTDaD), transmuted Dagum distribution (TDaD) and the conventional Dagum distribution (DaD)). These results are clearly confirmed by the estimated density plots and also the probability plots of the fitted distributions as shown in the figures 7, 10 and 13 above. Considering all these results, it has concluded that the proposed distribution (alpha power cubic transmuted Dagum distribution (APCTDaD)) is a more flexible distribution than the other existing distributions fitted in this study. This study is in line with the fact that all extended distributions perform better than their classical counterpart and the research is also in agreement with the study of [13] and [16], which are also based on the alpha power transformation family.

## 4. Conclusion

This research considered the alpha power transformation approach to define and study a Dagum distribution producing a new distribution called "alpha power cubic transmuted Dagum distribution (APCTDaD)". The research derived and studied the reliability functions of the proposed distribution which include survival and hazard functions with their graphical presentations and discussions on their usefulness and applications. It is concluded that the proposed distribution is better compared to other existing distributions based on our application of the model to a real life datasets.



## REFERENCES


[1] Dagum, C. (1977). A new model of personal income distribution: Specification and estimation. *Economie Appliquee,* 30, 413-437.

[2] Cordeiro, G. M., Ortega, E. M. M. & Ramires, T. G. (2015). A new generalized Weibull family of distributions: Mathematical properties and applications. *Journal of Statistical Distributions and Applications*, 2:13DOI 10.1186/s40488-015-0036-6

[3] Tahir, M. H., Cordeiro, G. M., Alzaatreh, A., Mansoor, M. & Zubair, M. (2016). The logistic-X family of distributions and its applications. *Communications in Statistics: Theory and Methods,* 45 (24): 7326-7349.

[4] Tahir, M. H., Zubair, M., Mansoor, M., Cordeiro, G. M. & Alizadeh, M. (2016). A New Weibull-G family of distributions. *Hacettepe Journal of Mathematics and Statistics*, 45(2): 629-647.

[5] Cakmakyapan, S. & Ozel, G. (2016). The Lindley Family of Distributions: Properties and Applications. *Hacettepe Journal of Mathematics and Statistics*, 46: 1-27.

[6] Alizadeh, M., Cordeiro, G. M., Pinho, L. G. B. & Ghosh, I. (2017). The Gompertz-G family of distributions. *Journal of Statistical Theory and Practice,* 11(1): 179–207, doi:10.1080/15598608.2016.1267668

[7] Gomes-Silva, F., Percontini, A., De Brito, E., Ramos, M. W., Venancio, R. & Cordeiro, G. M. (2017). The Odd Lindley-G Family of Distributions. *Austrian Journal of Statistics*, 46: 65-87.

[8] Mahdavi, A & Kundu, D. (2017). A new method for generating distributions with an application to exponential distribution, *Communications in Statistics-Theory and Methods*, 46(13): 6543–6557.

[9] Cordeiro, G. M., Afify, A. Z., Ortega, E. M. M., Suzuki, A. K. & Mead, M. E. (2019). The odd Lomax generator of distributions: Properties, estimation and applications. Journal of Computational and Applied Mathematics, 347: 222–237.

[10] Tahir, M. H., Zubair, M., Cordeiro, G. M., Alzaatreh, A. & Mansoor, M. The Poisson-X family of distributions, Journal of Statistical Computation and Simulation, 2016; 86:14, 2901-2921.

[11] Ieren, T. G. & Balogun, O, S. Exponential-Lindley Distribution: Theory and Application to Bladder Cancer Data. Journal of Applied Probability and Statistics, 2021; 16(2): 129-146.

[12] Ieren, T. G. & Abdullahi, J. A Transmuted Normal Distribution: Properties and Applications. Equity Journal of Science and Technology, 2020; 7(1): 16-35.

[13] Nassar, M., Alzaatreh, A., Mead, M. & Abo-Kasem, O. Alpha power weibull distribution: Properties and applications, Communications in Statistics - Theory and Methods, 2017; 46 (20): 10236–10252.

[14] Ieren, T. G., Abdulkadir, S. S. & Issa, A. A. Odd Lindley-Rayleigh Distribution: Its Properties and Applications to Simulated and Real Life Datasets. Journal of Advances in Mathematics and Computer Science, 2020; 35(1): 63-88.

[15] Umar, S. A., Bukar, A. B., Makama, M. S. & Ieren, T. G. Some Results on the Transmuted Odd Lindley-Rayleigh Distribution. Benin Journal of Statistics, 2021; 4:135– 153.

[16] Dey, S., Alzaatreh, A., Zhang, C. & Kumar, D. A new extension of generalized exponential distribution with application to ozone data, Ozone: Science & Engineering, 2017; 39 (4): 273–285.

[17] Ieren, T. G. & Abdullahi, J. Properties and Applications of a Two-Parameter Inverse Exponential Distribution with a Decreasing Failure Rate. Pakistan Journal of Statistics, 2020; 36(3): 183-206.

[18] Chama, A. F. (2021). Cubic Transmuted Dagum Distribution: Properties and Applications. *Mathematics Letters,* 7(1):7-18.

[19] Chen, G., Balakrishnan, N. (1995). A general purpose approximate goodness-of-fit test. *Journal of Quality Technology* 27: 154–161.

[20] Efron, B. (1988). Logistic regression, survival analysis and the Kaplan-Meier curve. *Journal of the American Statistical Association,* 83: 414-425.

[21] Shanker, R., Hagos, F. & Sujatha, S. (2015). On modeling of lifetimes data using exponential and Lindley distributions. *Biometrics and Biostatistics International Journal,* 2(5): 140-147.

[22] Oguntunde, P. E., Balogun, O. S., Okagbue, H. I. & Bishop, S. A. (2015). The Weibull-Exponential Distribution: Its properties and application. *Journal of Applied Sciences*, 15(11): 1305-1311.

[23] Lee, E. T. & Wang, J. W. (2003). Statistical Methods for Survival Data Analysis. 3rd Edn., John Wiley and Sons, New York, ISBN: 9780471458555, 2003; Pages: 534.

[24] Rady, E., A., Hassanein, W. A. & Elhaddad, T. A. (2016).The power Lomax distribution with an application to bladder cancer data. *Springer Plus (2016);5(1)1838.*

[25] Ieren TG, & Chukwu AU. Bayesian Estimation of a Shape Parameter of the Weibull-Frechet Distribution. Asian J. of Prob. and Stat., 2018; 2(1):1-19.

[26] Abdullahi J, Abdullahi UK, Ieren TG, Kuhe DA, & Umar AA. On the properties and applications of transmuted odd generalized exponential- exponential distribution, Asian J. of Prob. Stat., 2018; 1(4):1-14.

[27] Bjerkedal, T. (1960), "Acquisition of resistance in guinea pigs infected with different doses of virulent tubercle bacilli," *American Journal of Hygiene,* **72,** 130-148.