Decomposition with the Additive Model using Buys-Ballot Approach when Trend-Cycle Component is Quadratic in Time Series

Abstract:This study discusses decomposition with the additive model when trend-cycle component is quadratic. Decomposition is a method that can adequately estimate and investigate the trend parameters, seasonal indices and residual component of the series. The study indicates that, Buys-Ballot procedure is computationally simple when compared with other descriptive techniques. The estimates of the quadratic trend-cycle component and seasonal effects are easily computed from periodic and seasonal averages. Hence, the computations are reduce to $\hat{a} =$ 3.2051, $\hat{b} =$, 0.0218 and $\hat{c} =$ -0.0001. Therefore, the fitted additive decomposition model is

$\hat{X}_t = 3.2051 + 0.0218t - 0.0001t^2 + \hat{S}_t$ Under acceptable assumption, the article shows that additive model satisfies $\left(\sum_{j=1}^{s} S_j = 0 \right)$ asin equation (4). We also consider test for seasonality in the additive model in this study.

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Keywords: Time Series Decomposition,Additive Model, Quadratic Trend, Row Variance, Overall Sample Variance, Buys-Ballot table,

1.Introduction

Decomposition method involves the separation of an observed time series into components consisting of trend (long term direction), the seasonal (systematic, calendar related movements), cyclical (long term oscillations or swings about the trend) and irregular (unsystematic, short term fluctuations) components. Time series analysis includes the examination of trend, seasonality, cycles, turning points, changes in level, trend and scale and so on that may influence the series. This is an important preliminary to modelling, when it has to be decided whether and how to seasonally adjust, to transform, and to deal with outliers and whether to fit a model. In the examination of trend, seasonality and cycles, a time series is often described as having trends, seasonal effects, cyclic pattern and irregular or random component Iwueze and Nwogu[1].

The classical decomposition procedure is sometime called procedure of decomposing time series. Its applications is usually predicated on time series models. As stated in the literature, classical decomposition procedure has attracted so much research attention. The objectives of the classical decomposition method have been mentioned in several studies. The advantages of classical decomposition method are; it is used to investigate the presence of trend, seasonal, cyclical and error components in time series analysis. The time series models are;

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Additive Model: $X_t = T_t + S_t + C_t + I_t$ (1)

Multiplicative Model: $X_t = T_t \times S_t \times C_t \times I_t$ (2)

Mixed Model: $X_t = T_t \times S_t \times C_t + I_t$ (3)

For short term period in which cyclical and trend components are jointly combined Chatfield [2]and the observed time series $(X_t, t = 1, 2, ..., n)$ can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the irregular component (e_t). Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \tag{4}$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \tag{5}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \tag{6}$$

Using equations (4) or (5) or (6) we can estimate the three components of our model and therefore decompose the series into its component parts. A summary of the traditional method of decomposition of the time series will be presented in section 2

Iwueze and Nwogu[1] proposed two methods of estimating the parameter of a linear trend-cycle component from the periodic means of the Buys-Ballot table (table 1). This method was initially developed for short period of time in which the trend-cycle component (M_t) is jointly combined and can be represented by linear equation

$M_t = a + b_t$, t = 1, 2, ...n (7)

where a is the intercept, b is the slope and t is the time point.

They two methods are (i) the Chain Base Estimation (CBE) method which calculates the slop from the relative periodic mean changes and (ii) the Fixed Base Estimation (FBE) method which calculates the slope using the first period as the base period for the periodic mean changes.

