Buys-Ballot Estimates for Linear Model with the Expected Values in Time Series Decomposition

Abstract: The study discusses the Buys-Ballot estimates for linear trend cycle and seasonal indices with the expected values for mixed model in descriptive time series analysis. The emphasis is to derive the expected values of row, column and overall means of the of Buys-Ballot table for the mixed model.We use a real life data to determine the estimation of trend parameters, seasonal indices and choice of appropriate model of the Buys-Ballot table. Results indicate that, the expected values of row and column averages are; (1) row average of the mixed model mimic the shape of the trending parameters of the original series and contains seasonal effect in $C_1 = \sum_{j=1}^{S} js_j$ (2) column average also mimic the shape of the trending curves of the original series and contain seasonal effect (3) the appropriate model that best describe the pattern of the study series listed in the summary table (Table 5) is mixed.

Keywords: Time Series Decomposition, Mixed Model, Linear Trend, Buys-Ballot Estimates, Expected Values, Choice of Model

1 Introduction

[1] proposed two methods of estimating the parameter of a linear trend-cycle component from the periodic means of the Buys-Ballot table (table 1). This method was initially developed for short period of time in which the trend-cycle component (M_t) is jointly combined and can be represented by linear equation

$$M_t = a + b_t \text{, t = 1, 2, ...n} \quad (1)$$

where a is the intercept, b is the slope and t is the time point.

The two methods are (i) the Chain Base Estimation (CBE) method which calculates the slop from the relative periodic mean changes and (ii) the Fixed Base Estimation (FBE) method which calculates the slope using the first period as the base period for the periodic mean changes.

Decomposition method involves the separation of an observed time series into components consisting of trend (long term direction), the seasonal (systematic, calendar related movements), cyclical (long term oscillations or swings about the trend) and irregular (unsystematic, short term fluctuations) components. Time series analysis includes the examination of trend, seasonality, cycles, turning points, changes in level, trend and scale and so on that may influence the series. This is an important preliminary to modelling, when it has to be decided whether and how to seasonally adjust, to transform, and to deal with outliers and whether to fit a model. In the examination of trend, seasonality and cycles, a time series is often described as having trends, seasonal effects, cyclic pattern and irregular or random component [1]. Theoretically, a time series contains four components, namely, the trend, the seasonal variation, the cyclical and irregular variation. The trend may be loosely defined as the long term change in the mean and refers to the general direction in which the graph of the series appeared to be going over a long interval of time. Trend shows the presence of factors that persist for a considerable duration. These factors include changes in population, fluctuation in price level, improvements in technology and several conditions that are peculiar to individual investments or establishment. Abrupt or sudden changes in trend may be caused by introduction of new element into or elimination of an old factors from forces affecting the series [2].

[3] stated that, the successive periodic mean $\left(\bar{X}_i\right)$ gives a simple description of the underlying trend from the methods of monthly or quarterly means. It was given that the estimates of the seasonal indices can be obtained from the column means $\bar{X}_{.j}$. Hence, while the periodic means estimate the trend, the column means gives estimates of seasonal indices. It has been noted that a time series theoretically contains four components, However, for short series, the trend components is estimated into the cyclical and trend cycle component is obtained and denoted by m_t. Under these conditions, it can be stated that estimates of trend-cycle and seasonal components can be obtained from the row and column means, respectively, of the Buys-Ballot table. These estimates have been

designated “Buys-Ballot” estimates in this study the details of the procedure for estimation of the trend-cycle component (m_t) are presented in section 2.1 for the mixed model.

Iwueze, *et al,* [4] provided the use of joint plot of seasonal means and standard deviationsfor choice between additiveand multiplicative models in descriptive in time analysis.

Dozie [5] discussed the expression of the parameters of linear trend cycle and seasonal components with emphasis on the periodic, seasonal and overall means for the mixed model. In his summary, he demonstratedthat, the linear trend cycle and seasonal components when there is zero trend and b = 0 in time series decomposition.

