

Structure of the Atom

ABSTRACT

The current consensus on the atomic structure was formulated about a hundred years ago. In the enlightenment of the newly developed instrumentations and theories the electronic structure of the atoms is reconsidered. It has been concluded that in order to comply with experiments the electron must go through phase transformation from point to surface charge when captured by the nucleus. The surface charge electron shell model of the atoms is consistent with all the known features of the atoms and offers a physical explanation for the Schrodinger's wave equation.

Keywords: Structure of the atom; Stability of the electron; Phase transformation of the electron; Photon emission and absorption; Wave function.

1. INTRODUCTION

The first modern version of the atomic model was proposed by Rutherford [1]. Based on the scattering experiments of Hans Geiger and Ernest Marsden, it was concluded that most of the space inside the atom is empty, and there is a small, heavy positively charged core (nucleus) inside the atom, which consist of almost the entire mass of the atom [1]. The nucleus also contains neutrons, which have almost the same mass as the proton but have no charge [2, 3]. It is assumed that the attraction of the proton/s on the negatively charged electron/s is balanced out by the centrifugal force of the orbiting electron/s. Based on this planetary model [4], Bohr was able to reproduce the main emission lines of the hydrogen [5], derive the correct values for the Rydberg constant [6], the radius of the hydrogen atom, and the ionization energy [4]. Despite this enormous success, the model had many shortcomings [7].

De Broglie had proposed the wave-particle duality [8]. The wave nature of the electron has been confirmed by the experiments of Davisson and Germer [9]. This is the fundamental base of quantum mechanics, which uses the wave function to describe all the physical phenomena at atomic scale. The wave equation describing the particles had been proposed by Schrodinger [10, 11]. This purely mathematical approach is able to reproduce not just the results of the Bohr's model but all the known features of the Hydrogen atom. No physical reasoning or explanation has been proposed as to why this mathematical treatment is able to correctly describe the characteristic features of the electron. The general consensus is that "It works, so we just have to accept it." After almost a hundred years the scientific community still has not reached a consensus on what is the physical process behind the quantized wave behavior of the electrons in the atoms. An attempt is made here to reveal this mystery.

2. ELECTROMAGNETISM AT ATOMIC SCALE

Both the Bohr, and the current quantum mechanical electron cloud models assume that the attraction of the nucleus on the electrons is balanced out by the centrifugal force of the orbiting electron/s. Neither model offers explanations why the bounded electron, revolving around the nucleus, does not emit radiation. Bohr postulation, the laws of electromagnetism is not valid at atomic scale has no merit. He does not offer any theoretical explanation why electromagnetism ceases to exist at atomic scale. Additionally, he employs electrostatics to describe the attraction of the proton to the electron in his model. The four Maxwell's equations give a coherent description of electromagnetism. Selectively discrediting certain parts of a well-established physical theory, without any reasoning cannot be justified. Furthermore, his postulates contradict each other. If the accelerating or the decelerating charges at atomic scale do not emit radiation (postulate iii) then this postulate must be universal. On the contrary, in postulate V he allows electromagnetic radiation triggered by the transition of the electrons. Additional problem with postulation V is that radiation is allowed only between stationary states. These states are representing different orbital radius of the electron. Based on orbital energy calculations, the orbital radius of the electron (r_n) in energy level of n of a Hydrogen atom is

$$r_n = a_0 n^2 \quad n = 1, 2, 3, \dots \quad (1)$$

where a_0 is the Bohr's radius. Changing the orbital radius of the electron increases the size of the atom. This expansion or contraction of atoms has never been confirmed by experiments.

The existence of electromagnetism at atomic scale is also indicated by many observations. Like, free electrons deflected by an atomic nucleus emit electromagnetic radiation, which is known as Bremsstrahlung. There is no known physical reason why the free electron deflected by an atomic nucleus should behave differently than a captured orbiting electron. Electromagnetic radiation is also emitted when an electron jumps from one electron shell to another in the atom. This radiation is known as a characteristic x ray. These emitted radiations also show that photons are emitted or absorbed when charges accelerate or decelerate at atomic scale. Thus, discarding electromagnetism at atomic scale has no physical merit and contradicts with the known features of the atoms.

3. PHASE TRANSFORMATION OF THE ELECTRON

The validation of electromagnetism at atomic scale has serious consequences. One of the most important is that the non-emitting captured electron in the atom must be stationary. The captured stationary point charge electron under the attraction of the nucleus cannot be stable. Two static opposite charges can only be stable if one of the point charges, the electron, transforms to spherical surface charge around the nucleus [12] (Fig. 1).