2. Traditional Method of Decomposition

The major task of the analyst dealing with the series for descriptive purposes is to segregate each component in so far as this is possible. By isolating individual components, the impact of each may be assessed Chatfield [2]. Either of the models (4) or (5) or (6) may be employed to effect the decomposition. The first step will normally be to estimate and then to eliminate trend-cycle (M_t) for each time period from the original data either by subtraction or division. The resulting time series after elimination the trend-cycle (M_t) is the de-trended series and expresses the effects of the season and irregular components. The de-trended series is expressed mathematically as:

$X_t - \hat{M}_t$ (8)

for the additive model or

$X_t / \hat{M}_t$ (9)

for the multiplicative model or

$$X_t / \hat{M}_t \qquad (10)$$

for the mixed model

The seasonal effect is obtained by estimating the average of the de-trended series at each season. The de-trended, de-seasonalized series is obtained as

$$X_t - \hat{M}_t - \hat{S}_t \qquad (11)$$

for the additive model,

$$X_t / \left(\hat{M}_t \, \hat{S}_t\right) \qquad (12)$$

for the multiplicative model,

$$X_t / \left(\hat{M}_t \, \hat{S}_t\right) \qquad (13)$$

for the mixed model.

Oladugba, *et al*,[3] gave brief description on how to choose in time series analysis between additive and multiplicative models. They stated that, the seasonal fluctuation exhibits constant amplitude with respect to the trend in additive model. While amplitude of the seasonal fluctuation depends on trend in multiplicative model.

Using the Buys-Ballot procedure for a seasonal time series, Dozie [4], provided expression for estimation of trend parameters and seasonal indices using row, column and overall means for the mixed model in descriptive time series analysis. He also showed the estimated trend parameters and seasonal for mixed model, when there is no trend ie (b = 0).

3 Methodology

Decomposition with the additive model and test of seasonality in periodic and overall variances in this study is done using Buys-Ballot procedure often referred to in the literature. This method adopted in this study assumed that the series are arranged in a Buys-Ballot table with m rows and s columns. For details of this method see Wei [5], Nwogu*et.al* [6], Iwueze and Ohakwe [7], Dozie and Ijeomah [8], Dozie and Uwaezuoke [9], Dozie*et.al* [10],Dozie and Nwanya [11], Dozie [4], Dozie andIbebuogu [12], Dozie and Uwaezuoke [13], Dozie and Ihekuna [14],Dozie andIbebuogu [15], Dozie and Uwaezuoke [16], Dozie [17], and Dozie and Ihekuna[18]

3.1 Quadratic Trend Cycle and Seasonal Components

The expression of the quadratic trend is given by

$$\bar{X}_i = a + bt + ct^2 \qquad (14)$$

Iwueze and Nwogu [19] provided estimation of the trend and seasonal indices for an additive decomposition model when trend-cycle component is quadratic as;

$$\hat{a} = a^{|} + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \qquad (15)$$

$$\hat{b} = \frac{b^{|}}{s} + \hat{c}(s-1) \qquad (16)$$

$$\hat{c} = \frac{c^{|}}{s^2} \qquad (17)$$

$$\hat{S}_j = \bar{X}_{.j} - d_j \qquad (18)$$

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \hat{b} + \hat{c}(n-s)j + \hat{c}\, j^2 \qquad (19)$$

3.2Test for Seasonality in the Additive Model

Comment [MF3]: ??????

3.2.1 Periodic and Overall Sample Variances

The Buys-Ballot estimates for periodic variance is listed in equations (20) and that of overall variance is given in (21) and for quadratic trending curve for the purposes of detection of presence seasonal effects

3.2.2 Quadratic Trending Curve $\left(a+bt+ct^2\right)$

$$\sigma_{i.}^2=\left\{\begin{array}{l}\dfrac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right\}+\\ \dfrac{1}{s-1}\left\{\sum_{j=1}^{s}S_j^2+2\left[b-2cs\right]C_1+2cC_2\right\}\\ +\left\{\dfrac{s^2(s+1)}{3}\left[bc-c^2(s-1)+\dfrac{4csC_1}{s-1}\right]\right\}i+\left[\dfrac{s^2(s+1)c^2}{3}\right]i^2\end{array}\right\} \quad (20)$$

$$\sigma_{..}^2=\frac{nc^2}{n-1}\left\{\begin{array}{l}\dfrac{\left(n^2-s^2\right)(2n-s)(8n-11s)}{180}+\dfrac{\left(s^2-1\right)(2s+1)(8s-1)}{180}\\ +\dfrac{(n-s)(s+1)\left(6n^2+7ns-n+s^2+5s+6\right)}{36}\end{array}\right\} \quad (21)$$