2. Methodology

A vital aspect of the time series decomposition using Buys-Ballot approach is the arrangement of the observed series in the Buys-Ballot table as given in Table 1. This method is based on the row, column and overall means of the Buys-Ballot table with m rows and s columns. For details of this method see Wei [6], Nwogu *et.al* [7], Dozie and Ihekuna [8], Dozie *et.al* [9], Dozie and Nwanya [10], Dozie [5], Dozie and Ijeomah [11], Dozie and Ibebuogu (12), Dozie and Uwaezuoke [13], Dozie and Ihekuna [14] Dozie and Ibebuogu [15] Akpanta and Iwueze [16], Dozie and Uwaezuoke [17], Dozie [18].

All the derivations made in this section are based on the systematic component of the mixed model of the Buys-Ballot table. The results of the Buys-Ballot estimates for mixed model with the error terms are given in Table 2. From Table 2, we observed that, the expected value Buys-Ballot estimates is a function of the intercept and seasonal indices $\left(S_{j,} i = 1, 2, ..., m\right)$ depend on the estimate of the slope. The derivations of expected values of row, column and overall means for mixed model with the error terms are given in Table 2

Table 1: Buys - Ballot Tabular Arrangement of Time Series Data

Rows/ Period (i)	Columns (season) j								
	1	2	$\cdots$	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	$\cdots$	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	$\cdots$	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	$\cdots$	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$\cdots$	$X_{(i-1)s+j}$	$\cdots$	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	$\cdots$	$X_{(m-1)s+j}$	$\cdots$	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	$\cdots$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\cdots$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	$\cdots$	$\hat{\sigma}_{.j}$	$\cdots$	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

In this study, the main objective is to obtain the expected values of the Buys-Ballot Estimates of the trend-cycle component with the error terms when the trend-cycle component is linear equation given by (1).

3 Buys-Ballot Estimates

This section presents the Buys-Ballot parameter estimation procedure when the trend-cycle component is linear equation given by (1). Section 2.1 considers the procedure for the mixed model. Section 2.2 gives the derivation of the expected values of row, column and overall means with the error terms of the Buys-Ballot table.

3.1: Buys-Ballot Procedure for the Mixed Model

The mixed model is given by

$$X_t = M_t \times S_t + e_t . \quad (2)$$

where for time t, X_t is the value of the series, S_t is the seasonal component whose sum over a complete period is s, e_t is the error term which, for our discussion, is the Gaussian $N(1,\sigma^2)$ white noise and M_t is the linear trend-cycle component given in equation (1)

Method of obtaining the row, column and overall averages of the Buys-Ballot table for mixed model with the error terms are those of Dozie [5]. Only the results are given

$$\bar{X}_i = a - bs(i-1) + \frac{b}{s}\sum_{j=1}^{s} jS_j + \bar{e}_i, \quad i = 1, 2, ...m \quad (3)$$

$$\bar{X}_{.j} = \left[a + b\left(\frac{n-s}{2} \right) + bj \right] S_j + \bar{e}_{.j}, \quad j = 1, 2, \ldots s \ (4)$$

$$\bar{X}_{..} = a + b\left(\frac{n-s}{2} \right) + bc_1 + \bar{e}_{..} \quad i = 1, 2, \ldots m \text{ and } j = 1, 2 \ldots . s \ (5)$$

3.2 Expected Values of the Row, Column and Overall Means for Mixed Model

3.1.1 Expected Value of Row Mean with the Error Term

Using the expressions in Table 1 and equation (3), we obtain expected value of the row mean as thus;

$$E\left(\bar{X}_i \right) = E\left[a - bs(i-1) + \frac{b}{s} \sum_{j=1}^{s} jS_j \right] + E\left(\bar{e}_i \right) \qquad (6)$$