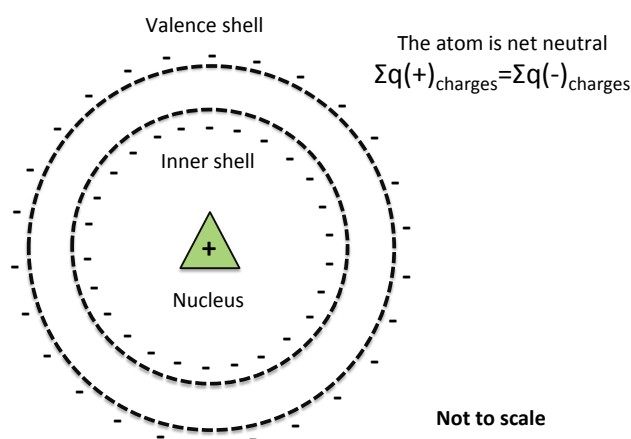


Fig. 1. Schematic figure of an atom is shown. Two static opposite charges remain stable only if one of the point charges (electron/s) transform to a static spherical surface charge around the nucleus. The electron shells built up in accordance with the periodic table.

The transformation does not require any additional energy because the electrostatic attraction on the point and surface charge at the same distance is identical. The surface charge electron shell configuration is consistent with the known features of the atom. The different behavior of the free and bounded electron is also an indicator of phase transformation [13]. One of the examples is that the bounded electrons have spin. This spin has never been detected for free electrons. Bounded electrons in the atom do not interact with the magnetic field contrarily to the free electrons.

The long-term stability of the atoms also supports the surface charge electron shell atom model. The orbit of a point charge electron around the nucleus in the time scale of the universe would be chaotic. Thus, collision between the electron/s and the proton/s should sometimes occur. In the time scale of the universe no or very few stable atoms should exist if the electrons are orbiting as point charges in the atom.

4. STABILITY REQUIREMENTS

The stability of the point and surface charge electrons is investigated through energy balance calculations [14]. The physical processes affecting the stability of the electron were not all of them known at the time when the current consensus on the atomic structure was formulated. Casimir predicted the existence of vacuum pressure in 1948 [15], and about half a century later had been experimentally verified in the late 1990s [16, 17], and confirmed by many experiments since then [e.g. 18]. The phase transformation of the point charge electron to the surface charge includes significant change in the size of the electron, which modifies the surface charge density. The change in the charge density indicates that the electron is not uniform and built up from smaller constituents. The electrons can be created from the quantum field with certain energy. Thus, the electron should build up from the basic building blocks of the field or more widely the universe. It is speculated that by organizing the orientation of these field elements results in the formation of the charge. The constituents of the electron can interact with electric and magnetic fields and by that block the quantum fluctuation of the field. Blocking the quantum field results in Casimir pressure, which is effective on the electron shell of the atoms. Thus, the Casimir effect should be taken into account in the energy balance calculations of the stability of the electron.

4.1 Free electron

Outside of the atom the electron can be depicted as a point charge. The destabilizing effects on this point charge electron are the electrostatic self-repulsion of the elementary charge (E_{rep}^{e-p}), the attraction of an opposite charge/s, proton/s or nucleus (E_{attr}^{e-p}), and the zero-point energy of the charge (E_{ZP}^{e-p}). The stability of a point charge electron is ensured by the Casimir effect (E_{Ca}^{e-p}), which exerts pressure on the surface of the electron. The kinetic energy of the point charge electron (E_{kin}^{e-p}) works against the transformation to surface charge since the surface charge electron in the atom possesses no kinetic energy. The stability requirement for the point charge electron³ then can be stated as:

$$E_{Ca}^{e-p} + E_{kin}^{e-p} > E_{rep}^{e-p} + E_{attr}^{e-p} + E_{ZP}^{e-p} \quad (2)$$

The superscript e-p refers to electron-point charge. The transformation from point charge to surface charge electron occurs when the overall destabilizing energies are equal or exceed the stabilizing energies. This condition allows calculating the distance where this destabilization occurs. Based on the equilibrium condition between the stabilizing and destabilizing energies, the point charge electron loses its stability in the vicinity of a proton at the Bohr's radius (a_0). Thus, the point charge electron in the vicinity of a proton loses its stability at the Bohr's radius. The destabilized charge under the attraction of the proton forms a static surface charge around the proton or nucleus.