$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s}S_j^2+2\left[b+c(n-s)\right]C_1+2cC_2\right\}$$

Where $C_1=\sum_{j=1}^{s}jS_j,\ C_2=\sum_{j=1}^{s}jS_j$

4.0 Empirical Example

The empirical example given in this section is based on short series in which the trend cycle

component is jointly estimated. The time series data was drawn from monthly dataof

General Hospital Owerri, Imo State, Nigeria over the period of January, 2008 to December,

2019, with row, column and overall means and standard deviation. The graphs of the series

listed as monthly data (s = 12) and for twelve years (m = 12) are shown in Figures 1,

2 and .3. As Figures.1, 2, and.3 and Appendix A indicate that the series is

seasonal with evidence of upward or downward trend. There is an upsurge of the series in

April, August, September, and October and drop in January, May and December. The

periodic standard deviations are stable, while the seasonal standard deviations differ, indicating that the series requires some transformation to make the seasonal indices additive.

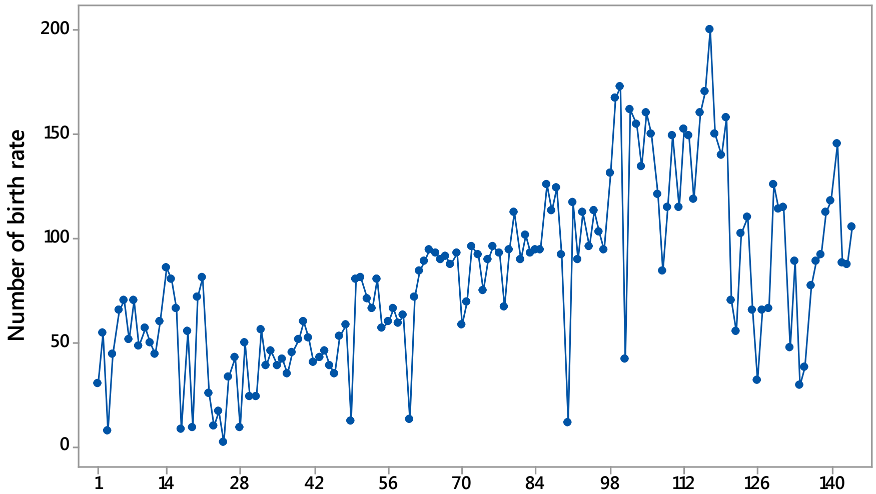

Fig 1 : original series of birth rate, between 2008 and 2019

Comment [MF4]: Rearrange the drawing

3.1: Buys-Ballot Estimates of Quadratic Trend and Seasonal Indices.

$$\bar{X}_{i} = 3.081 + 0.275i - 0.0121i^{2} \quad (22)$$

Using (15), (16), (17) and (18) we obtain,

$$\hat{c} = \frac{-0.0121}{144} = -0.0001$$

$$\hat{b} = \frac{0.275}{12} - 0.0001(12-1) = 0..0218$$

$$\hat{a} = 3.081 + \left(\frac{12-1}{2}\right)0.0218 - \left(\frac{(12-1)(24-1)}{6}\right)0.0001 = 3.2051$$

$$d_{j} = 3.2051 + \frac{0.0218}{2}(144-12) - \frac{0.0001(132-12)(288-12)}{6} + (0.0218) - 0.0001(132-12)j - 0.0001j^{2}$$

$$\bar{X}_{.j} = 4.0584 + 0.0132j + 0.0001j^{2}$$

Table 1: Buys-Ballot Estimates of Seasonal Indices for Additive Model

j	$\bar{X}_j$	$\hat{S}_j$
1	3.800	-0.2451
2	4.247	0.2154
3	4.188	0.1701
4	4.306	0.3020
5	4.108	0.1181
6	4.038	0.0624
7	4.162	0.2009
8	4.429	0.4826
9	4.477	0.5455
10	4.267	0.3506
11	4.203	0.3019
12	4.060	0.1744
$\sum_{j=1}^{s}\hat{S}_j$		

Comment [MF5]: ??