Hence, the expected value of row mean is

$$E\left(\bar{X}_i \right) = a - bs + bsi + \frac{b}{s} \sum_{j=1}^{s} jS_j \qquad (7)$$

where $E\left(\bar{e}_i \right) = 0$

3.1.2 Expected Value of Column Mean with the Error Term

Using the expressions in Table 1 and equation (4), we obtain expected value of the column mean as thus;

$$E\left(\bar{X}_{.j} \right) = E\left[a + b\left(\frac{n-s}{2} \right) + bj \right] E\left(S_j \right) + E\left(\bar{e}_{.j} \right) \qquad (8)$$

Therefore, the expected value of the column mean is

$$E\left(\bar{X}_{.j} \right) = \left[a + b\left(\frac{n-s}{2} \right) + bj \right] S_j \qquad (9)$$

where $E\left(\bar{e}_{.j} \right) = 0$

3.1.3 Expected Value of Overall Means with the Error Term

Using the expressions in Table 1 and equation (5), we obtain expected value of the overall

means as; $$E\left(\bar{X}_{..}\right) = E\left[a + b\left(\frac{n-s}{2}\right) + bc_1\right] + E\left(\bar{e}_{..}\right) \quad (10)$$

Hence , the expected value of the overall mean is

$$E\left(\bar{X}_{..}\right) = a + b\left(\frac{n-s}{2}\right) + bc_1 \quad (11)$$

where $E\left(\bar{e}_{..}\right) = 0$

3.3 Estimation of trend parameters

The row and overall means are applied to estimate parameters of trend line. The length of periodic interval is taken to be s. using the expressions in (2) and (4), we obtain mixed model as;

$$\bar{X}_{i.} = a - b(s - c_1) + (bs)i \quad (12)$$

$$\equiv \alpha + \beta_i \quad (13)$$

Hence, $\hat{a} = \alpha + \hat{b}(s - c_1)$ (14)

$$\hat{b} = \frac{\beta}{s} \quad (15)$$

For mixed model, when b=0, that is when there is no trend,

$$\bar{X}_{i.} = a + \bar{e}_i \quad (16)$$

3.4 Estimation of Seasonal Indices $S_j, \quad j = 1,2,...,s$

The column and overall means are used to estimate the seasonal indices. The length of periodic interval is also taken to be s. using the expression in (3) and (4), we obtain mixed model as;

$$\equiv \lfloor \alpha + \beta_j \rfloor S_j \tag{17}$$

Where. $\alpha = a + b\left(\frac{n-s}{2}\right)$ (18)

$$\beta = b \tag{19}$$

$$\therefore \quad S_j = \frac{\bar{X}_j}{a + b\left(\frac{n-s}{2}\right) + b_j} \tag{20}$$

For mixed model, where there is no trend (b = 0), we obtain from (19)

$$\hat{S}_j = \frac{\bar{X}_j}{a + \bar{e}_j} \tag{21}$$

3.5Test for Constant Variance

To test the null hypothesis that the variances are equal, that is

$$H_0 : \sigma_i^2 = \sigma_j^2$$

against the alternative

$$H_1 : \sigma_i^2 \neq \sigma_j^2 \text{ for } i \neq j$$

and at least one variance is different from others

the statistic is given as;

$$T = \frac{(N-k)\ln S_p^2 - \sum (N_i - 1)\ln S_i^2}{1 + \frac{1}{3(k-1)}\left[\sum_{i=1}^{k} \frac{1}{(N_i - 1)} - \frac{1}{N-k}\right]} \tag{22}$$

follows Chi-square distribution with (k – 1) degrees of freedom. Using the parameters of the Buys-Ballot table, $N = ms,\ k = s,\ N_i = m$, the statistic in (22) is then given as

$$T_c = \frac{(ms-s)\ln\hat{\sigma}_p^2 - \sum(m-1)\ln\hat{\sigma}_j^2}{1+\frac{1}{3(s-1)}\left[\sum_{j=1}^{s}\frac{1}{m-1}-\frac{1}{ms-s}\right]}$$

$$= \frac{(m-1)\left[s\ln\hat{\sigma}_p^2 - \sum\ln\hat{\sigma}_j^2\right]}{1+\frac{(s+1)}{3s(m-1)}} \qquad (23)$$

Where ms is the total number of observations, m is the number of observations in each column and s is length of the periodic interval.