4.2 Captured electron

The stability requirement of the captured surface charge electron around the nucleus slightly differs from the point charge free electron, because the attraction of the nucleus in this case works for the stability of the electron shell. Additionally, when the surface charge is destabilized then only the one-dimensional Casimir effect is active. Thus, during the transformation from surface to point charge the contribution of Casimir effect to the stability of a surface charge electron (E_C^{e-s}) is effective in one-dimension.

$$E_C^{e-s} = E_{C-1D}^e = \frac{1}{3} E_{Ca}^e \quad (3)$$

where superscript e-s refers to electron-surface charge. The stability requirement of the surface charge electron in an atom then can be given as:

$$E_C^{e-s} + E_{attr}^{e-s} > E_{rep}^{e-s} + E_{ZP}^{e-s} + E_{\gamma}^{e-s} + E_{kin}^{e-s} \quad (4)$$

where E_{γ}^{e-s} is the absorbed photon energy.

In the atoms the distance between the positive charge and the surface charge electron is constant. In terms of the Hydrogen atom this is equal with the Bohr's radius, resulting that the self-repulsion, and zero-point energies are identically compensating the attraction of the proton.

$$E_{attr}^{e-s} = E_{rep}^{e-s} + E_{ZP}^{e-s} \quad (5)$$

The electrostatic attraction and repulsions are balanced out in an atom; therefore, electrostatic attraction does not affect the stability of the atoms. The stability requirement for the surface charge electron surrounding the nucleus can be reduced then as:

$$E_C^{e-s} > E_{\gamma}^{e-s} + E_{kin}^{e-s} \quad (6)$$

The stability of the outer shell electron with zero kinetic energy is lost, when the energy of the absorbed photon is equal or exceeds the one-dimensional or effective

Casimir energy of the surface charge electron shell. For the Hydrogen atom the calculated one-dimensional Casimir energy is the same as the ionization energy of the Hydrogen atom. It is speculated that this relationship can be generalized to all elements as:

$$E_C^{e-s} = IE, \tag{7}$$

where IE is the ionization energy of the element. The schematic figure of the transition is shown in Figure 2.

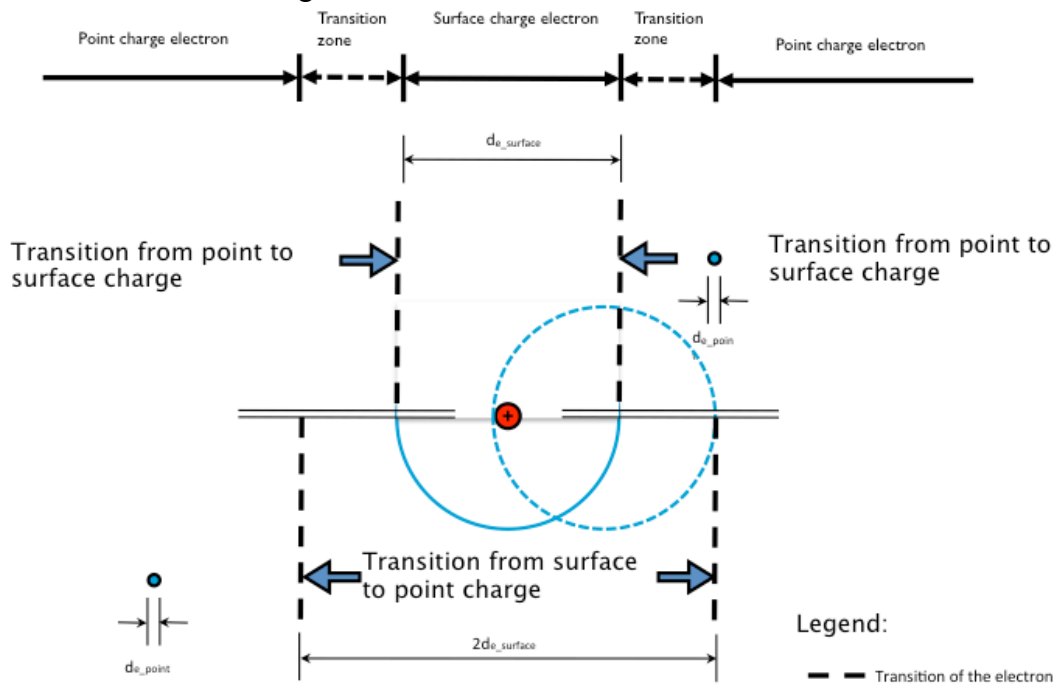


Fig. 2. Schematic figure of the transition of an electron from point to surface charge and vice versa in the vicinity of the nucleus is shown. For the Hydrogen atom the transition from point to surface charge occurs at the Bohr's radius. In reverse the transformation from surface to point charge occurs at the diameter of the Bohr's atom.