The Buys-Ballot estimates of quadratic trend parameters are

$\hat{a} =$ 3.2051, and $\hat{b} =$, 0.0218 and $\hat{c} =$ -0.0001

Therefore, the estimate of the trend-cycle component is

$$\hat{M}_t = 3.2051 + 0.0218t - 0.0001t^2$$

The estimates of seasonal indices are obtained by averaging the difference $X_t - \hat{M}_t$ at each season are given as $\hat{S}_1 =$ - 0.4683, $\hat{S}_2 =$ - 0.0078, $\hat{S}_3 =$ - 0.0531, $\hat{S}_4 =$ 0.0788, $\hat{S}_5 =$ - 0.1051, $\hat{S}_6 =$ - 0.1608, $\hat{S}_7 =$ - 0.0223, $\hat{S}_8 =$ 0.2594 , $\hat{S}_9 =$ 0.3223, $\hat{S}_{10} =$ 0.1274, $\hat{S}_{11} =$ 0.0787, $\hat{S}_{12} =$ -0.0492 listed in Table 3 for additive model. Hence, the fitted model is

$$\hat{X}_t = 3.2051 + 0.0218t - 0.0001t^2 + \hat{S}_t$$

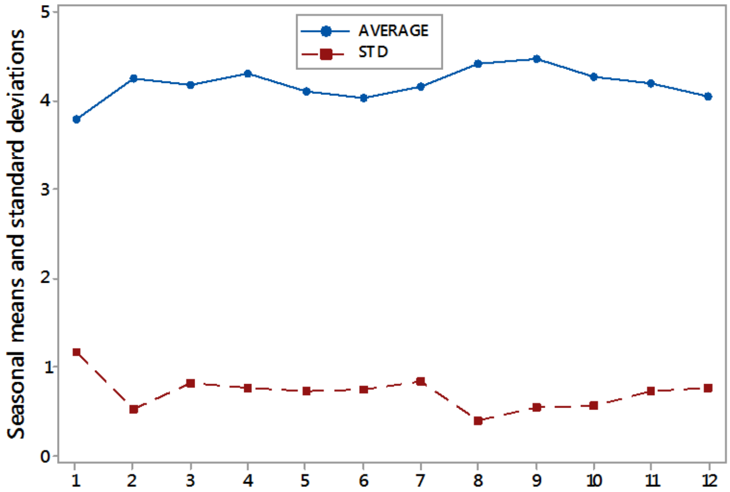

Fig 2 :Graphical presentation showing seasonal variation

Table 2: Estimates of Trend and Seasonal Indices

parameter	Additive Model
$\hat{a}$	3.2051
$\hat{b}$	0.0218
$\hat{c}$	-0.0001
$\hat{S}_1$	-0.2451
$\hat{S}_2$	0.2154
$\hat{S}_3$	0.1701
$\hat{S}_4$	0.3020
$\hat{S}_5$	0.1181
$\hat{S}_6$	0.0624
$\hat{S}_7$	0.2009
$\hat{S}_8$	0.4826
$\hat{S}_9$	0.5455
$\hat{S}_{10}$	0.3506
$\hat{S}_{11}$	0.3019

$\hat{S}_{12}$	0.1744

Table 3: Row Totals, Means and Standard Deviations

Periods i		Quadratic trend cycle		
	r_i	T_i	$\bar{X}_i$	$\sigma_{i.}$
1	12	45.33	3.7780	0.6230
2	12	40.43	3.5360	0.9460
3	12	39.34	3.2780	0.9500
4	12	45.87	3.8223	1.1836
5	12	47.21	3.9340	0.6690
6	12	53.14	4.4283	0.1557
7	12	54.09	4.5078	0.1319
8	12	53.75	4.4790	0.6660
9	12	57.77	4.8140	0.4080
10	12	59.82	4.9854	0.1644
11	12	51.75	4.3130	0.4230
12	12	52.91	4.4090	0.4600
Overall Total	144			

Comment [MF6]: ??