3.6 Chi-Squared Test

Nwogu, *et al,* [7] and Dozie *et al*[9] proposed chi-square test for choice between the multiplicative and mixed models is based on the column variances. According to them, the null hypothesis to be tested is

H_0**:** $\sigma_j^2 = \sigma_{0j}^2$

and the appropriate model is mixed, against the alternative

H_1**:** $\sigma_j^2 \neq \sigma_{0j}^2$

and the appropriate model is not mixed, where

$\sigma_j^2 = (j = 1, 2, ..., s)$ is the actual variance of the jth column.

$$\sigma_{0j}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (24)$$

and σ_1^2 is the error variance, assumed equal to 1.

They stated the statistic

$$\chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{0j}^2} \qquad (25)$$

follows the chi-square distribution with $m-1$ degrees of freedom, m is the number of observations in each column and s is the seasonal lag (number of columns). They also showed that under the null hypothesis, the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ contains the statistic (25) with 100 (1- α)% degree of confidence.

3.7 Choice of Transformation

Akpanta and Iwueze [16] have shown that, the slope of the regression equation of log of group standard deviation on log of group mean as given in equation (26) is what is needed for choice of appropriate transformation. Some of the values of slope β and their implied transformation are stated in Table 2

$$\log_e \hat{\sigma}_i = a + \beta \log_e \hat{X}_{i.} \qquad (26)$$

Table 2: Bartlett's Transformation for Some Values of β

S/No	**1**	**2**	**3**	**4**	**5**	**6**	**7**
β	**0**	$\frac{1}{2}$	**1**	$\frac{3}{2}$	**2**	**3**	**-1**
Transformation	**No transformation**	$\sqrt{X_t}$	$\log_e X_i$	$\frac{1}{\sqrt{X_t}}$	$\frac{1}{X_t}$	$\frac{1}{X_t^2}$	X_t^2

They stated that, for a method used in choosing an appropriate transformation, the natural logarithm of standard deviation will be applied to regress against the natural logarithm of periodic means and the result of the β - value will determine the type of transformation.

4. **Real Life Data**

This section presents time series based on short series in which the trend cycle component is jointly estimated. The time series data for the period 2013 to 2022, is used to determine the estimation of trend parameters, seasonal indices and suitable model for decomposition of the study series. The periodic standard deviations are stable while the seasonal standard deviations are different which suggest that the series needs transformation to stabilize the variance. The time series data was transformed by taking the inverse square root of the one hundred and twenty (120) observed values given in Appendix B. From the transformed series, the estimation of trend parameters and seasonal indices are given in Tables 3 and 4 respectively.

4.1 Results of Trend Parameters and Seasonal Indices

Using (14),(15) and (20), we obtain

$$\hat{b} = 0.02, \qquad \hat{a} = 2.58 - 0.02\left(\frac{120-12}{2}\right)$$

Table 3: Estimates of trend parameters

Parameter	Mixed model values
a	1.50
b	0.02

$$\hat{S}_j = \frac{\bar{X}_{.j}}{2.58 + 0.02_j}$$

Table 4: Estimates of Seasonal indices

j	$\bar{X}_j$	$\hat{S}_j$
1	2.24	0.86
2	2.84	1.09
3	2.60	0.99
4	2.70	1.01
5	2.87	1.07
6	2.71	1.00
7	2.79	1.02
8	2.94	1.07
9	2.72	0.98
10	2.53	0.91
11	2.88	1.03
12	2.75	0.97
$\sum_{j=1}^{s} \hat{S}_j$		12.00