5. POSITION STABILITY OF THE NUCLEUS

The uniformly distributed static spherical surface charges of the electron cannot secure the position of the nucleus. The proton inside the shell is attracted from all sides of the surrounding electron shell. If the nucleus gets closer to the electron shell then the attraction of the electron shell on the nucleus has quadratic increases as a function of distance. However, as the proton moves away from the center of the spherical surface charge shell then the surface area of the electron shell attracting the proton on the opposite side has quadratic increase too. These two effects cancel each other and the nucleus is stable anywhere inside the surface charge electron shell (Fig. 3).

Attraction of the surface charge electron on the proton

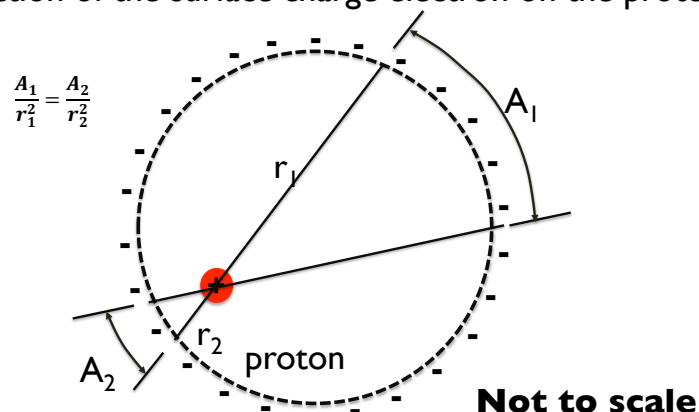


Fig. 3. The attraction from the electron shell on the proton is balanced out everywhere inside the atom. Thus, the position of the positive charge inside a static surface charge electron shell is not secured.

It is postulated that the vibration of the electron shell can hold the nucleus in the central position inside the atom. The developing stationary vibration of the electron shell is symmetrical. The generated forces by the vibration, which are acting on the proton, are symmetrical. This attraction holds the nucleus in the center of the electron shell. It is proposed that this physical process stabilizes the position of the proton in the center of the formed surface charge electron shell (Fig. 4). At absolute zero the vibration is generated by the zero point energy, which secures the position of the nucleus. The stabilizing force on the proton increases with the vibration energy of the electron shell. Thus up to the ionization energy, the higher energy of the atom results in stronger stabilizing forces on the nucleus.

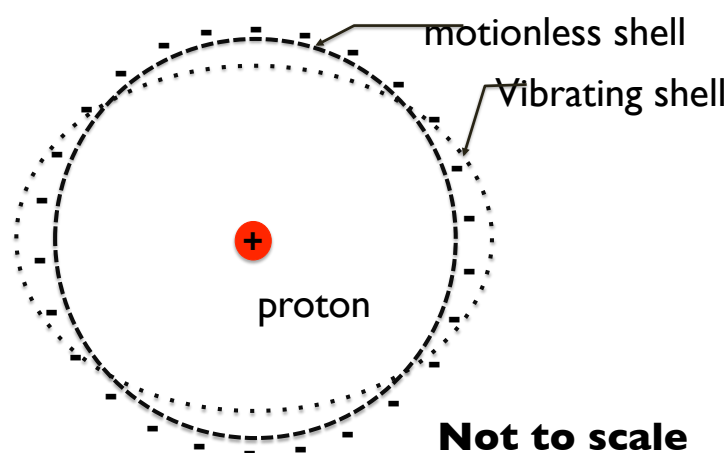


Fig. 4. The symmetrical vibration of the surface charge electron shell secures the position of the proton at the center of the atom.

6. TESTING THE MODEL

It has been shown that an electron in the valence shell loses its stability when the absorbed photon and kinetic energies exceed the one-dimensional Casimir energy. It is speculated that if the Casimir effect is not active on the entire surface of the atom, then electrons can be removed at lower energies than the ionization one. If this

hypothesis is correct, then the energy required to remove an outer shell electron from a neutral atom should be proportional to the active surface of the atom on which the Casimir effect is acting. The stability condition of the surface charge electrons, given in Eq. 6 then it should be modified as:

$$A_{\text{active}}^{\text{atom}} E_C^{e-s} > E_{\gamma}^{e-c} + E_{\text{kin}}^{e-c} \quad (8)$$

where $A_{\text{active}}^{\text{atom}}$ is the proportion of the active surface area of the atom, exposed by the Casimir pressure. Thus, the stability of an electron in the valence shell is also the function of the exposed surface of the atom on which the Casimir effect is active. Casimir pressure is effective on conducting plates. Tightly bond electrons can weaken the Casimir effect; therefore, equation 8 is applicable only to metals, which has zero band gap energy.