Comment [MF7]: ?

Comment [MF8]: ?

$$n = \sum_{j=1}^{r} c_j = \sum_{i=1}^{c} r_i = total\ \ number\ \ of\ \ observation$$

Where,

r_i = Number of observation in the rth row

c_j = Number of observation in the jth column.

Table 4: Seasonal Totals, Means and Standard Deviations

Seasons j		Quadratic trend cycle		
	c_j	$T._j$	$\bar{X}_{.j}$	$\sigma_{.j}$
1	12	45.60	3.800	1.173
2	12	50.97	4.247	0.528
3	12	50.26	4.188	0.826
4	12	51.68	4.306	0.772
5	12	49.30	4.108	0.727
6	12	48.46	4.038	0.749
7	12	49.94	4.162	0.831
8	12	53.15	4.429	0.390
9	12	53.72	4.477	0.544
10	12	51.20	4.267	0.565
11	12	50.43	4.203	0.719
12	12	48.72	4.060	0.755
Overall Total	144			

Table 5: Estimates of Trend, Seasonal and Residual Values for 2007 to 2019

Year	T	Y_t	$\hat{T}_t$	$\hat{S}_t$	$\hat{Y}_t = \hat{T}_t + \hat{S}_t$	$\hat{R}_t = Y_t - \hat{Y}_t$	$Adj\ \hat{R}_t$
2007	1	3.7780	3.2268	- 0.2451	2.9813	0.7967	0.1733
2008	2	3.5360	3.2483	0.2154	3.4637	0.0723	-0.5511
2009	3	3.2780	3.2696	0.1701	3.4397	- 0.1617	-0.7851
2010	4	3.8223	3.2907	0.3020	3.5927	0.2296	-0.3938
2011	5	3.9340	3.3116	0.1181	3.4297	0.5043	-0.1191
2012	6	4.4283	3.3323	0.0624	3.3947	1.0336	0.4102
2013	7	4.5078	3.3528	0.2009	3.5537	0.9541	0.3307
2014	8	4.4790	3.3731	0.4826	3.8557	0.6233	-0.0001
2015	9	4.8140	3.3932	0.5455	3.9387	0.8753	0.2519
2016	10	4.9854	3.4131	0.3506	3.7636	1.2218	0.5984
2017	11	4.3130	3.4328	0.3019	3.7637	0.5493	-0.0741
2018	12	4.4090	3.4523	0.1744	3.6267	0.7823	0.1589