The Buys-Ballot estimates of linear trend parameters are

$\hat{a} = 1.50$, and $\hat{b} =$, 0.02

Therefore, the estimate of the trend-cycle component is

$$\hat{M}_t = 1.50 + 0.02t$$

The estimates of seasonal indices are obtained by the ratio $X_t / \hat{M}_t$ at each season are given as $\hat{S}_1 =$ 0.86, $\hat{S}_2 =$ 1.09, $\hat{S}_3 =$ 0.99, $\hat{S}_4 =$ 1.01, $\hat{S}_5 =$ 1.07, $\hat{S}_6 =$ 1.00, $\hat{S}_7 =$ 1.02, $\hat{S}_8 =$ 1.07, $\hat{S}_9 =$ 0.98, $\hat{S}_{10} =$ 0.91, $\hat{S}_{11} =$ 1.03, $\hat{S}_{12} = 0.97$ listed in Table 3 and 4 respectively for mixed model. Hence, the fitted model is

$$\hat{X}_t = 1.50 + 0.02t \times \hat{S}_t$$

Note: mixed satisfies $\left(\sum_{j=1}^{s} S_j = s \right)$ as in equation (2)

4.2 Choice of Appropriate Model

The tests for Constant Varianceand Chi-Squared given in equations (23) and (25) respectivelyare applied to determine the choice of model for decomposition of the study data.To test the null hypothesis that the time series data admits additive model, the test statistic in equation (23) is used. The null hypothesis is rejected, if T_c is greater than the tabulated value, which for $\alpha = 0.05$ level of significance and $m - 1 = 9$ degrees of freedom equal to 19.7 or do not reject H_0 otherwise.From Appendix and Table 4

$$m = 10,\ s = 12, \hat{\sigma}_p^2 = 16.9707,\ \ln \hat{\sigma}_p^2 = 2.8315 \text{ and } \sum_{j=1}^{s} \ln \hat{\sigma}_j^2 = 30.9062$$

Hence,

$$T_c = \frac{(9)[12(2.8315) - (30.9062)]}{1 + \frac{13}{3(12)9}} = 26.5797$$

T_c is greater, when compared with critical value (19.7) suggesting that the data does not admit the additive model. Having confirmed that the data does not admit additive model, the choice now lies between mixed and multiplicative models.

In other to choose between mixed and multiplicative models, the test statistic in equation (25) is used. The null hypothesis that the data admits mixed model is rejected, if the statistic defined in equation (25) lies outside the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ which for m-1 = 9, equals (2.7 and 19.0) or do not reject H_0 otherwise.

Table 5: Seasonal effects (S_j), estimate of the column variance ($\hat{\sigma}_j^2$) and Calculated Chi-square $\left(\chi^2_{cal}\right)$

J	1	2	3	4	5	6	7	8	9	10	11	12
S_j	0.86	1.09	0.99	1.01	1.07	1.00	1.02	1.07	0.98	0.91	1.03	0.97
$\hat{\sigma}_j^2$	2.06	8.10	50.77	15.88	33.43	14.84	9.78	10.68	12.46	9.82	14.71	21.12
$\ln \hat{\sigma}_j^2$	0.72	2.09	3.93	2.77	3.51	2.70	2.28	2.37	2.52	2.28	2.69	3.05
χ^2_{cal}	4.9	7.8	18.6	8.4	14.4	13.0	12.3	17.5	13.4	12.1	14.2	8.3

From Table 5,

$\sigma_1^2 = 1, \quad b = 1.051, \quad n = 120, \quad s = 12, \quad m = 10$

Hence, from (5)

$$\sigma^2_{0j} = (1.051) \times 120 \left(\frac{120+12}{12}\right) s_j^2 + 1$$

and the calculated values, χ^2_{cal} given in Table 4 were obtained. When compared with the critical values (2.7 and 19.0), the calculated values of the statistic lie within the interval, indicating that the data admits mixed model.