6.1 Photoelectric effect

On the surface of a solid metal approximately only half of the surface of the atom is exposed to the Casimir effect because the neighboring atoms in the bulk of the metal shield the inner parts of the surface of the atom/s. Thus, the removal of an electron from the valence shell predictably should be about half the energy of the ionization. This prediction is consistent with measured work functions of the photoelectric effect (Fig. 5).

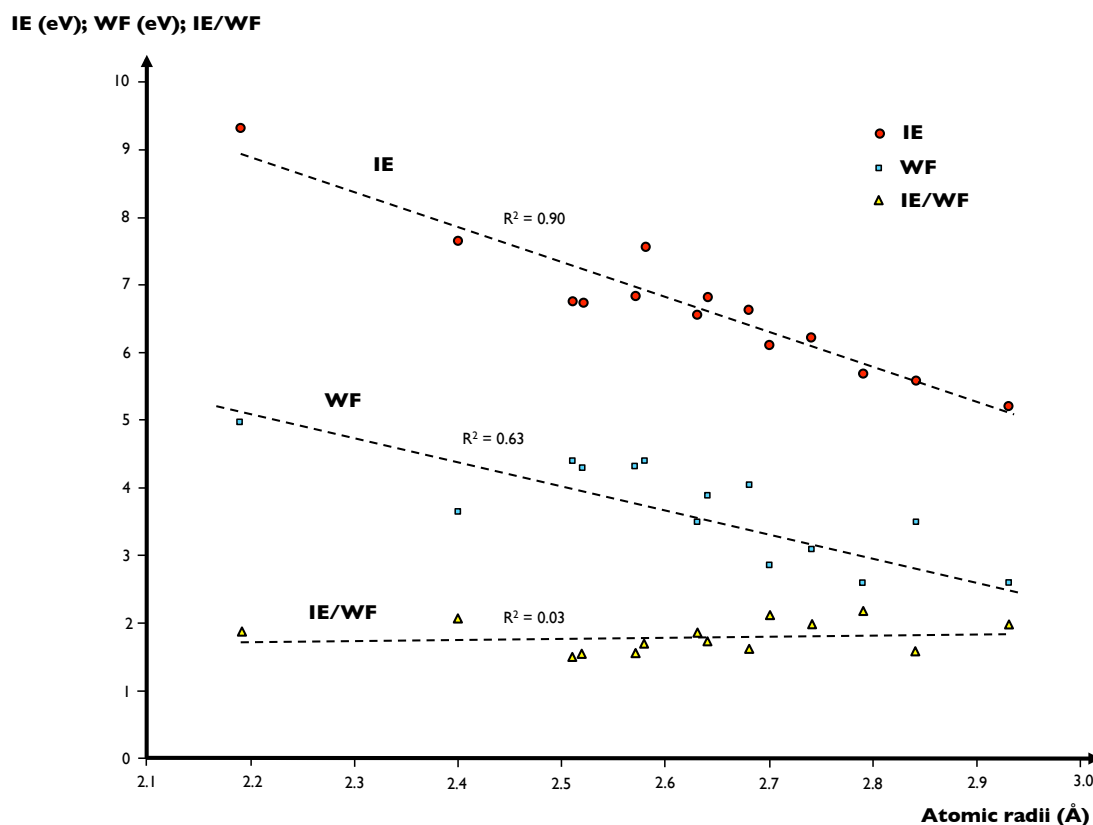


Fig. 5. Ionization energies and the work functions of the elements with closed packing crystal structure are shown as a function of atomic radii [19, 20]. The ratio between the ionization energy and the work function is averaging around two with no correlation to the atomic radii. Thus, the ratio is independent from the ionization energy and remains constant for the same crystal structure.

6.2 Sea of electrons

Inside a metal the proportion of the effective surface area is zero, since the atoms are completely shielded by the neighboring atoms. Thus, the Casimir effect is entirely blocked, and ineffective in the bulk of the metal. Lacking the Casimir effect inside the metal results in the destabilization of the outer shell electrons even in the absence of any photon absorption energy. The electrons inside the metal; therefore, are unstable and can move freely forming the “sea of electrons” [21-23]. The postulated shielding effect of the Casimir pressure on the electron shell of the atoms is consistent with experiments (Fig. 6).