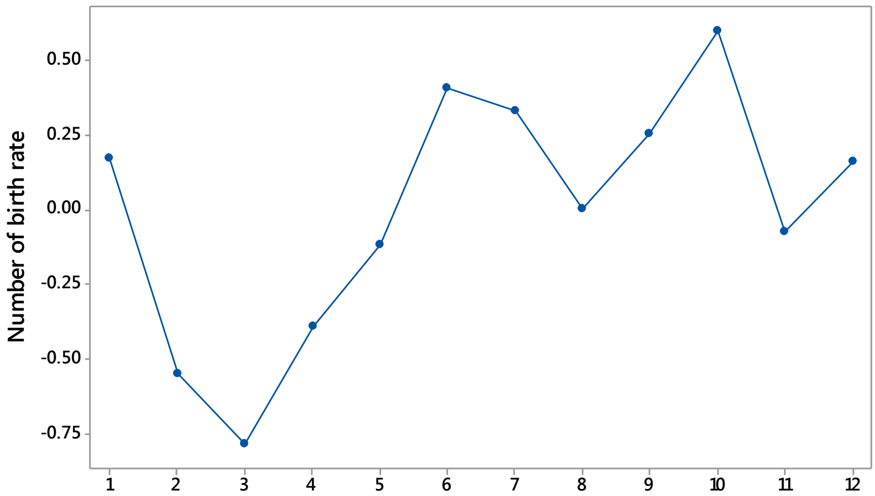

Fig 3: plot of residuals for birth rate between 2008 and 2019

The estimated trend line for these data is

$\hat{T}_t = 3.2051 + 0.0218t - 0.0001t^2$, with t = 1 in 2008 and estimated trend values given in Table 5. The irregular component obtained by subtracting the estimates of $\hat{M}_t$ and $\hat{S}_t$ from the X_t Therefore, the residual mean obtained is zero, while the variance is 0.1541. Hence, the fitted model becomes

$$\hat{X}_t = 3.2051 + 0.0218t - 0.0001t^2 + \hat{S}_t$$

3.2 Application of Test for Seasonality in the Additive Model

Matched pairs of data are applied to the periodic and overall variances of the Buys-Ballot table. For the matched pairs of data, $(X_i, Y_i), \quad i = 1, 2, ...n$, define $d_i = X_i - Y_i$. For identification of the presence and absence of seasonal indices in series. Let X_i represents periodic and overall variances in the presence of seasonal indices and denote Y_i represents periodic and overall variances in the absence of seasonal indices (Nwogu *et al.* [20])

3.3For Quadratic Trend, in the Presence of Seasonal Effect, the Periodic Variance is obtained as

$$X_i(Q) = \sigma_{i.}^2(Q) = \left\{ \begin{array}{l} \frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2 - 30(s-1)bc + 15b^2\right\} + \\ \frac{1}{s-1}\left\{\sum_{j=1}^{s} S_j^2 + 2\left[b - 2cs\right]C_1 + 2cC_2\right\} \\ + \left\{\frac{s^2(s+1)}{3}\left[bc - c^2(s-1) + \frac{4csC_1}{s-1}\right]\right\} i + \left[\frac{s^2(s+1)c^2}{3}\right] i^2 \end{array} \right\} \quad (23)$$

When there is no seasonal indices, $S_j = 0 \forall j = 1, 2, ..., s, \quad C_1 = C_2 = \sum_{j=1}^{s} S_j^2 = 0.$ thus

$$Y_i(Q)=\left\{\begin{array}{l}\frac{s(s+1)}{180}\left\{(2s-1)(8s-11)c^2-30(s-1)bc+15b^2\right\}+\\ +\left\{\frac{s^2(s+1)}{3}\left[bc-c^2(s-1)\right]i\right\}+\left[\frac{s^2(s+1)c^2}{3}\right]i^2\end{array}\right\} \quad (24)$$

$$d_i(Q)=X_i(Q)-Y_i(Q)\left\{\begin{array}{l}\frac{1}{s-1}\left\{\sum_{j=1}^{s}S_j^2+2\left[b-2cs\right]C_1+2cC_2\right\}\\ +\left\{\left[+\frac{4csC_1}{s-1}\right]\right\}i\end{array}\right\} \quad (25)$$

Which is zero under null hypothesis ($H_o:S_j=0$)

3.4The Overall Variance is obtained as

$$X_i(Q)=\sigma_{..}^2(Q)=\frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\ +\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\} \quad (26)$$
$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s}S_j^2+2\left[b+c(n-s)\right]C_1+2cC_2\right\}$$

When there is no seasonal indices, $S_j=0\forall j=1,2,...,s,\quad C_1=C_2=\sum_{j=1}^{s}S_j^2=0.$ thus

$$Y_i(Q)=\frac{nc^2}{n-1}\left\{\begin{array}{l}\frac{(n^2-s^2)(2n-s)(8n-11s)}{180}+\frac{(s^2-1)(2s+1)(8s-1)}{180}\\ +\frac{(n-s)(s+1)(6n^2+7ns-n+s^2+5s+6)}{36}\end{array}\right\} \quad (27)$$
$$+\frac{bcn(n+1)^2}{6}+\frac{b^2n(n+1)}{12}+$$

$$d_i(Q) = X_i(Q) - Y_i(Q) = \frac{n}{s(n-1)}\left\{\sum_{j=1}^{s} S_j^2 + 2\left[b + c(n-s)\right]C_1 + 2cC_2\right\} \quad (28)$$