5 Summary, Conclusions and Recommendations

This study presented the expected values of Buys-Ballot estimatesof row, column and overall meansfor mixed model with the error terms when trend-cycle component is linear. In this study, we derived the expected values of Buys-Ballot estimates with the error terms of row, column and overall means for multiplicative model.The method of estimation is based on the row, column and overall means of time series data arranged in a Buys-Ballot table. A real example for the linear case is applied to illustrate the estimation of trend parameters, seasonal indices, and choice of model of the Buys-Ballot table. Successful transformation is carried to stabilize the variance and make the distribution normal. Results indicate that, the expected values of row and column averages are; (1) row average of the mixed model mimic the shape of the trending parameters of the original series and contains seasonal effect in $C_1 = \sum_{j=1}^{S} js_j$ (2) column average also mimic the shape of the trending curves of the original series and contain seasonal effect (3) the appropriate model that best describe the pattern of the study series listed in the summary table (Table 5) is mixed.No attempt has been made to discuss this method when the trend-cycle component is not linear or when the cyclical component is separated from the trend. Hence, further investigations in these directions are recommended

References

[1] Iwueze, I. S. & Nwogu E.C. (2004). *Buys-Ballot estimates for time series decomposition,* Global Journal of Mathematics, 3(2), 83-98.

[2] Alder, H.L., & Roesslar, E.B., (1990). *Introduction to Probability and Statistics,* W.H. Freeman and Company, San Francisco.

[3] Chatfield, C. (2004). *The analysis of time Series: An introduction*. Chapman and Hall,/CRC Press, Boca Raton.

[4] Iwueze, I.S., Nwogu, E.C., Ohakwe, J. &Ajaraogu J.C. (2011). *Uses of the Buys – Ballot table in time series analysis*. Applied Mathematics Journal 2, 633 –645

[5] Dozie, K. C. N (2020). Estimation of seasonal variances in descriptive time series analysis. Asian Journal of Advanced Research and Reports 10(3), 37 – 47

[6] Wei, W. W. S (1989). *Time series analysis: Univariate and multivariate methods*, Addison-Wesley publishing Company Inc, Redwood City.

[7] Nwogu, E.C, Iwueze, I.S. Dozie, K.C.N. &Mbachu, H.I (2019). Choice between mixed and multiplicative models in time series decomposition. International Journal of Statistics and Applications 9(5), 153-159

[8] Dozie, K.C.N, Ihekuna S.O (2020). Buys-Ballot estimates of quadratic trend component and seasonal indices and effect of incomplete data in time series. International Journal of Science and Healthcare Research 5(2), 341-348

[9] Dozie, K.C.N, Nwogu, E.C &Ijeomah, M.A (2020). Effect of missing observations on Buys-Ballot estimates of time series components. Asian Journal of Probability and Statistics, 6(3), 13-24

[10] Dozie, K.C.N, Nwanya J.C (2020). Comparison of mixed and multiplicative models when trend-cycle component is liner. Asian Journal of Advanced Research and Report 12(4), 32-42

[11] Dozie, K.C.N &Ijomah M.A (2020). A comparative study on additive and mixed models in descriptive time series. American Journal of Mathematical and Computer Modelling 5(1), 12-17

[12] Dozie, K.C.N &Ibebuogu C.C (2021). Road traffic offences in Nigeria: an empirical analysis using buys-ballot approach. Asian Journal of Probability and Statistics 12(1), 68-78

[13] Dozie, K.C.N &Uwaezuoke M.U (2021). Procedure for estimation of additive time series model. International Journal of Research and Scientific Innovation 8(2), 251-256

[14] Dozie, K.C.N &Ihekuna, S.O (2022). Additive seasonality in time series using row and overall sample variances of the buys-ballot table. Asian Journal of Probability and Statistics 18 (3), 1-9

[15] Dozie, K.C.N &Ibebuogu C.C (2023). Decomposition with the mixed model in time series analysis using Buy-Ballot procedure. Asian Journal of Advanced Research and Report 17(2), 8-18