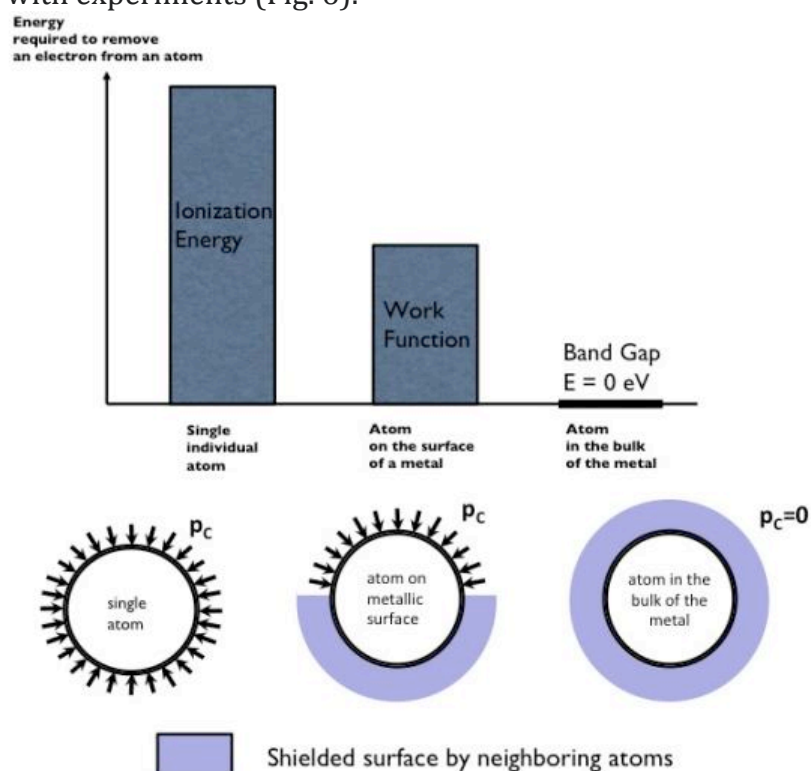


Fig. 6. The removal of the first valence electron from the surface of an individual atom requires overcoming the one-dimensional Casimir energy acting on the surface. For a free, single atom, the one-dimensional Casimir energy is equivalent with the ionization energy. If the surface of the atom is shielded from the Casimir effect then the energy required to remove the electron reduced proportionally with the shielded surface area. (a) free individual atom (b) atom on the surface of a metal (c) atom inside a metal

6.3 Compton scattering

The classical “Thomson” scattering treats the photons as waves. In this case the low energy photons scatter, as the forced oscillation of the electrons radiate the same energy that they receive. Thus, the outgoing photons have the same wavelength as the incoming ones. Compton found that the scattered high-energy photons have longer wavelengths than the incident ones [24]. Thus the photons lose energy in the interactions. This observation can be explained if the incident photon is treated as a particle. When the incoming photon collides with an outer electron, then the electron receives kinetic energy and recoils from the point of impact (Fig. 7). This model allows calculating the difference between the initial and final wavelength of the photons, which is in agreement with the experimental results.

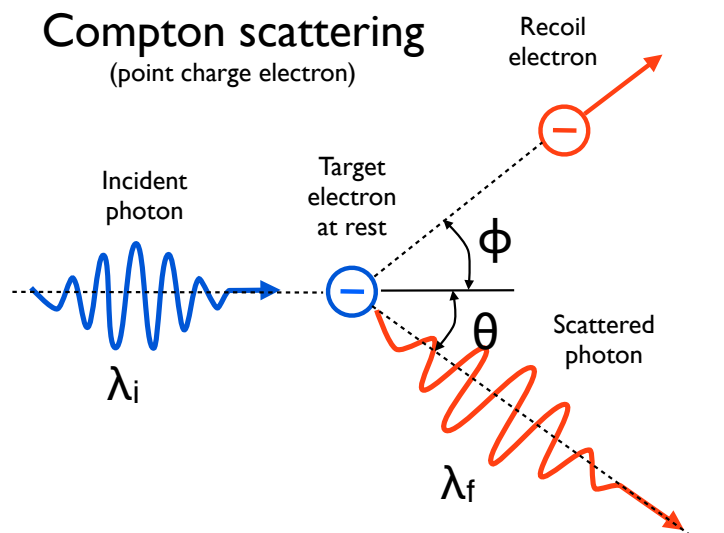


Fig. 7. The schematic figure of the interaction of Compton scattering is shown. The problem with this description is that the outer shell electron cannot be at rest under the electrostatic attraction of the nucleus.

The description of Compton scattering assumes that the outer electron in whom the photon collides is at rest. This assumption is not attainable for a point charge electron, which cannot be at rest under the attraction of the nucleus as discussed before. The electron can only be at rest if forms a surface charge shell around the nucleus. When the incoming photon destabilizes the electron in the shell then phase transformation of the electron occurs and the point charge electron recalls. Thus, the surface charge electron shell atom model with its stationary electron shell is consistent with the experimental results of Compton scattering (Fig. 8).