Which is zero under null hypothesis ($H_o : S_j = 0$)

Table 6: Estimates in the Presence of Seasonal Indices for Row Variance

Quadratic Trending Curve $\left(a+bt+ct^2\right)$	$\left\{\begin{array}{l}\frac{1}{s-1}\left\{\sum_{j=1}^{s} S_j^2 + 2\left[b-2cs\right]C_1 + 2cC_2\right\} \\ +\left\{\left[+\frac{4csC_1}{s-1}\right]\right\}i\end{array}\right\}$

Where $C_1 = \sum_{j=1}^{s} jS_j$, $C_2 = \sum_{j=1}^{s} jS_j$

Table 7: Estimates in the Presence of Seasonal Indices for Overall Variance

Quadratic Trending Curve $\left(a+bt+ct^2\right)$	$\frac{n}{s(n-1)}\left\{\sum_{j=1}^{s} S_j^2 + 2\left[b+c(n-s)\right]C_1 + 2cC_2\right\}$

Where $C_1 = \sum_{j=1}^{s} jS_j$, $C_2 = \sum_{j=1}^{s} jS_j$

Test of seasonality in time series is applied using matched pairs of data in the Buys-Ballot table for quadratic trending curve shown given in equations (20) and (21) respectively. The test was developed using the periodic and overall variances of the Buys-Ballot table. The estimates for the data in the presence of seasonal indices for periodic and overall variances are listed in Tables 6 and 7.The Buys-Ballot estimates obtained are listed in equations (25) and (28) are functions of the seasonal indices only when the trend parameters are removed, while that of equations (24) and (27) are products of trend parameters.

4.0 Summary, Conclusion and Recommendations

This study has examined decomposition with the additive model and test of seasonality in time series. The method adopted in this study is Buys-Ballot procedure developed for choice of modelamong other uses based on row, column and overall means and variances of the Buys- Ballot table. This study is limited to a series in when trend-cycle component is quadratic and admits additive model.The study shows that the periodic standard deviations are stable, while the seasonal standard deviations differ, indicating that the series requires some transformation to make the seasonal indices additive.Successful transformation given in appendix B was carried out to meet the constant variance and normality assumptions in the distribution. The adjusted residual mean obtained is zero, while the variance is 0.1541. Hence, the fitted model is inadequate.This study also consider a test for seasonality in the additive model using the Buys-Ballot table and the nature of trending curve is quadratic. The test is applied to the periodic and overall sample variances of the Buys-Ballot table to detect the presence of seasonal indices. Results indicate that, the Buys-Ballot estimates obtained and listed in equations (25) and (28) are functions of seasonal indices only, while that of equations (24) and (27) are functions of trend parameters.This study has provided decomposition with the additive model when trend cycle component is quadratic. Other trending curves yet to be considered, are therefore recommended for further investigation.