[16] Akpanta, A.C and Iwueze I.S (2009), “On applying the Bartlett transformation method to time series data. Journal of Mathematical Sciences, 20(5), 227-243

[17] Dozie, K.C.N &Uwaezuoke M.U (2023). The Proposed Buys-Ballot estimates for multiplicative model with the error variances. Journal of Engineering Research and Reports, 25(8), 94-106

[18] Dozie, K.C.N (2023). Buys-Ballot estimates for overall sample variances and their statistical properties. A Mixed Model Case. Asian Journal of Advanced Research and Reports, 17(10), 213-223

Appendix A. Original Data on number of Registered Baptism in St Paul Parish Owerri

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma_{i.}$
2013	15	35	20	11	22	25	13	28	12	10	40	27	21.8	9.00
2014	16	24	20	15	11	26	23	18	22	13	28	23	20.1	5.05
2015	8	18	11	14	26	14	24	18	21	19	25	10	17.3	6.02
2016	14	14	18	22	14	15	26	14	19	16	24	20	18.0	4.24
2017	17	21	20	23	16	11	27	22	14	12	5	21	17.4	6.14
2018	8	11	14	23	32	22	13	20	25	13	19	15	17.9	6.84
2019	5	21	9	10	18	15	9	20	18	10	15	16	13.8	5.10
2020	9	18	11	12	12	13	10	27	9	18	18	14	14.3	5.22
2021	6	10	10	11	18	12	11	10	11	8	11	10	10.7	2.81
2022	5	13	9	14	13	8	18	20	9	10	17	10	12.2	4.49
$\bar{X}_{.j}$	10.40	18.40	14.30	15.60	18.50	16.00	17.50	19.60	16.10	13.00	20.10	16.60	16.30	
$\sigma_{.j}$	4.84	7.17	4.95	5.10	6.57	5.96	6.98	5.21	5.69	3.62	9.49	5.97		41.4

Source: St Paul Parish, Owerri 2013-2022

Transformed Data on Number of Registered Baptism in St Paul Parish Owerri

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma_{i.}$
2013	2.8	3.5	3.0	2.3	3.1	3.2	2.6	3.3	2.6	2.4	3.7	3.3	3.0	0.18
2014	2.8	3.2	2.3	2.71	2.6	3.3	3.1	2.9	3.1	2.6	3.3	3.1	2.9	0.11
2015	2.1	2.8	2.4	2.6	3.3	2.6	3.2	2.8	3.0	2.9	3.2	2.3	2.8	0.15
2016	2.6	2.6	2.9	3.1	2.6	2.7	3.3	2.6	2.9	2.8	3.2	2.3	2.8	0.08
2017	2.8	3.0	2.3	3.1	2.7	2.4	3.3	3.1	2.6	2.5	1.6	3.0	2.7	0.22
2018	2.1	2.4	2.6	3.1	3.5	3.1	2.5	2.3	3.2	2.6	2.9	2.7	2.8	0.17
2019	1.6	3.0	2.2	2.3	2.8	2.7	2.2	2.3	2.1	2.3	2.7	2.8	2.4	0.16
2020	2.2	2.8	2.4	2.5	2.5	2.6	2.3	3.3	2.2	2.8	2.9	2.6	2.6	0.11
2021	1.8	2.3	2.3	2.4	2.9	2.5	2.4	2.3	2.4	2.1	2.4	2.3	2.3	0.06
2022	1.6	2.6	2.2	2.6	2.6	2.1	2.8	2.3	2.2	2.3	2.8	2.3	2.4	0.12

$\overline{X}_{.j}$	2.2	2.9	2.5	2.7	2.9	2.7	2.7	2.7	2.6	2.5	2.9	2.7	2.7	
$\sigma_{.j}$	0.2	0.1 S	0.1S	0.1	0.1	0.1	0.2	0.2	0.2	0.1	0.3	0.2		0.1 3

Source: St Pual Parish, Owerri 2013-2022