Compton scattering (surface charge electron shell)

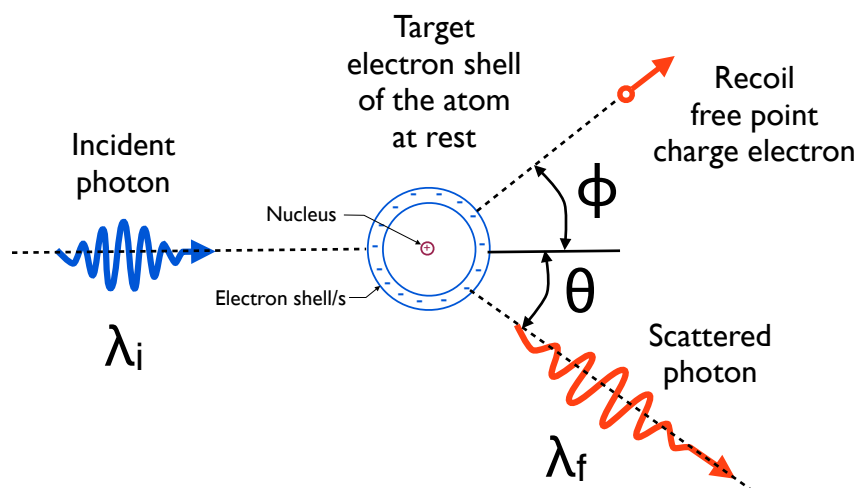


Fig. 8. The schematic figure of the physical process of Compton scattering is shown. Electrons at rest can only be stable in the atom if they form a surface charge electron shell. Thus, Compton scattering is an additional support for the phase transformation of the electron from surface to point charge.

6.4 Wave equation

The wave description of the electron is able to reproduce all of the features of the Hydrogen atom. The fundamental problem with the Bohr's planetary model is that the orbiting point charge electron is one-dimensional; therefore, it is only able to reproduce the main quantum numbers. Schrodinger wave equation describes the behavior of the electron in three-dimension [10, 11]. This quantum mechanical description is probabilistic, and predicts the odds of the location where the electron/s might be found. The wave function (ψ) for the Hydrogen atom in spherical coordinates can be written as the product of the radial $R(r)$ and angular $Y(\Theta, \phi)$ parts as:

$$\psi_{n,l,m}(R, \Theta, \phi) = R_{n,l}(r) Y_l^m(\Theta, \phi), \tag{9}$$

where r is the radius, Θ is the azimuthal angle, and ϕ is the polar angle,

$$r > 0; \quad 0 \leq \Theta \leq \pi, \quad 0 \leq \phi \leq 2\pi, \quad \text{and}$$

$$n = 1, 2, 3, \dots; \quad l = 0, 1, 2, \dots, n-1; \quad m = -l, \dots, l.$$

Expanding equation 1 gives the fully normalized wave function for Hydrogen as:

$$\psi_{n,l,m} = \sqrt{\left(\frac{2}{na_o^*}\right)^3 \frac{(n-1-l)!}{2n[(n+l)!]^3}} e^{\frac{r}{na_o^*}} \left(\frac{2r}{na_o^*}\right)^l L_{n-l-1}^{2l+1}\left(\frac{2r}{na_o^*}\right) Y_l^m(\Theta, \phi) \tag{10}$$

where L_{n-l-1}^{2l+1} is the generalized Legendre polynomial with degree of $n-l-1$, $Y_l^m(\Theta, \phi)$ is a spherical harmonic function of degree of l , and order of m , and a_o^* is the reduced Bohr radius, which can be calculated as

$$a_o^* = \frac{4\pi\epsilon_o \hbar^2}{\mu e^2} \quad \text{and} \quad \mu = \frac{m_p m_e}{m_p + m_e}, \tag{11}$$

where ϵ_o is the permittivity of free space, \hbar is the reduced Planck constant, μ is the reduced mass, e is the charge of the electron, and m_p and m_e are the mass of the proton and electron respectively. This three-dimensional wave equation describes a spherical surface density vibration (Eq. 10-12). The spherical harmonics are related to the quantum numbers. Thus, the presented three-dimensional surface charge electron shell model is consistent with the quantum mechanical treatment of the electron and offers a physical explanation for the wave equation.

6.5 Photon absorption and emission

The spherical surface charge vibrates upon induced disturbances. The speed of the propagation of the generated wave had been calculated for the Hydrogen atom. Assuming a simple stretched string model, the calculated speed of the propagating wave is the same as the orbiting velocity of the electron in the Bohr's model [12]. Thus, not the point charge electron orbits around the nucleus but rather the generated wave propagates in the static spherical surface charge halo. The induced vibration of the shell is constrained by the size of the electron shell. Thus, the wavelength of the triggered vibration and the related energy are quantized.