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Appendix A: Buys-Ballot table of Birth Rate atGeneral Hospital Owerri (2008-2019)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2008	30	54	7	44	65	70	51	70	48	57	50	44	590	49.17	17.60
2009	60	86	80	66	8	55	9	72	81	25	10	17	569	47.42	31.19
2010	2	33	43	9	50	24	24	56	39	46	39	42	407	33.92	16.32
2011	35	45	51	60	52	40	43	46	39	35	53	58	557	46.42	8.45
2012	12	80	81	71	66	80	57	60	66	59	63	13	708	59.00	23.27
2013	72	84	89	94	93	90	91	87	93	58	69	90	1016	84.67	11.93
2014	92	75	90	96	90	75	94	112	90	101	93	94	1097	91.42	11.38
2015	94	126	113	124	92	11	117	90	112	96	113	103	1191	99.25	30.39
2016	94	131	167	173	42	162	155	134	160	150	121	84	1573	131.10	39.80
2017	115	149	115	152	149	119	160	170	200	150	140	158	1777	148.08	24.38
2018	70	55	102	110	65	32	65	66	126	114	115	47	967	80.58	31.10
2019	89	29	38	77	89	92	112	118	145	88	87	105	1069	89.08	31.80
total	765	947	976	1076	864	842	978	1081	1199	979	953	861			
$\bar{y}_{.j}$	63.8	78.9	81.3	89.7	72.0	70.2	81.5	90.1	99.9	81.6	79.4	71.8			
$\sigma_{.j}$	36.3	39.2	42.5	45.7	35.1	42.7	48.5	36.8	50.7	41.9	38.6	42.3			

Source: General Hospital Owerri, Imo State,Nigeria

Appendix B: Transformed Data of Birth Rate at General Hospital Owerri (2008-2019)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{y}_{i.}$	$\sigma_{i.}$
2008	3.4012	3.9890	1.9459	3.7842	4.1744	4.2485	3.9318	4.2485	3.8712	4.0431	3.9120	3.7842	45.33	3.7780	0.6230
2009	4.0943	4.4544	4.3820	4.1897	2.0794	4.0073	2.1972	4.2767	4.3945	3.2189	2.3026	2.8332	42.43	3.5360	0.9460
2010	0.6932	3.4965	3.7612	2.1972	3.9120	3.1781	3.1781	4.0254	3.6636	3.8286	3.6636	3.7377	39.34	3.2780	0.9500
2011	3.5554	3.8067	3.9318	4.0943	3.9512	3.6888	3.7612	3.8286	3.6637	3.5554	3.9703	4.0604	45.87	3.8223	0.1836
2012	2.4849	4.3820	4.3946	4.2627	4.1897	4.3820	4.0431	4.0943	4.1897	4.0775	4.1431	2.5650	47.21	3.9340	0.6690
2013	4.2767	4.4308	4.4886	4.5433	4.5326	4.4988	4.5109	4.4657	4.5326	4.0604	4.2341	4.5644	53.14	4.4283	0.1557

2014	4.5218	4.3175	4.4998	4.5644	4.5326	4.2047	4.5433	4.7185	4.4998	4.6151	4.5326	4.5433	54.09	4.5078	0.1319
2015	4.5433	4.8363	4.7274	4.8203	4.5218	4.3979	4.7622	4.4998	4.7185	4.5644	4.7274	4.6347	53.75	4.4790	0.6660
2016	4.5433	4.8752	5.1180	5.1533	3.7377	5.0876	5.0434	4.8978	5.0752	5.0106	4.7958	4.4308	57.77	4.8140	0.4080
2017	4.7449	5.0040	4.7450	5.0239	5.0040	4.7791	5.0752	5.1358	5.2983	5.0106	4.9416	5.0626	59.82	4.9854	0.1644
2018	4.2485	4.0073	4.6250	4.7005	4.1744	3.4657	4.1744	4.1897	4.8363	4.7362	4.7449	3.8502	51.75	4.3130	0.4230
2019	4.4886	3.3673	3.6376	4.3438	4.4886	4.5218	4.7185	4.7707	4.9767	4.4773	4.4659	4.6540	52.91	4.4090	0.4600
total	45.60	50.97	50.26	51.68	49.30	48.46	49.94	53.15	53.72	51.20	50.43	48.72			
$\bar{y}_{.j}$	3.800	4.247	4.188	4.306	4.108	4.038	4.162	4.429	4.477	4.267	4.203	4.060			
$\sigma_{.j}$	1.172	0.528	0.826	0.772	0.727	0.749	0.831	0.390	0.544	0.565	0.719	0.755			

Source: General Hospital Owerri, Imo State,Nigeria