Photons are only emitted or absorbed in an atom when transition occurs between the stationary states of the electron. These stationary waves are vibrating symmetrically to the charge center. The induced electrical field of the vibration of the surface charge is interfering destructively. Consequently, as long as the frequency of the standing wave remains the same no emission occurs. When the vibration switches from one symmetrical mode (stationary state) to another one, then during the transition period, the vibration becomes asymmetric. The generated electric field in

the asymmetric vibration is not completely destructive resulting in a photon emission. In reverse, photon absorption can transfer the energy to the vibrating shell triggering a transition between the stationary states. (Fig. 9). The frequency or energy change of the vibration has no effect on the size of the shell. Thus, the size of the atom remains the same during the transition, which is consistent with experiments.

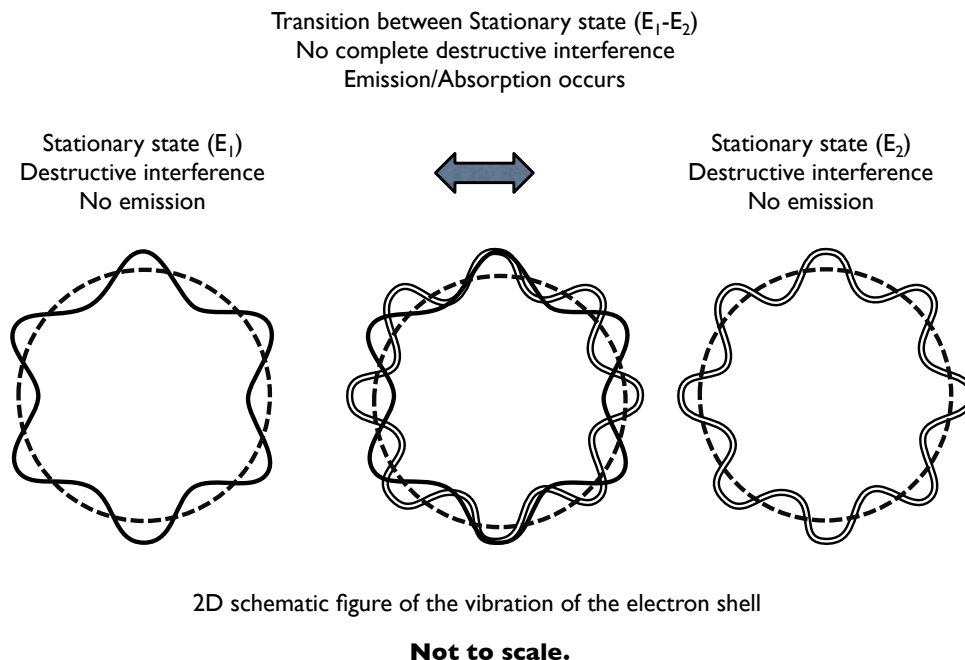


Fig. 9. Schematic 2D figure shows the occurrence of the emission and the absorption of the electromagnetic radiation at atomic scale. The vibration of the electron halo around the nucleus is symmetrical at the stationary energy levels. Thus, no emission occurs resulting from destructive interference. However, when the transition occurs between one symmetrical vibration state to another symmetric one then electromagnetic radiation is emitted or absorbed. The uniform surface charge distribution of the electron explains the emission and absorption of photons without violating classical laws. Thus, classical electromagnetism remains valid at atomic scale.

7. CONCLUSIONS

It has been shown that electromagnetism is valid at atomic scale. Thus, electrons in the atom do not orbit but rather remains stationary in the atom. The stationary electron can only be stable under the attraction of the nucleus if goes through a phase transformation at capturing, and forms a surface charge electron shell around the atom. The model fully complies with the quantum mechanical offering a physical explanation for the wave equation. The calculated velocity of the propagating wave in the surface charge electron shell for the Hydrogen atoms is identical with the Bohr's velocity. Thus the wave equation describes the vibration of the surface charge electron shell. Energy balance investigation, regarding to the stability of the electron shell, shows that the Casimir pressure maintains the stability of the electron shell in the atom. If the Casimir pressure is shielded from the electron shell than the electron can be removed with lower energy than the ionization energy. This prediction is consistent with the photoelectric effect and the "see of electrons" inside the metal, in which the half and the entire surface of the electron shell is shielded respectively by the neighboring atoms. The stationary electron shell is also consistent with the

Compton scattering. The vibrating surface charge electron shell offers a physical explanation why photon emission and absorption occurs only at transition between the stationary states. In conclusion, the presented surface charge electron shell atom model is a viable explanation for all the known features of the atoms.

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